Euler Circuit & Hamiltonian Cycle

A Presentation for *CSE* 300: Technical Writing and Presentation

Soumik Bhattacharjee Reduanul Imon Tusher Bhomik

Bangladesh University of Engineering & Technology

February 13, 2024

Euler Circuit & Hamiltonian Cycle

A Presentation for *CSE* 300: Technical Writing and Presentation

Soumik Bhattacharjee Reduanul Imon Tusher Bhomik

Bangladesh University of Engineering & Technology

February 13, 2024

Tale of Contents

Introduction to Euler Cycle

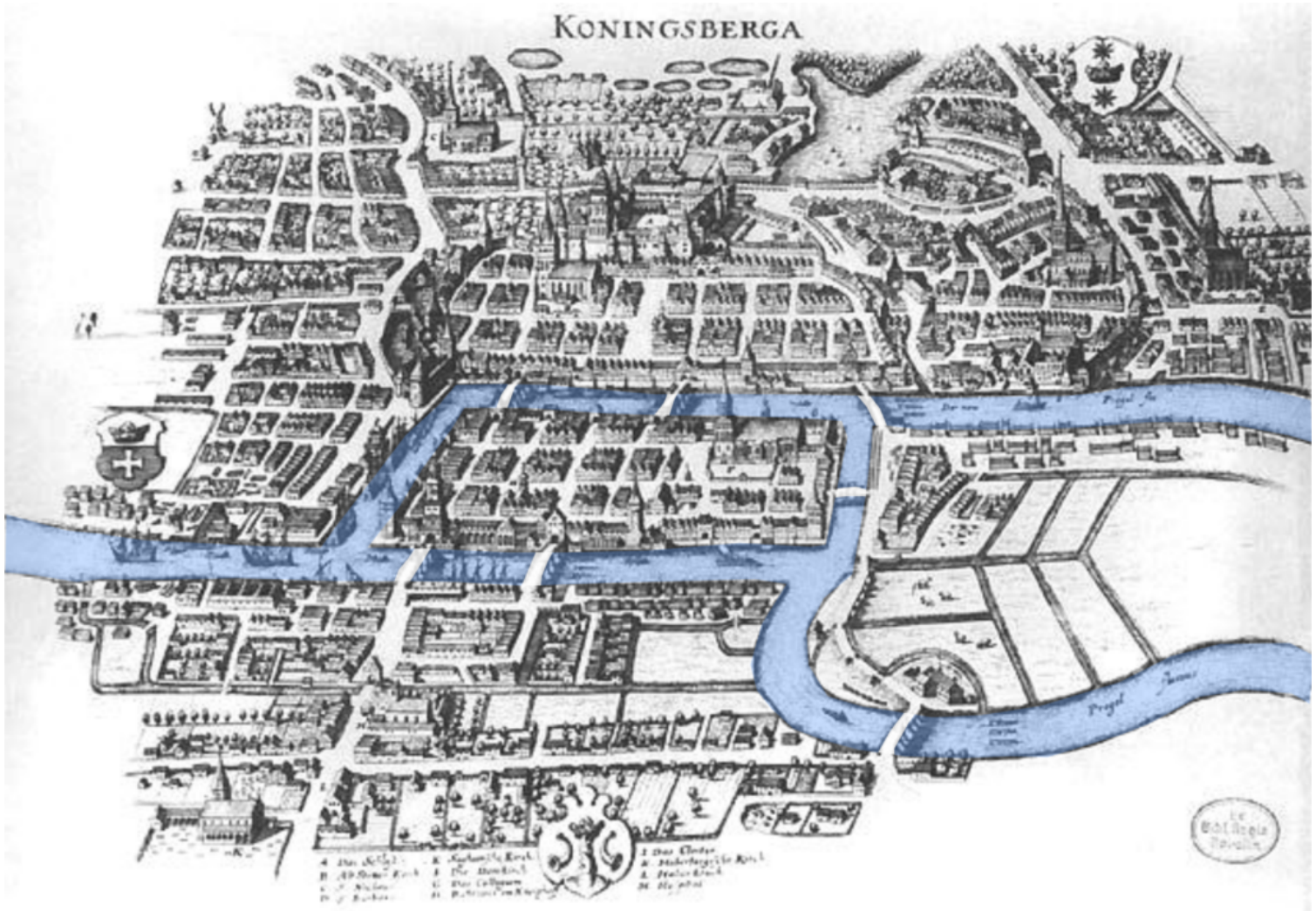
Euler Path

The path in a graph that uses every edge of a graph exactly once.

