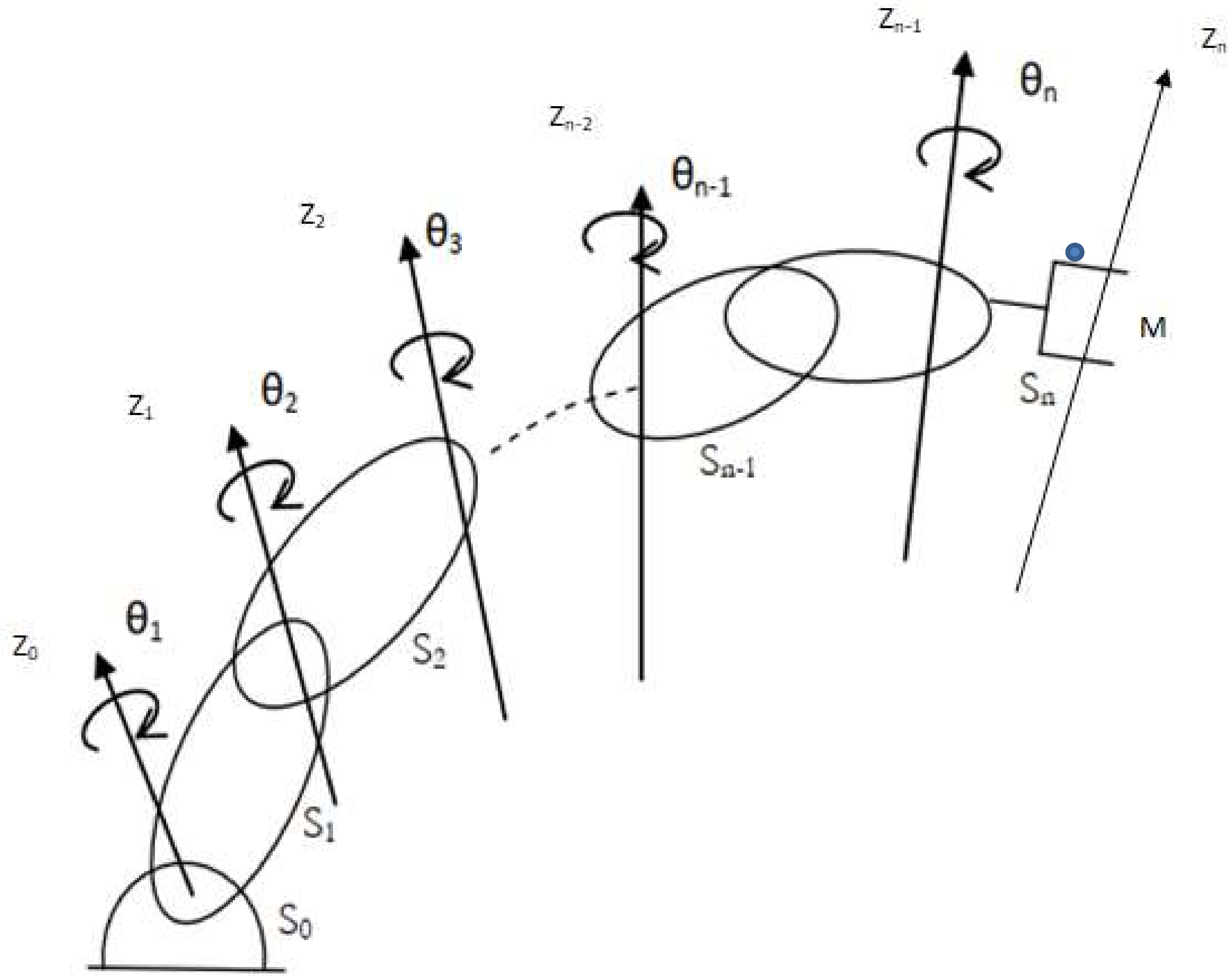
