CSE 551 Homework 2

27^{th} September, 2020

Submission Instructions: Deadline is **11:59pm on 10/04/2020**. Late submissions will be penalized, therefore please ensure that you submit (file upload is completed) before the deadline. Additionally, you can download the submitted file to verify if the file was uploaded correctly. **Please TYPE UP YOUR SOLUTIONS and submit a PDF** electronically, via *Canvas*.
Furthermore, please note that the graders will grade 2 out of the 4 questions randomly. Therefore, if the grader decides to check questions 1 and 4,and you haven't answered question 4, you'll lose points for question 4. Hence, please answer all the questions.

1. Design an algorithm to compute the 2nd smallest number in an unordered (unsorted) sequence of numbers $\{a_1, a_2, ..., a_n\}$ in $n + \lceil log_2(n) \rceil - 2$ comparisons in the worst case. If you think such an algorithm can be designed, then show how it can be done. If your answer is no, then explain why it cannot be done. **[25 points]**

2. Consider the product of matrices of size $n \times n$, where n is a power of 3. Using divide-and-conquer the problem can be reduced to the multiplication of 3×3 matrices. The conventional method requires 27 multiplications. In how many multiplications must one be able to multiply 3×3 matrices so that the resultant computing time is $O(n^{2.81})$? Do the same for the case when n is a power of 4 and the problem is reduced to the multiplication of 4×4 matrices. **Show all your work**. **[25 points]**

3. If k is a non-negative constant, then prove that the recurrence: **[25 points]**

$$T(n) = k \text{ , for } n = 1 \text{ and}$$
$$T(n) = 3T(\tfrac{n}{2}) + kn, \text{ for } n > 1$$

has the following solution (for n a power of 2):

$$T(n) = 3kn^{log_2 3} - 2kn$$

4. Let $n = 2p$, $V = (v_1, ..., v_n), W = (w_1, ..., w_n)$. Then we can compute the vector product VW by the formula:

$\sum_{1\leq i\leq p}(v_{2i-1}+w_{2i})\times(v_{2i}+w_{2i-1})-\sum_{1\leq i\leq p} v_{2i-1}\times v_{2i}-\sum_{1\leq i\leq p} w_{2i-1}\times w_{2i}$

which requires $3n/2$ multiplications. Show how to use this formula for the multiplication of two $n \times n$ matrices giving a method which requires $n^3/2 + n^2$ multiplications rather than the usual n^3 multiplications. [**25 points**]