Cosmology with the Gravitational-Waves from Binary Neutron Stars

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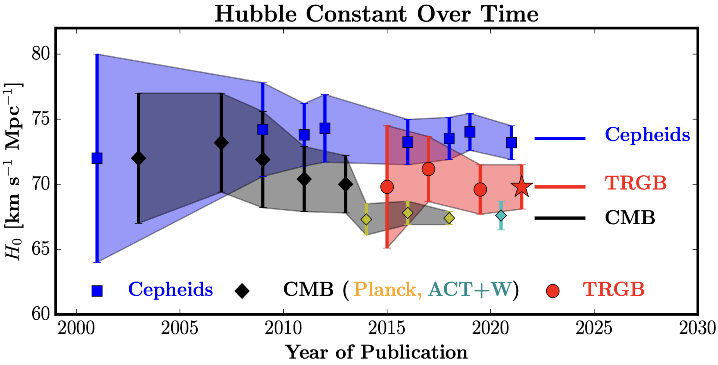
PAMU, ISI Kolkata
`https://github.com/SoumendraRoy/Seminars`

Measurement of Hubble Constant

Tension between Early and Late Universe Measurements

SNe IA calibrated with Cepheids: Riess et al. 2021 (arXiv:2112.04510)
Planck 18: Planck Collaboration 2018 (arXiv:1807.06209)
SNe IA calibrated with TRGB: Freedman 2021 (arXiv:2106.15656)

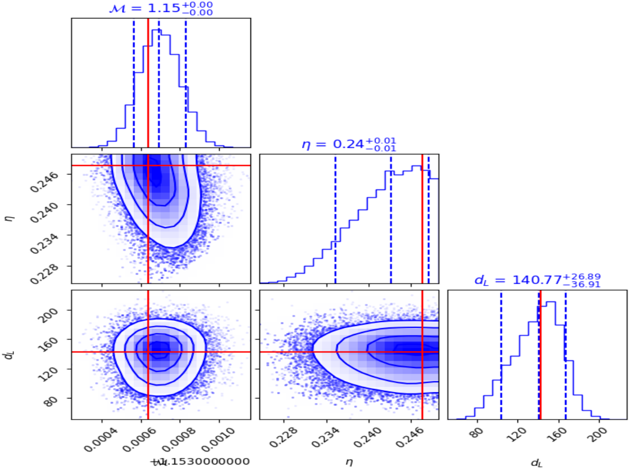


Taken from Freedman 2021 (arXiv:2106.15656)

GW Distances are Easy

Standard Sirens

$$L_{\mathrm{GW}} = \frac{c^5}{G} \approx 10^6 \mathrm{SNe}$$



Distance Ind. of Binary Masses: Distance comes only in GW Amplitude

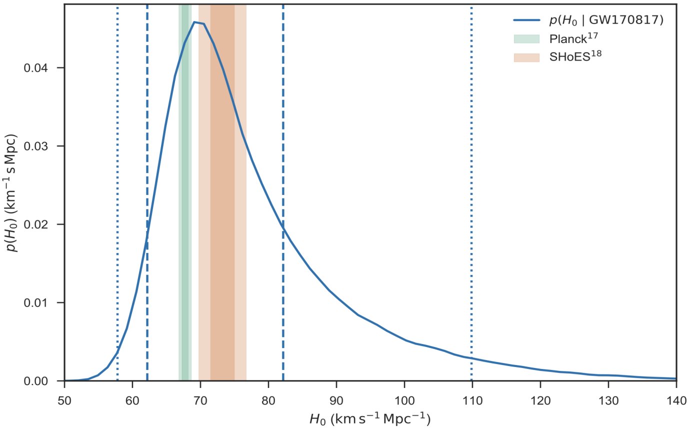
GW Redshifts are Hard

- **From Electromagnetic Counterparts**
- Others...

Bright Sirens

Redshift with Electromagnetic Counterpart

First BNS Event: GW170817 ⇒ GRB 170817A after 2s
Identify the Galaxy: NGC 4993 ⇒ Estimate Redshift



LSC et al. 2017 (arXiv:1710.05835)

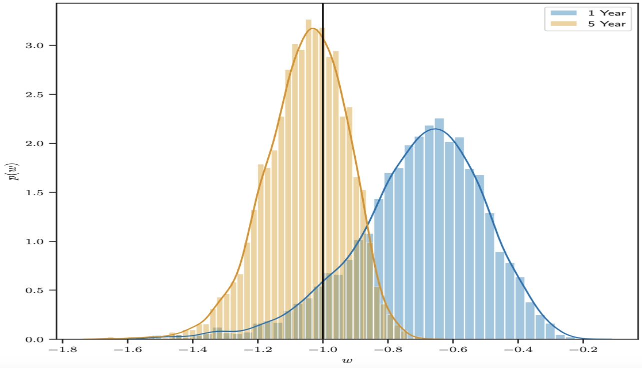
Dark Energy Equation of State

Impacts Hubble Parameter, and hence Distances as,

DE EoS w in CDM

$$H(z) = H_0[\Omega_M(1+z)^3 + (1-\Omega_M)(1+z^{3(1+w)}]^{\frac{1}{2}}$$

Constant w with BBHs (Farr et al. 2019):