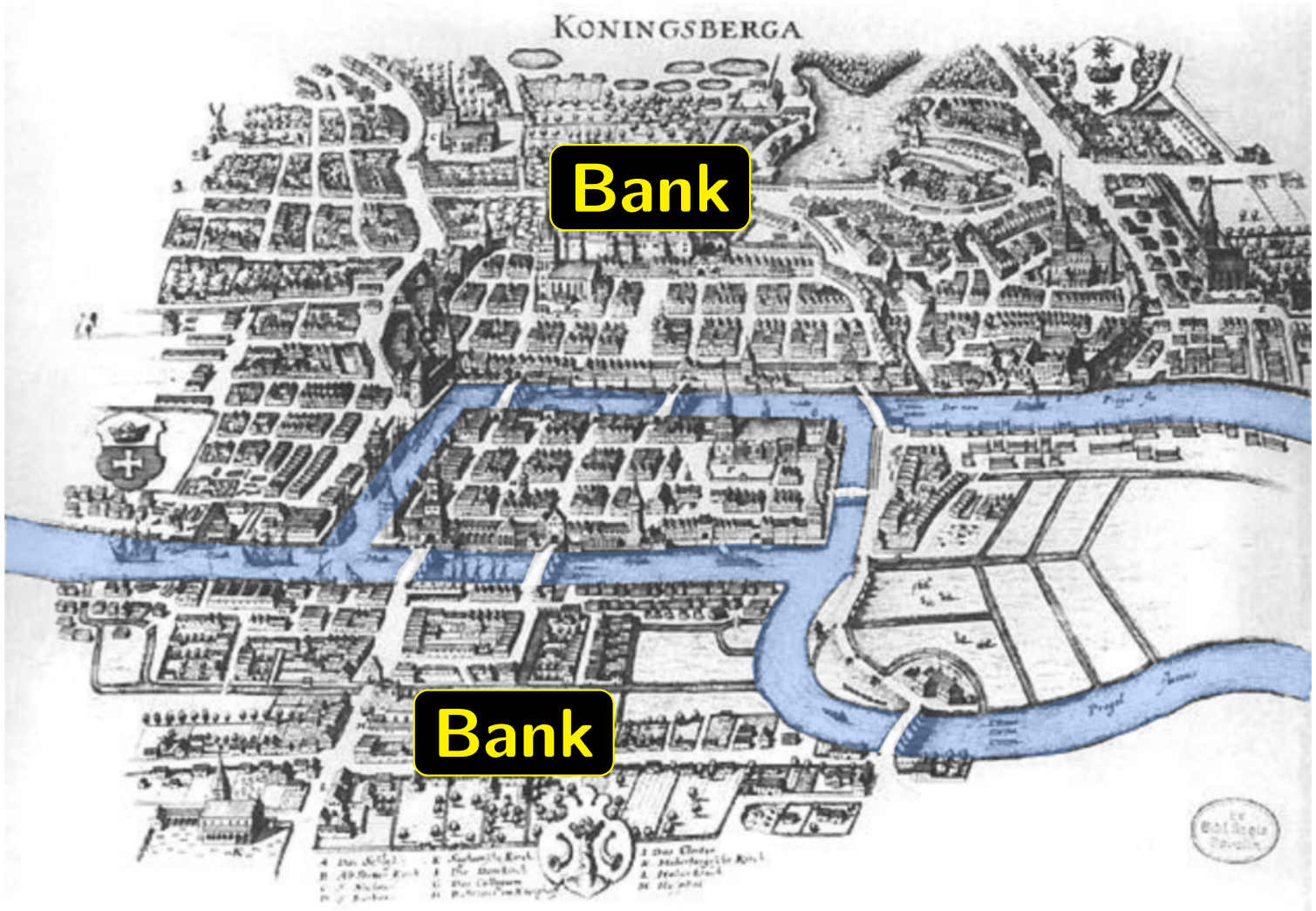
Euler Cycle

The euler path of which the starting and ending vertices are the same.

