Simultaneous Inference of Cosmology and Binary Neutron Star Population from Gravitational-Waves

Soumendra Kishore Roy

Stony Brook University

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PAMU, ISI Kolkata
https://github.com/SoumendraRoy/PAMU_seminar

Outline

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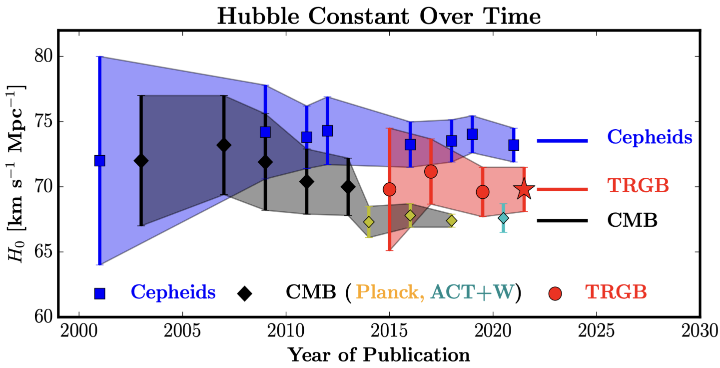
Measurement of Hubble Constant

Tension between Early and Late Universe Measurements

SNe IA calibrated with Cepheids: Riess et al. 2021 (arXiv:2112.04510)
Planck 18: Planck Collaboration 2018 (arXiv:1807.06209)
SNe IA calibrated with TRGB: Freedman 2021 (arXiv:2106.15656)



Taken from Freedman 2021 (arXiv:2106.15656)

Standard Sirens

Why Gravitational Binary is Special?

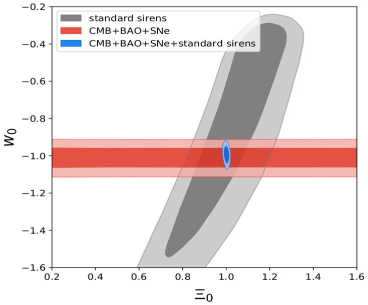
Standard Sirens vs. Standard Candles: (Schutz 1986 Nature)

- **Amplitude averaged over Orientation** $= \langle h \rangle \propto M^{2/3} \mu f^{2/3} r^{-1}$
  Time Scale in which frequency Changes $= \tau \propto M^{-2/3} \mu^{-1} f^{-8/3}$

Luminosity Distance

$\left(\frac{r}{100\text{Mpc}}\right) = 7.8 \left(\frac{f}{100Hz}\right)^{-2} \left(10^{23} \langle h \rangle \tau\right)^{-1}$ **: Distance Ind. of Mass**

- **Extremely Well-Modeled System:** Known System



- **Less Polluted:** Fine Structure Constant >> Gravitational Coupling

Beyond General Relativity:

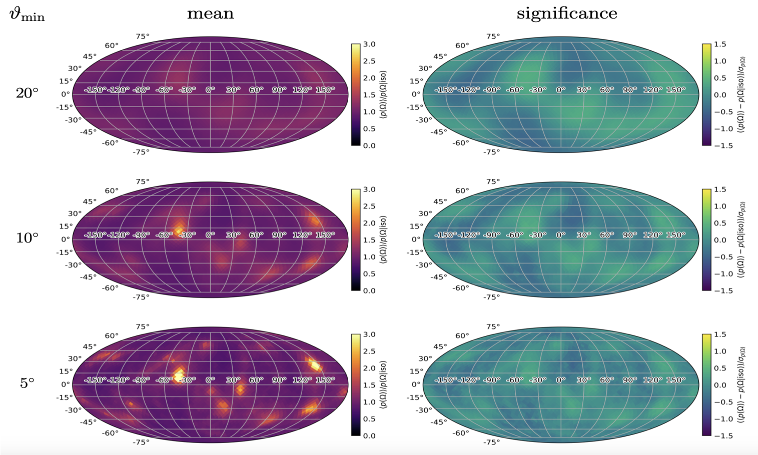
- $D_L^{\text{gw}} / D_L^{\text{em}} = \Xi_0 + a^n (1 - \Xi_0)$
  (Ξ_0, n) characterizes Modified Gravity Theories. Belgacem et al. 2018 (arXiv:1805.08731)

