Mathematics in the Modern World

Problem Solving & Reasoning

Logic Puzzles

Logic puzzles can be solved by using deductive reasoning and a chart that enables us to display the given information in a visual manner.

Example: *Solve a Logic Puzzle*

Brianna, Ryan, Tyler, and Ashley were recently elected as the new class officers (president, vice president, secretary, treasurer) of the sophomore class at Summit College. From the following clues, determine which position each holds.

1. Ashley is younger than the president but older than the treasurer.
2. Brianna and the secretary are both the same age, and they are the youngest members of the group.
3. Tyler and the secretary are next-door neighbors.

Solution:

From clue 1, we know that **Ashley is not the president or the treasurer**. In the following chart, write X1 (which stands for "ruled out by clue 1") in the President and Treasurer columns of Ashley's row.

CLUES:

1. Ashley is younger than the president but older than the treasurer.
2. Brianna and the secretary are both the same age, and they are the youngest members of the group.
3. Tyler and the secretary are next-door neighbors.

	Pres.	V. P.	Sec.	Treas.
Brianna				
Ryan				
Tyler				
Ashley	X1			X1

Solution:

From clue 2, **Brianna is not the secretary**. We know from clue 1 that the **president is not the youngest**, and we know from clue 2 that **Brianna and the secretary are the youngest members of the group**. Thus **Brianna is not the president**. In the chart, write X2 for these two conditions. Also we know from clues 1 and 2 that **Ashley is not the secretary**, because **she is older than the treasurer**. Write an X2 in the Secretary column of Ashley's row.

CLUES:

1. Ashley is younger than the president but older than the treasurer.
2. Brianna and the secretary are both the same age, and they are the youngest members of the group.
3. Tyler and the secretary are next-door neighbors.

	Pres.	V. P.	Sec.	Treas.
Brianna				
Ryan				
Tyler				
Ashley				

Solution:

At this point we see that **Ashley must be the vice president** and that none of the other members is the vice president. Thus we can update the chart as shown below.

CLUES:

1. Ashley is younger than the president but older than the treasurer.
2. Brianna and the secretary are both the same age, and they are the youngest members of the group.
3. Tyler and the secretary are next-door neighbors.

	Pres.	V. P.	Sec.	Treas.
Brianna				
Ryan				
Tyler				
Ashley				

Solution:

Now we can see that **Brianna must be the treasurer** and that **neither Ryan nor Tyler is the treasurer**. Update the chart as shown below.

CLUES:

1. Ashley is younger than the president but older than the treasurer.
2. Brianna and the secretary are both the same age, and they are the youngest members of the group.
3. Tyler and the secretary are next-door neighbors.

	Pres.	V. P.	Sec.	Treas.
Brianna				
Ryan				
Tyler				
Ashley				

Solution:

From clue 3, we know that **Tyler is not the secretary**. Thus we can conclude that **Tyler is the president** and **Ryan must be the secretary**. See the chart below.