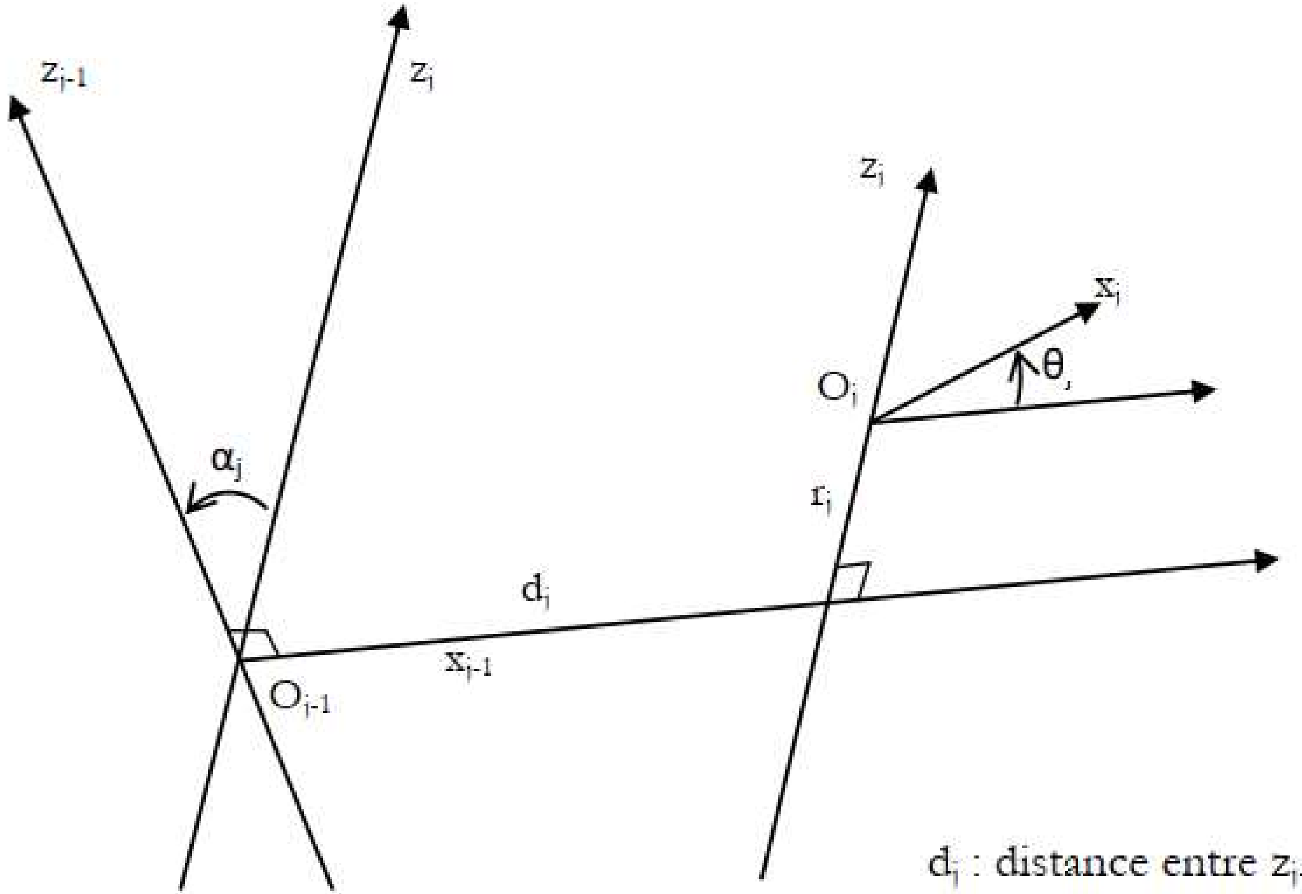
Denavit Hartenberg Representation