DE EoS measurement is subjected to the Redshift evolution model of $w(z)$

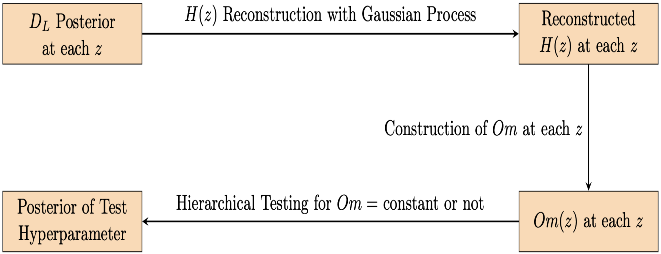
Null Diagnostic (Sahni et al. 2008)

$Om(x) = \frac{(\frac{H(z)}{H_0})^2 - 1}{(1+z)^3 - 1}$ (CDM and Spatially Flat)

1. $Om(z_1) = Om(z_2)$ for $z_1 \neq z_2$ in **ΛCDM**
2. $Om(z_1) > Om(z_2)$ for $z_1 > z_2$ in **Quintessence**
3. $Om(z_1) < Om(z_2)$ for $z_1 > z_2$ in **Phantom**

Om (Contd)

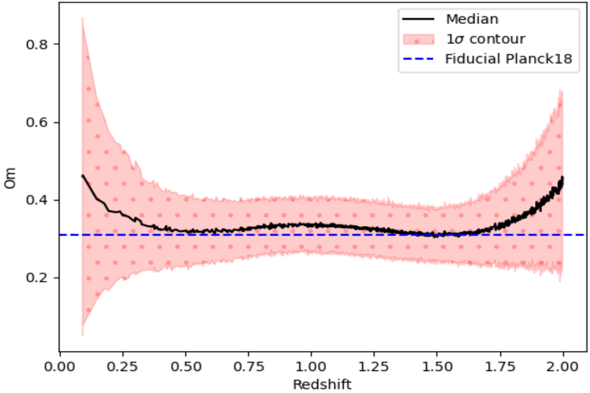
1000 NS Binaries with Gaussian Errorbars on Distance at Cosmic Explorer Sensitivity and Spectroscopic Redshifts Up to z=2:



Roy et al. (in preparation)

Om (Contd)

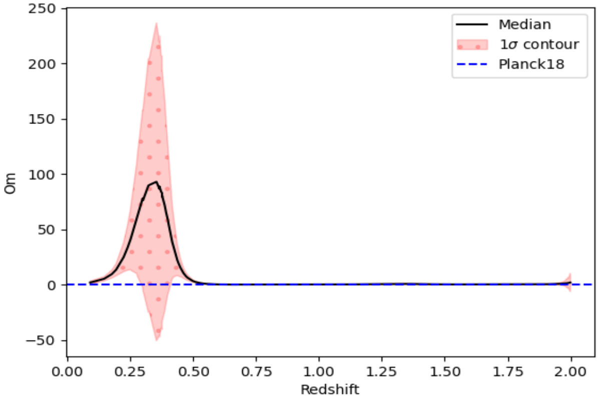
ΛCDM Injection:



Consistent with Constant *Om*

Om (Contd)

Injection Non-trivial w(z) at z$\sim$0.4:



Not Consistent with ΛCDM

With Only GWs:

Mass Scale

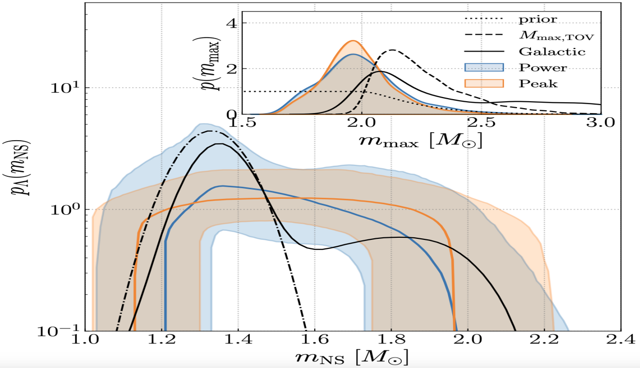
$$m_{\rm obs} = m(1+z)$$

- **We measure:** $m_{\rm obs}$
- **If we know:** m
- **we have:** z

Search for the Features in Astrophysical Mass Distribution!

