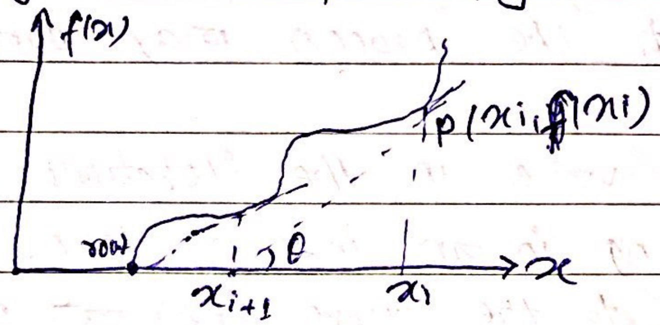
11. a. Newton - Raphson's method:-

In Newton-Raphson's method only one initial guess is taken further iteration, Let x_i be the initial guess then from fig.



$$\tan\theta = \frac{f(x_i)}{(x_i - x_{i+1})}$$

$$f'(x_i) = \frac{f(x_i)}{(x_i - x_{i+1})}$$

$$x_i - x_{i+1} = \frac{f(x_i)}{f'(x_i)}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \leftarrow \text{In general}$$

If $i = 0$, $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

The process will be terminated when the difference between two succesives values within the prescribe limit.

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Limitations of Newton Raphson Method:-

1. Division by zero may occur if $f'(x_0)$ is zero or very closed to zero.
2. If the initial guess is too far away from the required root, the process may converge to some other root.
3. A particular value in the iterations sequence may repeat, resulting in an infinite loop. This occurs when the tangent to the curve $f(x)$ at $x_i = x_{i+1}$ cuts the x-axis again at $x = x_i$.

Q. Calculate the root of $f(x) = x - 1.5\sin x - 2.5 = 0$. Correct to four significant digit using NR method. Also, calculate the relative error in each iteration.

Sol^n $f(x) = x - 1.5\sin x - 2.5 = 0$

$f'(x) = 1 - 1.5\cos x$ (±3 91)

Table:

x	-3	-2	-1	0	1	2	3
$f(x)$	-ve	-ve	-ve	-ve	-ve	-ve	+ve

Let the initial guess be 4.

It^n	X x_0	A $f(x_0)$	B $f'(x_0)$	C $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$	D $RE = \left(\frac{x_1 - x_0}{x_1}\right) \times 100\%$	shift
1	4	2.6352	1.9804	2.6694	49.84%	9f x
2.	2.6694	-0.5128	2.3358	2.8889	7.599%	..

$A = X - 1.5 \sin X - 2.5 : B = 1 - 1.5 \cos X :$

$C = X - A/B : D = (C - X)/C \times 100$

3	2.8889	0.0140	2.4583	2.8832	0.196%	$x_0 \leftarrow x_1$
4.	2.8832	6.1788×10^{-6}	2.4502	2.8832	1.27×10^{-3}%	"

$\therefore$ Root = 2.8832 Ans

Algorithm :-

1. Start
2. Define one initial guess x_0, and define stopping criteria E.
3. Compute $f_0 \leftarrow f(x_0)$
   $f_0' \leftarrow f'(x_0)$
4. Compare $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$
5. If $\left(\left|\frac{x_1 - x_0}{x_1}\right|\right) \le E$

   Display root. r
   Goto step (step 6)
   else
   $x_0 = x_1$ goto step 3.
6. stop.

IV. b Secant Method :-

secant method, like the ~~first~~ false posn and bisection method used two initial guesses but doesnot reqd that they must bracket the root.

Using N-R Method:

Q. 1) $f(x) = \cos x - x e^{x}$

$$f'(x) = -\sin x - e^{x}\cdot 1 - x\cdot e^{x}\cdot 1$$
$$= -\sin x - e^{x} - x e^{x}$$
$$= -\sin x - e^{x}(1 + x)$$

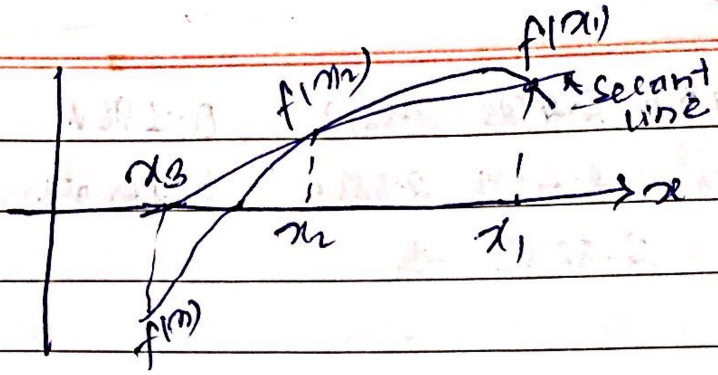
Initial guess, $x_0 = 0.5$

Q. 2) $f(x) = \sin x - e^{-x}$

$$f'(x) = \cos x - e^{-x}\cdot(-1)$$
$$= \cos x + e^{-x}$$

Range $[0, 1]$

lets take, $x_0 = 0.6$



$$\frac{f(x_1)}{x_1-x_3} = \frac{f(x_2)}{x_2-x_3}$$

$$f(x_1)(x_2-x_3) = f(x_2)(x_1-x_3)$$

Then: $$x_3 = \frac{f(x_2)\,x_1 - f(x_1)\,x_2}{f(x_2)-f(x_1)}$$

By adding and subtracting $f(x_2)\,x_2$ to the numerator and arranging the terms we get

$$x_3 = \frac{f(x_2)x_2 - f(x_1)x_2 - f(x_2)x_2 + f(x_2)x_1}{f(x_2)-f(x_1)}$$

$$= \frac{x_2\left(f(x_2)-f(x_1)\right)}{f(x_2)-f(x_1)} - \frac{f(x_2)(x_2-x_1)}{f(x_2)-f(x_1)}$$

$$\therefore x_3 = x_2 - \frac{f(x_2)(x_2-x_1)}{f(x_2)-f(x_1)}$$

Q. Calculate the root of $x^2 - 4x - 10 = 0$. Correct to 3-decimal places using secant method.

Sol: $f(x) = x^2 - 4x - 10 = 0$

Range of Root $= \pm 6$

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
$f(x)$	+ve	+ve	+ve	+ve	+ve	-ve	-ve	-ve	-ve	-ve	-ve	-ve	+ve

Let, the initial guesses are: $x_1 = 5$ & $x_2 = 4$.

Itn	x_1	x_2	$f(x_1)$	$f(x_2)$	$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$	I Left
1.	5	4	-5	-10	6	$x_1 \leftarrow x_2$ $x_2 \leftarrow x_3$
2.	4	6	-10	2	5.6667	"
3.	6	5.6667	2	-0.5553	5.7391	"
4.	5.6667	5.7391	-0.5553	-0.0191	5.7417	"
5.	5.7391	5.7417	-0.0191	0.003	5.7417	"

$\therefore$ Root $= 5.7417$ Ans

ii. Q. fixed-point method:-

- In this method the given eqn $f(x)$ is converted into different other eqn in the form $g(x)$. One initial guess is taken and iteration starts.
- Any function in the form of $f(x) = 0$ — (i) can be