

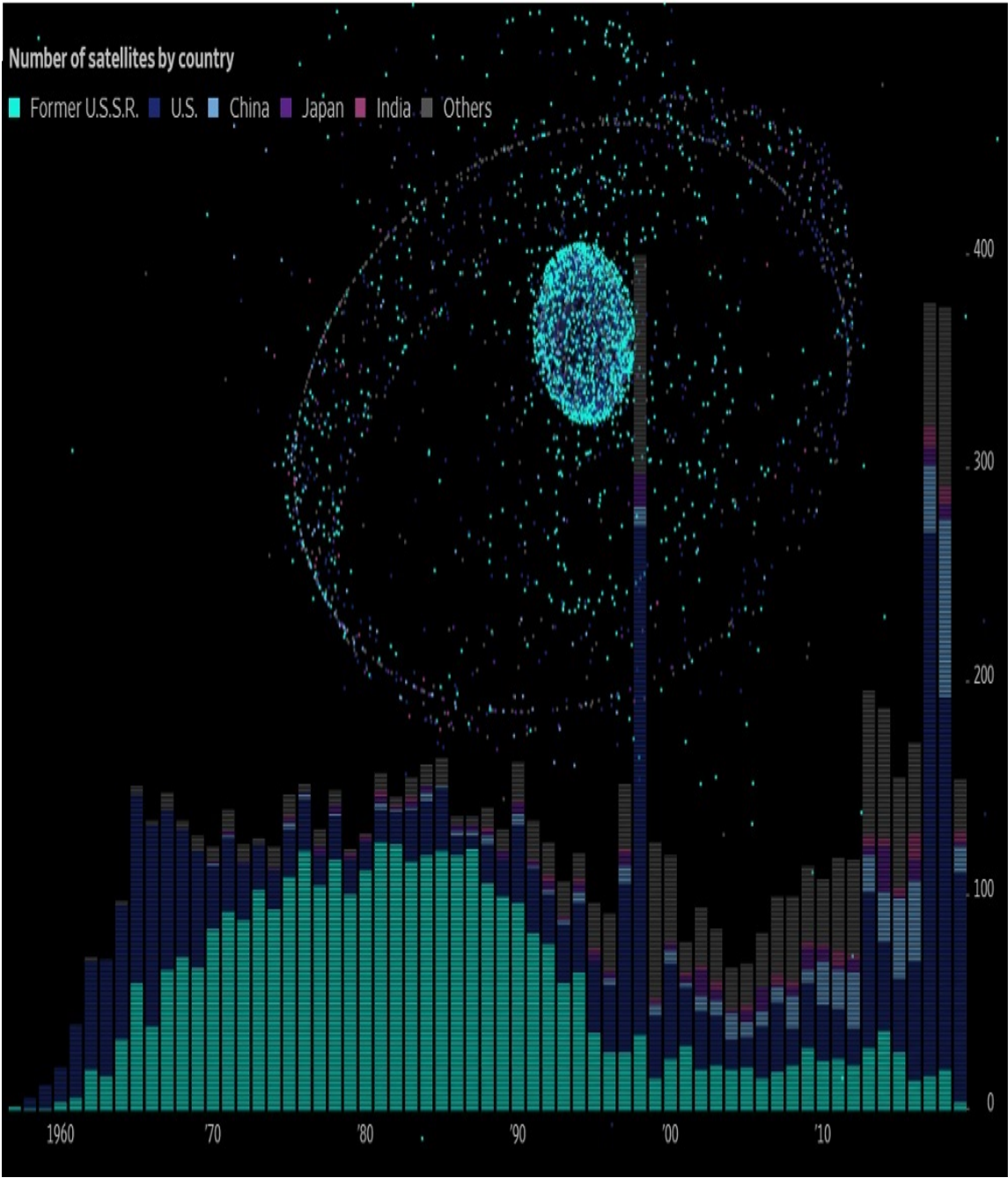
# Indian Institute of Technology, Madras - BS in Data Science and Applications

## Notations :

- 1.Options shown in green color and with ✓ icon are correct.
- 2.Options shown in red color and with ✗ icon are incorrect.

