A Supplementary Material: Complex Temporal Constraints Description

We present in what follows the complex constraints we defined and used in our explainable fact validation approach. Given two time sequences S_1 and S_2 of two time-dependent properties P_1 and P_2, the complex constraints combine several simple temporal relations (see an illustration in Figure 7) to represent the result of comparisons of the intervals of S_1 and S_2 than can not be expressed using simple temporal constraints (e.g., before, meets, overlap, etc.).

We defined ten complex constraints that can be either joint or disjoint constraints (i.e., without any intersection intervals between the two sequences). As we present in Figure 6, these complex constraints can be represented in an ordering tree (CT^2), in which the constraints are organised thanks to the hierarchical relation $\preceq$. It expresses that if a constraint c is valid for a pair of properties P_1 and P_2 then all the constraints on the path to the root of CT^2 are also valid for P_1 and P_2. Moreover, these constraints can also be distinguished on whether they are symmetrical or not. For instance the complex constraint `NAND` is symmetrical while `Sequence_Meets` is not.

In the following we will present in more details the different complex constraints we defined by following their organisation in CT^2 tree while giving their intuitive and the formal definition.

In Figure 8 we provide a graphical illustration for each complex constraint we describe below.

A.1 Joint Complex Temporal Constraints

Three constraints falls in this category: `overlapping`, `always_while` and `equality` that is specific case of the former constraint.

Overlapping Constraint. For two time sequences S and S', and their corresponding set of relevant comparisons $\Omega(S, S')$, an *overlapping* constraint expresses that every quadruplet overlaps a quadruplet of the other time sequence (except for the quadruplet that starts the latest).

More formally, let us consider IR the set of intersecting relations (section 3.2), the two time sequences S and S' of the properties P and P' respectively, and the matrix of inter-comparisons $M_{\triangleright}$ of S and S' fulfils the Overlapping constraint if:

$$M_{\triangleright}[overlapping][r(P, P')] + \\ M_{\triangleright}[overlapping][r(P', P)] = |S| + |S'| - 1$$

Always_while Constraint. For two time sequences S and S', and their corresponding set of relevant comparisons $\Omega(S, S')$, a *Always While* constraint expresses that all quadruplets of a time sequence shares an intersection with another quadruplet of the other time sequence that is equal to its temporal interval (i.e. $q.I \cap_T q'.I = q.I$).

More formally, given the pair of time sequences S and S' of the properties P and P' respectively, and the matrix of inter-comparisons $M_{\triangleright}$ of S and S' fulfils the Always While constraint if:

$$M_{\triangleright}[equals][r(P, P')] + M_{\triangleright}[during][r(P, P')] + \\ M_{\triangleright}[starts][r(P, P')] + M_{\triangleright}[finishes][r(P, P')] = |S|$$

Equality Constraint. For two time sequences S and S', and their corresponding set of relevant comparisons $\Omega(S, S')$, an *Equality* constraint expresses that every quadruplets of a time sequence has a quadruplet in the other time sequence that has the same interval.

More formally, let us consider the two time sequences S and S' of the properties P and P' respectively, and the matrix of inter-comparisons $M_{\triangleright}$ of S and S' fulfils the Always While constraint if:

$$M_{\triangleright}[equality][r(P, P')] + \\ M_{\triangleright}[equality][r(P', P)] = |S| + |S'|$$

A.2 Disjoint Complex Temporal Constraints

The other seven complex temporal constraints we defined falls in the disjointness case. We define them below while providing their formal definition.

NAND Constraint. For two time sequences S and S', and their corresponding set of relevant inter-comparisons $\Omega(S, S')$, a `NAND` constraint expresses that for every relevant inter-comparison (i, i') there is no intersected relation that is fulfilled.

More formally, let us consider IR the set of intersected relations (section 3.2), S and S' the two time sequences of the properties P and P' respectively, and the matrix of inter-comparisons $M_{\triangleright}$ of S and S' fulfils the NAND constraint if:

$$\left(\sum_{a \in IR} M_{\triangleright}[a][r(P, P')]\right) = 0$$

Closed_NAND Constraint. For two time sequences S and S', and their corresponding set of relevant comparisons $\Omega(S, S')$, a Closed NAND constraint expresses that no gap appears between the first and last quadruplets regardless of the time sequence.