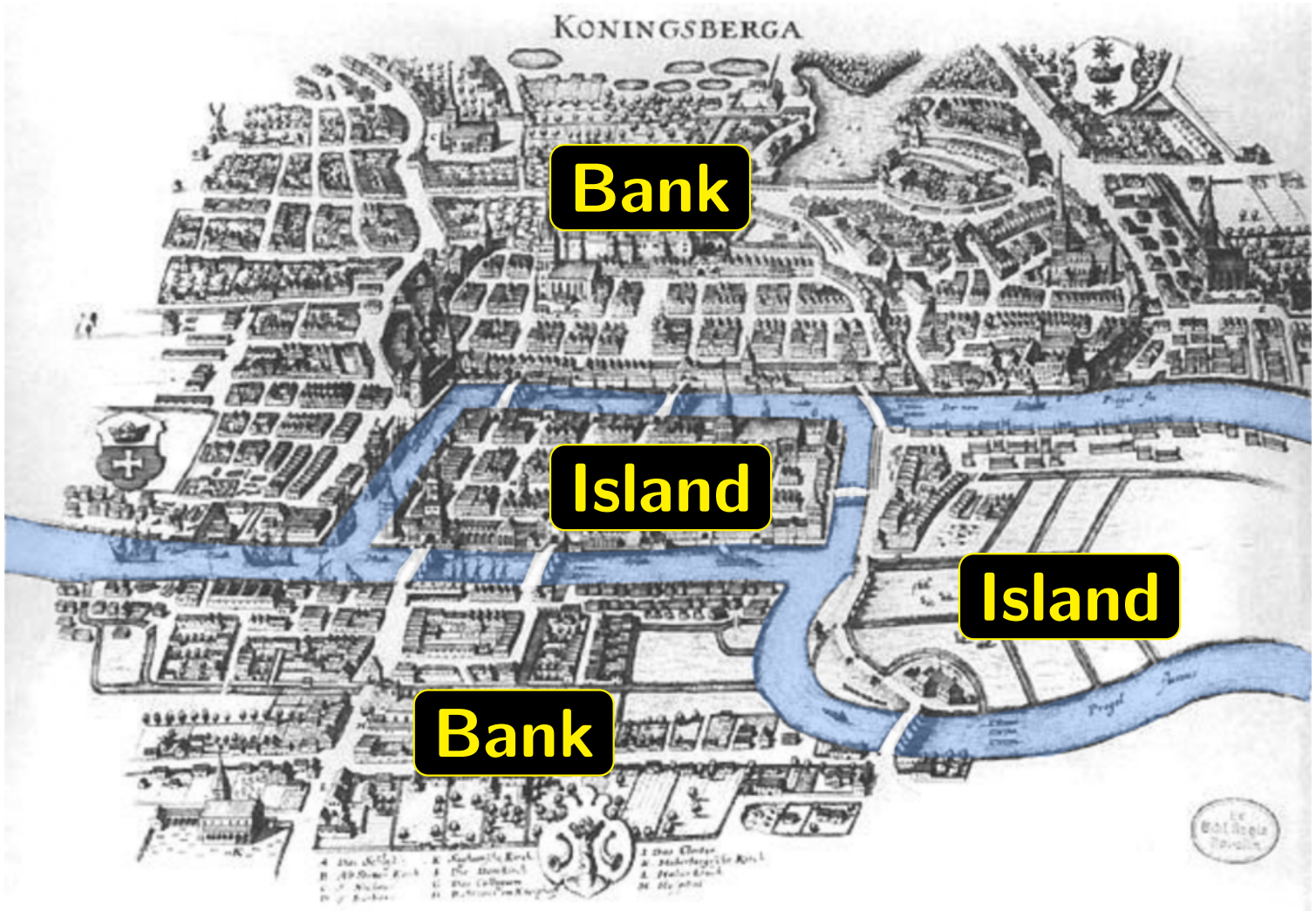
The Seven Bridges of Königsberg



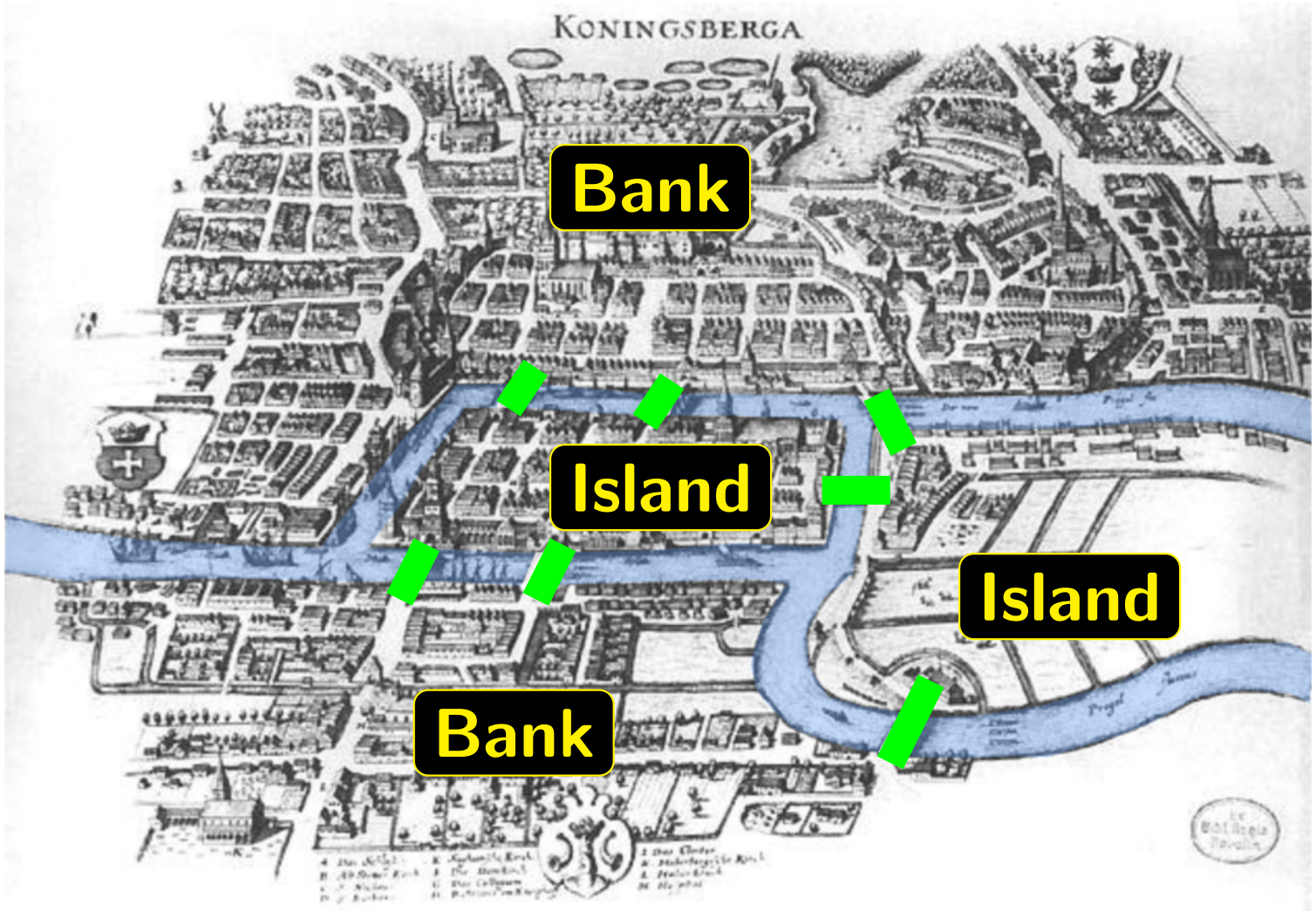
The Seven Bridges of Königsberg



The Seven Bridges of Königsberg



The Seven Bridges of Königsberg



The Seven Bridges of Königsberg

The Seven Bridges of Königsberg