Question Paper Name :	IIT M DEGREE AN2 EXAM QPE2 16 JULY 2023
Subject Name :	2023 July: IIT M DEGREE AN2 EXAM QPE2
Creation Date :	2023-07-10 17:54:46
Duration :	120
Total Marks :	575
Display Marks:	Yes
Share Answer Key With Delivery Engine :	Yes
Actual Answer Key :	Yes
Calculator :	Scientific
Magnifying Glass Required? :	No
Ruler Required? :	No
Eraser Required? :	No
Scratch Pad Required? :	No
Rough Sketch/Notepad Required? :	No
Protractor Required? :	No
Show Watermark on Console? :	Yes
Highlighter :	No
Auto Save on Console?	Yes
Change Font Color :	No
Change Background Color :	No

4. Critically analyze this visualization. Critique its effectiveness, and suggest at least one improvement.



Options :

6406531933192. ✓ I have written answers on the answer sheets

6406531933193. ✗ Not applicable

## Sem2 Statistics2

Section Id :

64065339135

Section Number :

11

<b>Section type :</b>	Online
<b>Mandatory or Optional :</b>	Mandatory
<b>Number of Questions :</b>	12
<b>Number of Questions to be attempted :</b>	12
<b>Section Marks :</b>	40
<b>Display Number Panel :</b>	Yes
<b>Group All Questions :</b>	No
<b>Enable Mark as Answered Mark for Review and Clear Response :</b>	Yes
<b>Maximum Instruction Time :</b>	0
<b>Sub-Section Number :</b>	1
<b>Sub-Section Id :</b>	64065382987
<b>Question Shuffling Allowed :</b>	No
<b>Is Section Default? :</b>	null

**Question Number : 191 Question Id : 640653579032 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 0**

Question Label : Multiple Choice Question

**THIS IS QUESTION PAPER FOR THE SUBJECT "FOUNDATION LEVEL : SEMESTER 2: STATISTICS FOR DATA SCIENCE II (COMPUTER BASED EXAM) "**

**ARE YOU SURE YOU HAVE TO WRITE EXAM FOR THIS SUBJECT?**

**CROSS CHECK YOUR HALL TICKET TO CONFIRM THE SUBJECTS TO BE WRITTEN.**

**(IF IT IS NOT THE CORRECT SUBJECT, PLS CHECK THE SECTION AT THE TOP FOR THE SUBJECTS REGISTERED BY YOU)**

**Options :**

6406531933194.  YES

6406531933195.  NO

Question Number : 192 Question Id : 640653579033 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 0

Question Label : Multiple Choice Question

Discrete random variables:

Distribution	PMF ( $f_X(k)$ )	CDF ( $F_X(x)$ )	$E[X]$	$\text{Var}(X)$
Uniform( $A$ ) $A = \{a, a + 1, \dots, b\}$	$\frac{1}{n}, \quad x = k$ $n = b - a + 1$ $k = a, a + 1, \dots, b$	$\begin{cases} 0 & x < 0 \\ \frac{k-a+1}{n} & k \leq x < k + 1 \\ & k = a, a + 1, \dots, b - 1, b \\ 1 & x \geq n \end{cases}$	$\frac{a+b}{2}$	$\frac{n^2-1}{12}$
Bernoulli( $p$ )	$\begin{cases} p & x = 1 \\ 1 - p & x = 0 \end{cases}$	$\begin{cases} 0 & x < 0 \\ 1 - p & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$	$p$	$p(1 - p)$
Binomial( $n, p$ )	${}^nC_k p^k (1 - p)^{n-k},$ $k = 0, 1, \dots, n$	$\begin{cases} 0 & x < 0 \\ \sum_{i=0}^k {}^nC_i p^i (1 - p)^{n-i} & k \leq x < k + 1 \\ & k = 0, 1, \dots, n \\ 1 & x \geq n \end{cases}$	$np$	$np(1 - p)$
Geometric( $p$ )	$(1 - p)^{k-1} p,$ $k = 1, \dots, \infty$	$\begin{cases} 0 & x < 0 \\ 1 - (1 - p)^k & k \leq x < k + 1 \\ & k = 1, \dots, \infty \end{cases}$	$\frac{1}{p}$	$\frac{1 - p}{p^2}$
Poisson( $\lambda$ )	$\frac{e^{-\lambda} \lambda^k}{k!},$ $k = 0, 1, \dots, \infty$	$\begin{cases} 0 & x < 0 \\ e^{-\lambda} \sum_{i=0}^k \frac{\lambda^i}{i!} & k \leq x < k + 1 \\ & k = 0, 1, \dots, \infty \end{cases}$	$\lambda$	$\lambda$

