CAT 2025

MBA PIONEER

Lecture- 01 | **Factors + Unit's Digit**

Number System - 1

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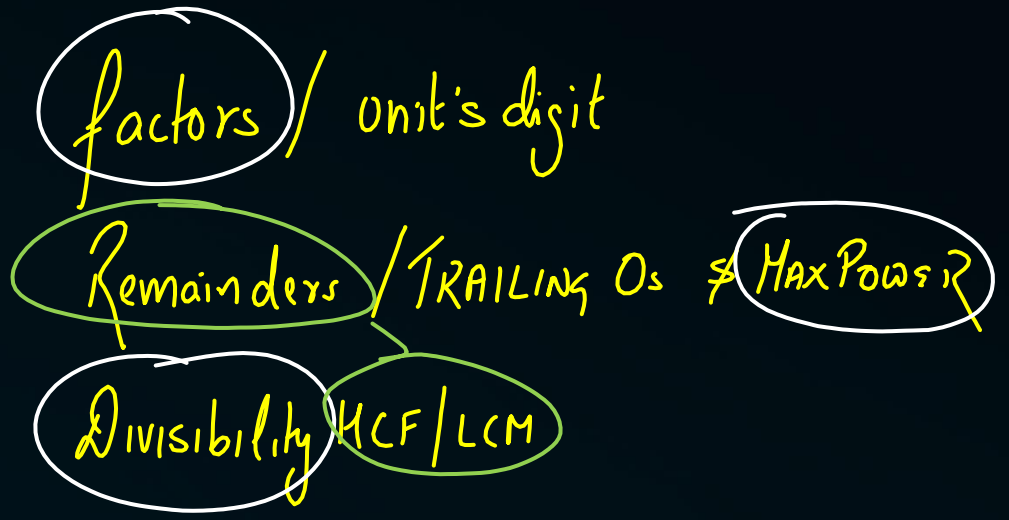
Recap of Previous Lecture :



TOPICS *to be covered*

1. Number of Factors
2. Sum of Factors

} formula based

3. From Factors to Number — Aptitude

4. Unit's Digit

Topic: Number of Factors

$24 = \boxed{1} \quad 24$

$2 \quad 12$

$\boxed{3} \quad 8$

$4 \quad 6$

$24 = 2^3 \times 3^1$

$4 \times 2 = 8$ factors

$3 \times 5 \times 7 \times 2$

Total Factors

$2^3 \times 3^1$

+1 kar ke ayega

+1 +1

$4 \times 2 = 8$

$2^0 \quad 3^0$

$2^1 \quad 3^1$

2^2

2^3

$4 \times 2 = 8$

Odd Factors

$\cancel{2^3} \times 3^1$

ayega nahi

+1

2 factors

3^0

3^1

$2 = 2$

Even Factors

TOTAL − ODD

$8 - 2$

$= 6$ factors

$2^1 \quad 3^0$

$2^2 \quad 3^1$

2^3

$3 \times 2 = 6$ f.

Total Factors = $N = p^a \times q^b \times r^c$

$N_0 = (a+1)(b+1)(c+1)$

# of factors

QUESTION – 1

#Q. Find the number of divisors of 144.

A 12

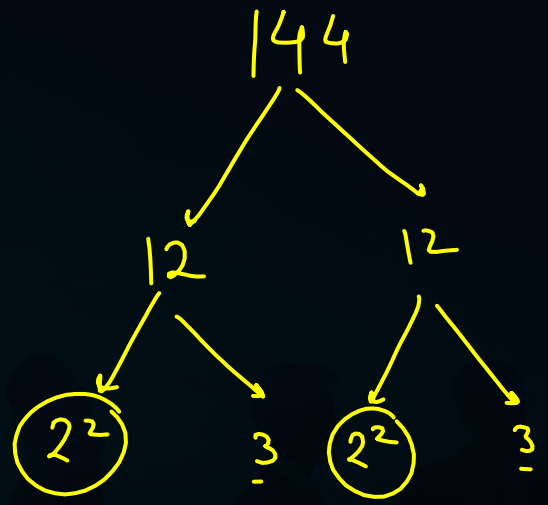
B 15

C 18

D *none*

$144 = 2^4 \times 3^2$

$5 \times 3 = 15$ ✓



QUESTION – 2

#Q. Find the number of divisors of 544 excluding 1 and 544.

