

**Definition:** A first-order differential equation is separable if it can be expressed in one of the following forms:

$$\frac{dy}{dx} = f(x, y) = g(x)h(y), \quad \text{or} \quad \frac{dy}{dx} = f(x, y) = h(y)g(x).$$

**Solving Separable Equations:** A separable equation is solved by isolating the variables; that is, rearranging the equation so that terms involving  $y$  are on one side and those involving  $x$  are on the other. The equation can then be integrated directly.

For an equation in the form:

$$\frac{dy}{dx} = g(x)h(y),$$

multiplying both sides by  $h(y)dx$  gives:

$$h(y)dy = g(x)dx,$$

which can be integrated directly:

$$\int h(y)dy = \int g(x)dx.$$

This integration process yields a solution for  $y(x)$ .

Similarly, an equation in the form:

$$\frac{dy}{dx} = h(y)g(x)$$

can be multiplied by  $dx/h(y)$  and then integrated:

$$\int \frac{dy}{h(y)} = \int \frac{dx}{g(x)},$$

providing a solution for  $y(x)$ .

*Note:* The solution obtained for  $y$  by computing these integrals may be implicitly defined. Rearranging the solution may be necessary to obtain an explicit expression for  $y$ , although in some cases, expressing  $y$  explicitly may not be possible.