## Practical\_2

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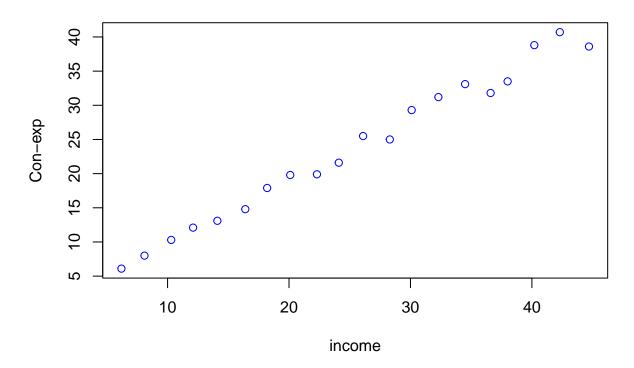
16/04/2022

## Problem 1

```
df<- read.csv("C:\\Users\\souma\\Dropbox\\Mstat_CU\\Sem 2\\Regression_analysis_1\\Data Sets\\con-exp_in
colnames(df)<-c('x_var','y_var')</pre>
lm_1<- lm(y_var~x_var,df)</pre>
summary(lm_1)
##
## Call:
## lm(formula = y_var ~ x_var, data = df)
##
## Residuals:
##
       Min
                 1Q
                     Median
## -2.44687 -0.94108 0.02916 1.19199 1.81151
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.84705 0.70335
                                   1.204
                                              0.244
## x_var
               0.89932
                           0.02531 35.534
                                             <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.314 on 18 degrees of freedom
## Multiple R-squared: 0.9859, Adjusted R-squared: 0.9852
## F-statistic: 1263 on 1 and 18 DF, p-value: < 2.2e-16
```

### Scatter Plot

### Scatter Plot of the data



#### #abline(lm\_1)

The model is not fitted well as the intercept term is insignificant. There may be several reason of this — the data may consist influential observations, the error variability may not be constant etc. In the following section we will try to find them.

#### Outlier

```
influence.measures(lm_1)
```

```
## Influence measures of
##
     lm(formula = y_var ~ x_var, data = df) :
##
##
        dfb.1_ dfb.x_vr
                          dffit cov.r cook.d
## 1
     -0.11552
                 0.0453 -0.1838 1.105 0.01727 0.0532
##
  2
      -0.03202
                 0.1450
                         0.2794 1.066 0.03889 0.0684
## 3
       0.18229
                -0.3787 -0.5417 0.925 0.13401 0.0978
## 4
       0.10051
                -0.0787
                         0.1050 1.253 0.00580 0.1142
## 5
      -0.34877
                 0.5588
                         0.6760 1.018 0.21157 0.1579
## 6
      -0.12527
                 0.1077 -0.1261 1.364 0.00839 0.1847
                -0.9594 -1.1173 0.804 0.50361 0.1904
## 7
       0.63252
## 8
                         0.2112 1.072 0.02250 0.0503
       0.07397
                 0.0154
## 9
       0.05756
                -0.0467
                         0.0591 1.290 0.00185 0.1329
                         0.5969 1.001 0.16594 0.1329
## 10 -0.27545
                 0.4715
## 11 -0.04557
                 0.0382 -0.0462 1.332 0.00113 0.1591
## 12 -0.06951
                 0.1808
                         0.2902 1.096 0.04222 0.0817
```

There are 2 influential obsn 6, 7 th obsn

SO we remove them from the model and refit the model below,

```
df_updated<- df[-c(6,7),]
lm_1updated<- lm(y_var~x_var,df_updated) # after removing influential observation
summary(lm_1updated)</pre>
```

```
##
## Call:
## lm(formula = y_var ~ x_var, data = df_updated)
## Residuals:
##
      Min
               10 Median
                               30
                                      Max
## -2.3578 -0.9786 0.4246 0.9694
                                  1.3282
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.46505
                          0.75112
                                    0.619
                                             0.545
                          0.02752 33.452 3.08e-16 ***
## x_var
               0.92057
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.216 on 16 degrees of freedom
## Multiple R-squared: 0.9859, Adjusted R-squared: 0.985
## F-statistic: 1119 on 1 and 16 DF, p-value: 3.082e-16
```

But still the model is inappropriate as, the intercept is still insignificant.

