2.5.1 Bernoulli Equation

The Bernoulli equation is given by:

$$\frac{dy}{dx} + P(x)y = Q(x)y^n.$$

Let $z = y^{1-n}$. Then,

$$\frac{dz}{dx} = (1-n)y^{-n}\frac{dy}{dx},$$

giving us

$$\frac{y^{-n}dy}{dx} + P(x)y^{1-n} = Q(x),$$

$$\frac{1}{1-n}\frac{y^{-n}dy}{dx} + \frac{P(x)}{1-n}y^{1-n} = \frac{Q(x)}{1-n},$$

which simplifies to

$$\frac{1}{1-n}\frac{dz}{dx} + P(x)z = Q(x),$$

$$\frac{dz}{dx} + (1-n)P(x)z = (1-n)Q(x),$$

which is linear in z.