

Linear differential equations of the first order are called equations that are presented in the following form:

$$y' + p(x) \cdot y = g(x)$$

To solve this differential equation, let's assume $y = u \cdot v$, where $u = u(x)$ and $v = v(x)$, with $v(x) \neq 0$.

The derivative of y is given by:

$$y' = u' \cdot v + u \cdot v'$$

Substitute y' and y into the original equation:

$$u' \cdot v + u \cdot v' + u \cdot p(x) \cdot v = g(x)$$

Factor out u from the brackets:

$$u' \cdot v + u \cdot (v' + p(x) \cdot v) = g(x)$$

Find v by solving equation, where constant C after integration $-\int p(x) dx$ equal 0 ($C=0$):

$$v = e^{-\int p(x) dx}$$

Find u by solving equation using integration by parts:

$$u = \int \frac{g(x)}{v} dx$$

Find y as the product of the found functions u and v :

$$y = u \cdot v$$