Continuous random variables:

Distribution	PDF ( $f_X(k)$ )	CDF ( $F_X(x)$ )	$E[X]$	$\text{Var}(X)$
Uniform $[a, b]$	$\frac{1}{b-a}, a \leq x \leq b$	$\begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & x \geq b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exp( $\lambda$ )	$\lambda e^{-\lambda x}, x > 0$	$\begin{cases} 0 & x \leq 0 \\ 1 - e^{-\lambda x} & x > 0 \end{cases}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Normal( $\mu, \sigma^2$ )	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right),$ $-\infty < x < \infty$	No closed form	$\mu$	$\sigma^2$
Gamma( $\alpha, \beta$ )	$\frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, x > 0$		$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$
Beta( $\alpha, \beta$ )	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$ $0 < x < 1$		$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

1. **Markov's inequality:** Let  $X$  be a discrete random variable taking non-negative values with a finite mean  $\mu$ . Then,

$$P(X \geq c) \leq \frac{\mu}{c}$$

2. **Chebyshev's inequality:** Let  $X$  be a discrete random variable with a finite mean  $\mu$  and a finite variance  $\sigma^2$ . Then,

$$P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

**Options :**

6406531933196.  Useful Data has been mentioned above.

6406531933197.  This data attachment is just for a reference & not for an evaluation.

**Sub-Section Number :**

2

**Sub-Section Id :**

64065382988

**Question Shuffling Allowed :**

Yes

**Is Section Default? :**

null

**Question Number : 193 Question Id : 640653579034 Question Type : MCQ Is Question**

**Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction**

Time : 0

Correct Marks : 3

Question Label : Multiple Choice Question

The joint PMF of two discrete random variables  $X$  and  $Y$  is given in the following table:

$Y \backslash X$	0	1	2	$f_Y(y)$
0	$\frac{1}{6}$	$a$	$b$	$\frac{1}{3}$
1	$c$	$d$	$\frac{1}{9}$	$\frac{2}{3}$
$f_X(x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	1

Joint PMF of  $X$  and  $Y$

Which of the following options is correct?

Options :

6406531933198. ✖  $2a = b = 6c = 4d$

6406531933199. ✖  $2a = b = 4c = 6d$

6406531933200. ✔  $6a = 12b = 2c = 3d$

6406531933201. ✖  $9a = 18b = 3c = 2d$

Question Number : 194 Question Id : 640653579039 Question Type : MCQ Is Question

Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction

Time : 0

Correct Marks : 3

Question Label : Multiple Choice Question

Let  $X \sim \text{Binomial}(n, p)$ . If the expected value and variance of  $X$  are 2 and  $\frac{3}{2}$ , respectively, find the value of  $P(X = 2)$ .

Options :

6406531933211. ✓  $\frac{7 \times 3^6}{4^7}$

6406531933212. ✖  $\frac{7 \times 3^6}{4^8}$

6406531933213. ✖  ${}^8C_2 \left( \frac{3^2}{4^8} \right)$

6406531933214. ✖  ${}^4C_2 \left( \frac{3^2}{4^4} \right)$

Sub-Section Number : 3  
Sub-Section Id : 64065382989  
Question Shuffling Allowed : No  
Is Section Default? : null

Question Id : 640653579035 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Question Numbers : (195 to 196)

Question Label : Comprehension

Suppose two fair dice are rolled. Let a random variable  $X$  denote the number obtained on the first die and let a random variable  $Y$  denote the number obtained on the second die. Define a new random variable  $U = X + Y - 1$ .