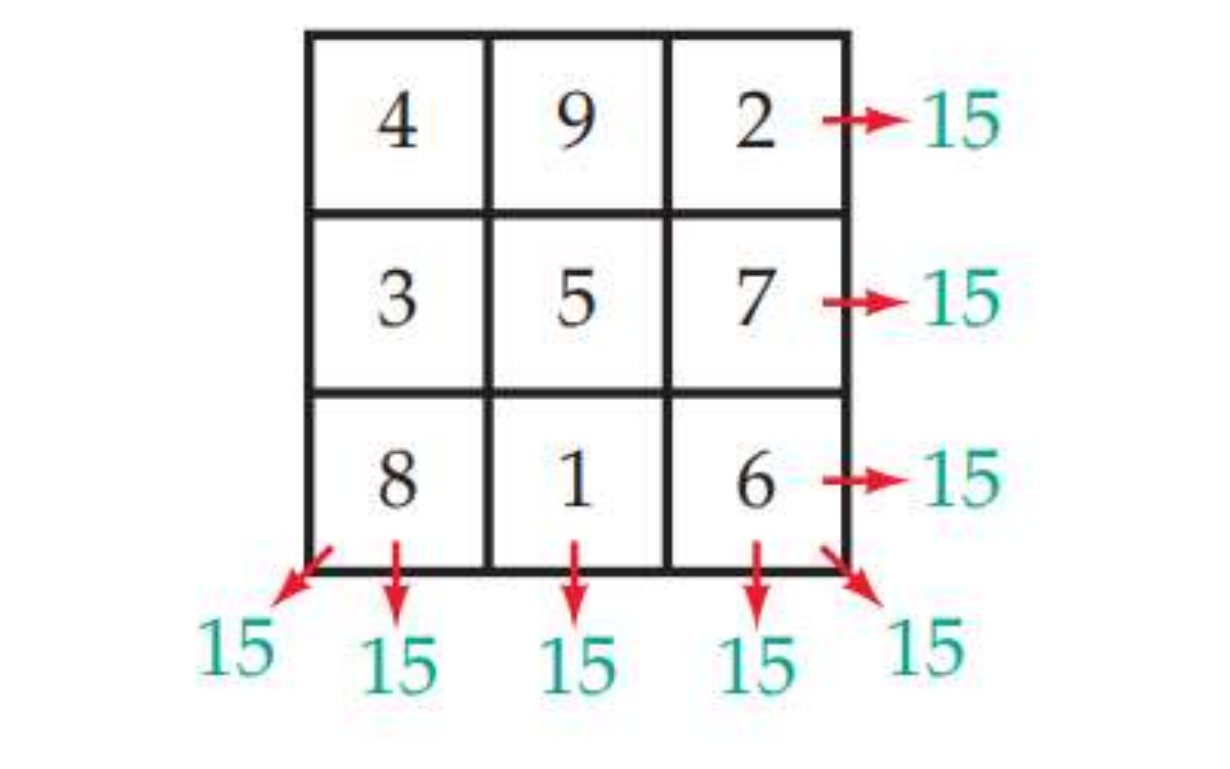
CLUES:

1. Ashley is younger than the president but older than the treasurer.
2. Brianna and the secretary are both the same age, and they are the youngest members of the group.
3. Tyler and the secretary are next-door neighbors.

	Pres.	V. P.	Sec.	Treas.
Brianna				
Ryan				
Tyler				
Ashley				

Magic Square

A magic square of order n is an arrangement of numbers in a square such that the sum of the n numbers in each row, column, and diagonal is the same number. The magic square below has order 3, and the sum of the numbers in each row, column, and diagonal is 15.



Example 1: *Magic Square*

Use deductive reasoning to determine the missing numbers in each magic square.

A magic square of order 4

		2	13
	10	11	
	6		12
4	15		1

Example 2: *Magic Square*

Use deductive reasoning to determine the missing numbers in each magic square.

A magic square of order 5

11		7		3
4			8	16
	5	13		
10	18	1		22
23	6		2	15

Problem Solving with Patterns

Sequences

- Sequence is an ordered list of numbers.
- Terms of the sequence are the numbers separated by *comma.*
- The three dots "…" indicate that the sequence continues beyond the last written term.
- x_n is used to designate the nth term of a sequence.

Arithmetic Sequence

- In an Arithmetic Sequence, the difference between one term and the next is constant.
- A kind of sequence that have a common difference between consecutive terms

Formula: $$x_n = a + d(n - 1)$$

where:

a = first term
d = common difference between the terms
n = term number

Example 1:

Calculate the 24^{th} term for this Arithmetic Sequence:

3, 10, 17, 24, 31, 38, …

Example 2:

Calculate the 38^{th} term for this Arithmetic Sequence:

71, 62, 53, 44, 35, …

Example 3:

Calculate the 140^{th} term for this Arithmetic Sequence:

2, 5, 8, 11, 14, …

Example 4:

Calculate the 21^{st} term for this Arithmetic Sequence:

10, 7, 4, 1, -2, -5, …

Example 5:

Calculate the 28^{th} term for this Arithmetic Sequence:

5, 2, -1, -4, -7, …

Geometric Sequence

- In a Geometric Sequence each term is found by multiplying the previous term by a constant.