First representation



d_j : distance entre z_{j-1} et z_j mesurée le long de x_{j-1}.

α_j : angle entre z_{j-1} et z_j mesuré autour de x_{j-1}.

r_j : distance entre x_{j-1} et x_j mesurée le long de z_j.

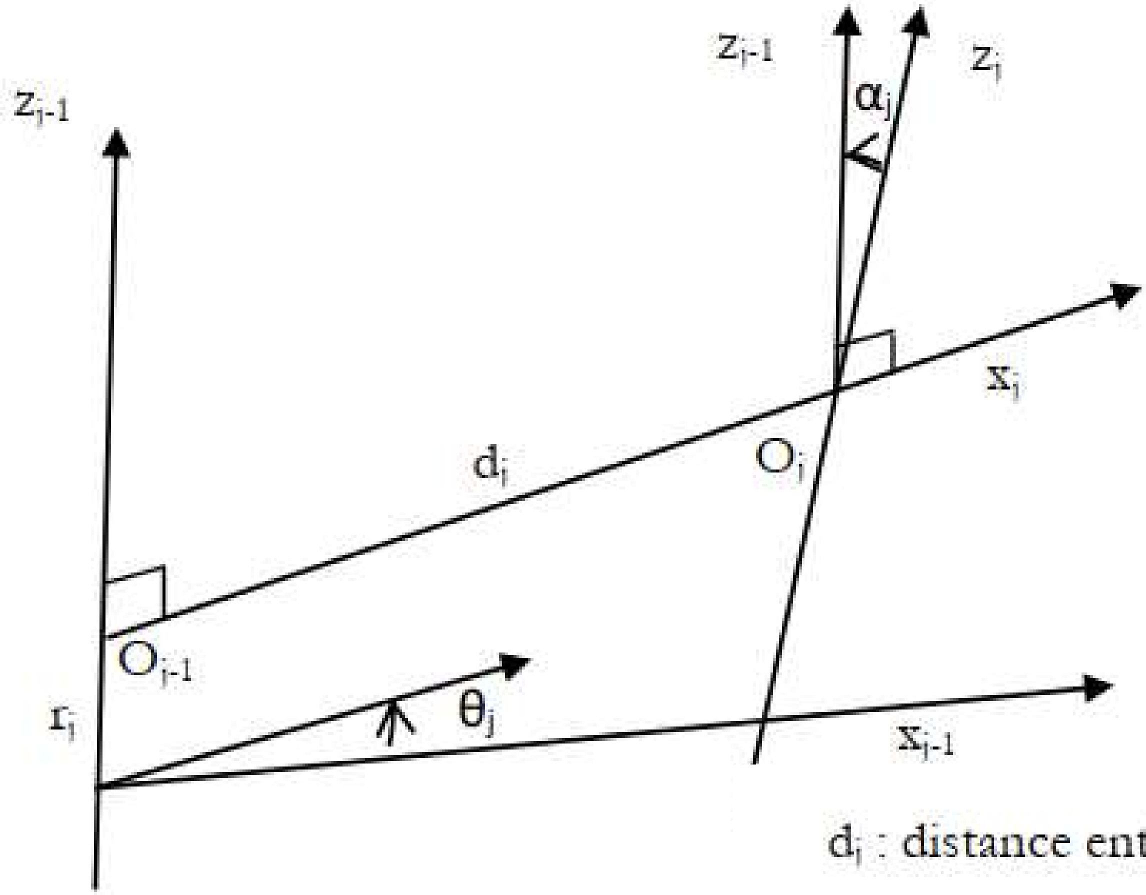
θ_j : angle entre x_{j-1} et x_j mesuré autour de z_j.

$$T^{j-1,j} = Rot(x,\alpha_j)\ Trans(x,d_j)\ Rot(z,\theta_j)\ Trans(z,r_j)$$

$$\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & c\alpha_j & -s\alpha_j & 0\\ 0 & s\alpha_j & c\alpha_j & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & d_j\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}c\theta_j & -s\theta_j & 0 & 0\\ s\theta_j & c\theta_j & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & r_j\\ 0 & 0 & 0 & 1\end{pmatrix}$$

$$T^{j-1,j} = \begin{pmatrix}c\theta_j & -s\theta_j & 0 & d_j\\ c\alpha_j s\theta_j & c\alpha_j c\theta_j & -s\alpha_j & -r_j\ s\alpha_j\\ s\alpha_j\ s\theta_j & s\alpha_j\ c\theta_j & c\alpha_j & r_j\ c\alpha_j\\ 0 & 0 & 0 & 1\end{pmatrix}$$

Second representation



d_j : distance entre z_{j-1} et z_j mesurée le long de x_j.

α_j : angle entre z_{j-1} et z_j mesurée autour de x_j.

r_j : distance entre x_{j-1} et x_j mesurée le long de z_{j-1}.

