



Colloquium

Lab: Concurrent and Distributed Systems Himeno Benchmark and Mandelbrot Set

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Agenda

The **Mandelbrot Set**

- Problem
- (Unoptimized) **Algorithm**
- Optimizations
- Evaluation

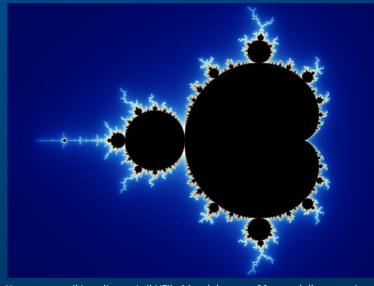
The **Himeno Benchmark**

- Problem
- (Unoptimized) **Algorithm**
- Optimizations
- Evaluation

Summary







https://commons.wikimedia.org/wiki/File:Mandel_zoom_00_mandelbrot_set.jpg





Problem

- Set of complex numbers
- $z_0 \coloneqq 0$
- $M = \{z_n \in \mathbb{C} \mid |z_n| \le 2 \ \forall n \in \mathbb{N} \}$
- *c* depends on pixel coordinate in image

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Unoptimized Algorithm

```
input: uint rows, uint cols, uint nn
output: char[]
begin
    let img = char[rows][cols]
    for r in 0..rows:
        for c in 0..cols:
            let z = complex(0, 0)
            let n = 0
            while |z| < 2.0 and n < nn:
                z = z*z + complex(c*2 / cols - 1.5, r*2 / rows - 1.0)
                n = n + 1
            imq[r][c] = (n == nn) ? '#' : '.'
    for r in 0...rows:
        for c in 0..cols: print(img[r][c])
        print( '\n' )
end
```





Optimizations

The Mandelbrot Set

General:

- Simplifying calculations
- Working directly with floats instead of std::complex<float>
- Storing and reusing interim results
- Working on continuous memory (as image)

Parallelization:

- Parallelize the pixel calculation (inner for loop)
- Provide and collect work dynamically via queues
- Aligning data structures to fit cache lines
- Each worker thread has one input- and output queue





Parallel Workflow

The Mandelbrot Set calculate pixel WORKER **INPUT OUTPUT** QUEUE **THREAD** QUEUE OUTPUT **INPUT INPUT** WORKER **OUTPUT** QUEUE **THREAD THREAD** QUEUE **THREAD** pull pixel push pixel results numbers **INPUT** WORKER **OUTPUT** QUEUE QUEUE **THREAD**





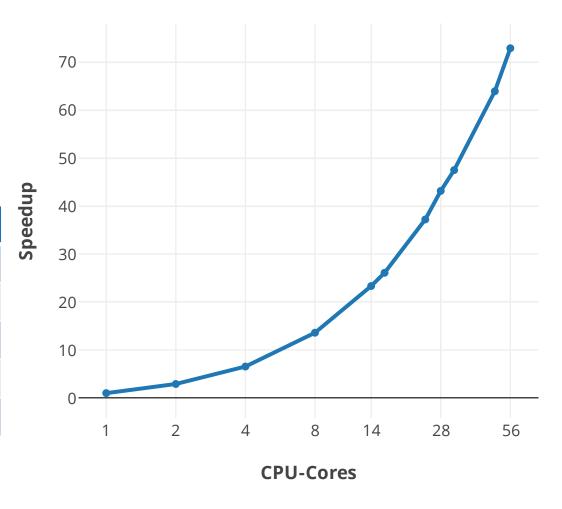
Evaluation

The Mandelbrot Set

- Unoptimized code as reference
- **C++** and **C#**
- Scaled very well
- No decline after **14** or **28** cores due to local queues
- Parallel code amount over 100% (not realistic)

Lang.	V	Optimization	Time	Abs.	Rel.
	0	Unmodified	207050	1x	1x
C++	1	I/O Threads	2182	86x	83x
	2	+ Simplified	263	716x	73x
C#	0	Unoptimized	63332	1x	1x
C#	2	Both	452	146x	1798x

Scaling of C++ Version 2









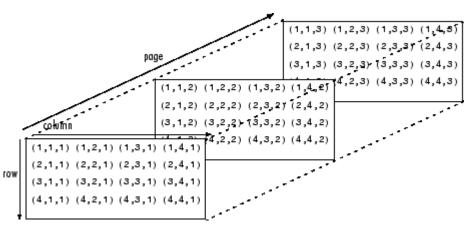
http://www.isum2017.udg.mx/es/contenido/dr-ryutaro-himeno





Problem

- A **benchmarking** program
- Extremely **memory intensive**
- Uses the Jacobi method for calculations
- Works on several 4D-Matrices, where first 3 dimensions are same size



http://www.isum2017.udg.mx/es/contenido/dr-ryutaro-himeno





Unoptimized Algorithm

```
input: uint rows, uint cols, uint deps, uint nn
output: float
begin
    initialize matrices()
    let gosa
    for n in 0..nn:
        gosa = 0.0
        for r in 1..rows-1: // iterate over every voxel
            for c in 1..cols-1:
                for d in 1..deps-1:
                    let ss = jacobi iteration(r, c, d)
                    qosa += ss*ss
                    wrk2[0][r][c][d] = p[0][r][c][d] + 0.8*ss
        wrk2.copy to(p) // copy content of p to wrk2
    print( gosa ) // print result (with a precision of 6)
end
```





Optimizations

The Himeno Benchmark

General:

- Drastically reducing memory usage:
 - **Substitute** 5 of 7 matrices by constants
 - From 33 to 9 matrix reads/writes (by 73%)
 - **Pointer swapping** instead of matrix copying (reduces (rows 2) * (cols 2) * (deps 2) to 1) per it.
 - **Reduce** matrix sizes by 2 each dimension
- Only calculate the gosa in last iteration