A. 12

B. 15

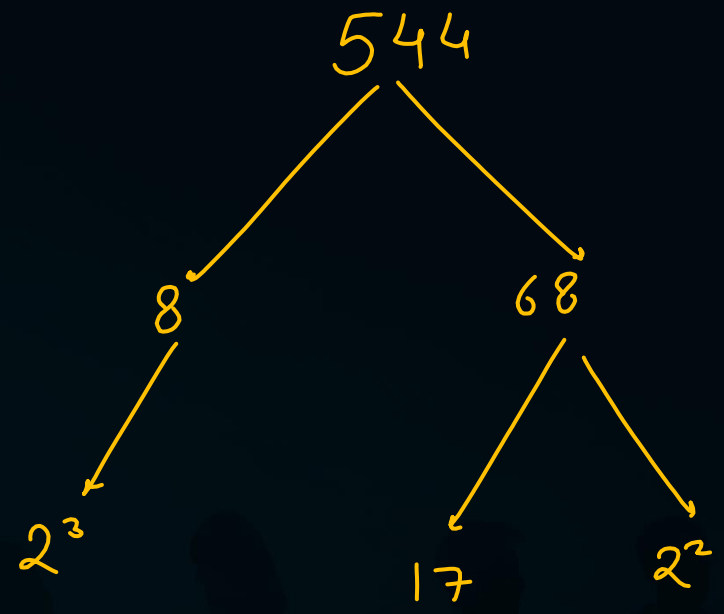
C. 18

D. *none*

$2^5 \times 17^1$

$6 \times 2 = 12$ factors

(1 & 544 inc.)



12 − 2 = 10 factors

(exc 1 & 544)

QUESTION – 3

#Q. Find the number of total factors, odd factors and even factors of the number $2^8 * 3^6 * 5^4 * 10^5$?

$10^5 = (2 \times 5)^5$

$N = 2^8 \times 3^6 \times 5^4 \times 10^5$??

$= 2^8 \times 3^6 \times 5^4 \times 2^5 \times 5^5$

$= 2^{13} \times 3^6 \times 5^9$

t. factors $= 14 \times 7 \times 10 = 980$

O. factors $= 7 \times 10 = 70$

E. factors $= 910$

72

1	2	3	4	6	8	9	12	18	24	36	72

Perfect Square

$72 = 2^3 \times 3^2$

$2^0 \quad 3^0$

$2^2 \quad 3^2$

$2 \times 2 = 4$

Perfect Cube

$72 = 2^3 \times 3^2$

$2^0 \quad 3^0$

2^3

$2 \times 1 = 2$

QUESTION – 4

#Q. How many factors of 1080 are perfect squares?

A. 8

B. 4 ✓

C. 6

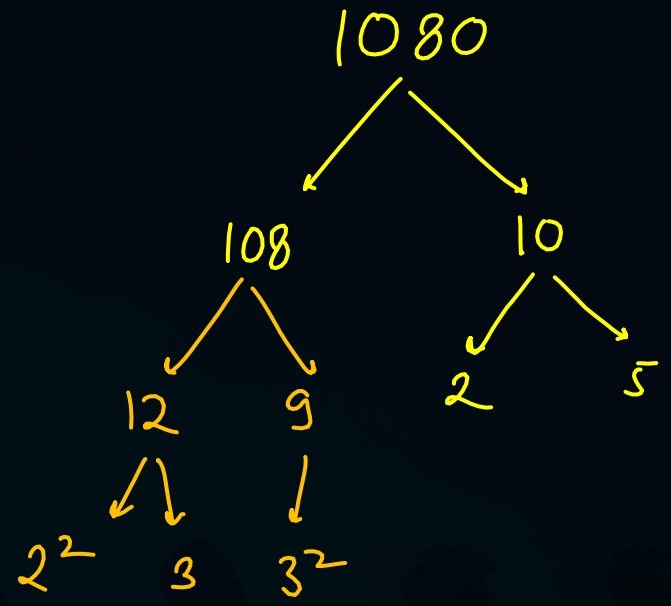
D. 2

$1080 = 2^3 \times 3^3 \times 5^1$

$2^0 \quad 3^0 \quad 5^0$

$2^2 \quad 3^2 \quad —$

$2 \times 2 \times 1 = 4$



QUESTION – 5

#Q. How many perfect cubes factor of 10648 is

1331

A 2

B 4

C 3

D 5

$N = 2^3 \times 11^3$

T. factor = $4 \times 4 = 16$

O. factor = 4

E. factor = $3 \times 4 = 12$

P.Sq. factor = $2 \times 2 = 4$

$2^0 \quad 11^0$

$\frac{2^3}{2} \times \frac{11^3}{2} = 4$ factor

72

1	2	3	4	6	8	9	12	18	24	36	72

Factors divisible by 8

$\frac{72}{8} = 9$

factors of $9 \longrightarrow 3^2$ → $2+1$ → 3

3 factors

Factors divisible by 6

$\frac{72}{6} = 12$

$2^2 \times 3^1$ → $(2+1) \times (1+1)$

$3 \times 2 = 6$ factors

$72 \longrightarrow 2^3 \times 3^2$

$6 \longrightarrow 2^1 \times 3^1$

$2^1, 2^2, 2^3$ — 3

$3^1, 3^2$ — 2

$3 \times 2 = 6$

QUESTION – 6

#Q. $N = 2^8 * 3^6 * 5^4 * 10^5$, Find

(i) factors divisible by 100 ⟶ 672

(ii) " " " 500 ⟶ 588

(iii) " " " 3000 ⟶ 462

(6)

