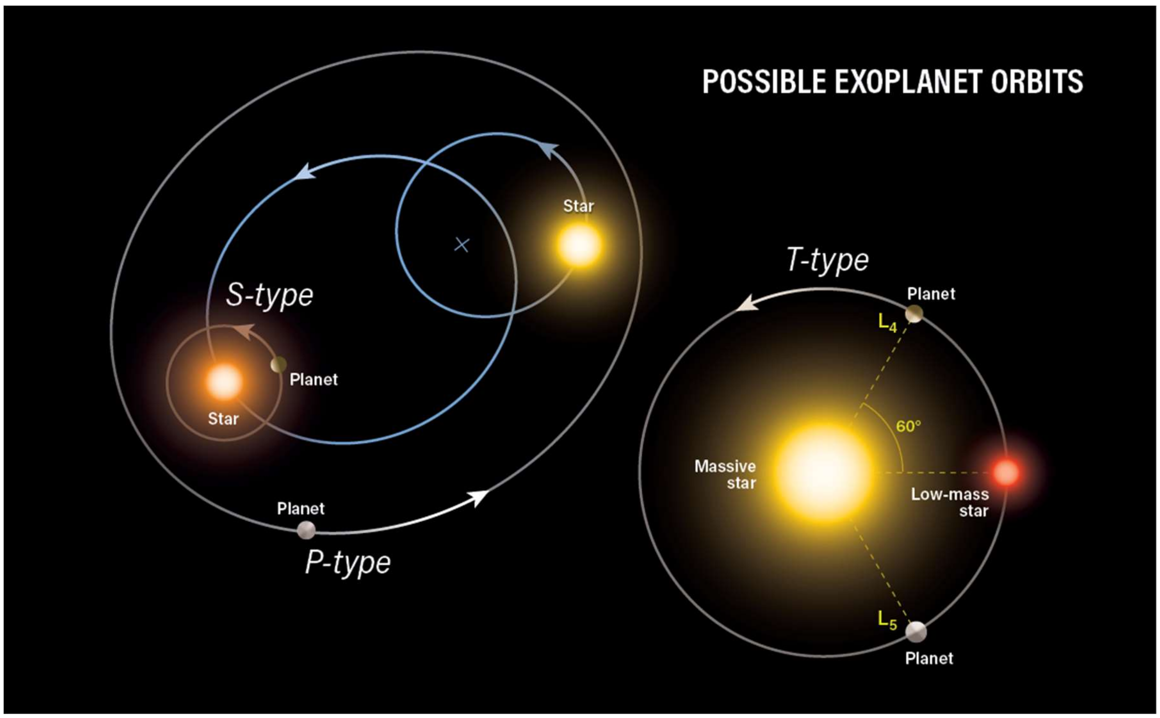


N Body System

STUDYING PROPERTIES OF MULTISTAR SYSTEM

Soumadip Saha | Computational Methods | April 10, 2021

1. ABSTRACT:

My objective is to analyze properties of a N body star system and to write a python code which gives an accurate approximation of the Newton's Law of motion for N bodies. I have studied multiple ODE solver in this course and I have found fourth order Runge Kutta Method gives us the best approximation in my known ODE solving techniques. My result is a useful, accurate simulation tool, which has many applications and also room for further improvements. To conclude, after 3 weeks of research I have assembled (all original otherwise stated) and animated using Matplotlib, provided a solution of a N body problem.

2. INDTRODUCTION:

The N body problem in classical mechanics which poses the question, given all the relevant data (Mass, Initial Position, Initial Velocity at time $t = 0$) what will be its position after time dt, under the gravitational force which is formulated by;

$$\vec{\mathrm{F}}_{12} = \frac{\mathrm{GM}_1\mathrm{M}_2}{|\mathrm{R}_{12}|^2}\hat{R}_{12}$$

Since solving the problem for $\mathbf{N} > \mathbf{3}$ is tough, we need computational help. So we need to choose a good numerical approximation so that our accuracy is high. As stated earlier I have used 4$^{\mathrm{th}}$ order Runge Kutta Method which gives error of order *O(h^5)*. I have chosen the idea of my paper which is partially based on this Project 14. I have taken some help about my code from this site.

For a N body system, the differential equations are:

$$\frac{d\vec{v}_i}{dt} = \sum_{j\neq i}^{N} \frac{Gm_j}{r_{ij}^3}\overrightarrow{r_{ij}}$$

$$\frac{d\vec{x}_i}{dt} = \vec{v}_i$$

We need to solve two differential equation simultaneously to get the solution of $\vec{x}_i$ for any time t.