- Leonhard Euler
- Swiss Mathematician
- 15 April 1707 – 18 September 1783

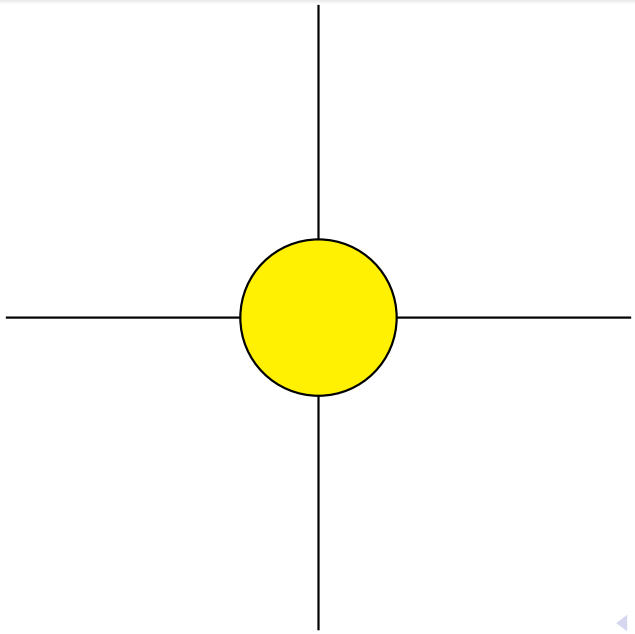
The Seven Bridges of Königsberg

Euler's Theorem

For an Euler path to exist, the graph must have at most two odd degree vertices
For an Euler cycle, the number must be zero