Test of Cosmological Principle

Isotropy Measurement with GWs

63 Confident Mergers from GWTC3: $\mathcal{B}^{\rm iso}_{\rm ani} = 3.7$

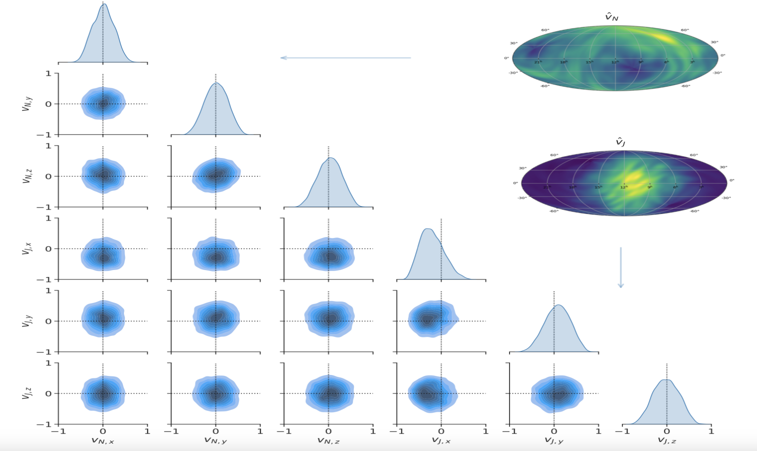
Essick et al. 2022 (arXiv:2207.05792)



$N_{\rm pix} = 3072$, θ_{min} smallest correlation angle

Sky Position and Total Angular Momentum

Are They Randomly Oriented? Isi et al. 2023 (arXiv:2304.13254)



No Evidence of Anisotropy Yet!

Cosmological Parameter Estimation with GWs

Waveform

Under Cosmological Principle

$D_L = c(1+z)\int_0^z \frac{dz'}{H(z')}$

CDM: $H(z') = [\Omega_M(1+z')^3 + (1-\Omega_M)(1+z')^{3(1+w)}]^{\frac{1}{2}}$

- **Waveform:** $h(f) = (\text{constant})\left(\frac{1\ \text{Mpc}}{D_{\text{eff}}}\right)\left(\frac{M_{cz}}{M_\odot}\right)^{5/6} f^{-7/6} e^{-i\Psi(f, M_{cz}, q)}$
- **Extrinsic Parameters:** $D_{\text{eff}} = D_L\left(F_+^2\left(\frac{1+cos^2\zeta}{2}\right)^2 + F_\times^2 cos^2\zeta\right)^{-1/2}$
  $\frac{D_eff}{D_L} = \Theta^2(\alpha, \delta, \theta_{jn}, \zeta)$ Contains **ALL** Extrinsic Parameters!
  True for (2,2) Mode; Finn and Chernoff 1993 (arXiv: 9301003)
- **Masses:** $M_{cz} = M_c(1+z)$, $M_c = \frac{m_1^{\frac{3}{5}} m_2^{\frac{3}{5}}}{(m_1+m_2)^{\frac{1}{5}}}$, $q = m_2/m_1$
- **Phase:** $\Psi = \Psi_{0\text{PN}} + \Psi_{1\text{PN}} + \ldots$; Allen et al. (arXiv:gr-qc/0509116)

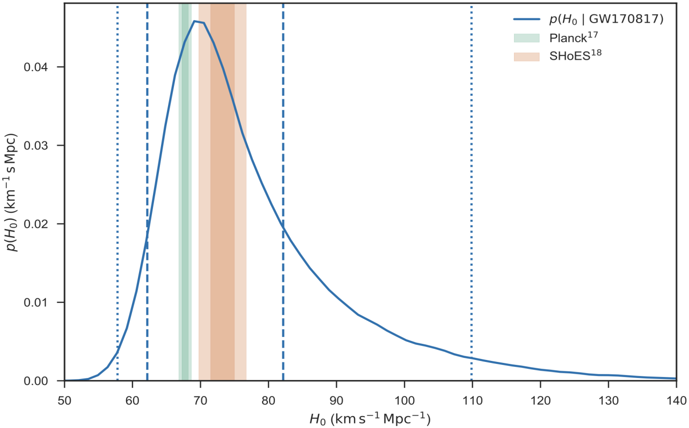
Infer Binary Parameters

$(D_L, M_{cz}, q, \ldots)$ Fit Them in Multi-Detectors!