① $2^8 \times 3^6 \times 5^4 \times 10^3$

$2^{11} \times 3^6 \times 5^7$

12 7 8

$12 \times 56 = 560 + 112$

$= 672$

② $2^{11} \times 3^6 \times 5^6$

12 7 7

$12 \times 49 = 588$

3 | $2^{10} \times 3^5 \times 5^6$

11 6 7

11×42

462

Topic: SUM OF FACTORS

60

1	2	3	4	5	6	10	12	15	20	30	60

$60 \rightarrow 2^2 \times 3^1 \times 5^1$

Even Factors

$2^1 \quad 3^0 \quad 5^0$

$2^2 \quad 3^1 \quad 5^1$

$3 \times 2 \times 2 = 8$ fac.

Odd Factors

$3^0 \quad 5^0$

$3^1 \quad 5^1$

$2 \times 2 = 4$ fac

Total Factors

$2^0 \quad 3^0 \quad 5^0$

$2^1 \quad 3^1 \quad 5^1$

2^2

$3 \times 2 \times 2 = 12$ fac.

Sum of Even Factors

$2^1 \quad 3^0 \quad 5^0$

$2^2 \quad 3^1 \quad 5^1$

$6 \times 4 \times 6 = 144$

Sum of Odd Factors

$3^0 \quad 5^0$

$3^1 \quad 5^1$

$4 \times 6 = 24$

Sum of Factors

$2^0 \quad 3^0 \quad 5^0$

$+ \; 2^1 \quad + \; 3^1 \quad + \; 5^1$

$+ \; 2^2$

$7 \times 4 \times 6$

$28 \times 6 = 168$

QUESTION – 7

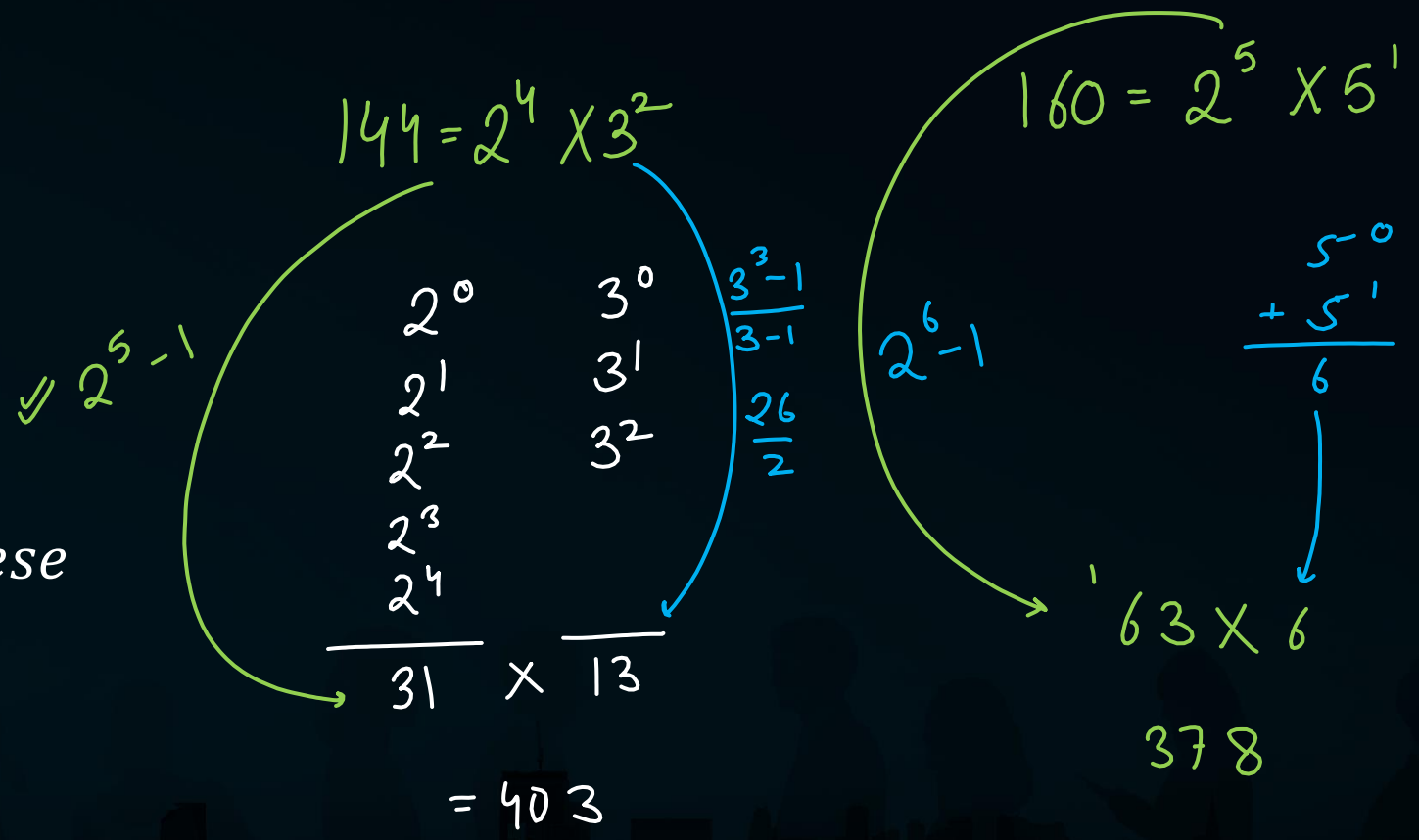
#Q. Find the sum of the sum of divisors of 144 and 160.

