Answers: Q.1)

3) * Show a) heat change and b) work done are not perfectly differentiable quantities.

Home work

a) Heat change, (From 1st law)

$$dq = \left(\frac{\partial u}{\partial T}\right)_V dT + \left[P + \left(\frac{\partial u}{\partial V}\right)_T\right] dV \quad \text{— (1)}$$

At const. Temp (T)

$$\left(\frac{\partial q}{\partial V}\right)_T = P + \left(\frac{\partial u}{\partial V}\right)_T$$

$$\frac{\partial^2 q}{\partial V \cdot \partial T} = \left(\frac{\partial P}{\partial T}\right)_V + \frac{\partial^2 u}{\partial V \cdot \partial T} \quad \text{— (2)}$$

Again at const volume, from (1)

$$\left(\frac{\partial q}{\partial T}\right)_V = \left(\frac{\partial u}{\partial T}\right)_V$$

$$\left(\frac{\partial^2 q}{\partial T \cdot \partial V}\right) = \left(\frac{\partial^2 u}{\partial T \cdot \partial V}\right) \quad \text{— (3)}$$

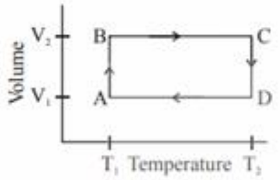
As, according to ~~Serceon~~ Scwarz's theorem / theorem of cross partial derivative

$$\frac{\partial^2 q}{\partial T \cdot \partial V} \neq \frac{\partial^2 q}{\partial V \cdot \partial T}$$

* q is not a perfectly differential.

Q 2.

62. CORRECT OPTION (D)




$W_{TOTAL} = W_{AB} + W_{BC} + W_{CD} + W_{DA}$

Process AB is isothermal

So, $W_{AB} = -nRT_1 \ln\frac{V_2}{V_1}$

Process BC is isochoric

So, $W_{BC}=0$

Process CD is isothermal

So, $W_{CD} = -nRT_2 \ln\left(\frac{V_1}{V_2}\right) = nRT_2 \ln\frac{V_2}{V_1}$

Process DA is isochoric

So, $W_{DA}= 0.$

$$W_{TOTAL} = -nRT_1 \ln\left(\frac{V_2}{V_1}\right) nRT \ln\frac{V_2}{V_1} = nR(T_2 - T_1) \ln\left(\frac{V_2}{V_1}\right)$$

Q.3)

73. Correct optino is (b)

For adiabatic process, $\Delta U = w = \frac{nR}{1-\gamma}(T_f - T_i)$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 500\left(\frac{5}{50}\right)^{0.4} = 199.05K$$

Now, $\Delta U = \frac{nR}{1-\gamma}(T_f - T_i)$

$$-\frac{2\times 8.314}{-0.4}(199.05 - 500) = -12510.3J = -12.5kJ$$

## Q.4

Show mathematically that the magnitude of the work involved in a reversible expansion of an ideal gas from volume $V_1$ and $V_2$ is larger than the corresponding work involved in an irreversible expansion against a constant pressure of $p_2$.

We have the relation

$$|w_{rev}| = nRT \ln \frac{V_2}{V_1} \tag{1.6.9}$$

$$= nRT \ln \left\{1 + \left(\frac{V_2}{V_1} - 1\right)\right\}$$

Expanding the logarithmic term, we have

$$|w_{rev}| = nRT\left\{\left(\frac{V_2}{V_1} - 1\right) + \text{higher terms}\right\} = \frac{nRT}{V_1}(V_2 - V_1) + \text{higher terms}$$

$$= p_1(V_2 - V_1) + \text{higher terms}$$

and $$|w_{irr}| = p_2(V_2 - V_1) \tag{1.6.10}$$

Therefore $$|w_{rev}| - |w_{irr}| = \{p_1(V_2 - V_1) + \text{higher terms}\} - p_2(V_2 - V_1)$$

$$= (V_2 - V_1)(p_1 - p_2) + \text{higher terms}$$

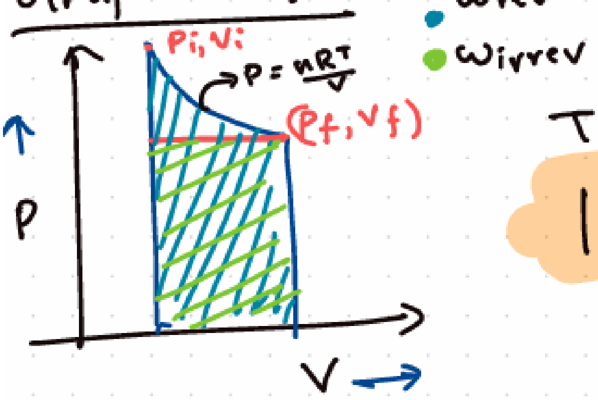
Since, in expansion $V_2 > V_1$ and $p_1 > p_2$, therefore

$$|w_{rev}| - |w_{irr}| = \text{positive}$$

that is, the magnitude of the work involved in a reversible expansion is larger than the corresponding work involved in an irreversible expansion.

**Q.5)**

Graphically




Thus

$$|w_{rev}| > |w_{irrev}|$$

0.101 3 MPa. What is the final temperature of the gas?

For an adiabatic irreversible process

$$dU = -p_{ext}\,dV$$

$$nC_{V,\mathrm{m}}(T_2 - T_1) = -p_{ext}(V_2 - V_1) = -p_{ext}\left(\frac{nRT_2}{p_2} - \frac{nRT_1}{p_1}\right)$$

Substituting $C_{V,\mathrm{m}} = 1.5\,R$ and simplifying, we get

$$1.5(T_2 - T_1) = -p_{ext}\left(\frac{T_2}{p_2} - \frac{T_1}{p_1}\right)$$

Substituting the values of $p_2$, $p_1$ and $T_1$, we get

$$1.5\,(T_2 - 298\text{ K}) = -(0.1013\text{ MPa})\left(\frac{T_2}{0.1013\text{ MPa}} - \frac{298\text{ K}}{1.013\text{ MPa}}\right)$$

$$2.5\,T_2 = 1.5\,(298\text{ K}) + \frac{298\text{ K}}{10}$$

$$T_2 = \frac{447.0\text{ K} + 29.8\text{ K}}{2.5} = 190.7\text{ K}$$