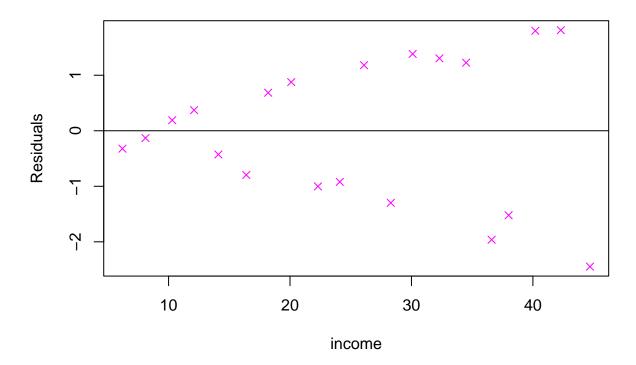
Lets check for the second reason.

As dropping the influential observations doesn't change the model so its better to work with the full model.

### Residuals

Now we will find the residual form the updated fit.

## **Residual Plot**



So from the residual plot we notice that there is a heteroscedasticity, but to confirm that statistically we should test for this.

An famous test is Goldfield Quant test

#### Goldfield Quant Test

```
library(lmtest)
## Warning: package 'lmtest' was built under R version 4.1.3
## Loading required package: zoo
## Warning: package 'zoo' was built under R version 4.1.3
## Attaching package: 'zoo'
## The following objects are masked from 'package:base':
       as.Date, as.Date.numeric
##
gqtest(lm_1,,fraction =6, order.by = ~df$x_var,data = df ) #alternative is more than
##
    Goldfeld-Quandt test
##
##
## data: lm_1
## GQ = 13.322, df1 = 5, df2 = 5, p-value = 0.006482
## alternative hypothesis: variance increases from segment 1 to 2
```

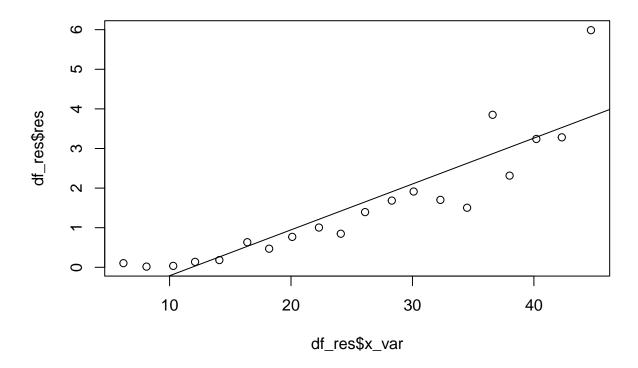
Since the p value is less than 0.05, we will reject the null hypothesis, so there is heteroscedasticity in the model.

So we need to workout with heteroscedasticity.

## Obtaining model for Residuals

We will fit an regression of  $residual^2$  on x\_var, where  $residual_i^2$  can be looked upon as a estimate of  $\sigma_i^2$ 

```
df_res<-cbind(df,res1^2) # add the (ei)^2 values
colnames(df_res)[3]<-"res"</pre>
# the model for the sigma
res_lm1<-lm(res ~ x_var, data= df_res)
summary(res_lm1)
##
## Call:
## lm(formula = res ~ x var, data = df res)
##
## Residuals:
##
       Min
                 1Q Median
                                 3Q
                                         Max
## -1.1212 -0.2607 -0.1955 0.1335 2.1811
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.37033
                            0.38860
                                    -3.526 0.00241 **
                0.11580
                            0.01398
                                       8.282 1.49e-07 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.7259 on 18 degrees of freedom
## Multiple R-squared: 0.7921, Adjusted R-squared: 0.7806
## F-statistic: 68.59 on 1 and 18 DF, p-value: 1.492e-07
Now from the above model, we will find an estimate of \sigma_i^2, by fitting this model, and given by
sigma_estimated<- predict(res_lm1)</pre>
sigma_estimated
##
                                       3
    1.21207079
                2.37009707
                             2.86804837
                                          0.03088399
                                                      3.52812335 -0.65235151
##
##
             7
                          8
                                       9
                                                  10
                                                               11
##
    3.80604966
                1.65212078 -0.17756074
                                          3.28493783 -0.43232652
                                                                   2.62486285
##
                         14
                                      15
                                                  16
                                                               17
                                                                            18
            13
    3.03017205
                0.26248925
                             0.52883529 1.42051552 2.11533129
##
            19
   0.73728002 0.95730501
The Plot of x and fitted(\sigma_i^2) (Unnecessary)
plot(df_res$x_var,df_res$res)
abline(res_lm1)
```



