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A Close Look into Black-Scholes Option Pricing Model

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Abstract: The Black Scholes Model is one of the most significant concepts in modern financial theory both in terms of approach as well as applicability. Attention in the theory of option pricing received a major motivation in 1973 with the publication of a pioneering paper titled "The Pricing of Options and Corporate Liabilities by Black and Scholes who developed closed-form formula to calculate the prices of European calls and puts, based on certain assumptions by showing how to hedge continuously the exposure on the short position of an option. This paper carefully studies the Black-Scholes Model of option pricing and makes a more detailed analysis of the assumptions of the model and the mathematical derivation process of the model and also analyses the inherent loopholes in the theory. The concepts behind the Black-Scholes analysis provide the framework for thinking about option pricing.

Keywords: Black-Scholes model, option price, implicit volatility, Brownian motion; Volatility.

1. Introduction:

The Black Scholes Model is one of the most significant concepts in modern financial theory both in terms of approach as well as applicability. Attention in the theory of option pricing received a major motivation in 1973 with the publication of a pioneering paper titled "The Pricing of Options and Corporate Liabilities", published in the Journal of Political Economy by Black and Scholes who developed closed-form formula to calculate the prices of European calls and puts, based on certain assumptions by showing how to hedge continuously the exposure on the short position of an option. Partial differential equation, derived by them, referred to as the Black–Scholes equation, governs the price of the option over time. The crucial idea behind the derivation was to hedge perfectly the option by buying and selling the underlying asset in just the right way and consequently "eliminate risk". Options theory is used to price derivatives and set up hedging strategies to cover the associated risks. Black-Scholes marked the beginning of a rapid expansion in derivatives markets, as it provides a fairly straightforward method for pricing and hedging derivatives based on various risk factors. The Black-Scholes mathematical model explains that the price of heavily traded assets follow a geometric Brownian motion that looks like a smile or smirk with constant drift and volatility. When applied to a stock option, the model incorporates the constant price variation of the stock, the time value of money, the option's strike price and the time to the option's expiry.

The model includes the Black–Scholes formula, which provides the price of European-style options. The formula was so thriving that it created inflation in options trading and thus the Chicago Board Options Exchange was formed. The Black-Scholes valuation is basically used by binary options market traders. Trading experience has proven that the Black-Scholes theory is "fairly accurate" to the observed prices, although there are distinguished inconsistencies such as the "option smirk". In essence, the Black-Scholes model states that by constantly adjusting the proportions of stocks and options in a portfolio, the investor can create a riskless hedge portfolio, where all market risks are eliminated. The ability to build such a portfolio relies on the assumptions of continuous trading and continuous sample paths of the asset price. In an efficient market, with no riskless arbitrage opportunities, any portfolio with a zero market risk must have an expected rate of return equal to the risk-free interest rate. This approach led to the differential equation, known in physics as the "heat equation". Its solution is the Black-Scholes formula for pricing European options on non-dividend paying stocks

In view of the above discussion, this paper carefully studies the Black-Scholes Model of option pricing and makes a more detailed analysis of the assumptions of the model and the mathematical derivation process of the model and also analyses the inherent loopholes in the theory.

2. Assumptions of Black Scholes Model:

The Black-Scholes Option Pricing Model is an approach used for computing the value of a stock option. It can be used to calculate values of both call and put option. To be acquainted with the assumptions of the Black-Scholes model is important for its correct application.In their seminal paper (1973), Black and Scholes made the following assumptions on the financial market.

(i)Constant volatility: Volatility, a measure of how much a stock can be expected to move in the near-term, is a constant over time. This means that the variance of the return is constant over the life of the option contract and is known to market participants.

While volatility can be relatively constant in very short term, it is never constant in longer term. Some advanced option valuation models substitute Black-Schole's constant volatility with stochastic-process generated estimates.

(ii) No dividends: Another assumption is that the underlying stock does not pay dividends during the option's life. In the real world, most companies pay dividends to their share holders. The basic Black-Scholes model was later adjusted for dividends, so there is a workaround for this. This assumption relates to the basic Black-Scholes formula. A common way of adjusting the Black-Scholes model for dividends is to subtract the discounted value of a future dividend from stock price.

(iii) Efficient markets: In Black Scholes model, the markets are assumed to be liquid, have price-continuity, be fair and provide all players with equal access to available information. This implies that zero transaction costs are assumed in the Black-Scholes analysis.This assumption of the Black-Scholes model suggests that people cannot consistently predict the direction of the market or an individual stock. The Black-Scholes model assumes stocks move in a manner referred to as a random walk. Random walk means that at any given moment in time, the price of the underlying stock can go up or down with the same probability. The price of a stock in time t+1 is independent from the price in time t.