Parallelization:

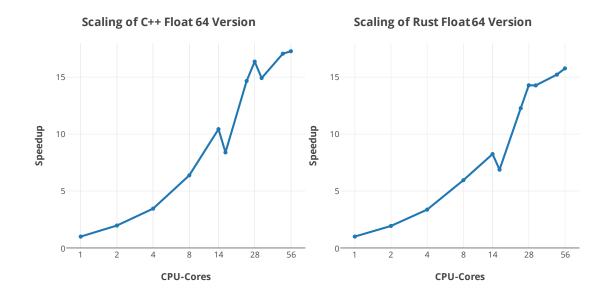
- Parallelize on the rows loop (outermost, parallelizable loop)
- Threads get work from a shared atomic variable containing the next row
- Calculate gosa sub-sums in parallel (only F64)





Evaluation

- Unoptimized code as reference
- C++ and Rust
- Scaled ok (w.r.t. the high memory usage)
- Strong decline at **16** and **32** cores
- Parallel code up to **97.37%**

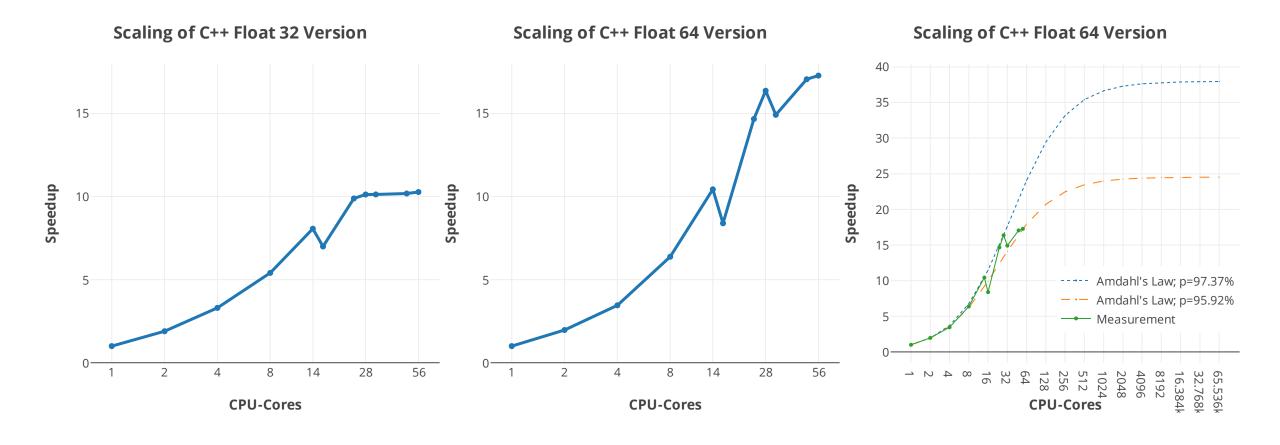


Lang.	FXX	Optimization	Time	Abs.	Rel.	p @ 28	p @ 56
	22	Unmodified	171978	1x	1x	0%	0%
C++	32	Parallel	34346	5x	10x	93.46%	91.91%
Стт	64	Unmodified	207050	1x	1x	0%	0%
	04	Parallel	19294	10x	17x	97.37%	95.92%
Duct	6.1	Unoptimized	289943	1x	1x	0%	0%
Rust	64	Parallel	7108	43x	16x	96.44%	95.36%





Evaluation







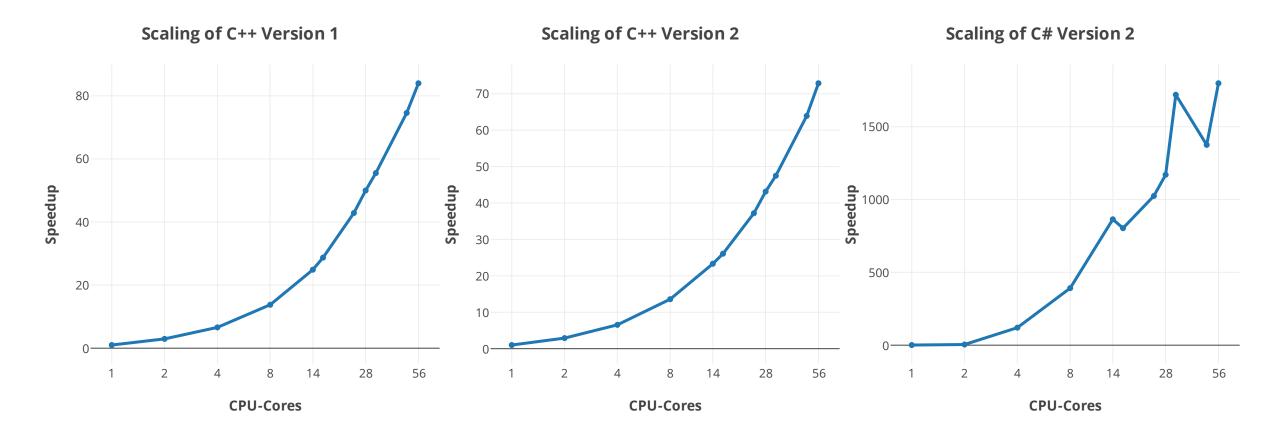
Summary

- Solved workload distribution with queues and atomic variables
- Optimized and simplified the algorithms
- Speedup of 1798 (716 abs.) for the Mandelbrot Set, optimal goal was 51
- Speedup of 17 (43 abs.) for the Himeno Benchmark, optimal goal was 13
- Learned about memory throughput and cache invalidation as limiting factor





Scaling of all Versions







Scaling with Amdahl's Law of all Versions