Bright Sirens

Redshift with Electromagnetic Counterpart

First BNS Event: GW170817 ⇒ GRB 170817A after 2s
Identify the Galaxy: NGC 4993 ⇒ Estimate Redshift

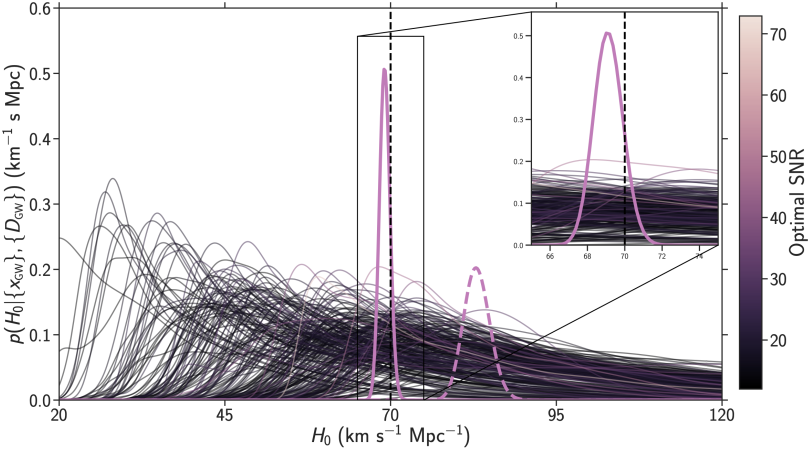


LSC et al. 2017 (arXiv:1710.05835)

Dark Sirens (GWCosmo)

Without Electromagnetic Counterpart

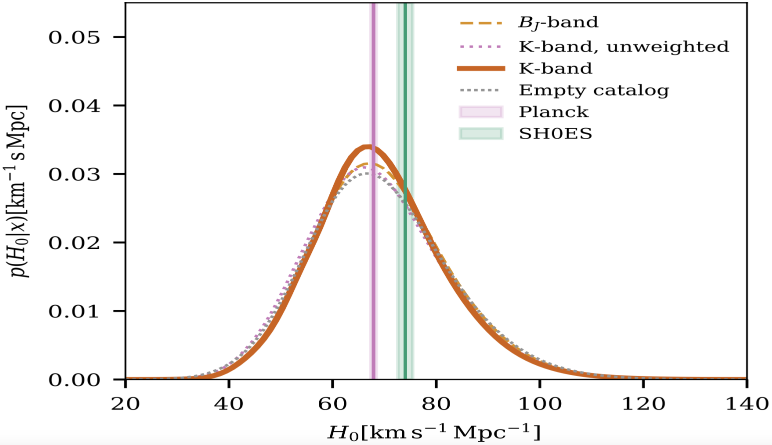
Identify Galaxy from GWs Only ⇒ Estimate Redshift



with 249 Events; Gray et al. 2019 (arXiv:1908.06050)

GWTC-3 Result

GWTC-3 with GLADE+



LVK Collaboration 2021 (arXiv:2111.03604)

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Why do We Need Spectral Sirens?

Bright Sirens:

- Only **One** Counterpart Yet!
- Small **Observed Volume**

Dark Sirens:

- Galaxy Catalogs: **Incomplete**
- Also Small **Observed Volume**

Early Warning in O4 is Online!

