Indian Institute of Technology, Madras - BS in Data Science and Applications

Notations :

1.Options shown in green color and with ✔ icon are correct.

2.Options shown in red color and with ✖ icon are incorrect.

Question Paper Name :	IIT M FOUNDATION AN4 EXAM QPF4 16 JULY 2023
Subject Name :	2023 July: IIT M FOUNDATION AN4 EXAM QPF4
Creation Date :	2023-07-10 18:54:05
Duration :	240
Total Marks :	705
Display Marks:	Yes
Share Answer Key With Delivery Engine :	Yes
Actual Answer Key :	Yes
Calculator :	Scientific
Magnifying Glass Required? :	No
Ruler Required? :	No
Eraser Required? :	No
Scratch Pad Required? :	No
Rough Sketch/Notepad Required? :	No
Protractor Required? :	No
Show Watermark on Console? :	Yes
Highlighter :	No
Auto Save on Console?	Yes
Change Font Color :	No

Sem2 Statistics2

Section Id :	64065339058
Section Number :	8
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	12
Number of Questions to be attempted :	12
Section Marks :	40
Display Number Panel :	Yes
Group All Questions :	No
Enable Mark as Answered Mark for Review and Clear Response :	Yes
Maximum Instruction Time :	0
Sub-Section Number :	1
Sub-Section Id :	64065382503
Question Shuffling Allowed :	No
Is Section Default? :	null

Question Number : 163 Question Id : 640653577601 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 0

Question Label : Multiple Choice Question

THIS IS QUESTION PAPER FOR THE SUBJECT "FOUNDATION LEVEL : SEMESTER 2: STATISTICS FOR DATA SCIENCE II (COMPUTER BASED EXAM) "

ARE YOU SURE YOU HAVE TO WRITE EXAM FOR THIS SUBJECT?

CROSS CHECK YOUR HALL TICKET TO CONFIRM THE SUBJECTS TO BE WRITTEN.

(IF IT IS NOT THE CORRECT SUBJECT, PLS CHECK THE SECTION AT THE TOP FOR THE SUBJECTS REGISTERED BY YOU)

Options :

6406531928898. ✔ YES

6406531928899. ✖ NO

Question Number : 164 Question Id : 640653577602 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 0

Question Label : Multiple Choice Question

Discrete random variables:

Distribution	PMF ($f_X(k)$)	CDF ($F_X(x)$)	$E[X]$	$\mathrm{Var}(X)$
Uniform(A) $A=\{a,a+1,\ldots,b\}$	$\frac{1}{n}, \quad x=k$ $n=b-a+1$ $k=a,a+1,\ldots,b$	$\begin{cases} 0 & x<0 \\ \frac{k-a+1}{n} & k\le x<k+1 \\ & k=a,a+1,\ldots,b-1,b \\ 1 & x\ge n \end{cases}$	$\frac{a+b}{2}$	$\frac{n^2-1}{12}$
Bernoulli(p)	$\begin{cases} p & x=1 \\ 1-p & x=0 \end{cases}$	$\begin{cases} 0 & x<0 \\ 1-p & 0\le x<1 \\ 1 & x\ge 1 \end{cases}$	p	$p(1-p)$
Binomial(n,p)	${}^nC_k p^k(1-p)^{n-k}$, $k=0,1,\ldots,n$	$\begin{cases} 0 & x<0 \\ \sum_{i=0}^{k} {}^nC_i p^i(1-p)^{n-i} & k\le x<k+1 \\ & k=0,1,\ldots,n \\ 1 & x\ge n \end{cases}$	np	$np(1-p)$
Geometric(p)	$(1-p)^{k-1}p$, $k=1,\ldots,\infty$	$\begin{cases} 0 & x<0 \\ 1-(1-p)^k & k\le x<k+1 \\ & k=1,\ldots,\infty \end{cases}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson(λ)	$\frac{e^{-\lambda}\lambda^k}{k!}$, $k=0,1,\ldots,\infty$	$\begin{cases} 0 & x<0 \\ e^{-\lambda}\sum_{i=0}^{k}\frac{\lambda^i}{i!} & k\le x<k+1 \\ & k=0,1,\ldots,\infty \end{cases}$	λ	λ

