Problem Set - 4

MA 1201 Spring Sem, 2025

1. *Describe the intersection of the three planes $u + v + w + z = 6$ and $u + w + z = 4$ and $u + w = 2$ (all in four-dimensional space). Is it a line or a point or an empty set? What is the intersection if the fourth plane $u = -1$ is included? Find a fourth equation that leaves us with no solution.

2. Sketch these three lines and decide if the equations are solvable:

$$\begin{aligned} x + 2y &= 2 \\ x - y &= 2 \\ y &= 1. \end{aligned}$$

   What happens if all right-hand sides are zero? Is there any nonzero choice of right-hand sides that allows the three lines to intersect at the same point?

3. Explain why the system

$$\begin{aligned} u + v + w &= 2 \\ u + 2v + 3w &= 1 \\ v + 2w &= 0 \end{aligned}$$

   has no solution by finding a combination of the three equations that adds up to $0 = 1$. What value should replace the last zero on the right side to allow the equations to have solutions, and what is one of the solutions?

4. Under what condition on y_1, y_2, $y_3 \in \mathbb{R}$ do the points $(0, y_1), (1, y_2), (2, y_3)$ lie on a straight line?

5. These equations are certain to have the solution $x = y = 0$. For which values of d is there a whole line of solutions?

$$\begin{aligned} dx + 2y &= 0 \\ 2x + dy &= 0. \end{aligned}$$

6. What multiple of equation 1 should be subtracted from equation 2?

$$\begin{aligned} 2x - 4y &= 6 \\ -x + 5y &= 0. \end{aligned}$$

   After this elimination step, solve the triangular system. If the right-hand side changes to $(-6, 0)$, what is the new solution?

7. *For which numbers d does elimination break down (i) permanently, and (ii) temporarily?

$$\begin{aligned} dx + 3y &= -3 \\ 4x + 6y &= 6. \end{aligned}$$

   Solve for x and y after fixing the second breakdown by a row exchange.

8. Apply elimination (circle the pivots) and back-substitution to solve

$$\begin{aligned} 2x - 3y &= 3 \\ 4x - 5y + z &= 7 \\ 2x - y - 3z &= 5. \end{aligned}$$

List the three row operations: Subtract __ times row __ from row __ .

9. *Apply elimination to the system

$$\begin{aligned} u + v + w &= -2 \\ 3u + 3v - w &= 6 \\ u - v + w &= -1. \end{aligned}$$

When a zero arises in the pivot position, exchange that equation for the one below it and proceed. What coefficient of v in the third equation, in place of the present -1, would make it impossible to proceed, and force elimination to break down?

10. *Suppose A commutes with every 2 by 2 matrix ($AB = BA$), and in particular

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ commutes with } B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } B_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Show that $a = d$ and $b = c = 0$.

If $AB = BA$ for all matrices B, then A is a multiple of the identity.

11. *Which of the following matrices are guaranteed to equal $(A+B)^2$?

$$A^2 + 2AB + B^2, \quad A(A+B) + B(A+B), \quad (A+B)(B+A), \quad A^2 + AB + BA + B^2.$$

12. *By trial and error find examples of 2 by 2 matrices such that

(i) $A^2 = -I$, A having only real entries.

(ii) $B^2 = 0$, although $B \neq 0$.

(iii) $CD = -DC$, but $CD \neq 0$.

(iv) $EF = 0$, although no entries of E or F are zero.

13. Find the powers A^2, A^3 (A^2 times A), and B^2, B^3, C^2, C^3. What are A^k, B^k, and C^k?

$$A = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{ and } C = AB = \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & -1/2 \end{bmatrix}.$$

14. Which three matrices E, F, G put A into triangular form U?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} \quad \text{and} \quad GFEA = U.$$

15. *Which elementary matrices make the following 4×4 matrix upper triangular?

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}.$$