BNS Mass Distribution

BNS m: Taylor et al. 2011 (arXiv:1108.5161)



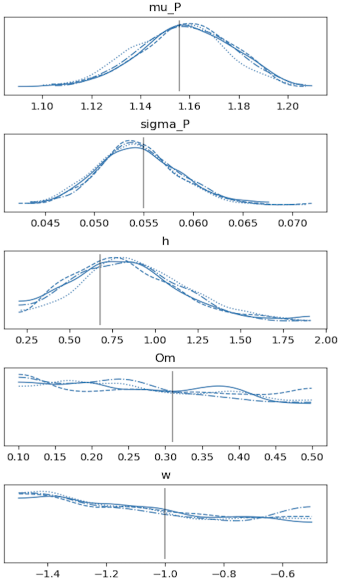
LVK Collaboration 2021 (arXiv:2111.03634)

Sharp Distribution $\Rightarrow$ Less Parameters $\Rightarrow$ Better Cosmology

- **Can be Approximated by a Gaussian:** $\vec{\Omega}_P = \{\mu_P, \sigma_P\}$

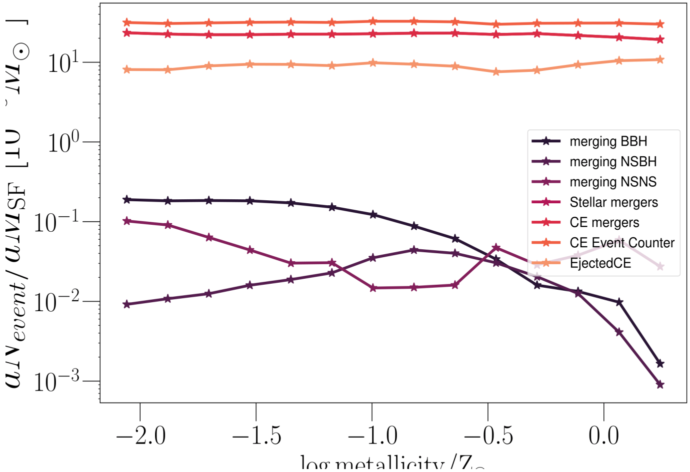
Use Features in BNS Population

62 BNS Injections in O4 LVK Noise



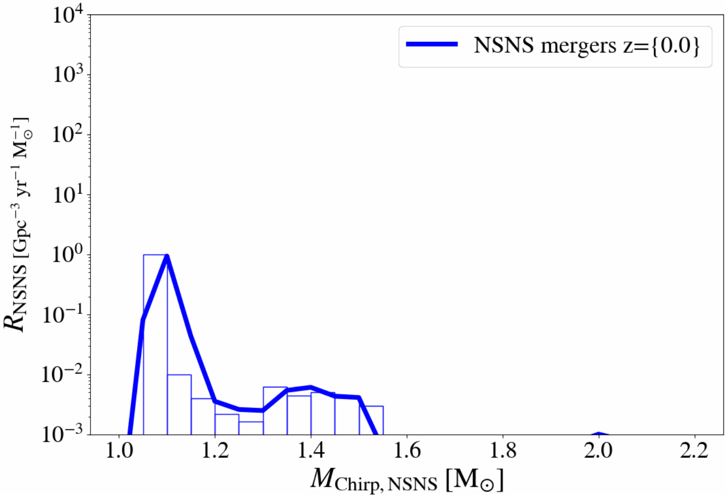
Systematics in Astrophysical Population

Cosmology is Possible if m does not depend on z.



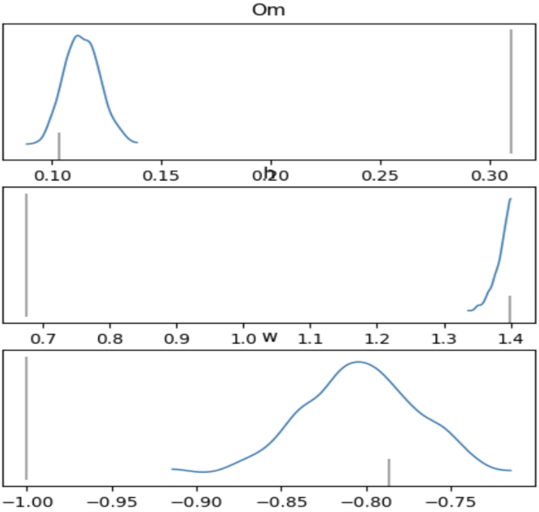
Credit: Lieke van Son

Redshift Dependence of m in BNS

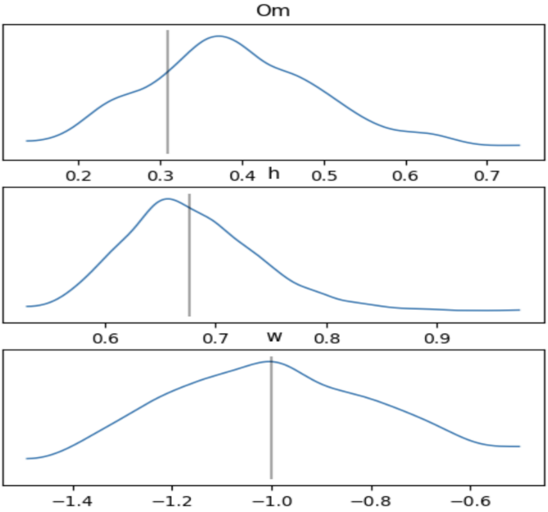


Biased

if z=0 Pop is assumed All z Pop



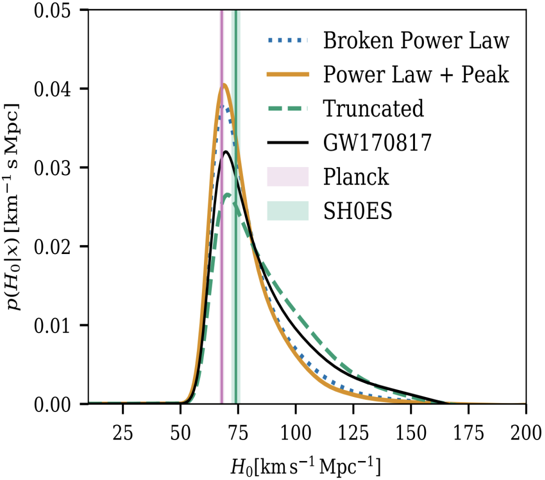
Fully Correlated Analysis



Cosmology from LVK

GWTC3 Result

with ∼70 events (47 in GWTC3) (Abbott et al. 2021)