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Tap on entry for detailed information

UID	Labels	FAR (Hz)	Created
S230611l	DQOK EM_READY LOW_SIGNIF_LOCKED PASTRO_READY EMBRIGHT_READY SKYMAP_READY LOW_SIGNIF_PRELIM_SENT	2.157e-06	2023-06-11 06:26:27 UTC
S230609u	DQOK SIGNIF_LOCKED LOW_SIGNIF_LOCKED EM_READY EMBRIGHT_READY PASTRO_READY SKYMAP_READY GCN_PRELIM_SENT DQR_REQUEST ADVOK PE_READY	1.004e-08	2023-06-09 06:50:15 UTC
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S230608aw	DQOK EM_READY LOW_SIGNIF_LOCKED SKYMAP_READY LOW_SIGNIF_PRELIM_SENT LLAMA_COMPLETE	2.444e-06	2023-06-08 23:48:25 UTC

https://gracedb.ligo.org/latest/

Features in Mass Distribution

Basic Idea

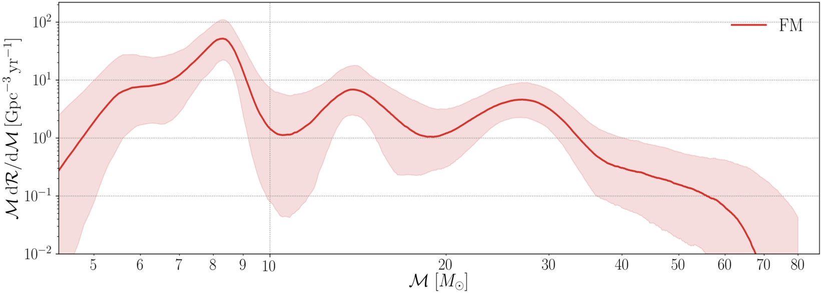
Observables:: $\{D_L, z\} \to \{D_L^{\mathrm{GW}}, m_{\mathrm{src}}(1+z)\}$

as $m_{\mathrm{src}} f_{\mathrm{src}} = m_{\mathrm{src}}(1+z)\frac{f_{\mathrm{src}}}{1+z} = m_{\mathrm{det}} f_{\mathrm{det}}$

How Do Features in m_{det} Evolve with Redshift?

Explain on chalk-board.

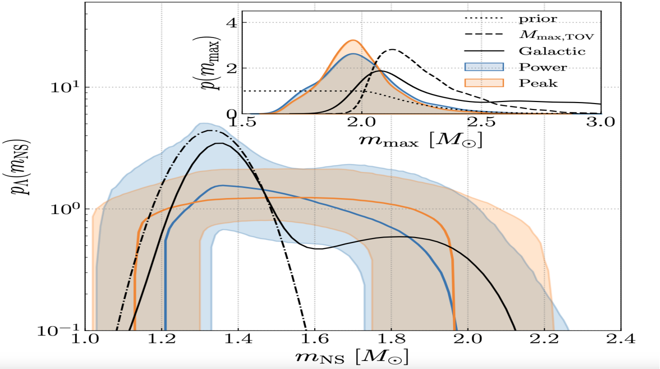
BBH m_{src}:



LVK Collaboration 2021 (arXiv:2111.03634)

BNS Mass Distribution

BNS $m_{\mathbf{src}}$**:** Taylor et al. 2011 (arXiv:1108.5161)



LVK Collaboration 2021 (arXiv:2111.03634)

Sharp Distribution ⇒ Less Parameters ⇒ Better Cosmology

- **Can be Approximated by a Gaussian:** $\vec{\Omega}_P = \{\mu_P, \sigma_P\}$

Simultaneous Inference of $\vec{\Omega}_P$ & $\vec{\Omega}_C$

Maths

$$\vec{\Omega}_P = \text{Pop Params} = \{\mu_P, \sigma_P\}$$
$$\vec{\Omega}_C = \text{Cosmo Params} = \{H_0, \Omega_M, w\}$$

Can We Infer $P(\vec{\Omega}_P, \vec{\Omega}_C | \vec{D})$**?**

$$P(\vec{\Omega}_P, \vec{\Omega}_C | \vec{D}) \propto P(\vec{\Omega}_P, \vec{\Omega}_C) \prod_i^{N_{\text{obs}}} \int_{\vec{\theta}} d\vec{\theta} P(d_i|\vec{\theta}) P_{\text{POP}}(\vec{\theta}|\vec{\Omega}_P, \vec{\Omega}_C)$$