Degree of a vertex

Number of edges associated with a vertex in a graph

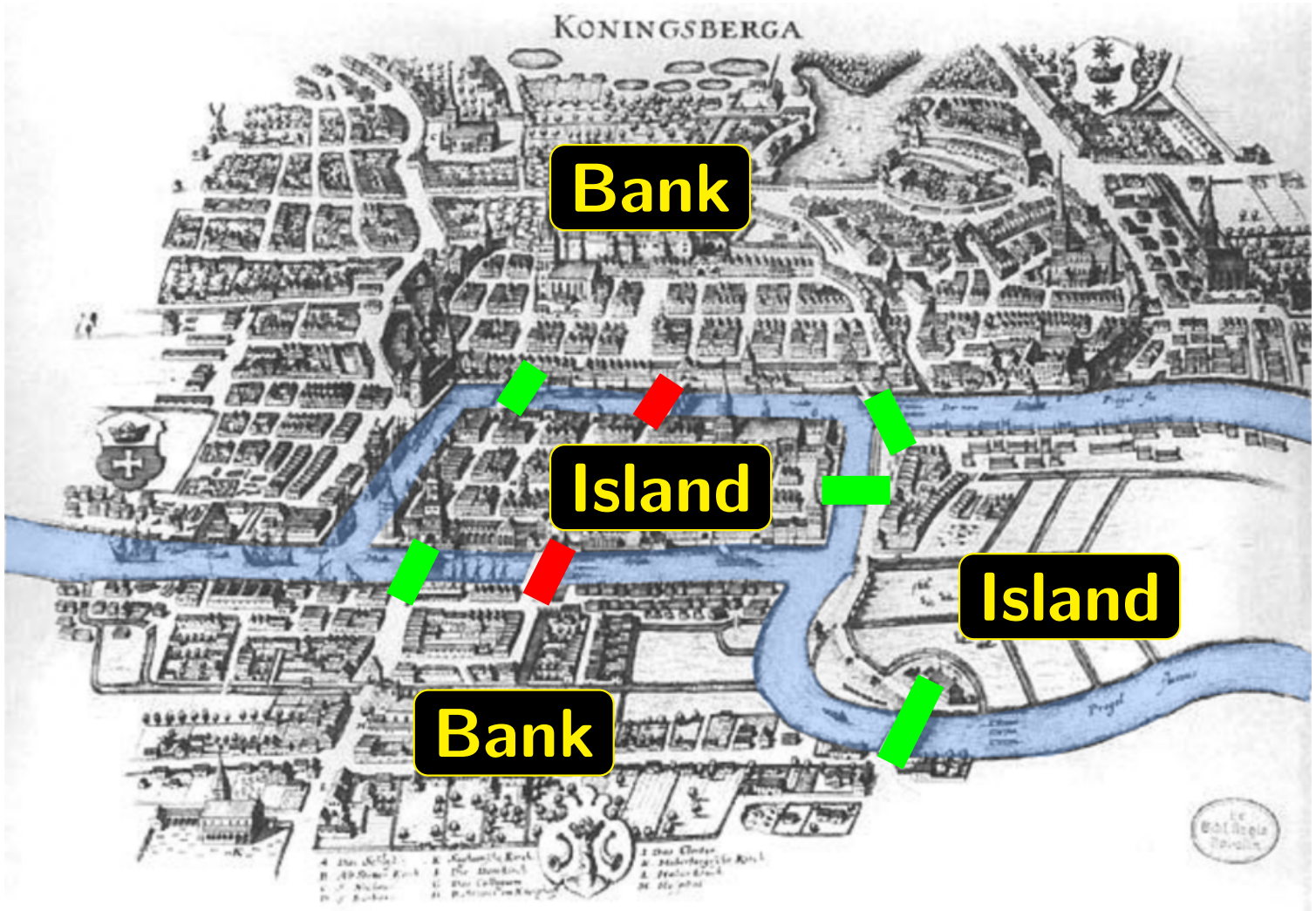


The Seven Bridges of Königsberg

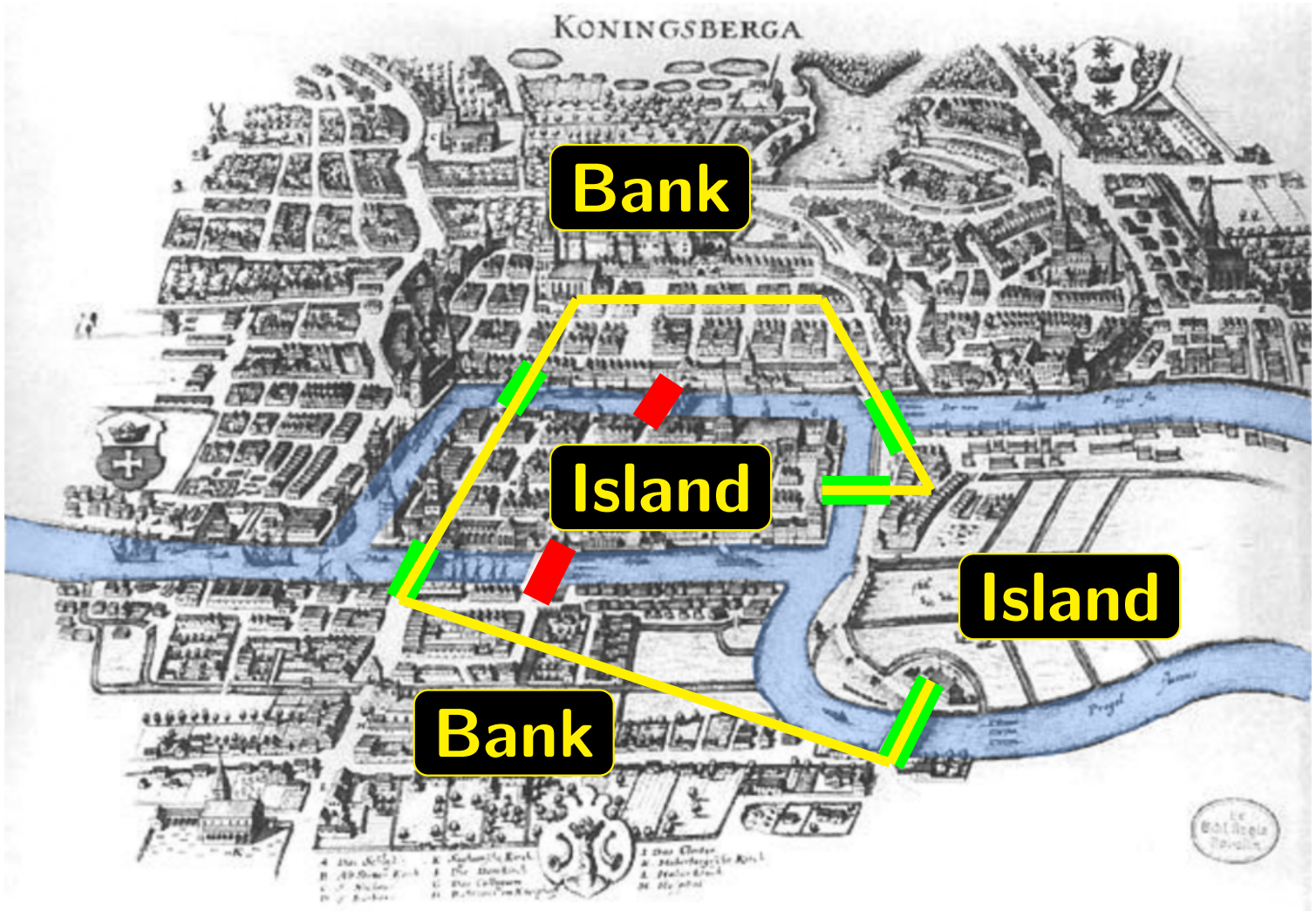
The Seven Bridges of Königsberg

The Seven Bridges of Königsberg

The Seven Bridges of Königsberg



The Seven Bridges of Königsberg



Introduction to Hamiltonian Cycle

Hamiltonian Path

The path in a graph that uses every vertex of a graph exactly once.