- A kind of sequence that have a common ratio between consecutive terms

Formula: $$x_n = ar^{(n-1)}$$

where:

a = first term
r = common ratio between the terms
n = term number

Example 1:

Calculate the 10^{th} term for this Geometric Sequence:

1, 3, 9, 27, 81, 243, …

Example 2:

Calculate the 12^{th} term for this Geometric Sequence:

$$32,\ 16,\ 8,\ 4,\ 2,\ 1,\ldots$$

Example 3:

Calculate the 15^{th} term for this Geometric Sequence:

$$2,\ 6,\ 18,\ 54,\ 162,\ 486, \ldots$$

Example 4:

Calculate the 11^{th} term for this Geometric Sequence:

3, 6, 12, 24, 48, …

Example 5:

Calculate the 9^{th} term for this Geometric Sequence:

$$1,\ 5,\ 25,\ 125,\ 625, \ldots$$

Example 6:

Calculate the 13^{th} term for this Geometric Sequence:

$$1,\ \ 4,\ \ 16,\ \ 64,\ \ 256, \ldots$$

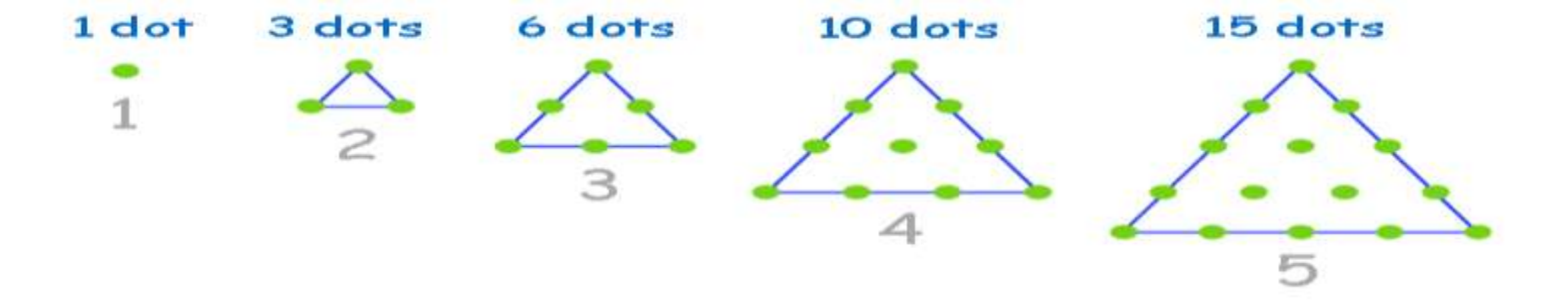
Example 7:

Calculate the 8^{th} term for this Geometric Sequence:

40, 20, 10, 5, 2.5, 1.25…

Triangular Number Sequence

- This is the Triangular Number Sequence: 1, 3, 6, 10, 15, 21, 28, 36, 45, ...



Formula: $$x_n = \frac{n(n+1)}{2}$$

where:

n = term number

Example 1:

Calculate the 27^{th} term in the Triangular Number Sequence.

Example 2:

Calculate the 39^{th} term in the Triangular Number Sequence.

Example 3:

Calculate the 52^{nd} term in the Triangular Number Sequence.

Example 4:

Calculate the 80^{th} term in the Triangular Number Sequence.

Example 5:

Calculate the 75^{th} term in the Triangular Number Sequence.

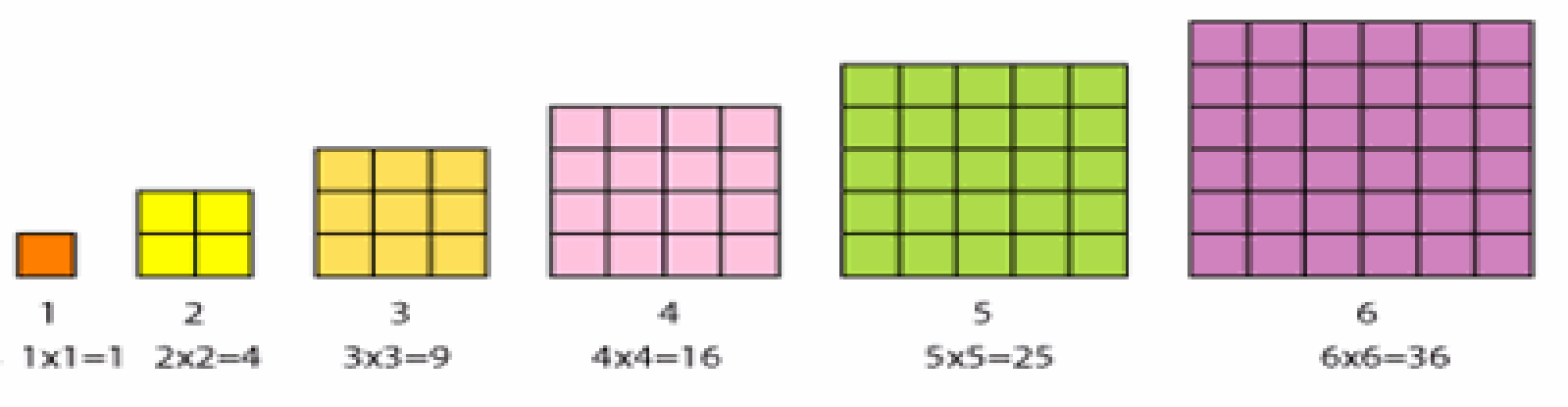
Example 6:

