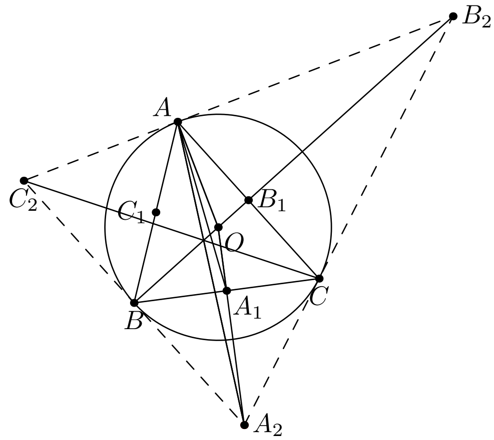
Problem 4.43 (USAMO 1995/3). Given a nonisosceles, nonright triangle ABC, let O denote its circumcenter, and let A_1, B_1, and C_1 be the midpoints of sides $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$, respectively. Point A_2 is located on the ray OA_1 so that $\triangle OAA_1$ is similar to $\triangle OA_2A$. Points B_2 and C_2 on rays OB_1 and OC_1, respectively, are defined similarly. Prove that lines AA_2, BB_2, CC_2 are concurrent.

Solution Our solution is based off the following lemma.



Lemma 1. Let X be the intersection of the tangents to (ABC) at B and C. Then line AX is a symmedian.

Proof. Since $\triangle OAA_1 \sim \triangle OA_2A$, we have

$$\frac{OA_1}{OA} = \frac{OA}{OA_2}$$

Which is

$$\frac{OA_1}{OB} = \frac{OB}{OA_2}$$

So

$$\triangle OBA_1 \sim \triangle OA_2B$$

Since A_1 is the midpoint of $\overline{BC}$, we have

$$\angle OA_1B = \angle OBA_2 = 90^\circ$$

So A_2B is tangent to (ABC). Similarly we have A_2C tangent to (ABC) too. According to the lemma, we claim that AA_2 is a symmedian. Similarly we claim that BB_2 and CC_2 are also symmedians. They concur at the symmedian point of $\triangle ABC$. $\square$