(iv)Log-normally distributed returns: The Black-Scholes model assumes that returns on the underlying stock are normally distributed. This assumption is reasonable in the real world.

(v) Interest rates constant and known: The interest rates are also assumed to be constant in the Black-Scholes model. The Black-Scholes model uses the risk-free rate to represent this constant and known rate. Therefore, the short term riskless interest rate is known and constant over time. Market participants can both borrow and lend at this rate. In other words, we assume that there exists a risk-free security which returns \$1 at time T when \$ $e^{-r(T-t)}$ is invested at time t. In the real world, there is no such thing as a risk-free rate.

(vi) No commissions and transaction costs: There are no transaction costs in buying or selling the asset or the option, no barriers to trading and no taxes. In other words, information is available to all without cost.

(vii)European-style options: The Black-Scholes model assumes European-style options that can be exercised only at expiration date. American-style options can be exercised at any time during the life of the option, making American options more valuable due to their greater flexibility.

(viii) Liquidity: The Black-Scholes model assumes that markets are perfectly liquid and it is possible to purchase or sell any amount of stock or options or their fractions at any given time.

It should be apparent that none of these principles can be entirely satisfied. Transaction costs exist in all markets, all securities come in discrete units, short selling with full use of proceeds is very rare, interest rates vary with time and there is evidence that the price of most stocks do not precisely follow a geometric Brownian process.

3. Black-Scholes model's inputs:

There are five basic factors that affect the Black Scholes Model inputs. The inputs include the spot price, the volatility of the stock, the time to expiration, the interest rate, and the exercise price. The intrinsic value of the stock is affected by the stock price increase or decrease. This determines if the stock option is valuable or not. The Black Scholes model's five main inputs are discussed below:

Spot Price:

The market price of the underlying asset on the valuation date is spot price. This can be a difficult input to estimate for options on illiquid assets; however under normal circumstances the closing market price can usually be used.

Strike Price:

This is the price level at which the option holder has the right to buy or sell the underlying asset. It is the most straightforward input as it will always be given in the option contract.

Time to Maturity: The time (in years) until the option expires and the holder is no longer entitled to exercise the option.

Risk free Interest Rate: The risk free interest rate for the period until the option expires. The risk free rate should typically be a zero coupon government bond yield.

Volatility:

Volatility is probably the most important single input to any option pricing model. There are several methods for estimating volatility. Historic volatility entails using historic price data for share price movements. A useful rule of thumb is to collect data from as far back as the options term (eg an option with a 5 year life would require an input of historic volatility calculated from the last 5 years of historic data). Historic volatility is often considered as flawed as it assumes the past will reflect the future – thus several forward-looking measures of volatility can be more powerful and

Accurate:

Implied Volatility is the volatility implied by the market price of traded options. As the price is already known and the volatility (which is typically an input) is unknown, the pricing model is reversed to determine the volatility. When using the implied volatility, it is important to be aware of the volatility surface. The volatility surface is the 3 dimensional representation of the relationship between volatility, option life and exercise price. Thus to use implied volatility the option from which the volatility is implied should have a similar life and exercise price (or ratio of market price to exercise price) as the option being valued. The most important Black Scholes Model inputs is volatility. There are several methods that are used to calculate this number. One of the most common methods is by using the historic volatility for the data. Smart investors collect data for the entire length of the term. The past does not always reflect the future however, so some investors chose to use implied volatility instead. This is done by reversing the pricing model to determine volatility.

The Black Scholes Model inputs factors are the value of the call option. They assist investors to decide the financial market and protect them against a negative investment through hedging. They can generate a pricing model that helps to reduce potential loss and increases potential profits. However, because it makes several assumptions, the parameters may not fit every situation and there is still an implied risk in investing in stock options. The Black Scholes Model inputs allow investors to determine the fair value of stock options in the market by calculating an established set of predetermined criteria. This is done in order to estimate the options value. Different expectations will yield different results and indicate whether the market price is higher or lower. Then, it can be decided if the market value is a rewarding investment or not. Finding this data input can be difficult though and a professional investment company may be needed to assess if the stock option's price is worthwhile or not.