More formally, let us consider the two time sequences S and S' of the properties P and P' respectively, the matrix of inter-comparisons $M_{\triangleright}$ of S and S', and the matrix of intra-comparisons $M_{\triangleleft}$ of S and S' fulfils the Closed NAND constraint if:

$$M_{\triangleright}[meets][r(P, P')] + M_{\triangleright}[meets][r(P', P)] + \\ M_{\triangleleft}[meets][P] + M_{\triangleleft}[meets][P'] = |S| + |S'| - 1$$

ALT_closed Constraint. For two time sequences S and S', and their corresponding set of relevant comparisons $\Omega(S, S')$, an ALT_closed expresses that after the apparition of a quadruplet of a time sequence a quadruplet of the other time sequence will happen (or nothing if at the end of time sequence).

More formally, let us consider the two time sequences S and S' of the properties P and P' respectively, and the matrix of inter-comparisons $M_{\triangleright}$ of S and S' fulfils the ALT_closed constraint if:

$$M_{\triangleright}[meets][r(P, P')] + M_{\triangleright}[meets][r(P', P)] \\ = |S| + |S'| - 1$$

Sequence_meets Constraint. For two time sequences S and S', and their corresponding set of relevant comparisons $\Omega(S, S')$, a *Sequence Meets* constraint expresses that the last quadruplet of S meets the first quadruplet of S'.

Let consider DR the set of disjoint relations (section 3.2), the two time sequences S and S' of the properties P and P' respectively, and the matrix of inter-comparisons $M_{\triangleright}$ of S and S' fulfils the Sequence Meets constraint if:

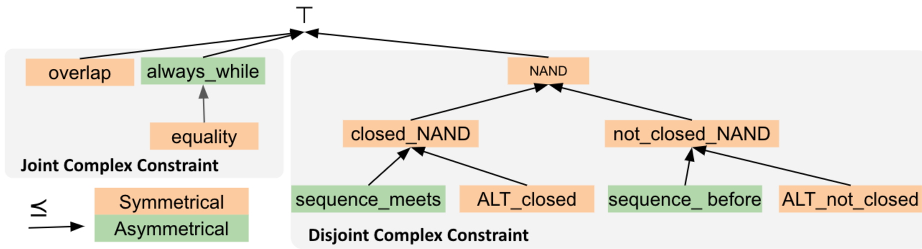

Figure 6: Complex Temporal Constraints Tree (CT^2)

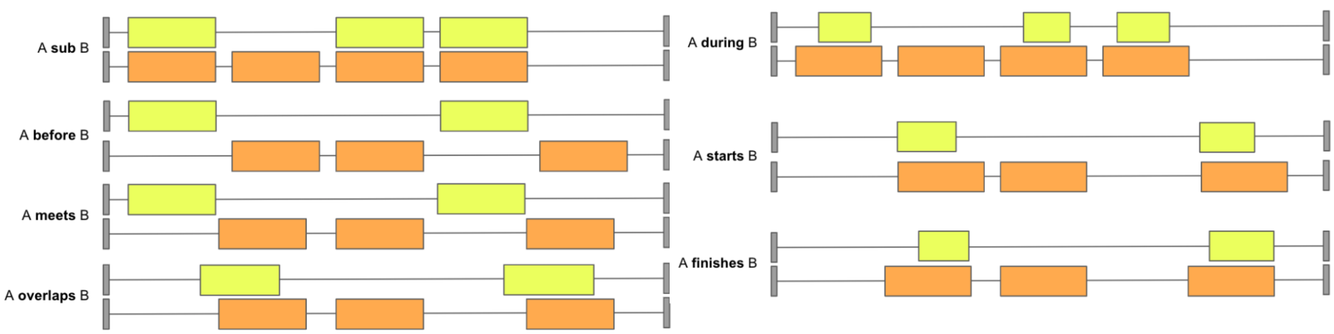

Figure 7: Simple Temporal Constraints Illustration

$$M_{\triangleright}[meets][r(P,P')] = 1 \land$$
$$(\sum_{a \in DR} M_{\triangleright}[a][r(P,P')] + M_{\triangleright}[a][r(P',P)]) = 1$$

Not_closed_NAND Constraint. For two time sequences S and S', and their corresponding set of relevant comparisons $\Omega(S,S')$, a *Closed NAND* constraint expresses that a gap always appear between any intervals (inter or intra-time sequence).