Hamiltonian Cycle

The cycle in a graph that uses every vertex of a graph exactly once.

Introduction to Hamiltonian Cycle

Ṡir William Rowan Hamilton
İrish mathematician
3̇/4 August 1805 – 2 September 1865

The Knight's Tour Problem

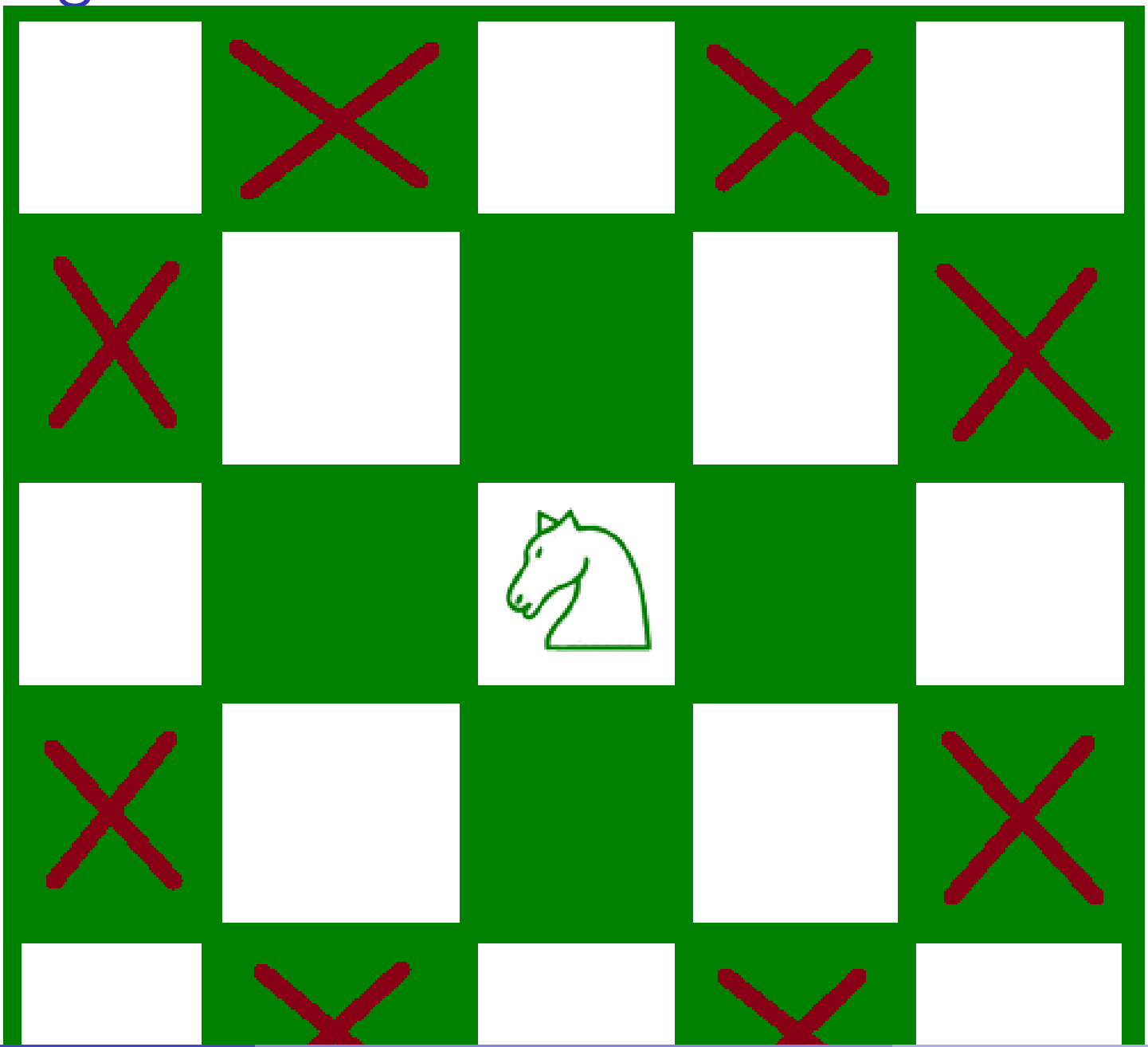
Problem 1

Can a knight traverse all cells of a chessboard only once?