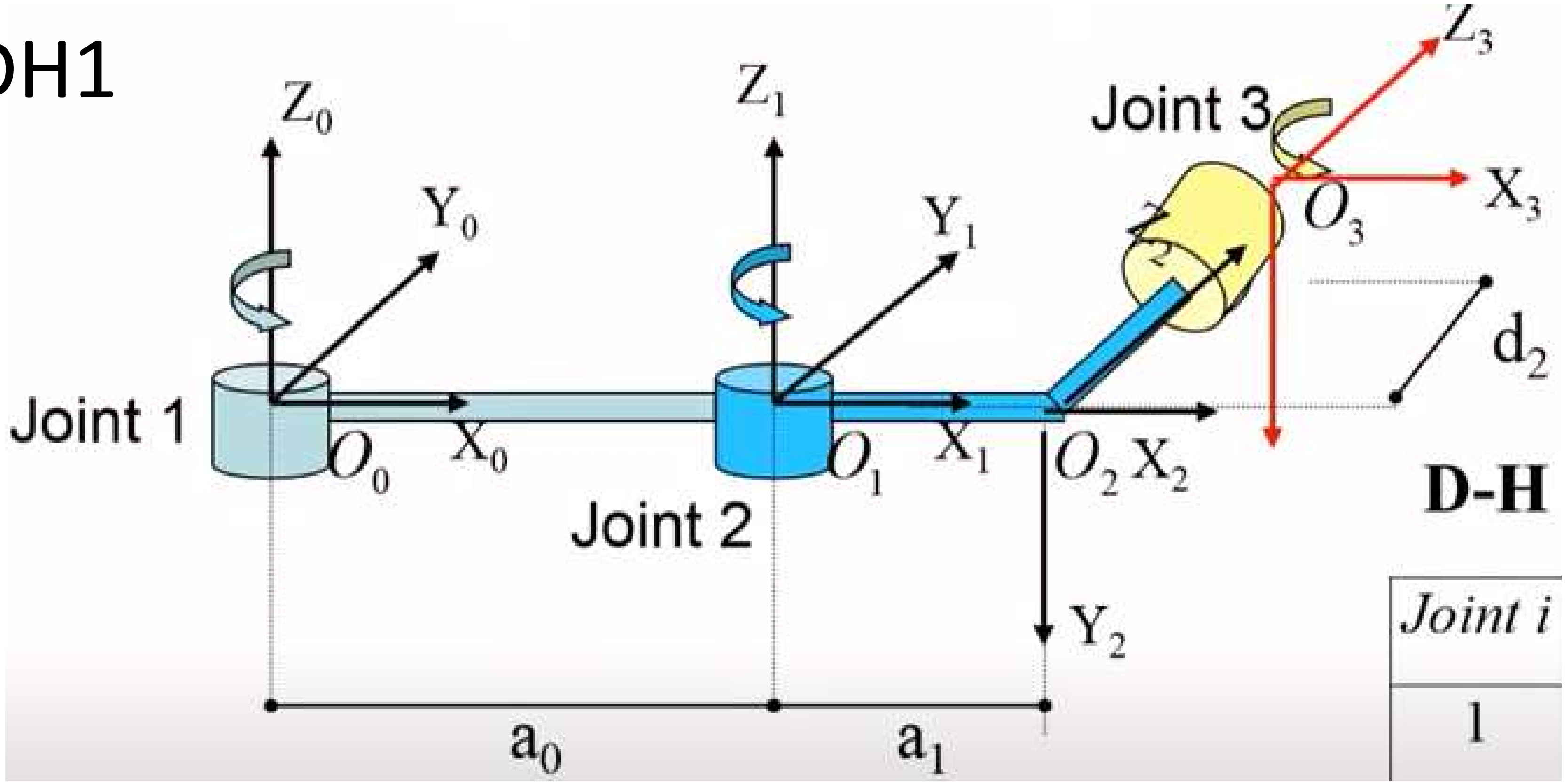
θ_j : angle entre x_{j-1} et x_j mesurée autour de z_{j-1}.

$$T^{j-1,j} = Rot(z,\theta_j)Trans(z,r_j)Trans(x,d_j)Rot(x,\alpha_j)$$

$$\begin{pmatrix} c\theta_j & -s\theta_j & 0 & 0 \\ s\theta_j & c\theta_j & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & r_j \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & d_j \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_j & -s\alpha_j & 0 \\ 0 & s\alpha_j & c\alpha_j & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T^{j-1,j} = \begin{pmatrix} c\theta_j & -c\alpha_j\, s\theta_j & s\alpha_j\, s\theta_j & d_j\, c\theta_j \\ s\theta_j & c\alpha_j\, c\theta_j & -s\alpha_j\, c\theta_j & d_j\, s\theta_j \\ 0 & s\alpha_j & c\alpha_j & r_j \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