A. 589

B. 735

C. 781 ✓✓

D. *none of these*



QUESTION – 8

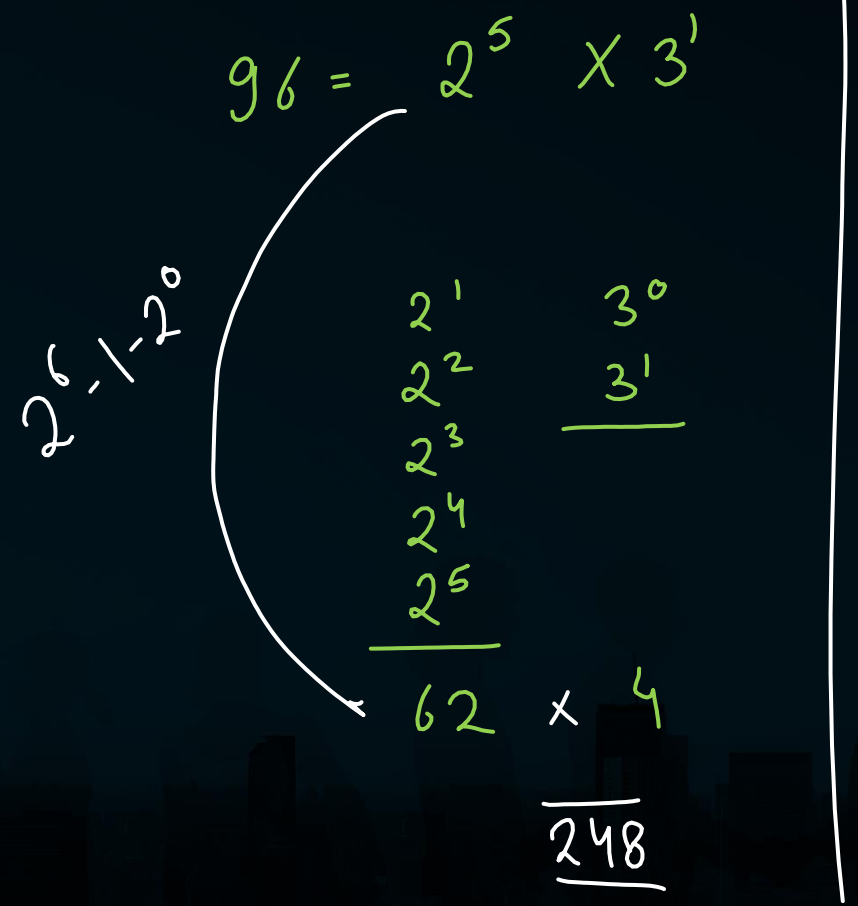
#Q. Find the sum of the sum of even divisors of 96 and the sum of odd divisors of 3600.