Continuous random variables:

Distribution	PDF ($f_X(k)$)	CDF ($F_X(x)$)	$E[X]$	$\mathrm{Var}(X)$
Uniform$[a,b]$	$\frac{1}{b-a}$, $a \le x \le b$	$\begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & x \ge b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exp(λ)	$\lambda e^{-\lambda x}$, $x > 0$	$\begin{cases} 0 & x \le 0 \\ 1-e^{-\lambda x} & x > 0 \end{cases}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Normal(μ, σ^2)	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$, $-\infty < x < \infty$	No closed form	μ	σ^2
Gamma(α, β)	$\frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$, $x > 0$		$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$
Beta(α, β)	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$, $0 < x < 1$		$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

1. **Markov's inequality:** Let X be a discrete random variable taking non-negative values with a finite mean μ. Then,
$$P(X \ge c) \le \frac{\mu}{c}$$

2. **Chebyshev's inequality:** Let X be a discrete random variable with a finite mean μ and a finite variance σ^2. Then,
$$P(|X - \mu| \ge k\sigma) \le \frac{1}{k^2}$$

Options :

6406531928900. ✔ Useful Data has been mentioned above.

6406531928901. ✖ This data attachment is just for a reference & not for an evaluation.

Sub-Section Number : 2

Sub-Section Id : 64065382504

Question Shuffling Allowed : Yes

Is Section Default? : null

Question Number : 165 Question Id : 640653577603 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction

Time : 0

Correct Marks : 3

Question Label : Multiple Choice Question

The joint PMF of two discrete random variables X and Y is given in the following table:

Y \ X	0	1	2	$f_Y(y)$
0	$\frac{1}{6}$	a	b	$\frac{1}{3}$
1	c	d	$\frac{1}{9}$	$\frac{2}{3}$
$f_X(x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	1

Joint PMF of X and Y

Which of the following options is correct?

Options :

6406531928902. ✖ $2a = b = 6c = 4d$

6406531928903. ✖ $2a = b = 4c = 6d$

6406531928904. ✔ $6a = 12b = 2c = 3d$

6406531928905. ✖ $9a = 18b = 3c = 2d$

Question Number : 166 Question Id : 640653577608 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 3

Question Label : Multiple Choice Question

Let $X \sim \text{Binomial}(n, p)$. If the expected value and variance of X are 2 and $\frac{3}{2}$, respectively, find the value of $P(X = 2)$.

Options :

6406531928915. ✔ $\frac{7 \times 3^6}{4^7}$

6406531928916. ✖ $\frac{7 \times 3^6}{4^8}$

6406531928917. ✖ ${}^8C_2\left(\frac{3^2}{4^8}\right)$

6406531928918. ✖ ${}^4C_2\left(\frac{3^2}{4^4}\right)$

Sub-Section Number :	3
Sub-Section Id :	64065382505
Question Shuffling Allowed :	No
Is Section Default? :	null

Question Id : 640653577604 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Question Numbers : (167 to 168)

Question Label : Comprehension

Suppose two fair dice are rolled. Let a random variable X denote the number obtained on the first die and let a random variable Y denote the number obtained on the second die. Define a new random variable $U = X + Y - 1$.

Based on the above data, answer the given subquestions.

Sub questions

Question Number : 167 Question Id : 640653577605 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1

Question Label : Multiple Choice Question

Find the range of U.

