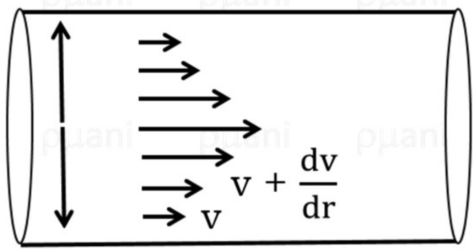
Lecture 20

In this lecture we will learn about “Viscosity”

When a liquid flows through a tube with a uniform flow rate which is not too high, then the speed of liquid at different location within the tube are not the same.

The layer of liquid that is in contact with the solid tube wall will feel maximum retarding force than the layer above it. Then the lower layer will try to retard the upper layer whereas the layer in the above will try to accelerate the layer below it. Thus, there will be a tangential force acting between two adjacent layers. This is the viscous force or viscous drag.



Newton showed that the force of internal friction (f) is directly proportional to area of contact (A) and the velocity gradient $\frac{dv}{dr}$, Thus, $f \propto A$ and $f \propto \frac{dv}{dr}$, $\Rightarrow f \propto (-) A \times \frac{dv}{dr}$

$$\Rightarrow \boxed{f = (-)\, \eta \times A \times \frac{dv}{dr}}$$

(–)ve sign is indicative of the force that acts in opposite direction to that of the liquid flow.

This is the Newton's law of viscosity. η is known as the viscosity coefficient of that liquid. The above equation is valid for laminar flow (flow rate is not too high) and does not hold for turbulent flow (very high flow rate).

$$f = (-)\,\eta \times A \times \frac{dv}{dr}$$

When $A = 1$, $\frac{dv}{dr} = 1$, then $f = (-)\,\eta$ or $\eta = (-)\,f$

Thus coefficient of viscosity is defined as tangential force required per unit area to maintain a unit velocity gradient, i.e. to maintain unit difference of velocity between two layers unit distance apart.

η values for different liquids at 25°C, 1 atm. are

Substances	C_6H_6	H_2O	CH_4	Glycerol
η (CentiPoise, cP)	0.60	0.89	0.011	954

η unit: dyne sec per square cm, poise, CentiPoise etc.

$N\ s\ m^{-2} \equiv kg\ s^{-1}\ m^{-1}$

$dyn\ .\ s\ .\ cm^{-2} \equiv$ poise

$1\ cP = 10^{-3}\ Pa\ .\ s$

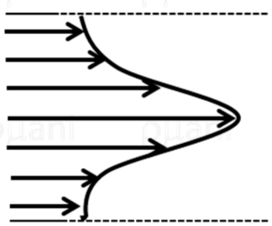
$1\ Pa = 1\ N/m^2$

Dimension of $\eta = \dfrac{f}{A \times \frac{dv}{dr}} = \dfrac{M\,L\,T^{-2}}{L^2 \times \frac{L\,T^{-1}}{L}} = M\ L^{-1}\ T^{-1}$

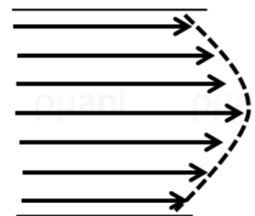
A fluid is said to be Newtonian for which η is independent of $\frac{dv}{dr}$. For those fluid when η depends on $\frac{dv}{dr}$, these are known as non-Newtonian.

A liquid that follows Newton's law of viscosity is a Newtonian fluid.

Gases and non-polymeric liquids are Newtonian whereas polymer solutions or colloidal suspensions are non-Newtonian in nature.