Based on the above data, answer the given subquestions.

Sub questions



**Question Number : 195 Question Id : 640653579036 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 1**

Question Label : Multiple Choice Question

Find the range of  $U$ .

**Options :**

6406531933202. ✖  $T_U = \{0, 1, 2, \dots, 12\}$

6406531933203. ✖  $T_U = \{1, 2, \dots, 12\}$

6406531933204. ✖  $T_U = \{0, 1, 2, \dots, 11\}$

6406531933205. ✔  $T_U = \{1, 2, \dots, 11\}$

**Question Number : 196 Question Id : 640653579037 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 2**

Question Label : Multiple Choice Question

Find the value of  $P(X = 4, U = 8)$ .

**Options :**

6406531933206. ✖  $\frac{1}{6}$

6406531933207. ✖  $\frac{2}{3}$

6406531933208. ✔



$$\frac{1}{36}$$

$$6406531933209. \times \frac{1}{3}$$

**Sub-Section Number :** 4  
**Sub-Section Id :** 64065382990  
**Question Shuffling Allowed :** Yes  
**Is Section Default? :** null

**Question Number : 197 Question Id : 640653579038 Question Type : SA Calculator : None**  
**Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**  
**Correct Marks : 3**

Question Label : Short Answer Question

The probability mass function of a random variable  $X$  is given as

$x$	$-3$	$6$	$9$
$P(X = x)$	$1/6$	$1/2$	$1/3$

Define  $Y = (2X + 1)^2$ . Find the expected value of  $Y$ .

**Response Type :** Numeric  
**Evaluation Required For SA :** Yes  
**Show Word Count :** Yes  
**Answers Type :** Equal  
**Text Areas :** PlainText  
**Possible Answers :**

209

**Question Number : 198 Question Id : 640653579040 Question Type : SA Calculator : None**  
**Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**  
**Correct Marks : 3**

Question Label : Short Answer Question

Let  $X$  be a continuous random variable with the following PDF:

$$f_X(x) = \begin{cases} \frac{k}{(1+x)^2}, & 0 \leq x \leq 4, \\ 0, & \text{otherwise.} \end{cases}$$

Find the value of  $k$ . Enter the answer correct to two decimal places.

Hint:  $\int \frac{1}{(a+bx)^2} dx = \frac{-1}{b(a+bx)}$

**Response Type :** Numeric

**Evaluation Required For SA :** Yes

**Show Word Count :** Yes

**Answers Type :** Range

**Text Areas :** PlainText

**Possible Answers :**

1.23 to 1.27

<b>Sub-Section Number :</b>	5
<b>Sub-Section Id :</b>	64065382991
<b>Question Shuffling Allowed :</b>	No
<b>Is Section Default? :</b>	null

**Question Id :** 640653579041 **Question Type :** COMPREHENSION **Sub Question Shuffling Allowed :** No **Group Comprehension Questions :** No **Question Pattern Type :** NonMatrix **Calculator :** None **Response Time :** N.A **Think Time :** N.A **Minimum Instruction Time :** 0

**Question Numbers :** (199 to 200)

**Question Label :** Comprehension

The joint PMF of two discrete random variables  $X$  and  $Y$  is

$$f_{XY}(x, y) = \begin{cases} \frac{1}{32}(x^2 + y), & x \in \{0, 1, 2, 3\}, y \in \{0, 1\}, \\ 0, & \text{otherwise.} \end{cases}$$

Based on the above data, answer the given subquestions.