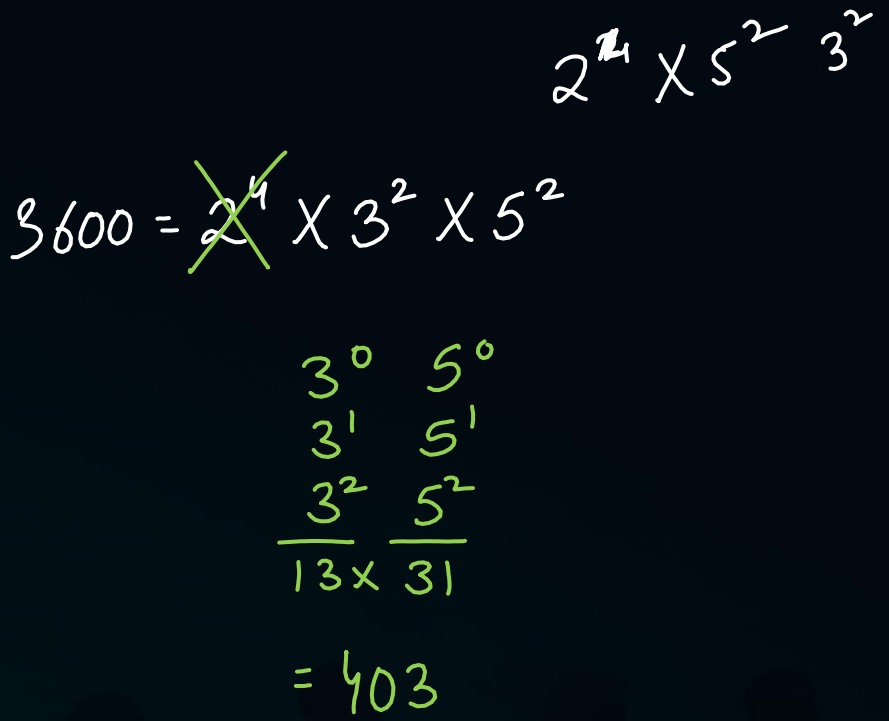
(A) 639

(B) 651

(C) 735

(D) 589





72

1	2	3	4	6	8	9	12	18	24	36	72

$72 = 2^3 \times 3^2$

Sum of Perfect Square

$2^0 \quad 3^0$

$2^2 \quad 3^2$

$5 \times 10 = 50$

Sum of Perfect Cube

$2^0 \quad 3^0$

2^3

$9 \times 1 = 9$

Sum of factors of 3600 is divisible by 100

$3600 = 2^2 \times 5^2 \times 3^2 \times 2^2$

$= 2^4 \times 3^2 \times 5^2$

$100 = 2^2 \times 5^2$

2-4

$2^2 \quad 3^0 \quad 5^2$

$2^3 \quad 3^1$

$2^4 \quad 3^2$

$28 \times 13 \times 25 = 9100$

700×13

Topic: Product Of Factors

72

1	2	3	4	6	8	9	12	18	24	36	72

1st Factor	1
2nd Factor	2
3rd Factor	3
4th Factor	4
5th Factor	6
6th Factor	8

12th Factor	72
11th Factor	36
10th Factor	24
9th Factor	18
8th Factor	12
7th Factor	9

$72 \times 72 \times 72 \times 72 \times 72 \times 72 = 72^6$

$\frac{f}{2} = \frac{12}{2} = 6$

Product of factors of $N = [N]^{\frac{f}{2}}$

QUESTION – 9

#Q. What is the product of all factors of 12 ?

A 144

B 512

C 1728 ✓

D 1331

$(12)^3$

QUESTION – 10

#Q. What is the product of all factors of 36 ?

A 36^8

B 36^9

C 6^8

D 6^9 ✓

$(36)^{9/2}$

$(6^2)^{9/2}$

6^9

$36 \rightarrow 2^2 \times 3^2$

$f = 9$

72

1	2	3	4	6	8	9	12	18	24	36	72

1st Factor	1
2nd Factor	2
3rd Factor	3
4th Factor	4
5th Factor	6
6th Factor	8

12th Factor	72
11th Factor	36
10th Factor	24
9th Factor	18
8th Factor	12
7th Factor	9

PARTNER factors → Sum of position = no. of factors + 1

13 12 + 1

QUESTION – 11

#Q. The 4th position factor of 384 from beginning (when arranged in ascending order) is 4. If 13th position factor from beginning is x. Find x

A 96

B 13

C 48

D None of these

$(4^{th})(13^{th}) = 384$

$4(x) = 384$

$x = 96$

$384 = 2^2 \times 96$

$= 2^7 \times 3^1$

$= 8 \times 2 = 16$

Topic: From Factors to Numbers

Number ⟶ factors.

⟵

QUESTION – 12

#Q. What is the smallest number which has total 15 factors ?

$72 \rightarrow 2^3 \times 3^2$

$4 \times 3 = 12$ factors

144 ✓

✓ $144 = 2^4 \times 3^2$

$5 \times 3 = 15$ factors

$N = 2^{14} \times$ ( )

$15 \times 1 = 15$ factors

QUESTION – 13

#Q. What is the minimum sum of two numbers which have total 12 factors ?

