Definition

A first-order differential equation is separable if it can be expressed in one of the following forms:

$$\frac{dy}{dx} = f(x,y) = g(x)h(y) \quad \text{or} \quad \frac{dy}{dx} = f(x,y) = h(y)g(x) \quad \text{or} \quad P_1(x)\cdot Q_2(y)\cdot dx + P_2(x)\cdot Q_1(y)\cdot dy = 0$$

Solving Separable Equations

if separable equation expressed in the forms:

$$\frac{dy}{dx} = f(x,y) = g(x)h(y) \quad \text{or} \quad \frac{dy}{dx} = f(x,y) = h(y)g(x)$$

then we solve it using Method 1.
if separable equation expressed in the forms:

$$P_1(x)\cdot Q_2(y)\cdot dx + P_2(x)\cdot Q_1(y)\cdot dy = 0$$

then we solve it using Method 2.

Method 1:

Equation in the forms:

$$\frac{dy}{dx} = f(x,y) = g(x)h(y) \quad \text{or} \quad \frac{dy}{dx} = f(x,y) = h(y)g(x)$$

can be multiplied by $\frac{dx}{h(y)}$ and then integrated:

$$\int \frac{dy}{h(y)} = \int g(x)\, dx$$

providing a solution for $y(x)$.

Method 2:

Equation in the forms:

$$P_1(x)\cdot Q_2(y)\cdot dx + P_2(x)\cdot Q_1(y)\cdot dy = 0 \quad \text{or} \quad P_2(x)\cdot Q_1(y)\cdot dy + P_1(x)\cdot Q_2(y)\cdot dx = 0$$

can be divided by $Q_2(y)\cdot P_2(x) \neq 0$ and then integrated:

$$\int \frac{P_1(x)}{P_2(x)}\cdot dx + \int \frac{Q_1(y)}{Q_2(y)}\cdot dy = C$$

Afterward, we find special solutions $Q_2(y)\cdot P_2(x) = 0$ verify whether they are included in the overall solution. If not, delineate them as a separate solution.

Note: The solution obtained for y by computing these integrals may be implicitly defined. Rearranging the solution may be necessary to obtain an explicit expression for y, although in some cases, expressing y explicitly may not be possible.