Presently

4th Observation (O4) Run is Ongoing. Interested People can keep an Eye on: `https://gracedb.ligo.org/superevents/public/O4/`

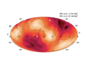
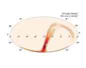
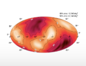
O4 Significant Detection Candidates: **81** (92 Total - 11 Retracted)

O4 Low Significance Detection Candidates: **1594** (Total)

Show All Public Events

Page 1 of 7. next last »

SORT: EVENT ID (A-Z)

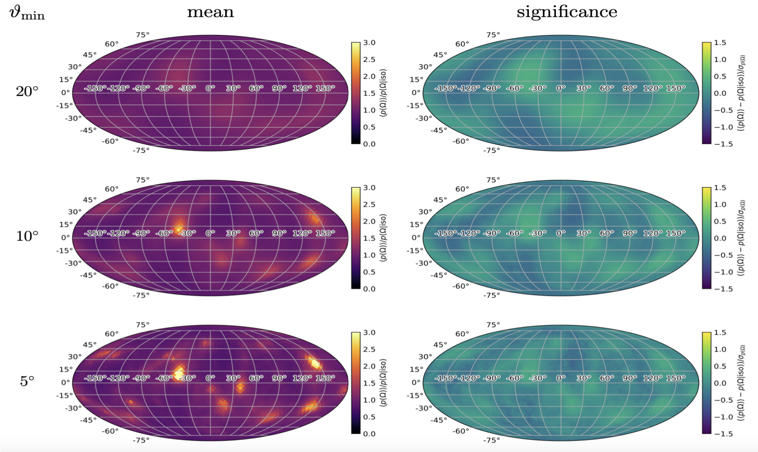
Event ID	Possible Source (Probability)	Significant	UTC	GCN	Location	FAR
S240109a	BBH (99%)	Yes	Jan. 9, 2024 05:04:31 UTC	GCN Circular Query Notices | VOE		1 per 4.3136 years
S240107b	BBH (97%), Terrestrial (3%)	Yes	Jan. 7, 2024 01:32:15 UTC	GCN Circular Query Notices | VOE		1.8411 per year
S240104bl	BBH (>99%)	Yes	Jan. 4, 2024 16:49:32 UTC	GCN Circular Query Notices | VOE		1 per 8.9137e+08 years

Back Up Slides

Test of Cosmological Principle

Isotropy Measurement with GWs

63 Confident Mergers from GWTC3: $\mathcal{B}^{\rm iso}_{\rm ani} = 3.7$
Essick et al. 2022 (arXiv:2207.05792)



$N_{\rm pix} = 3072$, θ_{min} smallest correlation angle

Cosmological Parameter Estimation with GWs

Waveform

Under Cosmological Principle

$D_L = c(1+z)\int_0^z \frac{dz'}{H(z')}$

CDM: $H(z') = H_0[\Omega_M(1+z')^3 + (1-\Omega_M)(1+z')^{3(1+w)}]^{\frac{1}{2}}$

- **Waveform:** $h(f) = (\text{constant})\left(\frac{1\ \text{Mpc}}{D_{\text{eff}}}\right)\left(\frac{M_{cz}}{M_\odot}\right)^{5/6} f^{-7/6} e^{-i\Psi(f,M_{cz},q)}$
- **Extrinsic Parameters:** $D_{\text{eff}} = D_L\left(F_+^2\left(\frac{1+cos^2\zeta}{2}\right)^2 + F_\times^2 cos^2\zeta\right)^{-1/2}$
  $\frac{D_eff}{D_L} = \Theta^2(\alpha, \delta, \theta_{jn}, \zeta)$ Contains **ALL** Extrinsic Parameters!
  True for (2,2) Mode; Finn and Chernoff 1993 (arXiv: 9301003)
- **Masses:** $M_{cz} = M_c(1+z),\ M_c = \frac{m_1^{\frac{3}{5}} m_2^{\frac{3}{5}}}{(m_1+m_2)^{\frac{1}{5}}},\ q = m_2/m_1$
- **Phase:** $\Psi = \Psi_{0\text{PN}} + \Psi_{1\text{PN}} + \ldots$; Allen et al. (arXiv:gr-qc/0509116)