**Sub questions**

Question Number : 199 Question Id : 640653579042 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 3

Question Label : Multiple Choice Question

Identify the correct joint PMF table of  $X$  and  $Y$ :

Options :

6406531933216. ✓

$Y \backslash X$	0	1	2	3
0	0	$\frac{1}{32}$	$\frac{4}{32}$	$\frac{9}{32}$
1	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{5}{32}$	$\frac{10}{32}$

6406531933217. ✖

$Y \backslash X$	0	1	2	3
0	0	$\frac{1}{32}$	$\frac{4}{32}$	$\frac{10}{32}$
1	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{4}{32}$	$\frac{10}{32}$

6406531933218. ✖

$Y \backslash X$	0	1	2	3
0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{5}{32}$	$\frac{10}{32}$
1	0	$\frac{1}{32}$	$\frac{4}{32}$	$\frac{9}{32}$

6406531933219. ✖

$X \backslash Y$	0	1	2	3
0	0	$\frac{1}{32}$	$\frac{4}{32}$	$\frac{9}{32}$
1	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{5}{32}$	$\frac{10}{32}$

**Question Number : 200 Question Id : 640653579043 Question Type : SA Calculator : None**

**Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 2**

Question Label : Short Answer Question

Find  $P\left(\frac{1}{2} < X < \frac{5}{2} \mid X > 1\right)$ . Enter

the answer correct to 2 decimal places.

**Response Type :** Numeric

**Evaluation Required For SA :** Yes

**Show Word Count :** Yes

**Answers Type :** Range

**Text Areas :** PlainText

**Possible Answers :**

0.30 to 0.34

**Question Id : 640653579044 Question Type : COMPREHENSION Sub Question Shuffling**

**Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix**

**Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Question Numbers : (201 to 202)**

Question Label : Comprehension

Ten students from classes 9 and 10 have been nominated to form the school committee.

The table below provides the number of boys and girls selected from each class:

	class 9	class 10
Boys	1	5
Girls	3	1

The committee will consist of four students, with two students selected from each class uniformly at random.

Based on the above data, answer the given subquestions.

### Sub questions

**Question Number : 201 Question Id : 640653579045 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 2**

Question Label : Multiple Choice Question

Let a random variable  $G$  represent the number of girls selected for the committee. Find the range of  $G$ .

**Options :**

6406531933221. ✖  $T_G = \{0, 1, 2, 3\}$

6406531933222. ✔  $T_G = \{1, 2, 3\}$

6406531933223. ✖  $T_G = \{1, 2, 3, 4\}$

6406531933224. ✖  $T_G = \{0, 1, 2, 3, 4\}$

**Question Number : 202 Question Id : 640653579046 Question Type : SA Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 3**

Question Label : Short Answer Question

Find the expected number of girls selected for the committee. Enter the answer correct to two decimal places.

**Response Type :** Numeric

**Evaluation Required For SA :** Yes

**Show Word Count :** Yes

**Answers Type :** Range

**Text Areas :** PlainText

**Possible Answers :**

1.81 to 1.85

**Question Id :** 640653579047 **Question Type :** COMPREHENSION **Sub Question Shuffling Allowed :** No **Group Comprehension Questions :** No **Question Pattern Type :** NonMatrix **Calculator :** None **Response Time :** N.A **Think Time :** N.A **Minimum Instruction Time :** 0  
**Question Numbers :** (203 to 204)

**Question Label :** Comprehension

In a bookstore, there are two book types: Type 1 and Type 2. Let  $X$  and  $Y$  be independent random variables representing the number of Type 1 and Type 2 books sold in a week, respectively. Suppose  $X$  and  $Y$  follow the Poisson distribution with averages of 2 and 3, respectively. Define a new random variable  $Z = X + Y$ .

Based on the above data, answer the given subquestions.

**Sub questions**

**Question Number :** 203 **Question Id :** 640653579048 **Question Type :** MSQ **Is Question Mandatory :** No **Calculator :** None **Response Time :** N.A **Think Time :** N.A **Minimum Instruction Time :** 0

**Correct Marks :** 3 **Max. Selectable Options :** 0

**Question Label :** Multiple Select Question

If  $Z = 5$ , then which of the following options are true?