## Estimated Omega hat

omega<- diag(sigma\_estimated)</pre>

## Model's Design Matrix

```
d_matrix <- model.matrix(lm_1)
d_matrix</pre>
```

```
##
      (Intercept) x_var
                   22.3
## 1
## 2
                   32.3
## 3
                    36.6
## 4
                    12.1
                    42.3
## 5
## 6
                     6.2
## 7
                    44.7
                    26.1
## 8
## 9
                    10.3
## 10
                    40.2
## 11
                 1
                     8.1
## 12
                    34.5
## 13
                    38.0
## 14
                    14.1
## 15
                    16.4
## 16
                    24.1
## 17
                 1 30.1
```

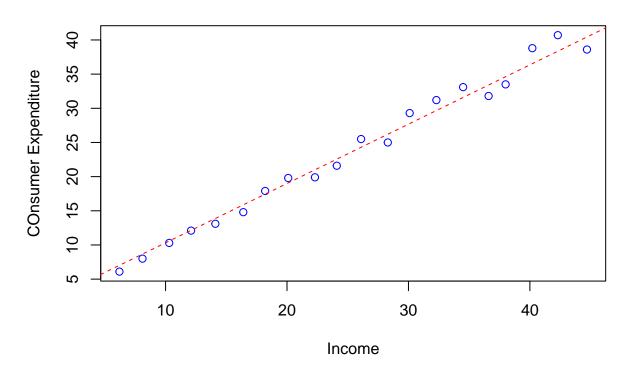
```
## 18
              1 28.3
## 19
              1 18.2
## 20
              1 20.1
## attr(,"assign")
## [1] 0 1
a1<- solve( t(d_matrix) %*% solve(omega) %*% d_matrix )
a2<- t(d_matrix) %*% solve(omega) %*% df$y_var
beta_egls<- a1 %*% a2
beta_egls
                  [,1]
## (Intercept) 1.638355
## x_var
             0.867986
```

## The Model

 $y = 1.638355 + 0.867986 \times x$ 

### The Scatter Plot

### The Ultimate Scatter Plot



### Problem 2

## 3 4 5

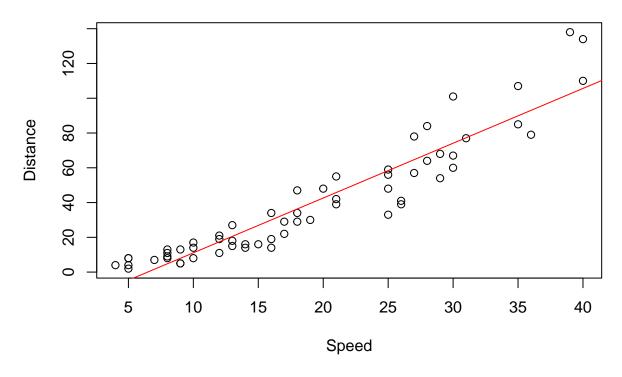
```
library(tidyverse)
## -- Attaching packages -----
## v ggplot2 3.3.5
                       v purrr
                                  0.3.4
## v tibble 3.1.6
                       v dplyr
                                 1.0.7
             1.1.4
## v tidyr
                       v stringr 1.4.0
## v readr
             2.1.1
                       v forcats 0.5.1
## -- Conflicts -----
                                                 ----- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()
                     masks stats::lag()
library(readr)
library(readxl)
df<- read.csv("C:\\Users\\souma\\Dropbox\\Mstat_CU\\Sem 2\\Regression_analysis_1\\Data Sets\\dis_sp_dat
colnames(df) \leftarrow c("y","x")
                            \# y=dis , x=sp
head(df)
##
     ух
## 1 4 4
## 2 2 5
```

```
## 4 8 5
## 5 8 5
## 6 7 7
```

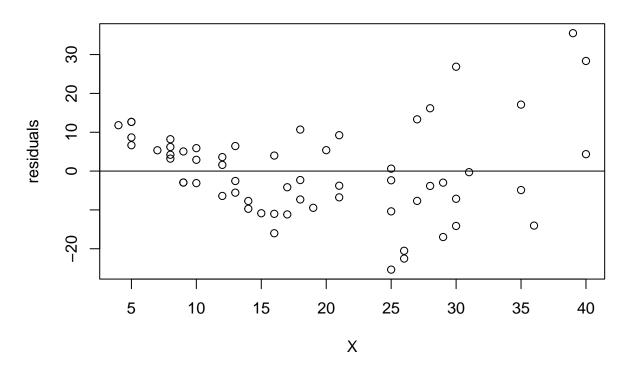
### Lets fit the regression

```
reg1<- lm(y~x,data=df)</pre>
summary(reg1)
##
## Call:
## lm(formula = y \sim x, data = df)
## Residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
## -25.371 -7.401 -2.340 6.266 35.508
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -20.4174 3.3446 -6.105 9.16e-08 ***
## x
                3.1515
                         0.1559 20.213 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 11.95 on 58 degrees of freedom
## Multiple R-squared: 0.8757, Adjusted R-squared: 0.8735
## F-statistic: 408.6 on 1 and 58 DF, p-value: < 2.2e-16
# Scatter Plot
plot(df$x,df$y,
    main="Regression Plot",
     xlab = "Speed", ylab="Distance")
abline(reg1, col="red")
```