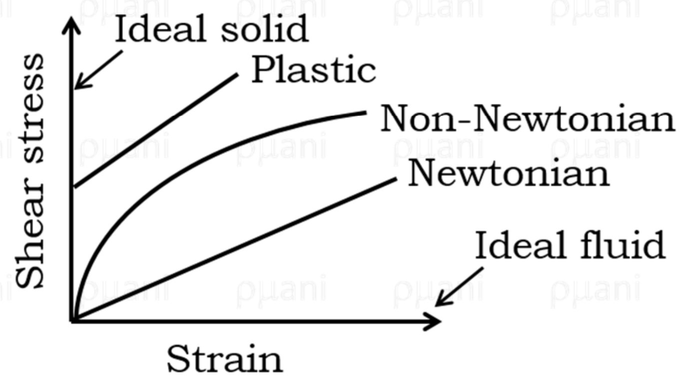


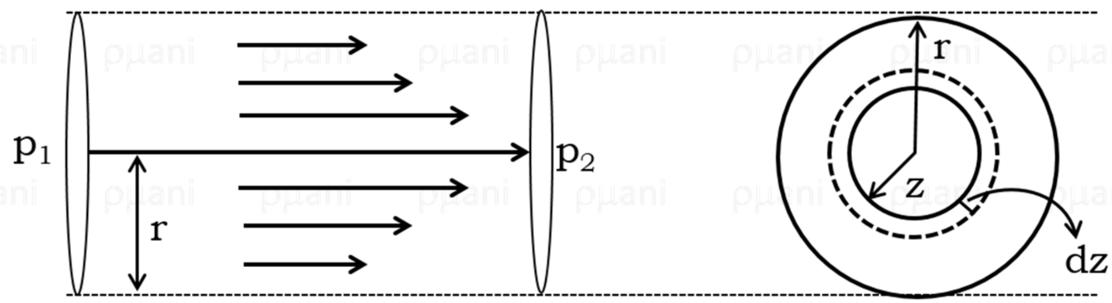
Newtonian



Non-Newtonian

Different possibilities:





Radius of the tube = r, length of the tube = L

Let v be the velocity at a distance 'z' from the centre of the tube then, from Newton's law of viscosity

$$f = (-)\,\eta \times (2\,\pi\,z\,L) \times \frac{dv}{dz} \quad \text{[Area of a cylinder} = (2\,\pi\,z \times L) + \pi\,r^2 + \pi\,r^2\text{, In present case} = (2\,\pi\,z \times L)]$$

For steady rate of flow, this force is equal to the driving force $\pi z^2 (p_1 - p_2)$ where p_1 and p_2 are pressures at two ends.

Thus, $(-)\,\eta \times (2\,\pi\,z\,L) \times \dfrac{dv}{dz} = \pi\,z^2\,(p_1 - p_2)$

$\Rightarrow dv = -\dfrac{z}{2\,\eta\,L}\,(p_1 - p_2)\,dz$

$\Rightarrow v = -\dfrac{(p_1 - p_2)}{4\,\eta\,L}\,z^2 + c$ (constant)

When v = 0, z = r, we have $v = \dfrac{(p_1 - p_2)}{4\,\eta\,L}\,(r^2 - z^2)$

Total volume of liquid flowing through the tube per unit time,

$$\frac{dV}{dt} = \int_0^r 2\,\pi\, z\, v\, dz = \int_0^r 2\,\pi\, z\, \frac{(p_1 - p_2)}{4\,\eta\, L}\,(r^2 - z^2)\, dz = \frac{\pi\,(p_1 - p_2)}{8\,\eta\, L}\, r^4$$

$$\Rightarrow \boxed{\eta = \frac{\pi\,(p_1 - p_2)\, r^4}{8\, L\,(\frac{dV}{dt})}}$$

This is Poiseuille's formula for incompressible fluid (and the flow is laminar)

Assumptions:

(a) Fluid is Newtonian

(b) Liquid in contact with the wall of the tube is at rest

(c) The flow is streamline and parallel to the axis of the tube

(d) Pressure difference is maintained just to balance the viscous drag

(e) Pressure is same at all points along the radius i.e. no radial flow

(f) No acceleration acts along the axis of the tube

For an ideal gas, it can be shown that

$$p_1\, V_1 = \frac{\pi\, r^4}{16\,\eta\, L}\,(p_1^2 - p_2^2)$$

How can we measure viscosity experimentally?

Viscosity of a liquid can be easily measured with reasonable accuracy by falling sphere method. It was shown by Stokes that when a spherical body of radius 'r' and density 'd' moves with a terminal velocity of 'u' through a medium of viscosity coefficient η and density d_l, then the driving force which just balances the frictional resistance,

$$f = 6 \pi \eta r u \quad \text{[Stokes law]}$$

If the spherical body falls under gravity then

$$f = \left(\frac{4}{3}\right) \pi r^3 (d - d_l) g \quad [d = \text{density of the solid sphere, } d_l = \text{density of the liquid}]$$

$$\Rightarrow 6 \pi \eta r u = \left(\frac{4}{3}\right) \pi r^3 (d - d_l) \times g \quad \text{[T remains constant]}$$

$$\Rightarrow \boxed{u = \left(\frac{2}{9}\right) \times \frac{r^2 (d - d_l) g}{\eta}}$$

'u' can be calculated by falling sphere method and thus η could be calculated.