Options :

6406531928906. ✖ $T_U = \{0, 1, 2, \ldots, 12\}$

6406531928907. ✖ $T_U = \{1, 2, \ldots, 12\}$

6406531928908. ✖ $T_U = \{0, 1, 2, \ldots, 11\}$

6406531928909. ✔ $T_U = \{1, 2, \ldots, 11\}$

Question Number : 168 Question Id : 640653577606 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 2

Question Label : Multiple Choice Question

Find the value of $P(X = 4,\ U = 8)$.

Options :

6406531928910. ✖ $\frac{1}{6}$

6406531928911. ✖ $\frac{2}{3}$

6406531928912. ✔

$\frac{1}{36}$

6406531928913. ✖ $\frac{1}{3}$

Sub-Section Number : 4

Sub-Section Id : 64065382506

Question Shuffling Allowed : Yes

Is Section Default? : null

Question Number : 169 Question Id : 640653577607 Question Type : SA Calculator : None

Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 3

Question Label : Short Answer Question

The probability mass function of a random variable X is given as

x	-3	6	9
$P(X = x)$	$1/6$	$1/2$	$1/3$

Define $Y = (2X + 1)^2$. Find the expected value of Y.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

209

Question Number : 170 Question Id : 640653577609 Question Type : SA Calculator : None

Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 3

Question Label : Short Answer Question

Let X be a continuous random variable with the following PDF:

$$f_X(x) = \begin{cases} \frac{k}{(1+x)^2}, & 0 \le x \le 4, \\ 0, & \text{otherwise.} \end{cases}$$

Find the value of k. Enter the answer correct to two decimal places.

Hint: $\int \frac{1}{(a+bx)^2} dx = \frac{-1}{b(a+bx)}$

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

1.23 to 1.27

Sub-Section Number :	5
Sub-Section Id :	64065382507
Question Shuffling Allowed :	No
Is Section Default? :	null

Question Id : 640653577610 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Question Numbers : (171 to 172)

Question Label : Comprehension

The joint PMF of two discrete random variables X and Y is

$$f_{XY}(x,y) = \begin{cases} \frac{1}{32}(x^2+y), & x \in \{0,1,2,3\}, y \in \{0,1\}, \\ 0, & \text{otherwise.} \end{cases}$$

Based on the above data, answer the given subquestions.

Sub questions

Question Number : 171 Question Id : 640653577611 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 3

Question Label : Multiple Choice Question

Identify the correct joint PMF table of X and Y:

Options :

6406531928920. ✔

Y \ X	0	1	2	3
0	0	$\frac{1}{32}$	$\frac{4}{32}$	$\frac{9}{32}$
1	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{5}{32}$	$\frac{10}{32}$

6406531928921. ✖

Y \ X	0	1	2	3
0	0	$\frac{1}{32}$	$\frac{4}{32}$	$\frac{10}{32}$
1	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{4}{32}$	$\frac{10}{32}$

6406531928922. ✖

Y \ X	0	1	2	3
0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{5}{32}$	$\frac{10}{32}$
1	0	$\frac{1}{32}$	$\frac{4}{32}$	$\frac{9}{32}$

6406531928923. ✖

X \ Y	0	1	2	3
0	0	$\frac{1}{32}$	$\frac{4}{32}$	$\frac{9}{32}$
1	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{5}{32}$	$\frac{10}{32}$

Question Number : 172 Question Id : 640653577612 Question Type : SA Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 2

Question Label : Short Answer Question

Find $P\left(\frac{1}{2} < X < \frac{5}{2} \mid X > 1\right)$. Enter the answer correct to 2 decimal places.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

0.30 to 0.34

Question Id : 640653577613 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Question Numbers : (173 to 174)

Question Label : Comprehension

Ten students from classes 9 and 10 have been nominated to form the school committee.

The table below provides the number of boys and girls selected from each class:

	class 9	class 10
Boys	1	5
Girls	3	1

The committee will consist of four students, with two students selected from each class uniformly at random.

Based on the above data, answer the given subquestions.