# **Regression Plot**



### **Residual Plot**



From the residual plot we can suspect that there is a heterosecdasticity within the data, so should test for that. So we should go for the **Glejser test** 

#### Now lets check for Influencial Observation

#### influence.measures(reg1)

```
Influence measures of
##
     lm(formula = y \sim x, data = df) :
##
##
         dfb.1_
                    dfb.x
                              dffit cov.r
                                            cook.d
                                                       hat inf
       0.244429 -0.205256
                            0.24572 1.057 3.02e-02 0.0551
##
##
  2
       0.129506 -0.106811
                            0.13069 1.078 8.64e-03 0.0502
##
  3
       0.168729 -0.139161
                            0.17027 1.069 1.46e-02 0.0502
##
       0.248027 -0.204562
                            0.25030 1.046 3.12e-02 0.0502
                            0.25030 1.046 3.12e-02 0.0502
## 5
       0.248027 - 0.204562
##
  6
       0.092346 -0.072912
                            0.09440 1.072 4.52e-03 0.0413
##
       0.051765 -0.039792
                            0.05345 1.073 1.45e-03 0.0374
## 8
       0.067948 -0.052231
                            0.07016 1.071 2.50e-03 0.0374
##
  9
       0.100395
                -0.077173
                            0.10366 1.065 5.44e-03 0.0374
## 10
       0.132995 -0.102233
                            0.13732 1.058 9.51e-03 0.0374
## 11
       0.076362 -0.056896
                            0.07991 1.065 3.24e-03 0.0338
## 12 -0.043535
                 0.031265 -0.04637 1.065 1.09e-03 0.0306
## 13
       0.040782 -0.029287
                            0.04344 1.066 9.59e-04 0.0306
##
  14
       0.083076 -0.059660
                           0.08848 1.059 3.97e-03 0.0306
     -0.076980
                 0.050113 -0.08649 1.051 3.79e-03 0.0251
                           0.02156 1.061 2.36e-04 0.0251
## 16
       0.019185 -0.012489
```

```
## 17 0.043209 -0.028128 0.04855 1.059 1.20e-03 0.0251
## 18 -0.061114 0.037181 -0.07142 1.052 2.59e-03 0.0229
## 21 -0.098289
              0.048687 -0.12898 1.025 8.34e-03 0.0194
## 22 -0.130108  0.054398 -0.18559  0.989  1.70e-02  0.0182
## 23 -0.088710 0.037089 -0.12654 1.023 8.02e-03 0.0182
     ## 25 -0.078879
              0.025187 -0.12512 1.022 7.84e-03 0.0174
## 26 -0.029202
              0.009325 -0.04632 1.049 1.09e-03 0.0174
## 27 -0.044255
              0.008344 -0.08033 1.039 3.26e-03 0.0168
              0.002629 -0.02531 1.052 3.26e-04 0.0168
## 28 -0.013944
     30 -0.048105 0.000349 -0.10362 1.030 5.40e-03 0.0167
  31 -0.011699 -0.008163 -0.04187 1.050 8.90e-04 0.0173
## 32
     0.028827
              0.021391 -0.174762 -0.33842 0.898 5.36e-02 0.0227
## 33
     0.008445 -0.068998 -0.13361 1.032 8.96e-03 0.0227
     0.001918 -0.015673 -0.03035 1.058 4.68e-04 0.0227
  36 -0.000509 0.004158 0.00805 1.059 3.30e-05 0.0227
##
  37
     0.041777 -0.179925 -0.31253 0.933 4.66e-02 0.0249
     0.037853 -0.163023 -0.28317 0.954 3.87e-02 0.0249
     0.021473 -0.068294 -0.10890 1.049 5.99e-03 0.0275
## 39
              ## 40 -0.037569
     0.082879 -0.193023 -0.27199 0.996 3.63e-02 0.0336
     0.014277 -0.033250 -0.04685 1.069 1.12e-03 0.0336
     0.083110 -0.176406 -0.23759 1.022 2.80e-02 0.0371
## 43
     0.041537 -0.088165 -0.11875 1.062 7.13e-03 0.0371
  45 -0.163666 0.347392 0.46788 0.889 1.01e-01 0.0371
     0.001913 -0.003782 -0.00491 1.080 1.22e-05 0.0410
     0.054118 -0.089980 -0.10586 1.095 5.68e-03 0.0601
## 48 -0.192927  0.320774  0.37737  1.020  6.97e-02  0.0601
     0.172818 -0.279509 -0.32357 1.052 5.19e-02 0.0657
              0.918090 1.02463 0.786 4.45e-01 0.0845
## 50 -0.604751
## 51 -0.044473
              0.033136 -0.04654 1.069 1.10e-03 0.0338
              0.054319 -0.11980 1.033 7.22e-03 0.0210
## 52 -0.097447
## 53 -0.044473
              0.033136 -0.04654 1.069 1.10e-03 0.0338
## 54 -0.077196
              0.043031 -0.09490 1.042 4.55e-03 0.0210
              0.022019
## 56 -0.021063 -0.014696 -0.07538 1.042 2.87e-03 0.0173
     0.014499 -0.038326 -0.05707 1.064 1.65e-03 0.0304
## 58 -0.062272 0.164607
                       0.24513 0.999 2.96e-02 0.0304
## 59 -0.072962 0.108962 0.12049 1.134 7.37e-03 0.0915
## 60 -0.501905 0.749552 0.82882 0.909 3.12e-01 0.0915
```