$72 = 2^3 \times 3^2$

$4 \times 3 \longrightarrow 12$

$60 = 2^2 \times 3^1 \times 5^1$

$3 \times 2 \times 2 \longrightarrow 12$

$96 = 2^5 \times 3^1$

$6 \times 2 \longrightarrow 12$

$60 + 72 = 132$ ✓

$N = 2^{11} \times 3^{6}$

$12 \times 1 \longrightarrow 12$

QUESTION – 14

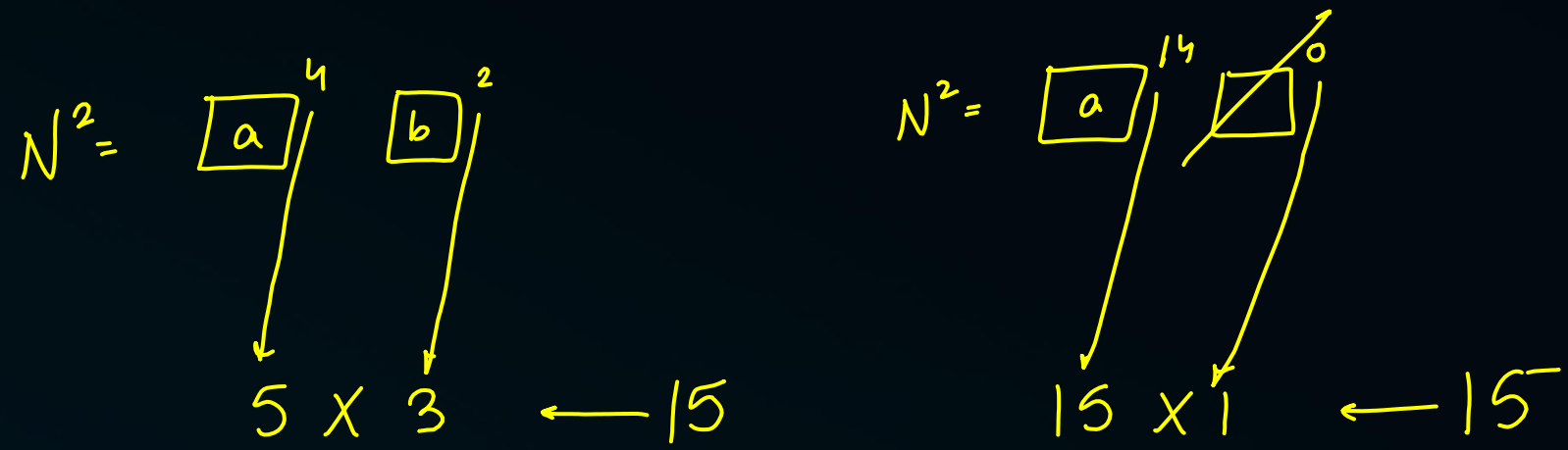
#Q. A number N^2 has 15 factors. How many factors can N have?