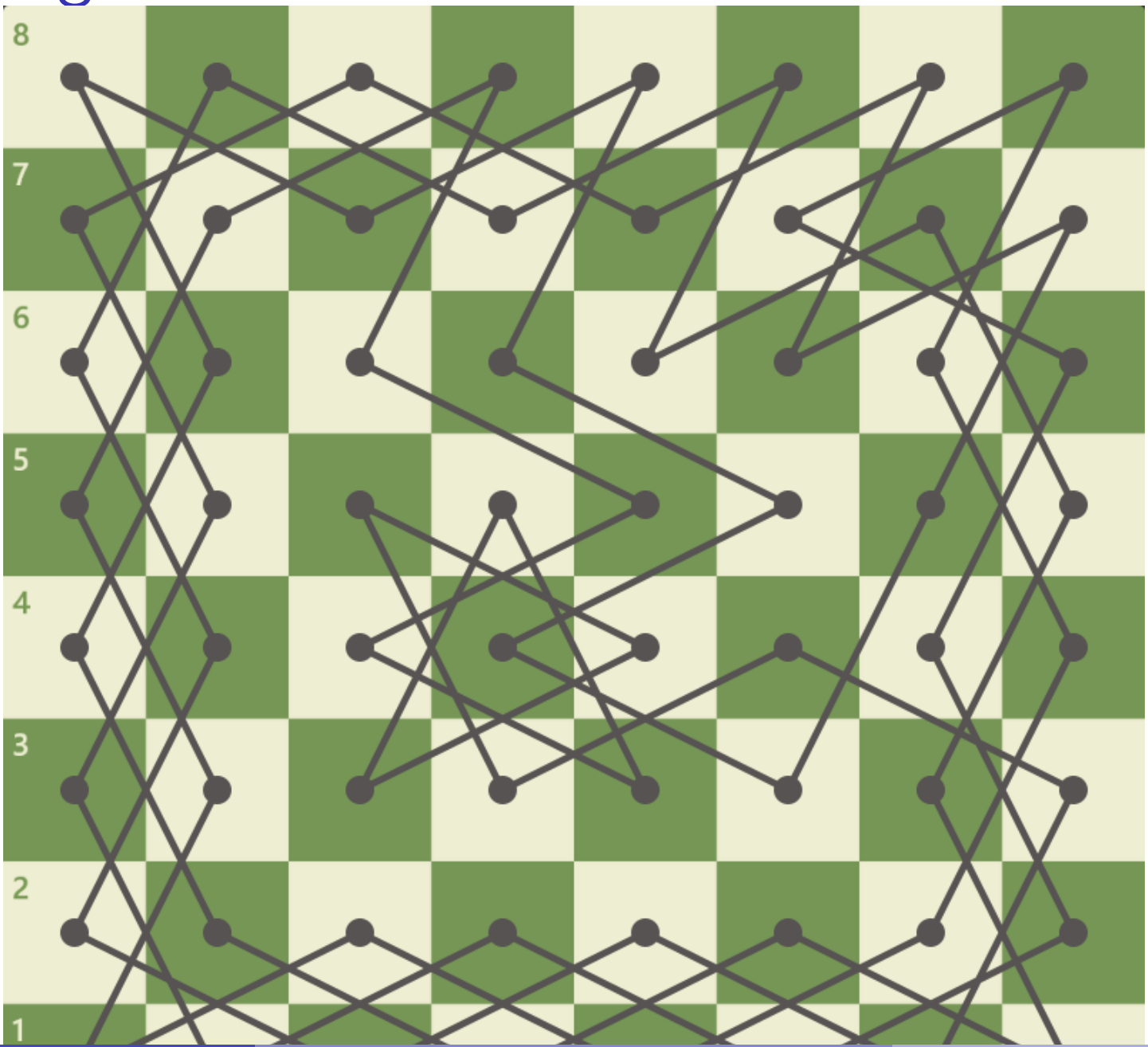
Problem 2

Can a knight traverse all cells of a chessboard only once and can return to the initial cell in the end?