Calculate the 105^{th} term in the Triangular Number Sequence.

Square Number

Sequence

- This is the Square Number Sequence: **1, 4, 9, 16, 25, 36, 49, 64, 81, 100, …**



Formula: $x_n = n^2$

where:

n = term number

Example 1:

Calculate the 33^{rd} term in the Square Number Sequence.

Example 2:

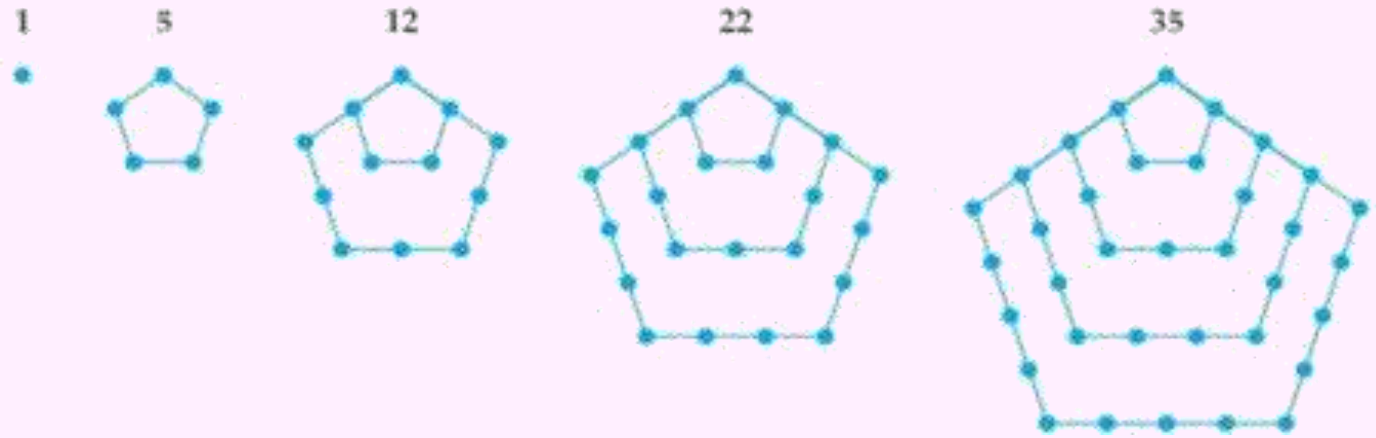
Calculate the 56^{th} term in the Square Number Sequence.

Example 3:

Calculate the 74^{th} term in the Square Number Sequence.

Pentagonal Number **Sequence**

- This is the Pentagonal Number Sequence: **1, 5, 12, 22, 35, 51, 70, 92, 117, 145,...**



Formula: $$x_n = \frac{n(3n-1)}{2}$$

where:

n = term number

Example 1:

Calculate the 19^{th} term in the Pentagonal Number Sequence.

Example 2:

Calculate the 32^{nd} term in the Pentagonal Number Sequence.

Example 3:

Calculate the 50^{th} term in the Pentagonal Number Sequence.

Example 4:

Calculate the 120^{th} term in the Pentagonal Number Sequence.

Example 5:

Calculate the 43^{rd} term in the Pentagonal Number Sequence.