PE Samples: $P(d_i|\vec{\theta}) \propto P_{\text{PE}}(\vec{\theta}) P(\vec{\theta}|d_i)$

$\vec{\theta} = \{M_{cz}, q, D_L\}$; Relevant Params for $\vec{\Omega}_P$ & $\vec{\Omega}_C$

Altogether,

Hierarchical Posterior

$$P(\vec{\Omega}_P, \vec{\Omega}_C | \vec{D}) \propto$$
$$P(\vec{\Omega}_P, \vec{\Omega}_C) \prod_i \int_{D_L} dD_L P(d_i|D_L) P(D_L) \left(\frac{1}{1+z(D_L, \vec{\Omega}_C)}\right) . N_{\left(\frac{M_{cz}^{obs,i}}{1+z(D_L, \vec{\Omega}_C)}\right)} (\vec{\Omega}_P)$$

Derive if time permits.

How To Generate PE Samples?

I. Pedagogical Set-up

We Generate Mock Samples of $P(\vec{\theta}|d_i)$ **with Fiducial** $\vec{\Omega}_P$ **&** $\vec{\Omega}_C$**.**

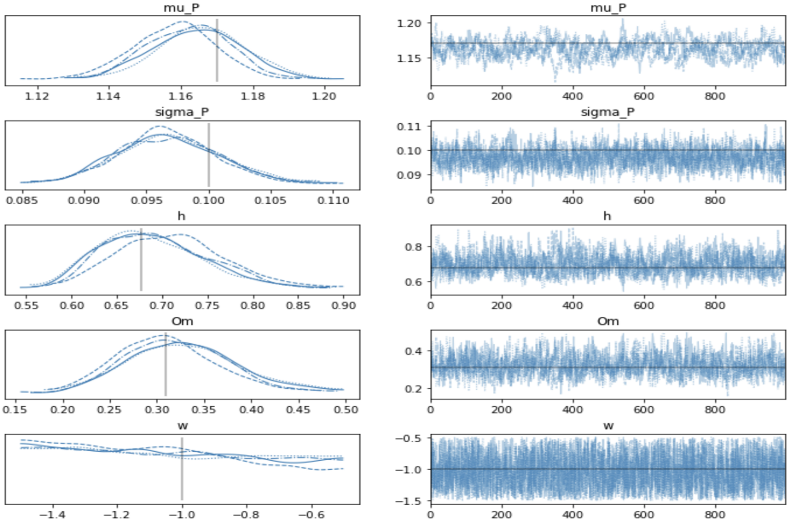
1. $N_{obs} = 500$
2. $z \sim 10\beta(3,9)$ Madau & Dickinson 2014 (arXiv:1403.0007)
3. $M_c \sim N(1.17M_\odot, 0.1M_\odot)$ (approx), for $m_1, m_2 \sim N(1.4M_\odot, 0.15M_\odot)$: Galactic Pop
4. $M_{cz}^{\rm obs} \approx M_{cz}^{\rm true}$: Width of $M_{cz}^{\rm obs}$ Pos Samples is Small
5. $D_L = D_L(z, \vec{\Omega}_C^{\rm Planck18})$, $d_o = D_L + \sigma_{DL} \times U(0,1)$
   $\frac{\sigma_{DL}}{DL} = 0.1 + 0.3\frac{DL}{DL(z=10)}$
   (No snr cut: if LIGO Can See the Whole Universe)
   d_o Samples $\sim N(d_o, \sigma_{DL}, N_S = 4000)$

If Hierarchical Pos Works:

1. **Unbiased:** $P(\vec{\Omega}_P, \vec{\Omega}_C|\vec{D})$ Contains Fiducial $\vec{\Omega}_P$ & $\vec{\Omega}_C$
2. **Pos Width:** is Ind. of Prior Width