Sub questions

Question Number : 173 Question Id : 640653577614 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 2

Question Label : Multiple Choice Question

Let a random variable *G* represent the number of girls selected for the committee. Find the range of *G*.

Options :

6406531928925. ✖ $T_G = \{0, 1, 2, 3\}$

6406531928926. ✔ $T_G = \{1, 2, 3\}$

6406531928927. ✖ $T_G = \{1, 2, 3, 4\}$

6406531928928. ✖ $T_G = \{0, 1, 2, 3, 4\}$

Question Number : 174 Question Id : 640653577615 Question Type : SA Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 3

Question Label : Short Answer Question

Find the expected number of girls selected for the committee. Enter the answer correct to two decimal places.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

1.81 to 1.85

Question Id : 640653577616 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Question Numbers : (175 to 176)

Question Label : Comprehension

In a bookstore, there are two book types: Type 1 and Type 2. Let X and Y be independent random variables representing the number of Type 1 and Type 2 books sold in a week, respectively. Suppose X and Y follow the Poisson distribution with averages of 2 and 3, respectively. Define a new random variable $Z = X + Y$.

Based on the above data, answer the given subquestions.

Sub questions

Question Number : 175 Question Id : 640653577617 Question Type : MSQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 3 Max. Selectable Options : 0

Question Label : Multiple Select Question

If $Z = 5$, then which of the following options are true?

Options :

6406531928930. ✖ $(Y|Z) \sim \text{Binomial}(5, \frac{2}{5})$.

6406531928931. ✔ $(X|Z) \sim \text{Binomial}(5, \frac{2}{5})$.

6406531928932. ✖ $(X|Z) \sim \text{Binomial}(5, \frac{3}{5})$.

6406531928933. ✔ $(Y|Z) \sim \text{Binomial}(5, \frac{3}{5})$.

Question Number : 176 Question Id : 640653577618 Question Type : SA Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 2

Question Label : Short Answer Question

Find the value of $P(X = 1|Z = 5)$.

Enter the answer correct to two decimal places.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

0.24 to 0.28

Question Id : 640653577619 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Question Numbers : (177 to 178)

Question Label : Comprehension

Suppose $X \sim \text{Binomial}\left(n, \frac{1}{2}\right)$.

Based on the above data, answer the given subquestions.

Sub questions

Question Number : 177 Question Id : 640653577620 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 3

Question Label : Multiple Choice Question

Find the value of n for which

$\frac{1}{30}P(X = 3) = P(X = 2)$.

Options :

6406531928935. ✖ 90

6406531928936. ✔ 92

6406531928937. ✖ 30

6406531928938. ✖ 32

Question Number : 178 Question Id : 640653577621 Question Type : SA Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 2

Question Label : Short Answer Question

Using the Chebyshev's inequality, find a lower bound for $P(-2\sigma \leq X - \mu \leq 2\sigma)$, where μ and σ^2 are mean and variance of X. Enter the answer correct to 2 decimal places

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

0.75

Question Id : 640653577622 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Question Numbers : (179 to 180)

Question Label : Comprehension

Sruthi throws a dart onto a circular board. Let a random variable X denote the distance from the center to the point where the dart hits the board. Suppose the PDF of X is

$$f_X(x) = \begin{cases} kx(1-x^2), & 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

Based on the above data, answer the given subquestions.

Sub questions

Question Number : 179 Question Id : 640653577623 Question Type : SA Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 2

Question Label : Short Answer Question

Find the value of *k*.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

4

Question Number : 180 Question Id : 640653577624 Question Type : SA Calculator : None

Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 3

Question Label : Short Answer Question

Find the value of $P\left(\frac{1}{4} \leq X \leq \frac{3}{4}\right)$. Enter the answer correct to two decimal places.

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

0.66 to 0.72

DBMS

Section Id :	64065339059
Section Number :	9
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	18
Number of Questions to be attempted :	18
Section Marks :	50
Display Number Panel :	Yes
Group All Questions :	No