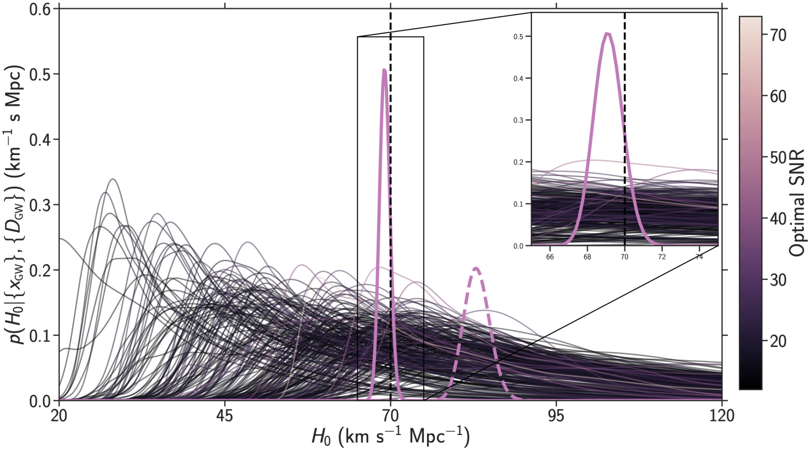
Infer Binary Parameters

$(D_L, M_{cz}, q, \ldots)$ Fit Them in Multi-Detectors!

Dark Sirens (GWCosmo)

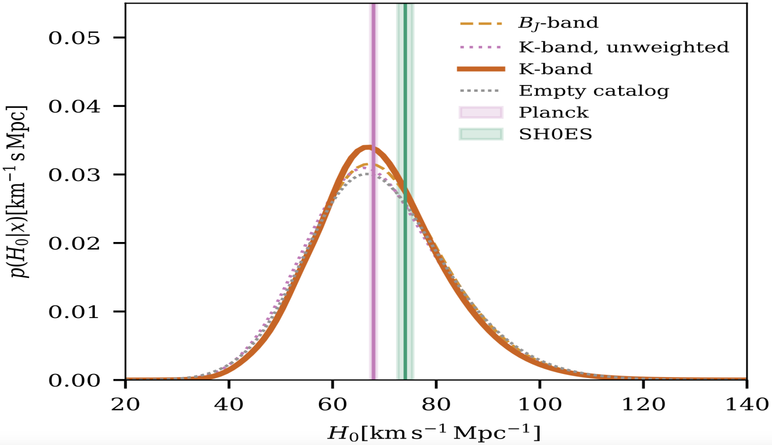
Without Electromagnetic Counterpart

Identify Galaxy from GWs Only ⇒ Estimate Redshift



with 249 Events; Gray et al. 2019 (arXiv:1908.06050)

GWTC-3 with GLADE+



LVK Collaboration 2021 (arXiv:2111.03604)

Why do We Need Spectral Sirens?

Bright Sirens:

- Only **One** Counterpart Yet!
- Small **Observed Volume**

Dark Sirens:

- Galaxy Catalogs: **Incomplete**
- Also Small **Observed Volume**

Early Warning in O4 is Online!

GraceDB Public Alerts Latest Search Documentation Login

Please log in to view full database contents.

Tap on entry for detailed information

UID	Labels	FAR (Hz)	Created
S230611l	DQOK EM_READY LOW_SIGNIF_LOCKED PASTRO_READY EMBRIGHT_READY SKYMAP_READY LOW_SIGNIF_PRELIM_SENT	2.157e-06	2023-06-11 06:26:27 UTC
S230609u	DQOK SIGNIF_LOCKED LOW_SIGNIF_LOCKED EM_READY EMBRIGHT_READY PASTRO_READY SKYMAP_READY GCN_PRELIM_SENT DQR_REQUEST ADVOK PE_READY	1.004e-08	2023-06-09 06:50:15 UTC
S230609a	DQOK LOW_SIGNIF_LOCKED EM_READY EMBRIGHT_READY PASTRO_READY SKYMAP_READY LOW_SIGNIF_PRELIM_SENT	7.991e-08	2023-06-09 01:08:37 UTC
S230608aw	DQOK EM_READY LOW_SIGNIF_LOCKED SKYMAP_READY LOW_SIGNIF_PRELIM_SENT LLAMA_COMPLETE	2.444e-06	2023-06-08 23:48:25 UTC

https://gracedb.ligo.org/latest/

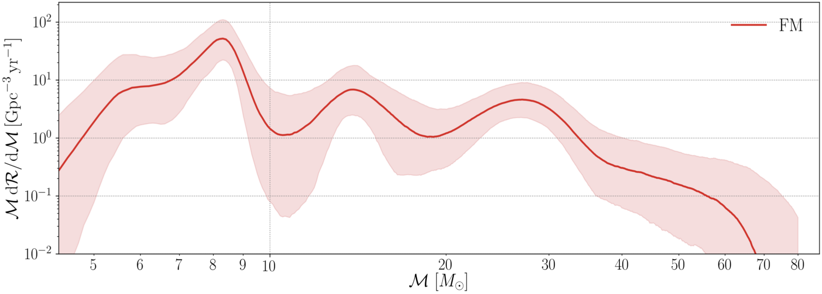
Observables:: $\{D_L, z\} \to \{D_L^{\rm GW}, m_{\rm src}(1+z)\}$

as $m_{\rm src} f_{\rm src} = m_{\rm src}(1+z)\frac{f_{\rm src}}{1+z} = m_{\rm det} f_{\rm det}$

How Do Features in $m_{\rm det}$ Evolve with Redshift?