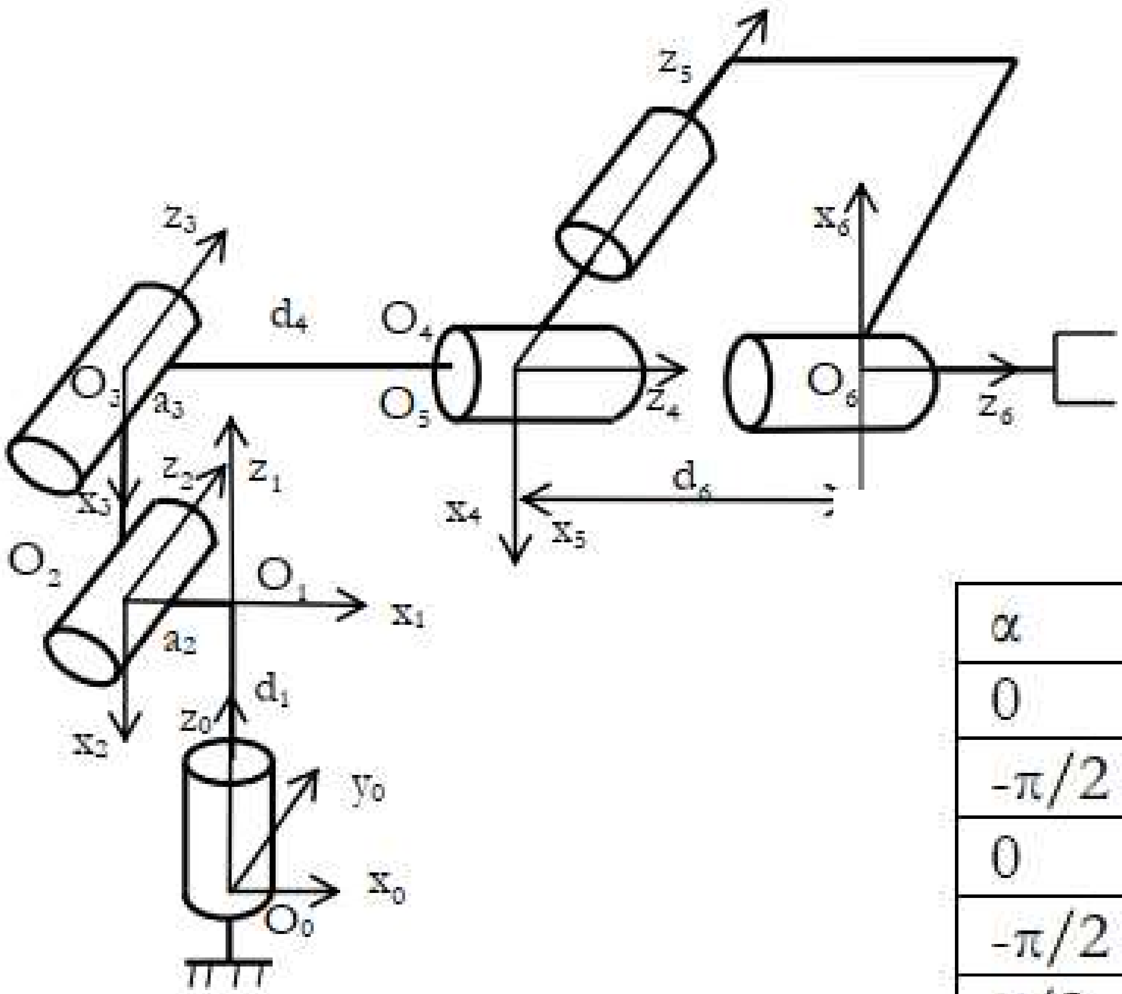
DH1



D-H Link Parameter Table

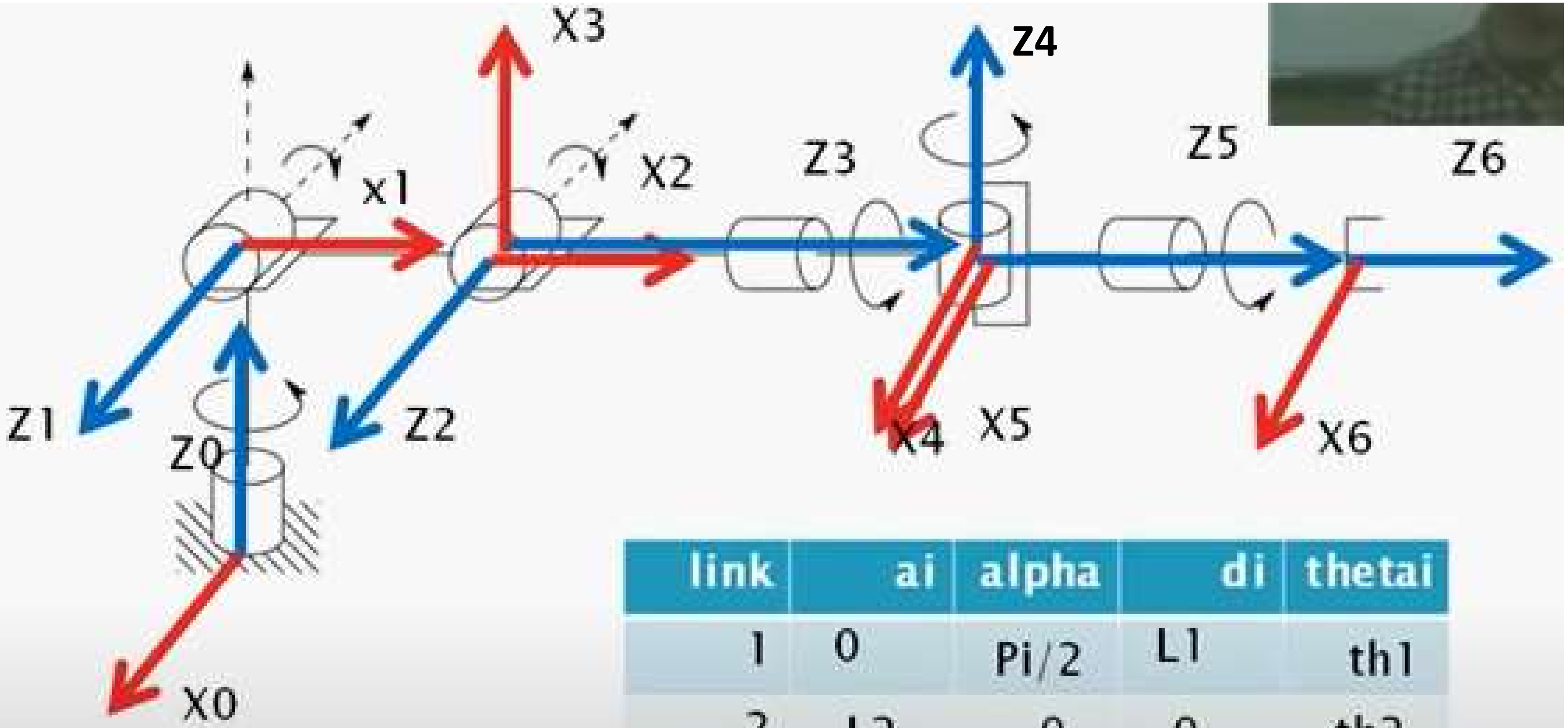
	Joint i	α_i	a_i	d_i	θ_i
0-1	1	0	a_0	0	θ_0
1-2	2	-90	a_1	0	θ_1
2-3	3	0	0	d_2	θ_2

