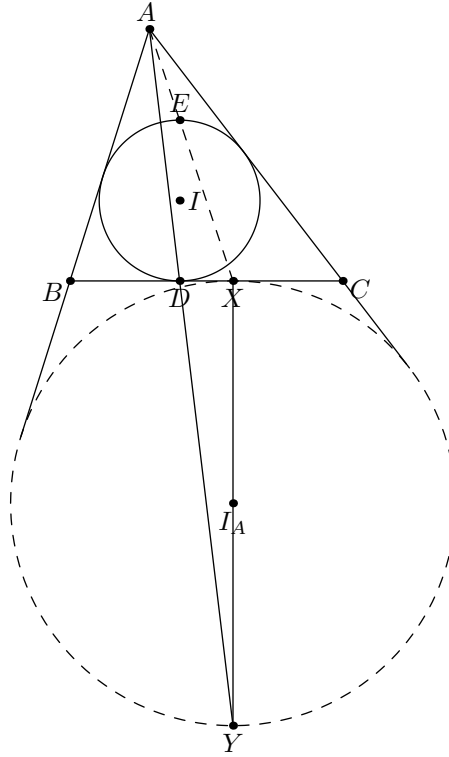


**Lemma 4.10 (Diameter of the Excircle).** *In the notation of Lemma 4.9, suppose  $\overline{XY}$  is a diameter of the  $A$ -excircle. Show that  $D$  lies on  $\overline{AY}$ .*

**Solution** Our solution is based off the following lemma.

**Lemma.** Let  $(ABC)$  be a triangle whose incircle is tangent to  $\overline{BC}$  at  $D$ . If  $\overline{DE}$  is a diameter of incircle and ray  $AE$  meets  $\overline{BC}$  at  $X$ , then  $X$  is the tangency point of the  $A$ -excircle to  $\overline{BC}$ .



*Proof.* According to the lemma, the homothety centered at  $A$ , sends  $E$  to  $X$ . So it also sends diameter  $ED$  to diameter  $XY$ . This tells us  $D$  lies on  $\overline{AY}$ .  $\square$