A. 5 or 8 factors

B. 6 or 8 factors

C. 4 or 6 factors

D. *none*



$N^2 = a^4 b^2$

$N = a^2 b^1$

$3 \times 2 = 6$

$N^2 = a^{14}$

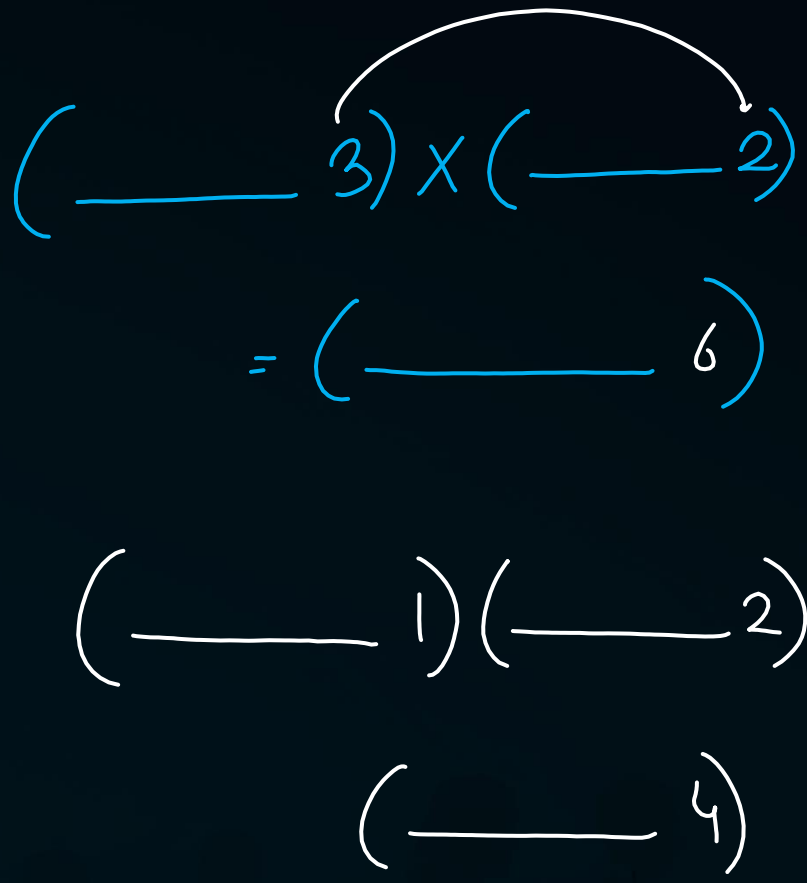
$N = a^7$

$+1$

8

Topic: UNIT's DIGIT

- Units' digit only depends upon Unit's Digit of all the numbers



- Power of __ 1

$$(-1)^n = (-1)(-1)\cdots(-1) = (-1)$$

- Power of __ 5

$$(-5)^n = (-5)(-5)(-5)\cdots(-5) = (-5)$$

- Power of __ 6

$$(-6)^n = (-6)(-6)\cdots(-6) = (-6)$$

- Power of __ 0

$$(-0)^n = (-0)(-0)\cdots(-0) = (-0)$$

Power of __4

$(\_\_4)(\_\_4) = (\_\_\_\_6)$

$(\_\_4)(\_\_4)(\_\_4) = (\_\_\_\_4)$

$(\_\_4)(\_\_4)(\_\_4)(\_\_4) = (\_\_\_\_6)$

4, 6, 4, 6, 4, 6 . . . .

$(\_\_\_\_4)^{odd} \rightarrow (\_\_4)$

$(\_\_\_\_4)^{even} \rightarrow (\_\_\_\_6)$

Power of __9 ——————— 9, 1, 9, 1 . . .

$(\_\_\_\_9)^{odd} = (\_9)$

$(\_9)^{even} = (\_1)$

Power of __ 2

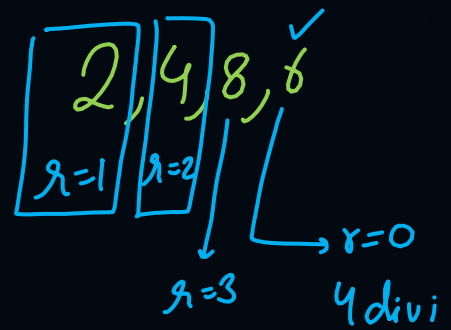
$(\_\_2)^1 = (\_\_2)$

$(\_\_2)^2 = (\_\_4)$

$(\_\_2)^3 = (\_\_8)$

$(\_\_2)^4 = (\_\_6)$

$\frac{P}{4} = r$



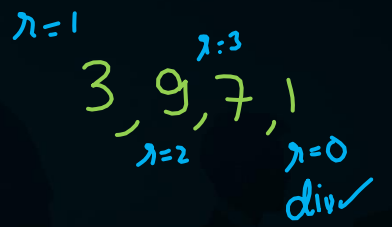
Power of __ 3

$(\_\_3)^1 = (\_\_3)$

$(\_\_3)^2 = (\_\_9)$

$(\_\_3)^3 = (\_\_7)$

$(\_\_3)^4 = (\_\_1)$



Power of _ _ 7

7, 9, 3, 1 ✓

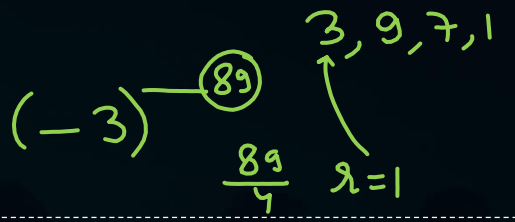
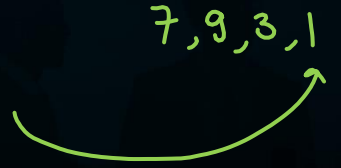
7 → r = 1
9 → r = 2
3 → r = 3

Power of _ _ 8

8, 4, 2, 6 ✓

8 → r = 1
4 → r = 2
2 → r = 3

Power of number ending with 0, 1, 5, 6	Power of number ending with 4, 9	Power of number ending with 2,3,7,8
Cyclicity - 1	Cyclicity - 2 odd EVEN	Cyclicity → 4 $\frac{\text{Power}}{4}$ $r = 1$ $r = 2$ $r = 3$ div ✓

Expression	Cyclicity	Answer	Remarks
26^{55}	1	6	6, 6, 6 - - - -
19^{173}	2	9	19^{odd}
24^{56}	2	6	24^{even}
$43^{123456789}$	4	3	$(-3)^{89}$ 
37^{532}	4	1	$\frac{32}{4}$ div 