So, we can see there are 4 influencial observation, viz 45,50,59,60 th observation. Lets remove these

#### Regression fit after removing influencial

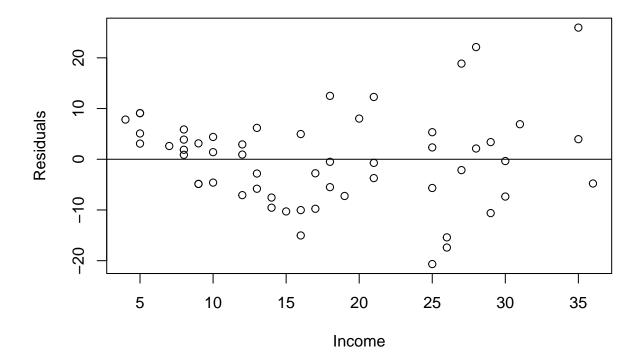
```
df1<- df[-c(45,50,59,60),]
reg2<- lm(y~x,data = df1)
summary(reg2)</pre>
```

```
##
## Call:
## lm(formula = y \sim x, data = df1)
##
## Residuals:
##
       Min
                                    3Q
                  1Q
                      Median
                                           Max
   -20.6646 -5.7017
                      0.8999
                                5.0026
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -14.7762
                           2.7524 -5.369 1.71e-06 ***
                 2.7376
                           0.1389 19.710 < 2e-16 ***
## x
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 9.194 on 54 degrees of freedom
## Multiple R-squared: 0.878, Adjusted R-squared: 0.8757
## F-statistic: 388.5 on 1 and 54 DF, p-value: < 2.2e-16
```

#### Lets see the residual plot

```
# Model after removing influential observation
res2<- resid(reg2)
plot(df1$x,res2,main="Residual plot after removing Influencial Observation",
     xlab = "Income",ylab = "Residuals")
abline(h=0)
```

## Residual plot after removing Influencial Observation



Still we can see that, there is a heteroscedasticity in the residuals. And the adjusted  $R^2$  for the new model doesn't change much

So we will stick in the actual data set and previous model(reg1 model), as removing data point causes information loss from our hand.

#### Test For Heteroscedasticity

So we till test for heteroscedasticity by Glejser Test

```
library(skedastic)

## Warning: package 'skedastic' was built under R version 4.1.3

# Glejser Test
glejser(reg1)

## # A tibble: 1 x 4

## statistic p.value parameter alternative

## <dbl> <dbl> <dbl> <dr>
## 1 12.5 0.000409 1 greater
```

It results that the p value is less than 0.05, so, the presence of Heteroscedasticity statistically significant under 5% level of significant.

So now we will remove the heteroscedasticity from the model from the data set , assuming the error variance as a linear function of speed with an intercept term.