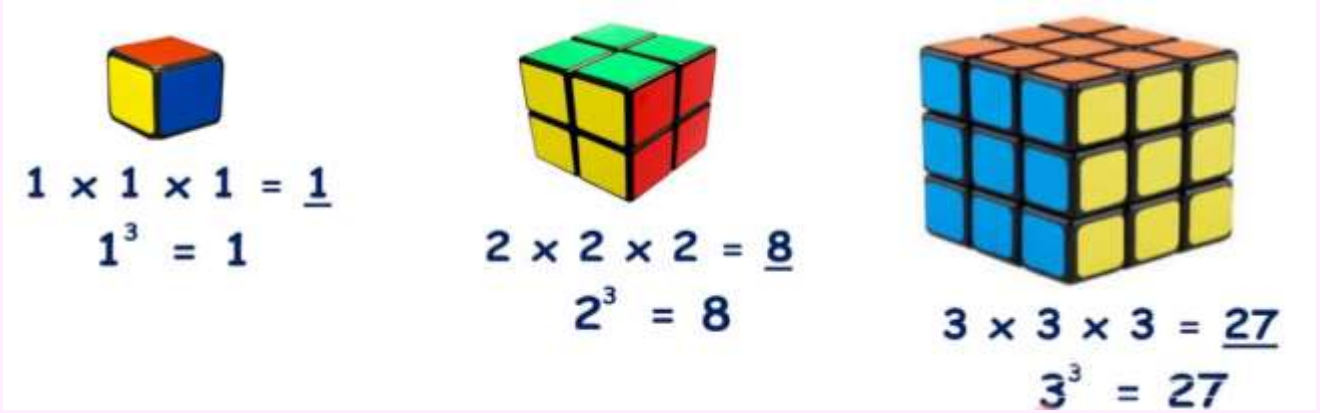
Example 6:

Calculate the 70^{th} term in the Pentagonal Number Sequence.

Cube Number **Sequence**

- This is the Cube Number Sequence:

  1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, …



Formula: $x_n = n^3$

where:

n = term number

Example 1:

Calculate the 18^{th} term in the Cube Number Sequence.

Example 2:

Calculate the 27^{th} term in the Cube Number Sequence.

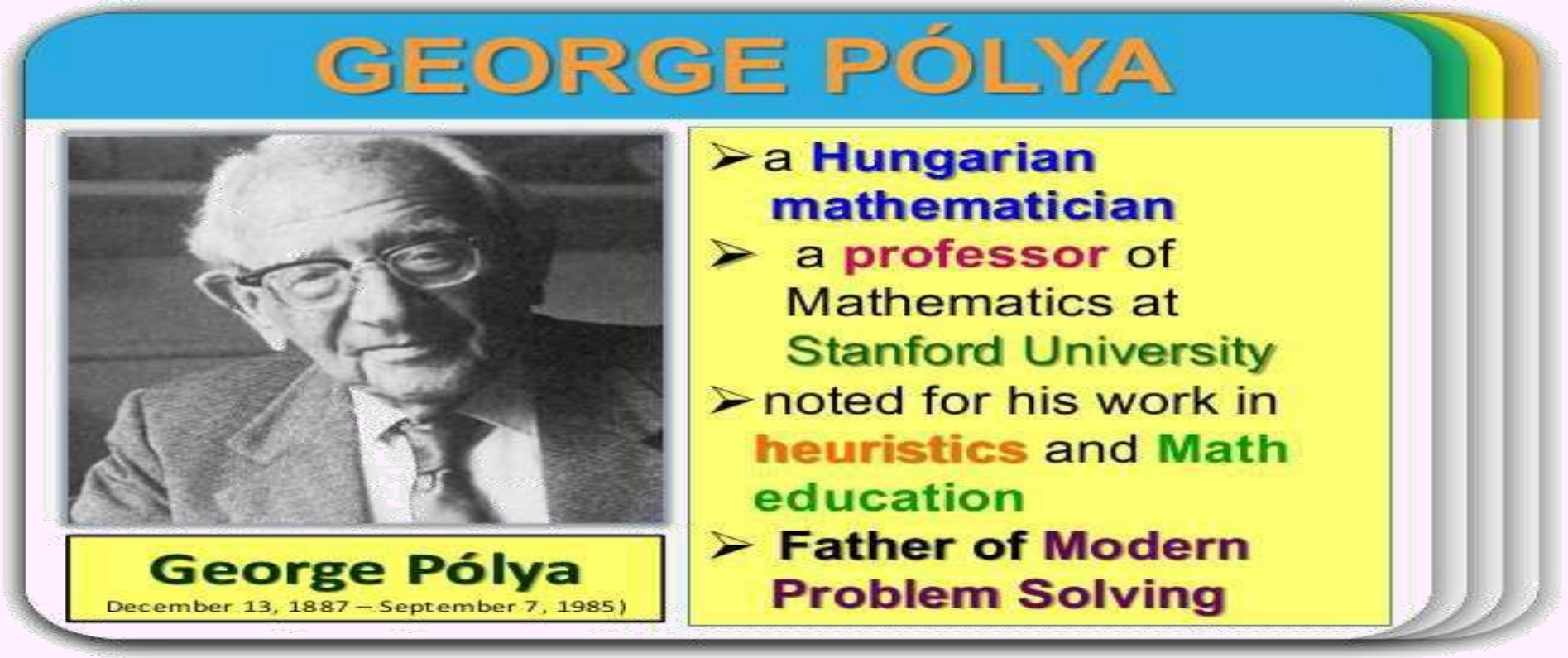
Example 3:

Calculate the 62^{nd} term in the Cube Number Sequence.

Problem-Solving Strategies

George Polya

One of the foremost recent mathematicians to make a study of problem solving was George Polya (1887–1985). He was born in Hungary and moved to the United States in 1940.



Polya's Problem-Solving Strategy

The basic problem-solving strategy that Polya advocated consisted of the following four steps.

Polya's Four-Step Problem-Solving Strategy

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Review the solution.

Polya's First Principle: Understand the Problem

To help you focus on understanding the problem, consider the following questions:

- Can you restate the problem in your own words?
- Can you determine what is known about these types of problems?
- Is there missing information that, if known, would allow you to solve the problem?
- Is there extraneous information that is not needed to solve the problem?
- What is the goal?

Polya's Second Principle: Devise a Plan

Here are some frequently used procedures/techniques when solving a problem:

- Make a list of the known information.
- Make a list of information that is needed.
- Draw a diagram.
- Make an organized list that shows all the possibilities.
- Make a table or a chart.
- Work backwards.
- Try to solve a similar but simpler problem.
- Look for a pattern.
- Write an equation. If necessary, define what each variable represents.
- Perform an experiment.
- Guess at a solution and then check your result.

Polya's Third Principle: Carry out the Plan

Once you devise a plan, you must carry it out:

- Work carefully.
- Keep an accurate and neat record of all your attempts.
- Realize that some of your initial plans will not work and that you may have to devise another plan or modify your existing plan.

Polya's Fourth Principle: Review the Solution

Once you have found a solution, always have it checked:

- Ensure that the solution is consistent with the facts of the problem.
- Interpret the solution in the context of the problem.
- Ask yourself whether there are generalizations of the solution that could apply to other problems.

Example 1:

A parking lot has motorcycles and cars. How many motorcycles and cars are there in a parking lot with 13 vehicles and 40 tires.

Solution: We apply the Polya's 4-step rule.

Step 1: Understand the Problem

Solution: We apply the Polya's 4-step rule.

Step 2: Devise a Plan

Solution: We apply the Polya's 4-step rule.

Step 3: Carry out the Plan

Solve the equations simultaneously by solving one variable in terms of the other.

Solution: We apply the Polya's 4-step rule.

Step 4: Review the Solution

Example 2:

A carpenter and a helper worked for a certain project. A carpenter is paid P800 and a helper P500 for each day of work. Together they earned a sum of P15,000 for 24 days in a work. How many days did the carpenter work?

Solution: We apply the Polya's 4-step rule.

Step 1. Understand the Problem

List the unknown and given data:

Let:
x = number of days that the carpenter work
$24 - x$ = number of days that the helper work
24 = number of days worked by the carpenter and the helper
$P15{,}000$ = total amount earned by the carpenter and the helper

Solution: We apply the Polya's 4-step rule.

Step 2: Devise a Plan

Make a guess based on the total wage. As indicated in the table, (closer to P15,000) the number of days worked by the carpenter less than the number of days worked by the helper.

No. of days (carpenter)	No. of days (helper)	Total amount earned
4	20	4(800) + 20(500) = 13,200
6	18	6(800) + 18(500) = 13,800
20	4	20(800) + 4(500) = 18,000
18	6	18(800) + 6(500) = 17,400

} Closer to P15,000

Solution: We apply the Polya's 4-step rule.

Step 3: Carry out the Plan

Use formula based on the total wage. Form the equation:

$$Wage\ earned\ by\ carpenter\ + Wage\ earned\ by\ helper\ = Total\ wages$$

Solution: We apply the Polya's 4-step rule.

Step 4: Review the Solution