Question: A steel ball of specific gravity 7.9 and diameter 4 mm took 50 seconds to fall a distance of 1 metre through an oil of specific gravity 1.1. Calculate the viscosity of the liquid.

Temperature dependence of viscosity:

Viscosity of a liquid and gas changes in a different manner against change of temperature.

Viscosity of gases increases with increase in temperature and it can be shown that viscosity increases proportionately with increase in square root of temperature.

However, for most liquids viscosity decreases with increase in temperature.

$$\boxed{\log \eta = \frac{a}{T} + b}$$ [where a and b are constants]

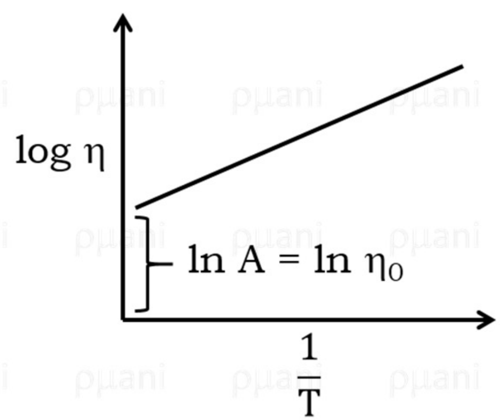
At temperature T, those molecules which will slide past the relatively stationary layer must have to possess kinetic energy greater than (or equal to)

E_{vis} (energy of activation)

$$\eta \propto \frac{1}{e^{-\frac{E_{vis}}{RT}}}$$

$$\eta \propto e^{\frac{E_{vis}}{RT}}$$

$$\Rightarrow \boxed{\ln \eta = \ln A + \frac{E_{vis}}{RT}}$$ [A is a constant]



Arrhenius model

With increase of pressure the free space within liquid decreases and hence viscosity increases. However, at 20°C, for a change of 500 atm. pressure viscosity change is only 60%. For water the situation is quite complicated, upto increase of a few hundred atm. pressure viscosity decreases and then increases.

A few terminologies:

Streamline/stationary flow:

If a liquid flows in such a way that the velocity at any point remains same in magnitude and direction.

Laminar flow:

When the streamline flow occurs through a capillary (of uniform cross-section), then the flow can be considered as a collective flow of thin liquid concentric cylinders having radii varying from zero to radius of the capillary.

When a liquid flows over a surface in streamline way, then it can be considered as the flow of thin layers parallel to the surface.

Such a flow is called a laminar flow. This situation occurs only when the speed is low (high viscous liquid at high velocity may flow in a streamline fashion)

Turbulent flow:

When the velocity of particles/molecules at a given point varies with time both in magnitude and direction in a irregular way, then that flow is said to be a turbulent flow.

Question. How does the viscosity of gas change with temperature ?

Ans: Generally gas viscosity $\eta = \frac{1}{3} \rho <c> \lambda$

Assuming λ to be independent of T, $\eta = \frac{1}{3} \rho \sqrt{\frac{8RT}{\pi M}} \lambda$

$\Rightarrow \eta \propto \sqrt{T}$ [for moderate temperature range]

But actually λ depends on T, Sutherland showed $\eta = \frac{k' \sqrt{T}}{1+\frac{c}{T}}$

In gas the layer concept of liquid for viscosity does not hold very well. Instead viscosity in gas can be understood in terms of rate of change of momentum/unit area, higher temperature means higher momentum transfer which means higher drag which higher viscosity.

At moderate pressure $\eta = \frac{1}{3} \rho <c> \lambda$ holds. But at low pressure and at high pressure this equation is not valid. Thus, $\rho \propto \frac{1}{\lambda}$ and hence, $\rho \propto n$, because $\lambda = \frac{1}{\sqrt{2}\pi\sigma^2 n}$ where, $\lambda \rightarrow$ mean free path.

Note: Pressure dependence of viscosity is quite complicated !