**Options :**

6406531933226. ✖  $(Y|Z) \sim \text{Binomial}(5, \frac{2}{5})$ .

6406531933227. ✔  $(X|Z) \sim \text{Binomial}(5, \frac{2}{5})$ .

6406531933228. ✖  $(X|Z) \sim \text{Binomial}(5, \frac{3}{5})$ .

6406531933229. ✔  $(Y|Z) \sim \text{Binomial}(5, \frac{3}{5})$ .

**Question Number : 204 Question Id : 640653579049 Question Type : SA Calculator : None**

**Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 2**

Question Label : Short Answer Question

Find the value of  $P(X = 1|Z = 5)$ .

Enter the answer correct to two decimal places.

**Response Type : Numeric**

**Evaluation Required For SA : Yes**

**Show Word Count : Yes**

**Answers Type : Range**

**Text Areas : PlainText**

**Possible Answers :**

0.24 to 0.28

**Question Id : 640653579050 Question Type : COMPREHENSION Sub Question Shuffling**

**Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix**

**Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Question Numbers : (205 to 206)**

Question Label : Comprehension



Suppose  $X \sim \text{Binomial} \left( n, \frac{1}{2} \right)$ .

Based on the above data, answer the given subquestions.

### Sub questions

**Question Number : 205 Question Id : 640653579051 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 3**

Question Label : Multiple Choice Question

Find the value of  $n$  for which

$$\frac{1}{30}P(X = 3) = P(X = 2).$$

**Options :**

6406531933231. ✖ 90

6406531933232. ✔ 92

6406531933233. ✖ 30

6406531933234. ✖ 32

**Question Number : 206 Question Id : 640653579052 Question Type : SA Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 2**

Question Label : Short Answer Question

Using the Chebyshev's inequality, find a lower bound for  $P(-2\sigma \leq X - \mu \leq 2\sigma)$ , where  $\mu$  and  $\sigma^2$  are mean and variance of  $X$ . Enter the answer correct to 2 decimal places

**Response Type : Numeric**

**Evaluation Required For SA :** Yes

**Show Word Count :** Yes

**Answers Type :** Equal

**Text Areas :** PlainText

**Possible Answers :**

0.75

**Question Id :** 640653579053 **Question Type :** COMPREHENSION **Sub Question Shuffling Allowed :** No **Group Comprehension Questions :** No **Question Pattern Type :** NonMatrix **Calculator :** None **Response Time :** N.A **Think Time :** N.A **Minimum Instruction Time :** 0  
**Question Numbers :** (207 to 208)

**Question Label :** Comprehension

Sruthi throws a dart onto a circular board. Let a random variable  $X$  denote the distance from the center to the point where the dart hits the board. Suppose the PDF of  $X$  is

$$f_X(x) = \begin{cases} kx(1 - x^2), & 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Based on the above data, answer the given subquestions.

**Sub questions**

**Question Number :** 207 **Question Id :** 640653579054 **Question Type :** SA **Calculator :** None  
**Response Time :** N.A **Think Time :** N.A **Minimum Instruction Time :** 0

**Correct Marks :** 2

**Question Label :** Short Answer Question

Find the value of  $k$ .

**Response Type :** Numeric

**Evaluation Required For SA :** Yes

**Show Word Count :** Yes

**Answers Type :** Equal

**Text Areas :** PlainText

**Possible Answers :**

4

**Question Number :** 208 **Question Id :** 640653579055 **Question Type :** SA **Calculator :** None

**Response Time :** N.A **Think Time :** N.A **Minimum Instruction Time :** 0

**Correct Marks :** 3

**Question Label :** Short Answer Question

Find the value of  $P\left(\frac{1}{4} \leq X \leq \frac{3}{4}\right)$ . Enter

the answer correct to two decimal places.

**Response Type :** Numeric

**Evaluation Required For SA :** Yes

**Show Word Count :** Yes

**Answers Type :** Range

**Text Areas :** PlainText

**Possible Answers :**

0.66 to 0.72