4. Black Scholes equation:

This section presents a derivation of the Black-Scholes equation. We consider a general derivative f whose value is a function of the value of the underlying security S.We assume that the underlying asset follows a geometric Brownian motion process. S is assumed to follow the stochastic process :

$$dS = \phi Sdt + \sigma SWdt$$

where ϕ (the average growth rate of the underlying security) and σ (the volatility) are constants and dt is a Wiener process, i.e. has zero mean and unit variance and increments are independent.. Using Ito's lemma, we see that

$$\partial f = \frac{\partial f}{\partial S} dS + \left(\frac{\partial f}{\partial t} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 f}{\partial S^2} \right) dt = \left(\phi S \frac{\partial f}{\partial S} + \frac{\partial f}{\partial t} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 f}{\partial S^2} \right) dt + \sigma S W \frac{\partial f}{\partial S} dt$$

We cannot value this directly as there is a stochastic term. To eliminate the stochastic term, we consider the portfolio $\pi = f - \frac{\partial f}{\partial S} S$. We see that

$$d\pi = df - \frac{\partial f}{\partial S} dS = \left(\frac{\partial f}{\partial t} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 f}{\partial S^2} \right) dt = r\pi dt = r\left(f - \frac{\partial f}{\partial S} S \right) dt$$

With the last equality following from the no-arbitrage condition (since there is no stochastic term, π is a risk-free investment and hence must offer the same return as any other risk-free investment). Simplifying the above equation, we obtain the Black-Scholes equation

$$\frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

The initial (or, in finance, usually final) conditions determine the kind of derivative that we are pricing. For a call option, the final condition we have to use is $f = \max(S - K, 0)$. We see that the principle of risk-neutral valuation is clearly satisfied in this case since the Black-Scholes equation is independent of ϕ, the expected rate of growth of the underlying security price.

It is significant to note that the portfolio π represents a self-financing, replicating, hedging strategy. It replicates a risk-free investment and it is hedged since it has no stochastic component.

5. Loopholes in the Black-Scholes Pricing Model:

The Black-Scholes formula provides the price of the option, in terms of other quantities, which are assumed known. These include the exercise price and the current price of the stock. The formula is derived under the assumption that the time interval between observations is very small, and that the log prices follow a random walk with normally distributed innovations. The formula is not affected by any linear drift in the random walk. The model for the stock prices themselves is called geometric Brownian motion. Over the past three decades the shortcomings of the Black-Scholes model have become increasingly clear, with some academic observers continually ringing the "death-knell" of the formula as its weaknesses become more obvious. Therefore, despite their popularity and wide spread use, the model is built on some non-real life assumptions about the market. One problem with the Black-Scholes analysis, however, is that the mathematical skills required in the derivation and solutions of the model are fairly advanced and probably unfamiliar to many economists. The shortcomings of the model are depicted below:

(i).The Black Scholes theorem assumes stocks move in a random walk; random walk means that at any given moment in time, the price of the underlying stock can go up or down with the same probability. However, this assumption does not hold as stock prices are determined by many factors that cannot be assigned the same probability in the way they will affect the movement of stock prices.

(ii). While volatility can be comparatively constant in very short term, it is never constant in longer term. In other words, it is often established that for financial time series, after taking logs (if needed) and first differences, the level of volatility (i.e., fluctuation) appears to change with time. Often, periods of high volatility follow immediately after a large change (often downward) in the level of the original series. It may take rather some time for this heightened volatility to subside. Large price changes tend to be followed by large price changes, and vice versa leading to a property called volatility clustering. But measures of volatilities are negatively correlated with asset price returns (leverage effect), while trading volumes or the number of trades are positively correlated, hence volatility cannot be a constant over time.

(iii). The assumption that returns of log normally distributed underlying stock prices are normally distributed is rational in the actual world, though not fitting observed financial data exactly. Asset returns have a finite variance and semi-heavy tails contrary to stable distributions like log normal with infinite variance and heavy tails (Clark, 1973). As the time scale over which return of assets are calculated increases, the distribution of asset prices look more like the normal distribution with heavy tails despite the fact that autocorrelation of asset prices are often insignificant.

(iv). The model assumes that the underlying stock does not pay dividends during the option's life. However, this assumption does not apply in all, or in fact, most cases since most public companies pay dividends to their shareholders.

This assumption relates to the basic Black-Scholes formula and typically the model is adjusted by subtracting the discounted value of a future dividend from stock prices to account for dividends.