More formally, let us consider DR the set of disjoint relations (section 3.2), the two time sequences S and S' of the properties P and P' respectively, the matrix of inter-comparisons $M_{\triangleright}$ of S and S', and the matrix of intra-comparisons $M_{\triangleleft}$ of S and S' fulfils the Closed NAND constraint if:

$$M_{\triangleleft}[meets][P] + M_{\triangleleft}[meets][P'] = 0 \land$$
$$((\sum_{a \in DR/\{before\}} M_{\triangleright}[a][r(P,P')] + M_{\triangleright}[a][r(P',P)]) = 0)$$

ALT_not_closed Constraint. For two time sequences S and S', and their corresponding set of relevant comparisons $\Omega(S,S')$, an *ALT_not_closed* constraint expresses that after the apparition of a quadruplet of a time sequence a quadruplet of the other time sequence will happen after a gap (or nothing if at the end of time sequence).

More formally, let us consider the two time sequences S and S' of the properties P and P' respectively, and the matrix of inter-comparisons $M_{\triangleright}$ of S and S' fulfils the *ALT_not_closed* constraint if:

$$M_{\triangleright}[before][r(P,P')] +$$
$$M_{\triangleright}[before][r(P',P)] = |S| + |S'| - 1$$

Sequence_Before Constraint. For two time sequences S and S', and their corresponding set of relevant comparisons $\Omega(S,S')$, a *Sequence Before* constraint expresses that the last quadruplet of S happens before every other quadruplets of S'.

More formally, let us consider DR the set of disjoint relations (section 3.2), the two time sequences S and S' of the properties P and P' respectively, and the matrix of inter-comparisons $M_{\triangleright}$ of S and S' fulfils the Sequence Before constraint if:

$$M_{\triangleright}[before][r(P,P')] = 1 \land$$
$$(\sum_{a \in DR} M_{\triangleright}[a][r(P,P')] + M_{\triangleright}[a][r(P',P)]) = 1$$

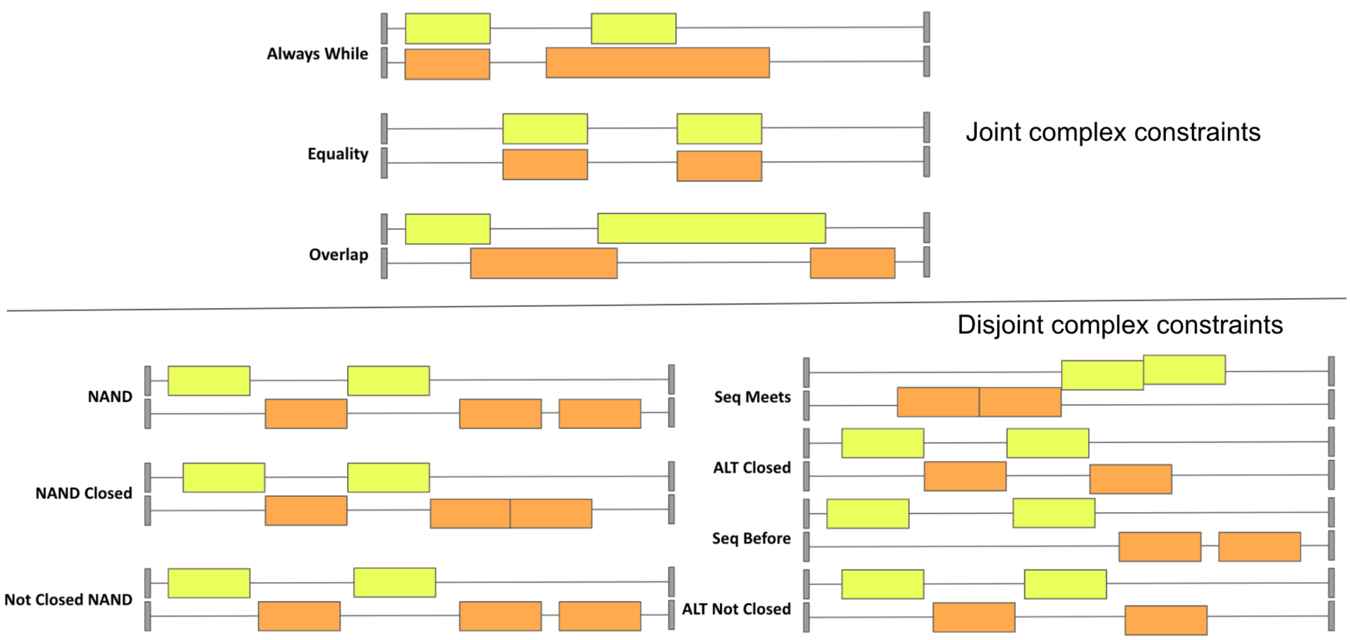

Figure 8: Complex Temporal Constraints Illustration