#### Remedial Measure

```
res1_sq<- res1^2 # ei Square
rem_lm<- lm(res1_sq~df$x)
summary(rem_lm)
##
## Call:
## lm(formula = res1_sq ~ df$x)
##
## Residuals:
##
      Min
                1Q Median
                                3Q
                                       Max
##
  -352.27 -97.17 -22.72
                             24.55
                                   900.68
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -73.672
                            56.369
                                   -1.307
                                              0.196
## df$x
                 11.123
                             2.628
                                     4.233 8.33e-05 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 201.4 on 58 degrees of freedom
## Multiple R-squared: 0.236, Adjusted R-squared: 0.2228
## F-statistic: 17.92 on 1 and 58 DF, p-value: 8.327e-05
```

But, the intercept term is insignificant, so we can drop this from the model. But here we are asked to work with an intercept term.

So the model is

```
e_i^2 = -73.672 + 11.123 \times x
```

#### So, the estiamted sigma\_i sq hats are

```
sigma_estimated<- predict(rem_lm)
omega<- diag(sigma_estimated)</pre>
```

## Lets find the design Matrix

```
d_matrix<- model.matrix(reg1)
d_matrix</pre>
```

## (Intercept) x ## 1					
## 2	##		(Intercept)		
## 3					
## 4					
## 5	##			5	
## 6	##				
## 7					
## 8	##				
## 9 1 8 ## 10 1 8 ## 11 1 9 ## 12 1 10 ## 13 1 10 ## 15 1 12 ## 16 1 12 ## 18 1 13 ## 19 1 13 ## 19 1 13 ## 20 1 13 ## 21 1 15 ## 22 1 16 ## 23 1 16 ## 24 1 16 ## 25 1 17 ## 26 1 17 ## 26 1 17 ## 27 1 18 ## 28 1 18 ## 29 1 18 ## 29 1 18 ## 30 1 19 ## 31 1 21 ## 32 1 21 ## 32 1 25 ## 34 1 25 ## 35 1 25 ## 36 1 25 ## 37 1 26	##	7		8	
## 10	##			8	
## 11	##				
## 12	##	10		8	
## 13	##	11		9	
## 14	##	12			
## 15	##	13	1	10	
## 16	##	14	1	10	
## 17	##	15	1	12	
## 18	##	16	1	12	
## 19	##	17	1	12	
## 20	##	18	1	13	
## 21	##	19	1	13	
## 22	##	20	1	13	
## 23	##	21	1	15	
## 24	##	22	1	16	
## 25	##	23	1	16	
## 26	##	24	1	16	
## 27	##	25	1	17	
## 28	##	26	1	17	
## 29	##	27	1	18	
## 30 1 19 ## 31 1 21 ## 32 1 21 ## 33 1 25 ## 34 1 25 ## 35 1 25 ## 36 1 25 ## 37 1 26	##	28	1	18	
## 31 1 21 ## 32 1 21 ## 33 1 25 ## 34 1 25 ## 35 1 25 ## 36 1 25 ## 37 1 26	##	29	1	18	
## 32 1 21 ## 33 1 25 ## 34 1 25 ## 35 1 25 ## 36 1 25 ## 37 1 26	##	30	1	19	
## 33 1 25 ## 34 1 25 ## 35 1 25 ## 36 1 25 ## 37 1 26	##	31	1	21	
## 34 1 25 ## 35 1 25 ## 36 1 25 ## 37 1 26	##	32	1	21	
## 34 1 25 ## 35 1 25 ## 36 1 25 ## 37 1 26	##	33	1	25	
## 36 1 25 ## 37 1 26	##	34	1		
## 36 1 25 ## 37 1 26	##	35	1	25	
## 37 1 26	##		1		
	##		1		
	##		1		

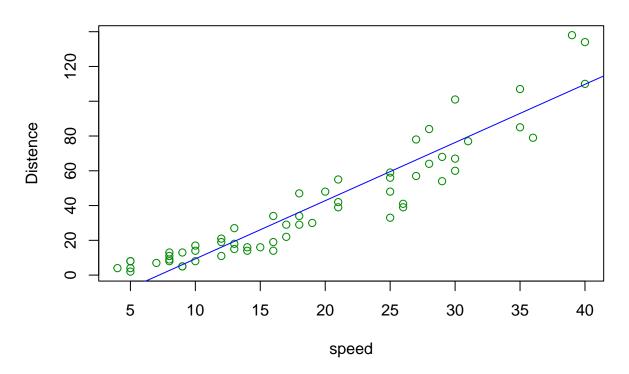
```
## 39
                1 27
## 40
                1 27
## 41
                1 29
## 42
                1 29
## 43
                1 30
## 44
                1 30
## 45
                1 30
                1 31
## 46
## 47
                1 35
                1 35
## 48
## 49
                1 36
## 50
                1 39
## 51
                1 9
## 52
                1 14
## 53
                1 9
## 54
                1 14
## 55
                1 20
## 56
                1 21
## 57
                1 28
                1 28
## 58
## 59
                1 40
## 60
                1 40
## attr(,"assign")
## [1] 0 1
a<- t(d_matrix) %*% solve(omega) %*% d_matrix
b<- t(d_matrix) %*% solve(omega) %*% df$y
beta_egls2 <- solve(a) %*% b
beta_egls2
## (Intercept) -24.118582
## x
                 3.345985
```