Explain on chalk-board.

BBH $m_{\rm src}$:



LVK Collaboration 2021 (arXiv:2111.03634)

Simultaneous Inference of $\vec{\Omega}_P$ & $\vec{\Omega}_C$

Maths

$$\vec{\Omega}_P = \text{Pop Params} = \{\mu_P, \sigma_P\}$$
$$\vec{\Omega}_C = \text{Cosmo Params} = \{H_0, \Omega_M, w\}$$

Can We Infer $P(\vec{\Omega}_P, \vec{\Omega}_C|\vec{D})$**?**

$$P(\vec{\Omega}_P, \vec{\Omega}_C|\vec{D}) \propto P(\vec{\Omega}_P, \vec{\Omega}_C) \prod_i^{N_{\mathrm{obs}}} \int_{\vec{\theta}} d\vec{\theta} P(d_i|\vec{\theta}) P_{\mathrm{POP}}(\vec{\theta}|\vec{\Omega}_P, \vec{\Omega}_C)$$

PE Samples: $P(d_i|\vec{\theta}) \propto P_{\mathrm{PE}}(\vec{\theta}) P(\vec{\theta}|d_i)$

$\vec{\theta} = \{M_{cz}, q, D_L\}$; Relevant Params for $\vec{\Omega}_P$ & $\vec{\Omega}_C$

Altogether,

Hierarchical Posterior

$$P(\vec{\Omega}_P, \vec{\Omega}_C|\vec{D}) \propto$$
$$P(\vec{\Omega}_P, \vec{\Omega}_C) \prod_i \int_{D_L} dD_L P(d_i|D_L) P(D_L) \left(\frac{1}{1+z(D_L, \vec{\Omega}_C)}\right) . N_{\left(\frac{M_{cz}^{obs,i}}{1+z(D_L, \vec{\Omega}_C)}\right)}(\vec{\Omega}_P)$$

Derive if time permits.

We Generate Mock Samples of $P(\vec{\theta}|d_i)$ **with Fiducial** $\vec{\Omega}_P$ **&** $\vec{\Omega}_C$**.**

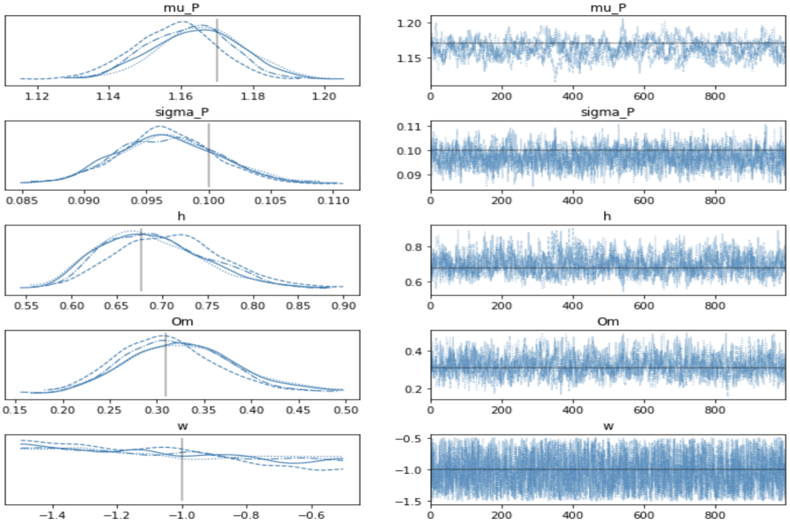
1. $N_{obs} = 500$
2. $z \sim 10\beta(3,9)$ Madau & Dickinson 2014 (arXiv:1403.0007)
3. $M_c \sim N(1.17M_\odot, 0.1M_\odot)$ (approx), for $m_1, m_2 \sim N(1.4M_\odot, 0.15M_\odot)$: Galactic Pop
4. $M_{cz}^{\rm obs} \approx M_{cz}^{\rm true}$: Width of $M_{cz}^{\rm obs}$ Pos Samples is Small
5. $D_L = D_L(z, \vec{\Omega}_C^{\rm Planck18})$, $d_o = D_L + \sigma_{DL} \times U(0,1)$
   $\frac{\sigma_{DL}}{DL} = 0.1 + 0.3\frac{DL}{DL(z=10)}$
   (No snr cut: if LIGO Can See the Whole Universe)
   d_o Samples $\sim N(d_o, \sigma_{DL}, N_S = 4000)$

If Hierarchical Pos Works:

1. **Unbiased:** $P(\vec{\Omega}_P, \vec{\Omega}_C|\vec{D})$ Contains Fiducial $\vec{\Omega}_P$ & $\vec{\Omega}_C$
2. **Pos Width:** is Ind. of Prior Width