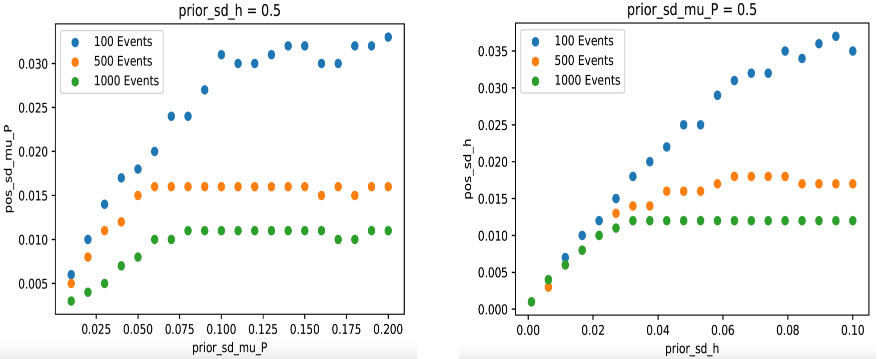
Unbiasedness

All Params:



Prior Width vs. Pos Width

Try to Infer $\{\mu_P, H_0\}$.



IT Works!

How to Generate PE Samples?

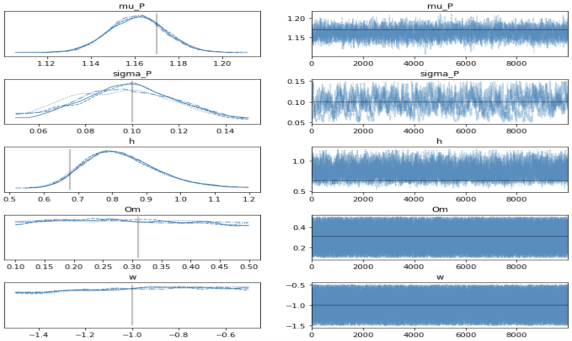
II. Fisher Matrix

Approximate log of PE Likelihood as

$$logP(d_i|\vec{\theta}) \approx -\frac{1}{2}\sum_{ab}(\theta_a - \theta_a^{\rm mean})F_{ab}(\theta_b - \theta_b^{\rm mean}),\ F_{ab} = \text{Fisher } (\theta_a,\ \theta_b).$$

$\vec{\theta}^{\rm mean} = \vec{\theta}^{\rm true}$ (no noise injections)

$\vec{\theta}^{\rm mean} = multiN(\vec{\theta}^{\rm true}, cov = F^{-1})$ (with noise injections)



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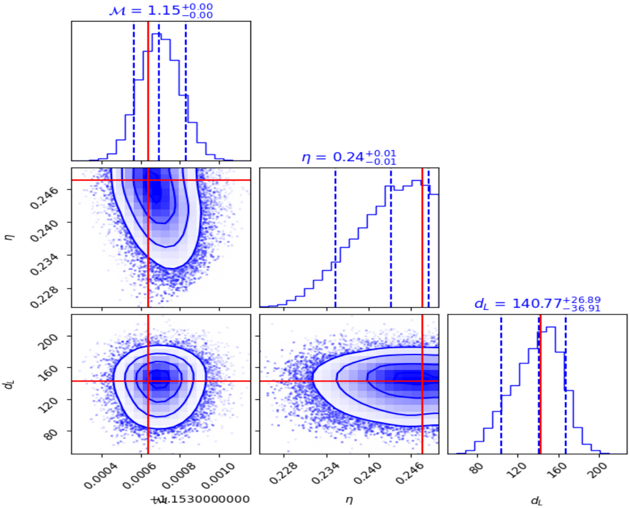
How to Generate PE Samples?