### The Final Model

So, after removing the heteroscedasticity, and find the final fitted model as,

```
y = -24.118582 + 3.345985 \times x
```

### The Scatter Plot with the final Fitted Model



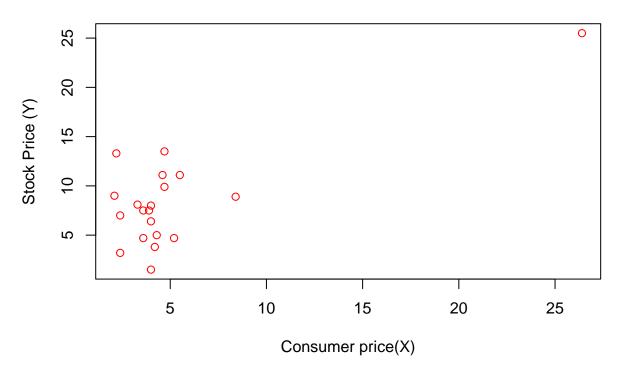
### Problem 3

```
library(tidyverse)
library(readr)
library(readxl)
```

```
## i..Country y x
## 1 Australia 5.0 4.3
## 2 Austria 11.1 4.6
## 3 Belgium 3.2 2.4
## 4 Canada 7.0 2.4
## 5 Chile 25.5 26.4
## 6 Denmark 3.8 4.2
```

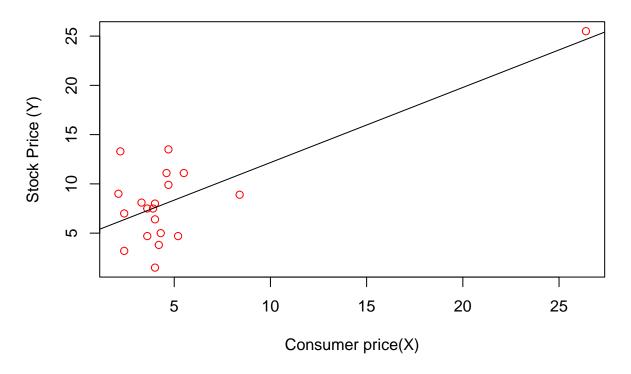
## Plot the data and fit a suitable Regression

### Scatter Plot of the data



```
# regission fit
reg3<- lm(y~x,data=df)</pre>
summary(reg3)
##
## Call:
## lm(formula = y ~ x, data = df)
##
## Residuals:
##
       Min
                1Q Median
                                3Q
                                       Max
##
   -6.0893 -2.6428 0.3132 1.9246
                                   7.0829
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
                                     4.204 0.000533 ***
                 4.5400
                            1.0799
## (Intercept)
## x
                 0.7623
                            0.1493
                                     5.108 7.36e-05 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 3.375 on 18 degrees of freedom
## Multiple R-squared: 0.5917, Adjusted R-squared: 0.569
## F-statistic: 26.09 on 1 and 18 DF, p-value: 7.361e-05
plot(df$x,df$y, main="Regression Plot of the data",
     xlab="Consumer price(X)",ylab = "Stock Price (Y)", col='red')
abline(reg3)
```

## **Regression Plot of the data**



## Lets check for influencial

influence.measures(reg3)