QUESTION – 15

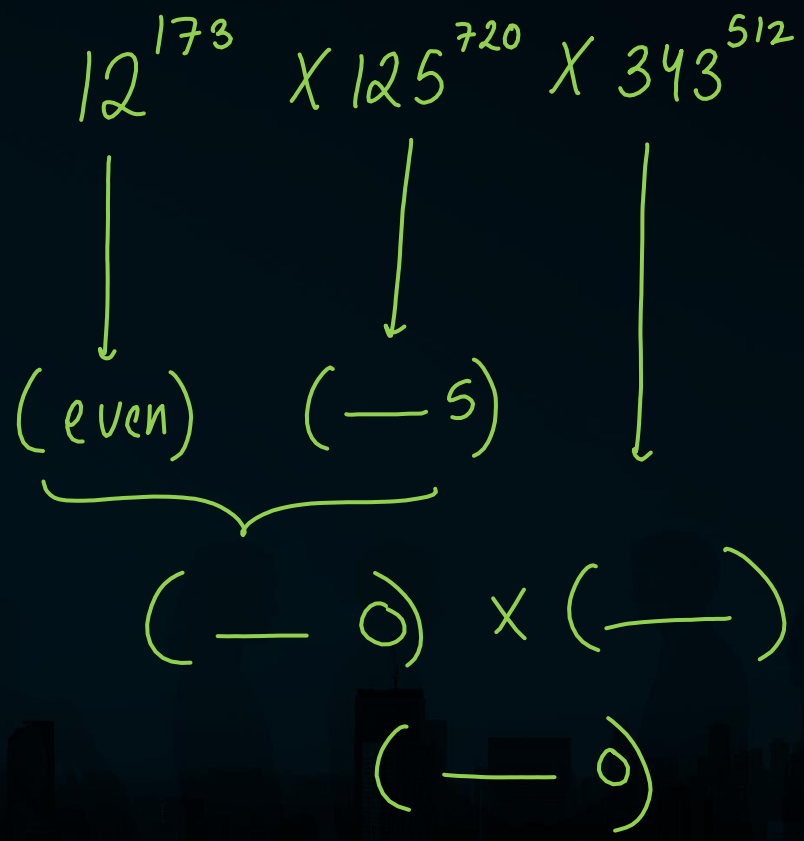
#Q. $12^{173} \times 125^{720} \times 343^{512} = x$. Find the unit's place digit of x.

A) 7

B) 5

C) 8

D) none ✓



QUESTION – 16

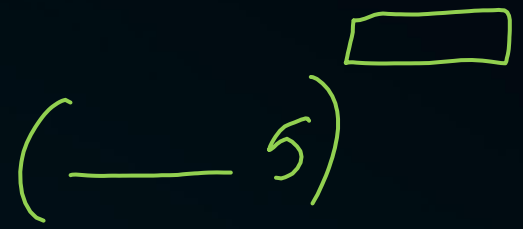
#Q. $215^{215^{215}}$ = z , find the unit digit of z is

A 0

B 5

C 1

D $none$



QUESTION – 17

#Q. $\frac{6^{33}}{3^{11}} + \frac{8^{18}}{32^{6}}$, find the unit digit of the resulting number

A) 4

B) 2

C) 5

D) 3

HW

QUESTION – 18

#Q. Find the unit digit of $[115^{117} \times 202^{159}]^4$

(— 5) × (even)

A 0 ✓

B 6

C 5

D $none$

QUESTION – 19

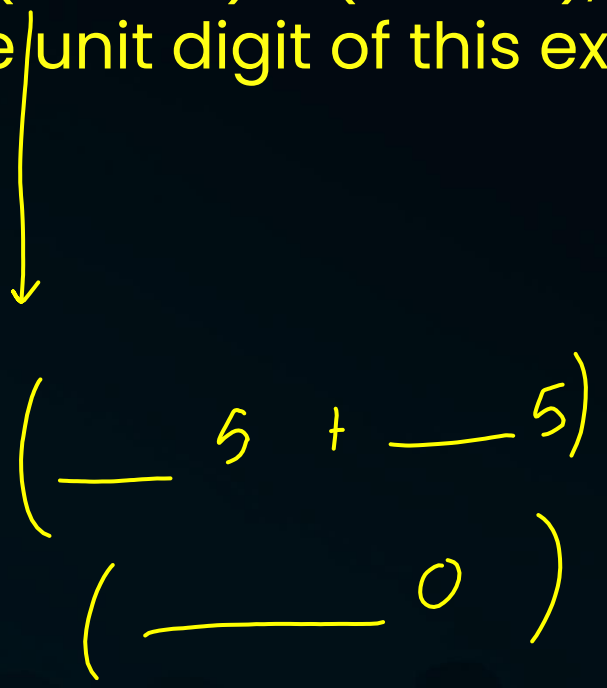
#Q. If $(3^m + 3^n) \times (4^o + 4^p) \times (5^q + 5^r) \times (6^s + 6^t)$, where each power is any natural number. Find the unit digit of this expression

A 7

B 1

C 3

D 0 ✓



QUESTION – 20

#Q. What is the sum of all the possible unit digits of $(3^x + 5^y + 6^z)$, if x, y and z are natural numbers ?

A 14

B 12

C 10

D $none$

HW

QUESTION – 21

#Q. What is the remainder when 42^{123} is devided by 10 ?

A 6

B 8

C 4

D 2

HW

QUESTION – 22

#Q. What is the remainder when 42^{123} is devided by 5 ?

A 2

B 3

C 4

D 1

HW

SUMMARY

factors →

UNIT'S DIGIT


THANK
You
52%
64%
85%
26,465.54
28,161.92
26,465.54
198.45
37,492.43
201.55
1,097.95
0.00
26,717.43
7,534.44
12,512.54
28,875.56