III. Inject in LVK Noise

1. $N_{obs} = 62$ (95% Limit of Pos of Exp. BNS No. in O4)
2. Use Old Pop Distributions
3. Generate $h(f)$ Using TaylorF2 + Tidal for Each Event, MacDonald et al. 2011 (arXiv:1102.5128)
4. PE with $logP(d_i|\vec{\theta}) = -\frac{1}{2}\langle d(f) - h(f,\vec{\theta})|d(f) - h(f,\vec{\theta})\rangle$
   with $\langle a(f)|b(f)\rangle = 2\int_0^\infty df\, \frac{a^*(f)b(f)+a(f)b^*(f)}{S_n(f)}$
   $S_n(f)$ One-Sided Noise Psd
5. PE Done in Bilby (Ashton et al. 2018, arXiv:1811.02042) with adaptive frequency resolutions (Morisaki 2021, arXiv:2104.0781)
6. snr cut $= 8$

Typical PE Contours

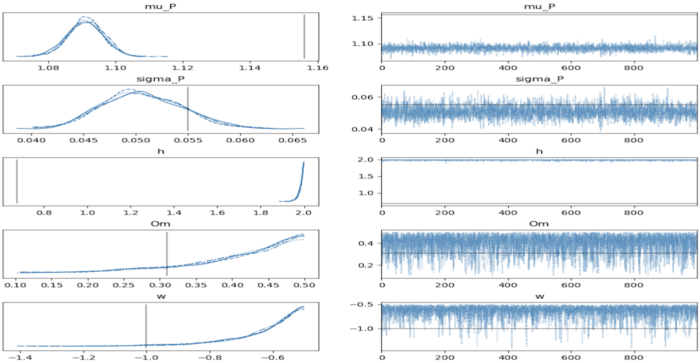
Example



Selection Function

Hierarchical Pos

$$P(\vec{\Omega}_P, \vec{\Omega}_C | \vec{D}) \propto P(\vec{\Omega}_P, \vec{\Omega}_C) \prod_i^{N_{\rm obs}} \int_{\vec{\theta}} d\vec{\theta} P(d_i|\vec{\theta}) P_{\rm POP}(\vec{\theta}|\vec{\Omega}_P, \vec{\Omega}_C)$$



Biased!

Selection Function (Contd)

Normalization of PE Likelihood:

$$\int_{\vec{\theta}} d\vec{\theta} P_{\text{POP}}(\vec{\theta}|\vec{\Omega}_P, \vec{\Omega}_C) \int_{d_i:\text{snr}>8} dd_i P(d_i|\vec{\theta})$$

with Sel Function

$$P(\vec{\Omega}_P, \vec{\Omega}_C|\vec{D}) \propto P(\vec{\Omega}_P, \vec{\Omega}_C) \prod_i^{N_{\text{obs}}} \frac{\int_{\vec{\theta}} d\vec{\theta} P(d_i|\vec{\theta}) P_{\text{POP}}(\vec{\theta}|\vec{\Omega}_P, \vec{\Omega}_C)}{\int_{\vec{\theta}} d\vec{\theta} P_{\text{POP}}(\vec{\theta}|\vec{\Omega}_P, \vec{\Omega}_C) \int_{d_i:\text{snr}>8} dd_i P(d_i|\vec{\theta})}$$

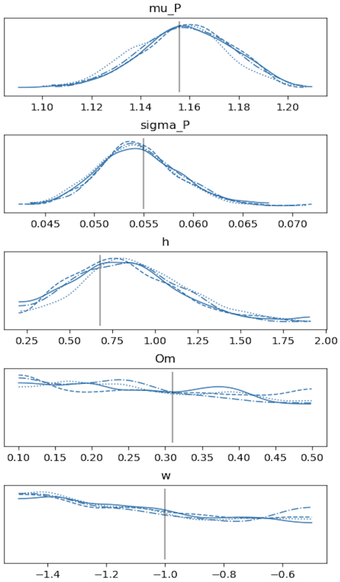
Assumptions:

1. No Background Term.
2. Merger Rate is Constant in Comoving Volume.
3. $T^{\text{nt}} >> T^{\text{tr}}$

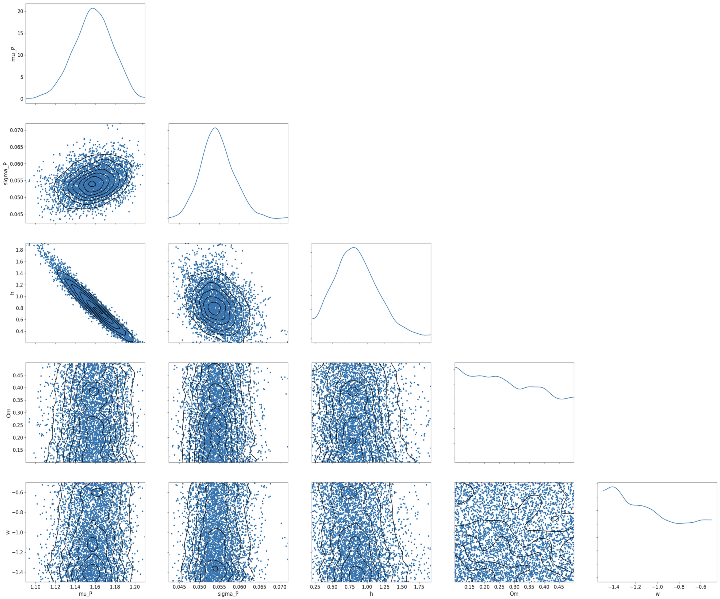
Full Hierarchical Posterior: Vitale et al. 2020 (arXiv:2007.05579)

Calculating $\int_{d_i:\text{snr}>8} dd_i P(d_i|\vec{\theta})$: Farr 2019 (arXiv:1904.10879)

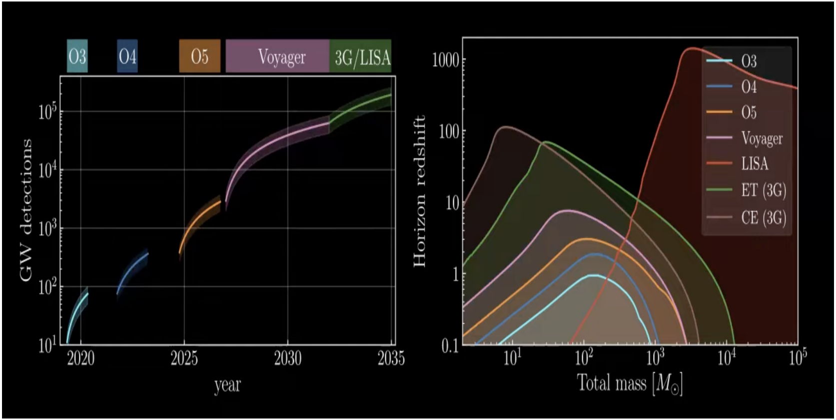
Hierarchical Pos



Contour Plots



Precision GW-Cosmology within the Next Decade

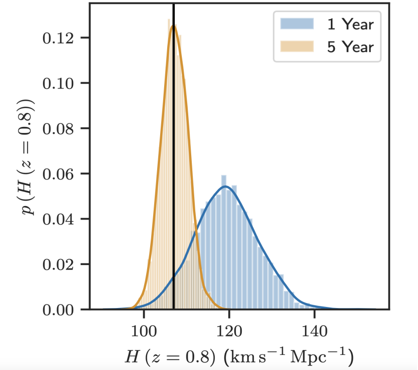


Taken from Jose Maria Ezquiaga, IAS seminar, 2021

$N \sim 10^5 \Rightarrow \frac{\sigma_h}{h} = 1\%$

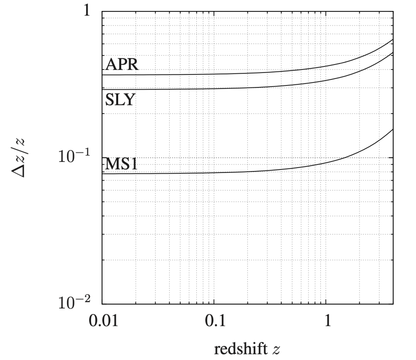
Other Thoughts

Spectral Siren Cosmo with BBH Mass Spectrum:
Farr et al. 2019 (arXiv:1908.09084)



Subjected to Mass Systematic

Finding Source Frame Mass from Tidal Deformability: Messenger et al. 2011 (arXiv:1107.5725)



Subjected to NS EoS and Mass Dist