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Homework 1: Written Problems

1. Let's assume there is a 10,000 people in the population, 1% of those people will have the given blood type therefore a 100 people will have the given blood type. The probability that the suspect has that blood type given that he is innocent would be 1%. Given that defendant as the blood type, what is the probability that he is guilty, that is the question we are trying to answer. Since its given that he has the blood type, the question we should be asking then is, out of the 100 people who have the blood type, what is the probability that he is the one that committed the crime, which would be 1/100, therefore 1%. So the probability that he is guilty is 1%.
2.
   a. We first take the log of both side
   $P(D|\theta) = \theta^{N1} * (1- \theta^{N0})$
   $\ln(P(D|\theta)) = \ln(\theta^{N1} * (1- \theta^{N0}))$
   $= N1 * \ln(\theta) + N0 * \ln(1- \theta) = 0$
   Now we take the derivate
   $N1 * \frac{1}{\theta} + N0 * \frac{-1}{1-\theta} = 0$
   $$\frac{N1}{\theta} = \frac{N0}{1-\theta}$$
   $$\frac{N1 * (1-\theta) - \theta * (N0)}{\theta * (1-\theta)} = 0$$
   $$N1 - N1 * \theta - \theta * (N0) = 0$$
   $$\theta * (-N0 - N1) = -N1$$
   $$\theta = -\frac{N1}{-(N1 + N0)}$$
   $$\theta = \frac{N1}{(N1 + N0)}$$
   b. $P(D|\theta) = \frac{P(\theta|D) * P(\theta)}{P(D)}$
   $P(D|\theta) = P(\theta|D) * P(\theta)$
   $P(D|\theta) = \theta^{N1} * (1-\theta)^{N0} * \theta^{\alpha} * (1-\theta)^{\alpha}$

   We first Take the log

   $$\ln\big(P(D|\theta)\big) = N1 * \ln(\theta) + N0 * \ln(1-\theta) + \alpha * \ln(\theta) + \alpha * \ln(1-\theta)$$
   We now take the derivative and set it equal to 0
   $$0 = N1 * \frac{1}{\theta} + N0 * \frac{1}{1-\theta} + \alpha * \frac{1}{\theta} + \alpha * \frac{1}{1-\theta}$$
   $$0 = (N1 + \alpha) * \frac{1}{\theta} + (N0 + \alpha) * \frac{1}{1-\theta}$$
   $$0 = \frac{(1-\theta) * (N1 + \alpha) + \theta * (N0 + \alpha)}{\theta * (1-\theta)}$$
   $$0 = \frac{N1 + \alpha - \theta * (N1 + N0)}{\theta * (1-\theta)}$$

Our Final answer is:

$$\frac{N1 + \alpha - \theta * (N1 + \mathrm{N0})}{\theta * (1 - \theta)}$$