Gaussian Integration Method:-

or Gaussian Quadrature formula:-

or Gaussian Legendre Method:-

Table: Parameters for Gaussian Integration:

n	i	w_i	z_i
2	1	1.00000	-0.57735
	2	1.00000	0.57735
3	1	0.55556	-0.77460
	2	0.88889	0.00000
	3	0.55556	0.77460
4	1	0.34785	-0.86114
	2	0.65215	-0.33998
	3	0.65215	+0.33998
	4	0.34785	+0.86114

Let, $I = \int_a^b f(x)\,dx$

The $I_g = \left(\frac{b-a}{2}\right) \sum_{i=1}^{n} w_i\, g(z_i)$

Where, $g(z_i)$ is the function of z_i so,

$$x_i = \left(\frac{b-a}{2}\right) z_i + \left(\frac{b+a}{2}\right)$$

Ex: Use Gauss legendre method to evaluate $\int_2^4 x^4+1\,dx$ for $n=3$.

Sol: Given, $n=3$, $a=2$, $b=4$ Hence

$$I_g = \frac{b-a}{2}\sum_{i=1}^{3} w_i g(z_i)$$

$$= \left(\frac{4-2}{2}\right)\left(w_1 g(z_1) + w_2 g(z_2) + w_3 g(z_3)\right)$$

$$= w_1 g(z_1) + w_2 g(z_2) + w_3 g(z_3) \quad \text{——— (1)}$$

Since, $x_i = \left(\frac{b-a}{2}\right) z_i + \left(\frac{b+a}{2}\right)$

$$= z_i + 3$$

$$\therefore g(z_i) = (z_i+3)^4 + 1$$

$$g(z_1) = (z_1+3)^4 + 1$$

$$g(z_2) = (z_2+3)^4 + 1$$

$$g(z_3) = (z_3+3)^4 + 1$$

$\therefore$ Eqn (1) becomes:

$$I_g = w_1[(z_1+3)^4+1] + w_2[(z_2+3)^4+1] + w_3[(z_3+3)^4+1]$$

$$= 200.4013 \text{ Ans}$$

A:
4.685

HW
Ex: Use integral $I = \int_{-2}^{2} e^{-x/2}\,dx$, Using Gaussian integration method for $n=2$.

Sol^n: Given, $n=3$, $a=-2$ and $b=2$

Now, $I_g = \frac{b-a}{2}\sum_{i=1}^{2} w_i\, g(z_i)$

$= \frac{2-(-2)}{2}\left[w_1 g(z_1) + w_2 g(z_2)\right]$

$\therefore I_g = 2\left[w_1 g(z_1) + w_2 g(z_2)\right]$ ----- (i)

Since, $x_i = \left(\frac{b-a}{2}\right) z_i + \left(\frac{b+a}{2}\right) = 2z_i$

Where, $g(z_i) = e^{-2z_i/2}$

$i=1$, $g(z_1) = e^{-2z_1/2}$

$= e^{-2\times(-0.57735)/2} = 1.78131$

$g(z_2) = e^{-2z_2/2}$

$= e^{-2\times 0.57735/2} = 0.56138$

Equation (i) becomes

$I_g = 2\left[1\times 1.78131 + 1\times 0.56138\right]$

$= 4.6853$ Ans

Q. Solve by Gauss Quadrature 3-point method. $\int_0^1 \frac{1}{1+x^3} dx$.

Ans: 0.8359