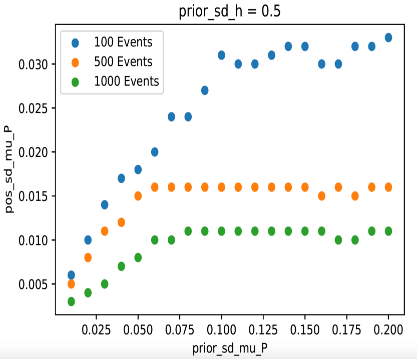
Unbiasedness

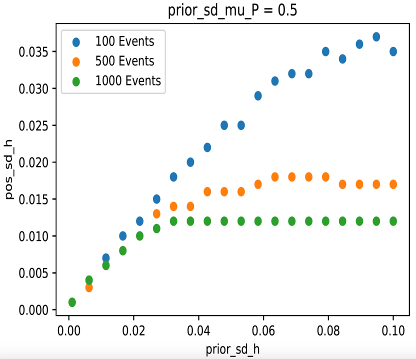
All Params:



Prior Width vs. Pos Width

Try to Infer $\{\mu_P, H_0\}$.





IT Works!

How to Generate PE Samples?

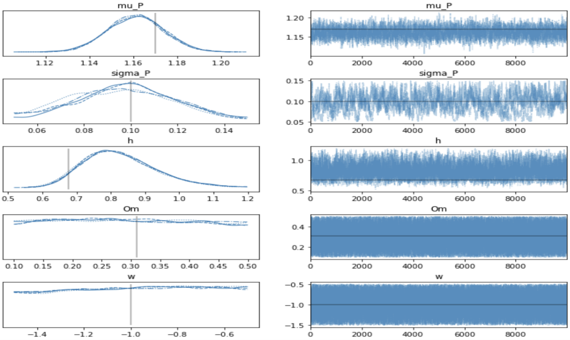
II. Fisher Matrix

Approximate log of PE Likelihood as

$$logP(d_i|\vec{\theta}) \approx -\frac{1}{2}\sum_{ab}(\theta_a - \theta_a^{\text{mean}})F_{ab}(\theta_b - \theta_b^{\text{mean}}),\ F_{ab} = \text{Fisher }(\theta_a,\ \theta_b).$$

$\vec{\theta}^{\text{mean}} = \vec{\theta}^{\text{true}}$ (no noise injections)

$\vec{\theta}^{\text{mean}} = multiN(\vec{\theta}^{\text{true}}, cov = F^{-1})$ (with noise injections)



How to Generate PE Samples?

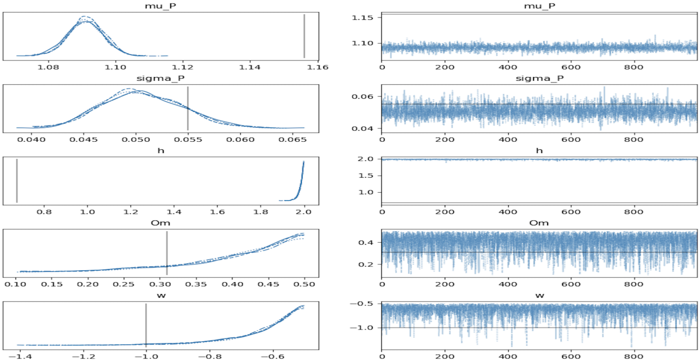
III. Inject in LVK Noise

1. $N_{obs} = 62$ (95% Limit of Pos of Exp. BNS No. in O4)
2. Use Old Pop Distributions
3. Generate $h(f)$ Using TaylorF2 + Tidal for Each Event, MacDonald et al. 2011 (arXiv:1102.5128)
4. PE with $logP(d_i|\vec{\theta}) = -\frac{1}{2}\langle d(f) - h(f, \vec{\theta})|d(f) - h(f, \vec{\theta})\rangle$
   with $\langle a(f)|b(f)\rangle = 2\int_0^{\infty} df \frac{a^*(f)b(f)+a(f)b^*(f)}{S_n(f)}$
   $S_n(f)$ One-Sided Noise Psd
5. PE Done in Bilby (Ashton et al. 2018, arXiv:1811.02042) with adaptive frequency resolutions (Morisaki 2021, arXiv:2104.0781)
6. snr cut $= 8$

Selection Function

Hierarchical Pos

$$P(\vec{\Omega}_P, \vec{\Omega}_C | \vec{D}) \propto P(\vec{\Omega}_P, \vec{\Omega}_C) \prod_i^{N_{\rm obs}} \int_{\vec{\theta}} d\vec{\theta} P(d_i|\vec{\theta}) P_{\rm POP}(\vec{\theta}|\vec{\Omega}_P, \vec{\Omega}_C)$$



Biased!

