Written Problems

1.

a. The logical functions that can be expressed as a linear classifier of the form $f(x; w) = sign(w_1x_1 + w_2x_2 + b)$ are $AND(x_1, x_2)$ and $OR(x_1, x_2)$.
The w_1, w_2, and bias b that mimic AND are: $w_1 = 1$, $w_2 = 1$ and $b = -1$.
The w_1, w_2, and bias b that mimic OR are: $w_1 = 1$, $w_2 = 1$ and $b = 1$.

XOR does not work, because for any value of w and b not all the conditions can be satisfied.

b. The weight vectors and biases that satisfies the XOR logical functions are:
$W_1 = [1\ \ 1]$ $W_2 = [1\ \ 1]$ $W_{out} = [-1\ \ 3]$
$b_1 = -1$ $b_2 = 1$ $b_{out} = -3$

for $x_1 = 1$ and $x_2 = -1$, $X = [1\ \ -1]$:
$h_1 = W_1X + b_1 = -1$ so $sign(h_1) = -1$
$h_2 = W_2X + b_2 = 1$ so $sign(h_2) = 1$
so $H = [-1\ \ 1]$, therefore $f(X;W)$ is:
$sign(W_{out}H + b_{out}) = sign(4-3) = +1$

for $x_1 = -1$ and $x_2 = 1$, $X = [-1\ \ 1]$:
$h_1 = W_1X + b_1 = -1$ so $sign(h_1) = -1$
$h_2 = W_2X + b_2 = 1$ so $sign(h_2) = 1$
so $H = [-1\ \ 1]$, therefore $f(X;W)$ is:
$sign(W_{out}H + b_{out}) = sign(4-3) = +1$

for $x_1 = -1$ and $x_2 = -1$, $X = [-1\ \ -1]$:
$h_1 = W_1X + b_1 = -3$ so $sign(h_1) = -1$
$h_2 = W_2X + b_2 = -1$ so $sign(h_2) = -1$
so $H = [-1\ \ -1]$, therefore $f(X;W)$ is:
$sign(W_{out}H + b_{out}) = sign(-2-3) = -1$

for $x_1 = 1$ and $x_2 = 1$, $X = [1\ \ 1]$:
$h_1 = W_1X + b_1 = 1$ so $sign(h_1) = 1$
$h_2 = W_2X + b_2 = 3$ so $sign(h_2) = 1$
so $H = [1\ \ 1]$, therefore $f(X;W)$ is:
$sign(W_{out}H + b_{out}) = sign(2-3) = -1$

so it does satisfy XOR conditions.