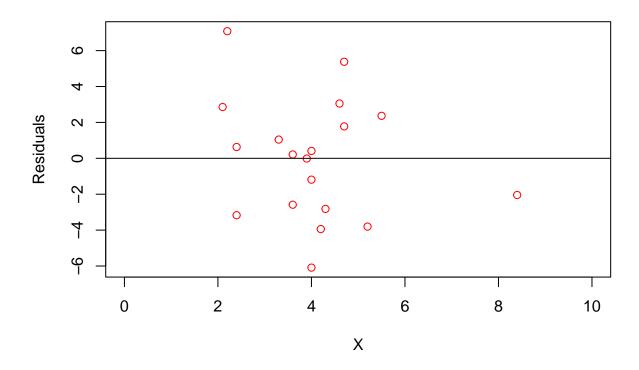
```
## Influence measures of
##
     lm(formula = y \sim x, data = df) :
##
##
                   dfb.x
                             dffit
                                    cov.r
                                            cook.d
## 1
     -0.16066
               0.033792 -0.198197
                                    1.087 1.99e-02 0.0515
##
       0.16556 -0.024128
                          0.213567
                                    1.071 2.30e-02 0.0506
##
  3
      -0.24469
               0.123032 -0.255743
                                    1.077 3.28e-02 0.0651
##
      0.04743 -0.023846
                          0.049568
                                    1.194 1.30e-03 0.0651
##
  5
      -1.83474
               3.343802
                          3.437398 14.652 5.95e+00 0.9308
      -0.23349
                0.053797 -0.284163
                                    1.001 3.93e-02 0.0519
## 7
               0.010464
                          0.163152
                                    1.113 1.37e-02 0.0502
       0.10631
## 8
       0.09306 -0.011410
                          0.122009
                                    1.143 7.75e-03 0.0504
       0.63731 -0.334860
                          0.660395
                                    0.654 1.70e-01 0.0673
## 10 -0.39799
               0.106895 -0.472310
                                    0.775 9.56e-02 0.0527
  1.166 3.65e-03 0.0527
## 12 -0.03474 -0.091250 -0.169706
                                    1.154 1.49e-02 0.0703
      0.06892 -0.026507
## 13
                          0.076246
                                    1.175 3.06e-03 0.0569
      0.30265 -0.037109
                          0.396805
                                    0.856 7.10e-02 0.0504
## 15 -0.18631 -0.001325 -0.267936
                                    1.011 3.52e-02 0.0500
## 16
      0.01354 -0.004575
                          0.015387
                                    1.186 1.25e-04 0.0549
## 17 -0.16511 0.055796 -0.187634
                                    1.106 1.80e-02 0.0549
      0.02416 -0.006488
                          0.028669
                                    1.181 4.35e-04 0.0527
## 19 -0.00078 0.000224 -0.000915
                                    1.184 4.43e-07 0.0532
```

```
## 20 0.22899 -0.122812 0.236377 1.103 2.83e-02 0.0685
The 5th and 9th obsn are influential
df1 < - df[-c(5,9),]
reg3_up<- lm(y~x,data=df1)</pre>
summary(reg3_up)
##
## Call:
## lm(formula = y \sim x, data = df1)
## Residuals:
## Min
            1Q Median
                           3Q
                                 Max
## -5.672 -2.325 0.484 2.060 5.892
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 4.6791
                        2.3140 2.022 0.0602 .
## x
                0.6232
                           0.5284
                                    1.179 0.2554
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 3.063 on 16 degrees of freedom
## Multiple R-squared: 0.07999,
                                   Adjusted R-squared: 0.02249
## F-statistic: 1.391 on 1 and 16 DF, p-value: 0.2554
So its worst than the previous.
```

### Residual Plot

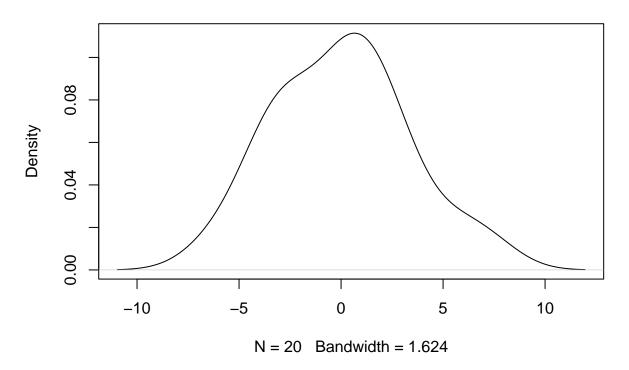
So, its better to stick with the original model

# **Residual Plot**



plot(density(res3), main = "Density of the residuals")

## **Density of the residuals**



## Goldfield Quant test

```
library(lmtest)
gqtest(reg3,fraction = 6,order.by = ~x, data = df)

##
## Goldfeld-Quandt test
##
## data: reg3
## GQ = 1.0387, df1 = 5, df2 = 5, p-value = 0.4839
## alternative hypothesis: variance increases from segment 1 to 2
Since the p value is more than 0.05, So we fail to reject the null hypothesis, so the data is homoscedastic.
```

## Parameter of the Model

So OLS works good here.

```
reg3$coefficients

## (Intercept) x
## 4.5400120 0.7623165
```

## Final Model

## The Final Fit