3. NUMERICAL METHOD:

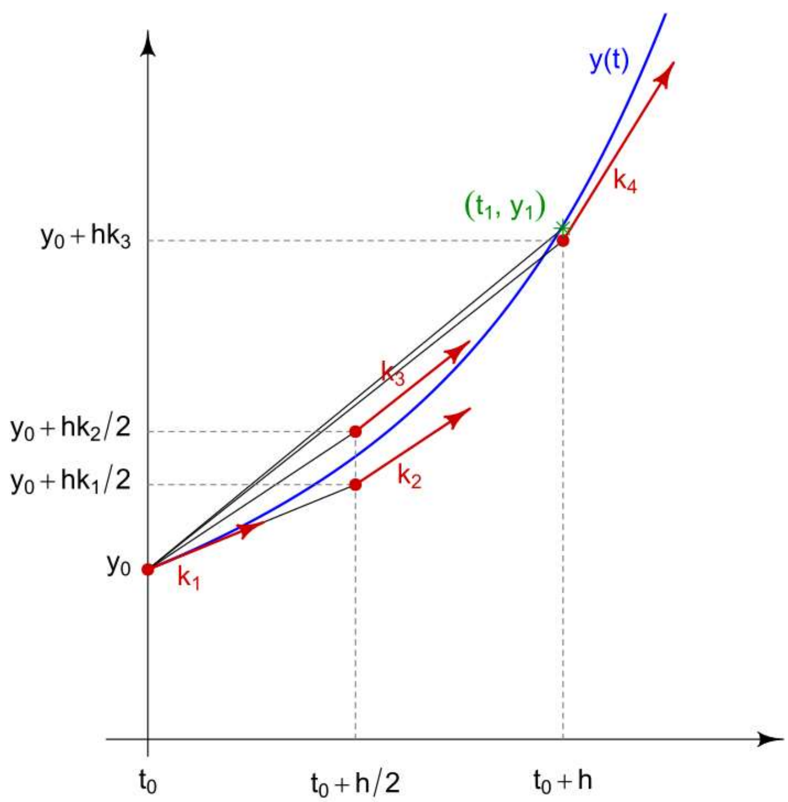
Fourth order Runge Kutta method is basically some steps to perform in order to find out the function value at time *t+dt* if we know the function value at time t.

Let's say, we know function value y_n at t_n.

If we take time step h. Now in order to calculate y_{n+1} at time t_n:

- $K_1 = f(t_n, y_n)$
- $K_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}K_1\right)$
- $K_3 = f\left(t_n + \frac{h}{2}, y_n + \frac{hK_2}{2}\right)$
- $K_4 = f(t_n + h, y_n + hK_3)$

Where $f(t, y) = \frac{dy}{dt}$.