DH1



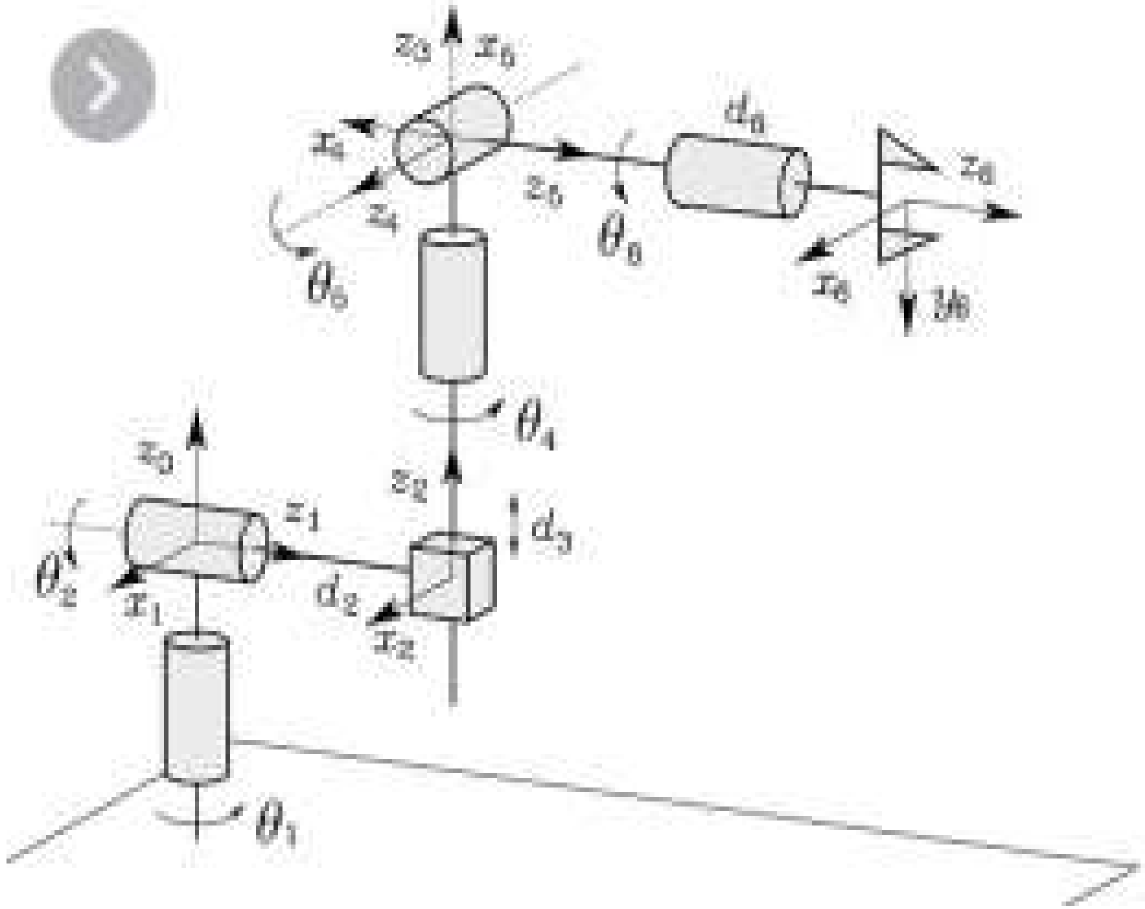
α	a	d	θ
0	0	352	θ_1
$-\pi/2$	70	0	$\theta_2-\pi/2$
0	360	0	θ_3
$-\pi/2$	0	380	θ_4
$\pi/2$	0	0	θ_5
$-\pi/2$	0	65	$\theta_6+\pi$

DH2



link	ai	alpha	di	thetai
1	0	Pi/2	L1	th1
2	L2	0	0	th2
3	0	Pi/2	0	th3
4	0	Pi/2	L3	th4
5	0	-pi/2	0	th5
6	0	0	L6	th6