Selection Function (Contd)

Normalization of PE Likelihood:
$\int_{\vec{\theta}} d\vec{\theta} P_{\mathrm{POP}}(\vec{\theta}|\vec{\Omega}_P, \vec{\Omega}_C) \int_{d_i:\mathrm{snr}>8} dd_i P(d_i|\vec{\theta})$

with Sel Function

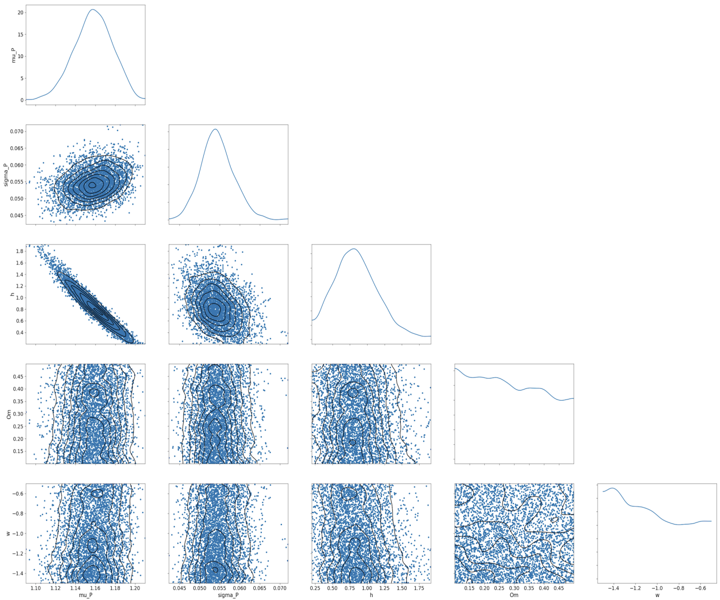
$$P(\vec{\Omega}_P, \vec{\Omega}_C|\vec{D}) \propto P(\vec{\Omega}_P, \vec{\Omega}_C) \prod_i^{N_{\mathrm{obs}}} \frac{\int_{\vec{\theta}} d\vec{\theta} P(d_i|\vec{\theta}) P_{\mathrm{POP}}(\vec{\theta}|\vec{\Omega}_P, \vec{\Omega}_C)}{\int_{\vec{\theta}} d\vec{\theta} P_{\mathrm{POP}}(\vec{\theta}|\vec{\Omega}_P, \vec{\Omega}_C) \int_{d_i:\mathrm{snr}>8} dd_i P(d_i|\vec{\theta})}$$

Assumptions:

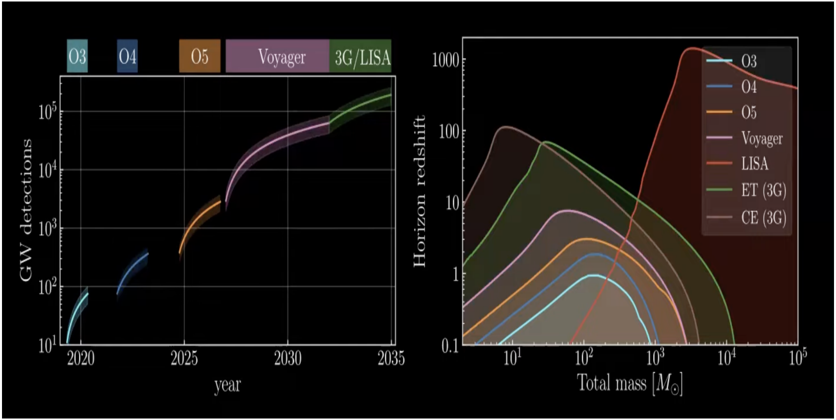
1. No Background Term.
2. Merger Rate is Constant in Comoving Volume.
3. $T^{\mathrm{nt}} >> T^{\mathrm{tr}}$

Full Hierarchical Posterior: Vitale et al. 2020 (arXiv:2007.05579)
Calculating $\int_{d_i:\mathrm{snr}>8} dd_i P(d_i|\vec{\theta})$: Farr 2019 (arXiv:1904.10879)

Contour Plots



Precision GW-Cosmology within the Next Decade

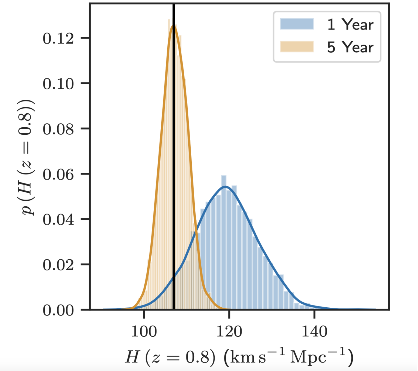


Taken from Jose Maria Ezquiaga, IAS seminar, 2021

$N \sim 10^5 \Rightarrow \frac{\sigma_h}{h} = 1\%$

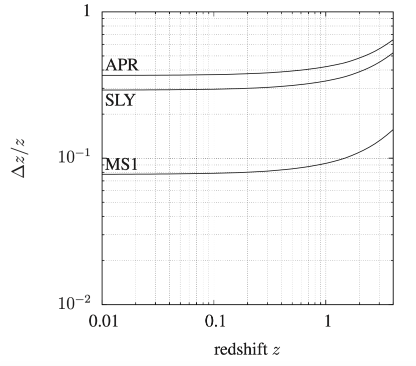
Other Thoughts

Spectral Siren Cosmo with BBH Mass Spectrum:
Farr et al. 2019 (arXiv:1908.09084)



Subjected to Mass Systematic

Finding Source Frame Mass from Tidal Deformability: Messenger et al. 2011 (arXiv:1107.5725)



Subjected to NS EoS and Mass Dist