(v). The model assumes that there are no fees for buying and selling options and stocks and no barriers to trading. Usually it is not true because stock brokers charge rates based on spreads and other criteria. However, this is hardly the case in the real world. More significantly, the model assumes that markets are perfectly liquid and it is possible to purchase or sell any amount of stock or options or their fractions at any given time. This assumption is not only implausible but can be critical.

(vi). Similar to the Black-Scholes theorem's assumption vis-à-vis constant volatility, the model assumes that interest rates are constant and known. This assumption is also unrealistic. In the real world, there is no such thing as a risk free rate, but it is possible to use the U.S. Government Treasury Bills 30-day rate since the U. S. government is deemed to be credible enough. However, these treasury rates can change in times of increased volatility.

In sum, the drawback to Black-Scholes is that it is based on simplistic assumptions such as constant volatility and a normal distribution function for the underlying asset return. This can lead to substantial discrepancies between actual market prices and prices calculated using the model. These discrepancies between market and theoretical prices are most evident in the observation of different implied volatilities according to exercise prices (smile or skew) and maturities (term structure). Several empirical studies have confirmed this phenomena, including those by Rubinstein [1994] and Dumas et al. [1998]. The implied volatility surface is not flat and varies with time. Implied volatility has subsequently become a preferred variable among traders who often quote options in terms of volatility rather than price.

Another area where Black-Scholes clearly diverges from actual market behaviour is in the distribution of the underlying returns. This distribution differs from the normal density assumed by the Black-Scholes model, and is usually characterized by a negative skewness and excess kurtosis. Several studies have documented this, such as those by Aït-Sahalia and Lo [1998], Bliss and Panigirtzoglou [2002], and Beber and Brandt [2006].

In addition, it can be said that the model was developed in 1973, when calls were traded on only 16 stocks, and there were no puts at all. The market was so young that many of today's strategies did not yet exist. In addition, the "population" of options trading was extremely limited. This means the assumptions in use for the model do not apply in the more complex modern options industry. With thousands more options to trade and with faster, more detailed, and more widely used formulas for tracking value, the whole options market is a different creature nowadays than it was in 1973. Even open contracts levels have changed, growing in the billions since 1973 to a volume nowadays that was unimaginable in the past. This also affects valuation very directly. The use of delta, gamma and vega are far more consistent measurements of implied volatility and option pricing than the more obscure Black-Scholes model with its unfeasible variables.

6. Conclusion:

Black -Scholes Options Pricing model was seen as a significant endeavor at articulating pricing of options and corporate bonds based on the assumption that a risk-free interest rate existed. It is still used today for estimating what options should be worth, but it is applied mostly in institutional portfolio management departments and in academia.

The Black-Scholes paper represents a landmark in the option pricing literature for several reasons: first, it was the first realistic general equilibrium model of option pricing; but it is equally vital in the sense that it has produced much subsequent literature on the valuation of many types of contingent claim. . Finally, it had significant implications for empirical work. Since the price of a call option depends on only five quantities (all of which are either directly observable or easily measurable), the empirical calculation of option prices can be a relatively uncomplicated task. The prevalent strong point of the model is the possibility of estimating market volatility of an underlying asset usually as a function of price and time without direct reference to expected yield, risk aversion measures or utility functions. The second strongest facet is its self-replicating strategy or hedging: explicit trading strategy in underlying assets and risk-less bonds whose terminal payoff, which equals the payoff of a derivative security at maturity. This replicating strategy therefore provides a kind of insurance against loss in the sense that if loss is incurred on one side of the portfolio at payoff, it is exactly compensated by a gain on the other side still at payoff (dynamic hedging as it involves continuous trading). In other words, theoretically an investor can continuously buy and sell derivatives by the strategy and never incur loss. It is also simple and mathematically tractable as compared to some of its more recent variations

Most of the above recognized restrictions of the Black-Scholes model are fundamental and, thus, it is crucial to come up with models that will take into consideration some of the assumptions not addressed by Black-Scholes models. And, not unexpectedly, there is no shortage in the academic literature, which proposes alternative models, all attempting to imitate the characteristics of the market fully.

Inspite of several loopholes in Black Scholes option pricing model, there are several reasons for wide use of this model. The most important one is that the concepts behind the Black-Scholes analysis provide the framework for thinking about option pricing. All the research in option pricing since the Black-Scholes analysis has been done either to extend it or to generalize it. Another important reason for studying the Black-Scholes theory is that the financial world uses it as a standard. In fact, traders quote Black-Scholes volatility to each other, not the actual price of the options. Further, Black-Scholes prices still give very good approximations to the prices of options.

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