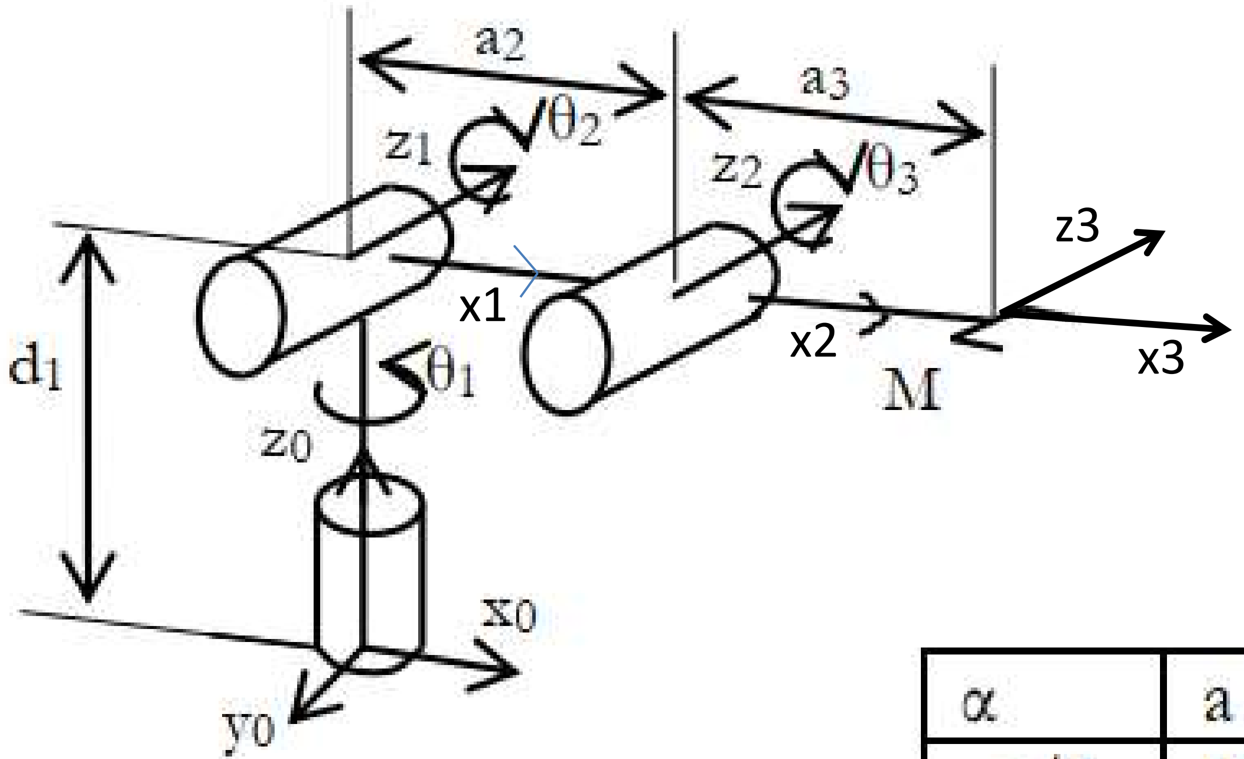
DH2



D-H parameters

Link	a_i	α_1	d_i	θ_i
1	0	$-\pi/2$	0	θ_1
2	0	$\pi/2$	d_2	θ_2
3	0	0	d_3	0
4	0	$-\pi/2$	0	θ_4
5	0	$\pi/2$	0	θ_5
6	0	0	d_6	θ_6

DH2



α	a	d	θ
$-\pi/2$	0	d_1	θ_1
0	a_2	0	θ_2
0	a_3	0	θ_3

Matlab function first representation

```
function T = Denavit1(a,d,t,r)

digits(2);
a=sym(a);
T=[cos(t) -sin(t) 0 d;cos(a)*sin(t) cos(a)*cos(t) -sin(a)
    - r*sin(a);...
    sin(a)*sin(t)  sin(a)*cos(t) cos(a)  r*cos(a);...
    0 0 0 1];
T=vpa(T);
```

Matlab function second representation

```
function T = Denavit2(a,d,t,r)

a=sym(a);
digits(2);
T=[cos(t) ,-cos(a)*sin(t), sin(a)*sin(t) ,d*cos(t);sin(t),
   cos(a)*cos(t), -sin(a)*cos(t), d*sin(t);...
   0 , sin(a), cos(a),  r;...
   0 ,0, 0, 1];
T=vpa(T);
```

Position and orientation of the end effector

$$T^{0,n} = \begin{pmatrix} A^{0,n} & O_0O_n{}^0 \\ 0\ 0\ 0 & 1 \end{pmatrix}$$

```
function [A,P,L]= matrices_TH1(alpha,a,teta,r)

  n=size(alpha);
  n=n(1,2);
  H=eye(4,4);
  for i=1:n
    T=Denavit1(alpha(i),a(i),teta(i),r(i));
    L(:,:,i)=T;
    H=H*T;
  end
  A=H(1:3,1:3);
  P=H(1:3,4);
```

```
function [A,P,L]= matrices_TH2(alpha,a,teta,r)

n=size(alpha);
n=n(1,2);
H=eye(4,4);
for i=1:n
   T=Denavit2(alpha(i),a(i),teta(i),r(i));
   L(:,:,i)=T;
   H=H*T;
end
A=H(1:3,1:3);
P=H(1:3,4);
```