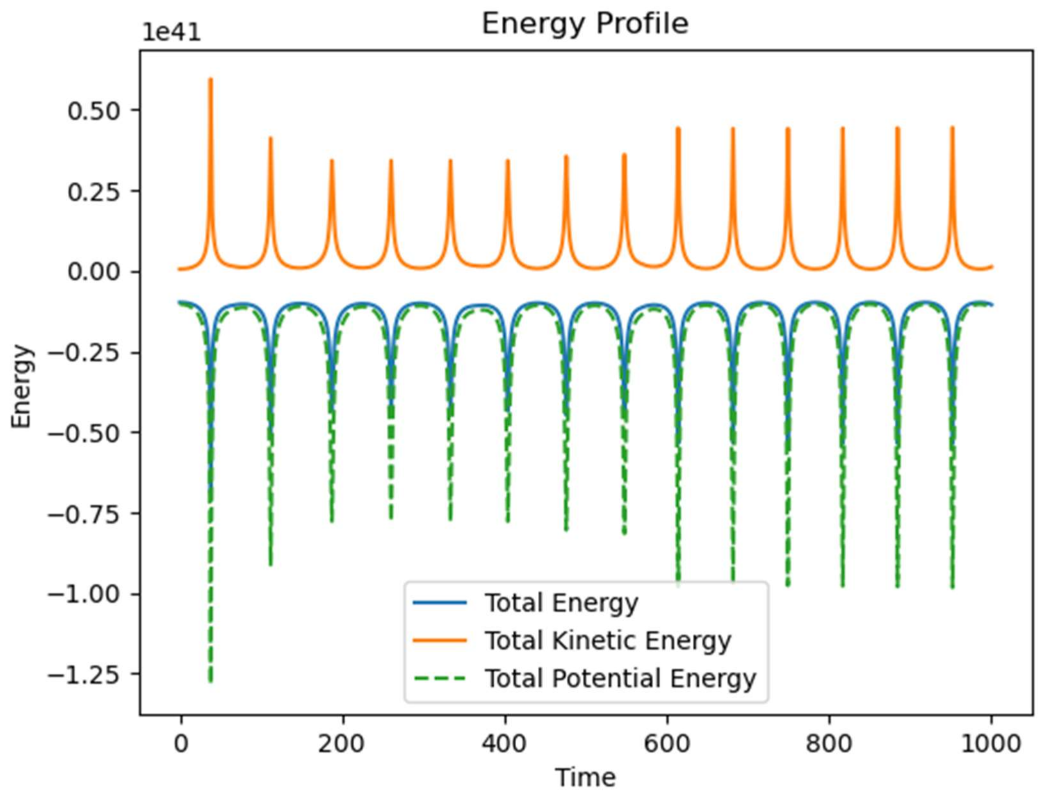


This method gives us local truncation error of order *O(h^5)* and total accumulated error of order *O(h^4)*. Whereas Euler Method global error bound of order *O(h)* or 2nd order Runge Kutta method gives us local truncation error of order *O(h^3)* and total accumulated error of order *O(h^2)*.

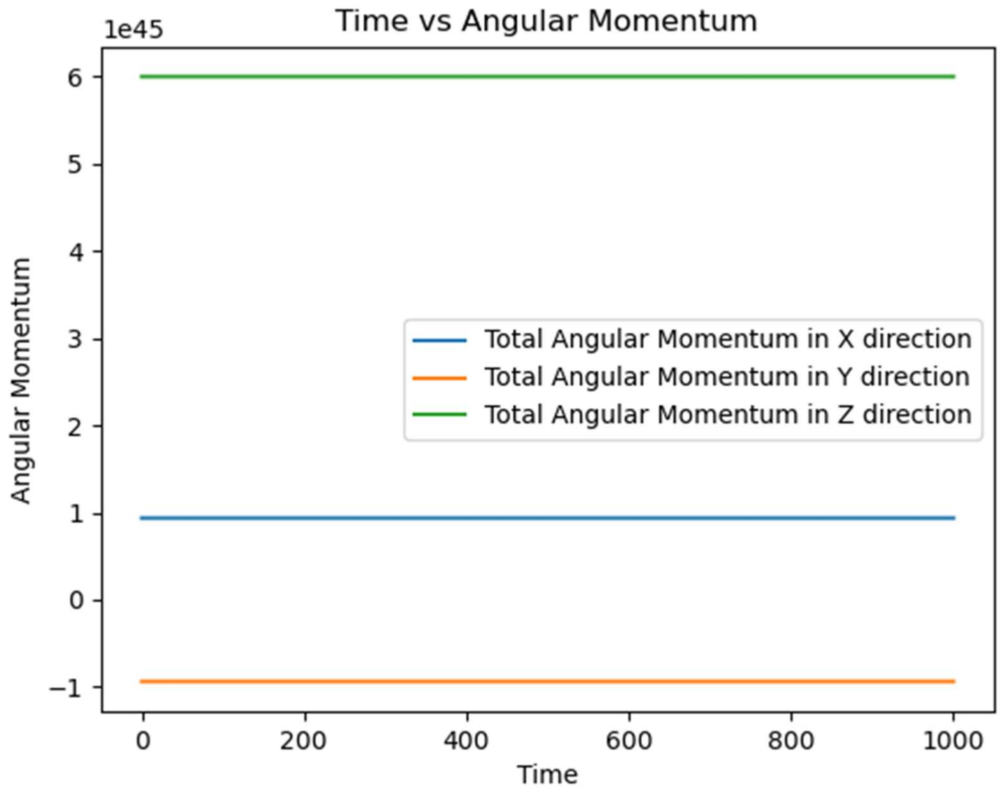
4. RESULTS:

Since our system is isolated so there are no external forces the energy, momentum and angular momentum of the system should be conserved.

I have analyzed the total energy conservation and angular momentum conservation of the system for 3-star system.



As we can see when the absolute value of our potential energy of the system goes up (which means that the stars are very close), the kinetic energy of the system goes up (which means their velocity is increasing) and our system's energy is conserved (Note that the numerical value of the potential energy is negative because our reference potential energy is at infinity).



As we can see the total angular momentum of the system in x, y, z direction is conserved because there are no external torque working on the system.

5. DISCUSSION:

Although the total energy if the system is almost constant but there are sudden dips in the total energy when the distance between the stars is low. It means that our numerical approximation is not well enough at those times. Since the velocity is increasing when stars are close to each other we may get better approximation if we reduce the time step *h* furthermore which will lead us to better approximation. (Note that although the Y axis is in power of 10^{41} the relative change is in order of 0.86.

While working on this topic I have reviewed some classical theories and understood very crucial techniques of python programming language and understood numerical techniques in a better way.

6. CONCLUSION:

As I have run the simulation of my problem I have realized vector implementation of array rather than using loops, makes a program much faster. Although after using vectors as much as possible still my code takes 300 seconds of time to execute the simulation entirely for 3 body problem. If we increase the number of bodies or time of simulation to calculate the trajectory of the stars it will take more and more time. Still from my perspective it was a very good learning opportunity for me.