The Knight's Tour Problem



The Knight's Tour Problem



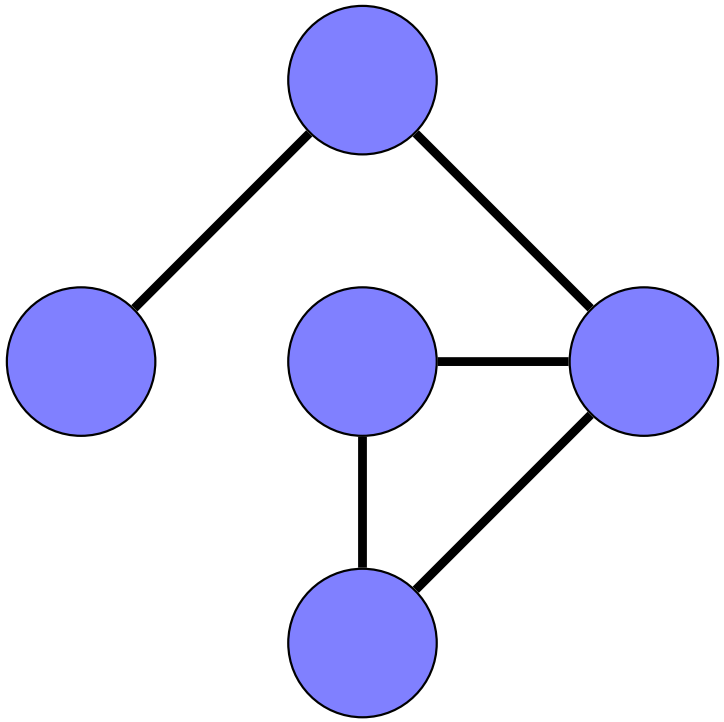
Over? It hasn't even begun

Eulerian Path

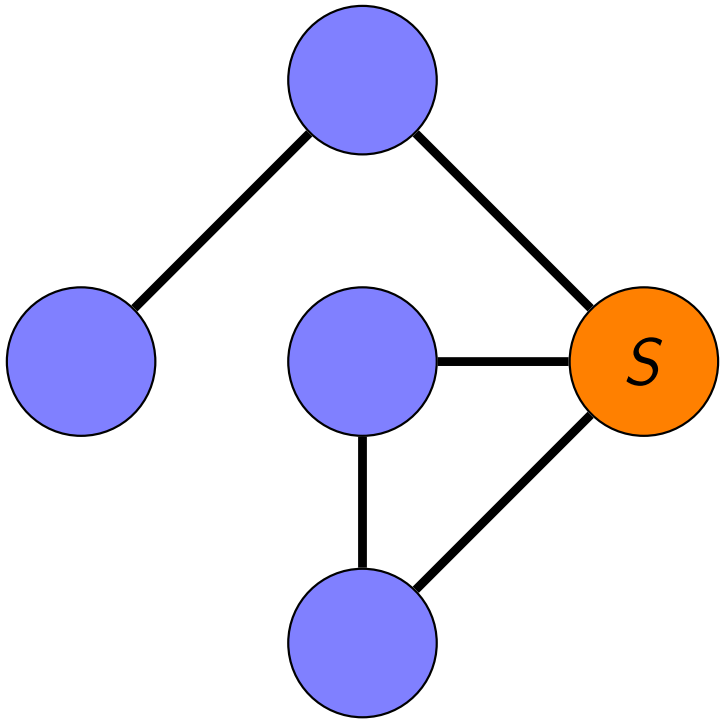
Definition

An **Eulerian Path** is a path of edges in the graph that visits every edge exactly once.

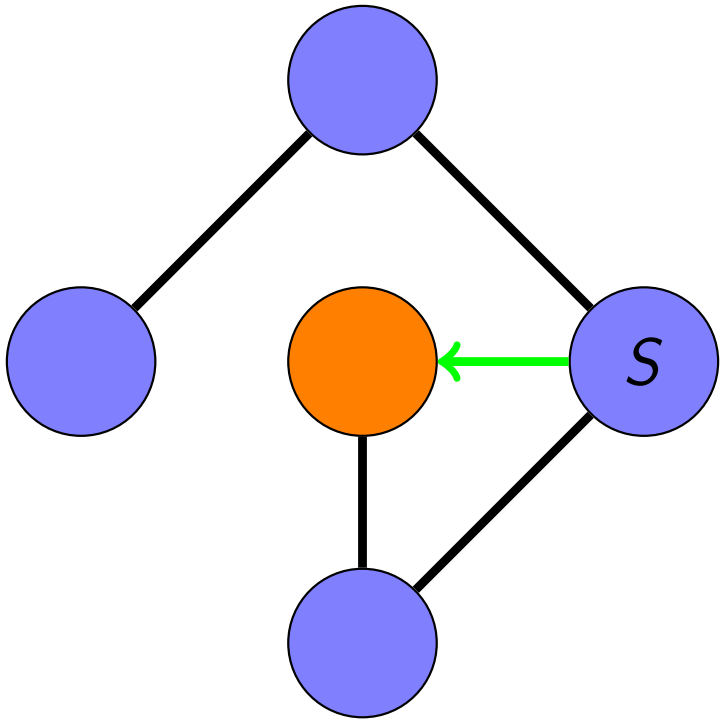
Finding Eulerian Path



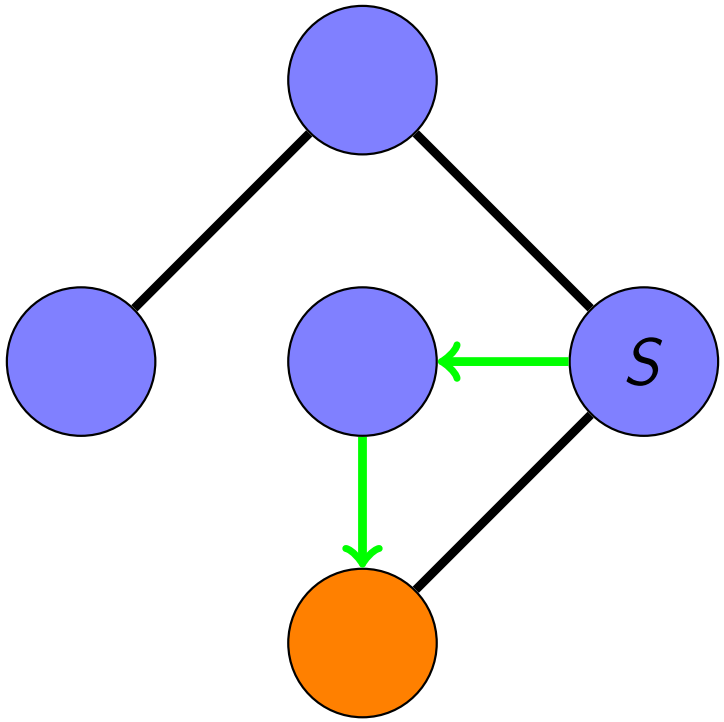
Finding Eulerian Path continue...



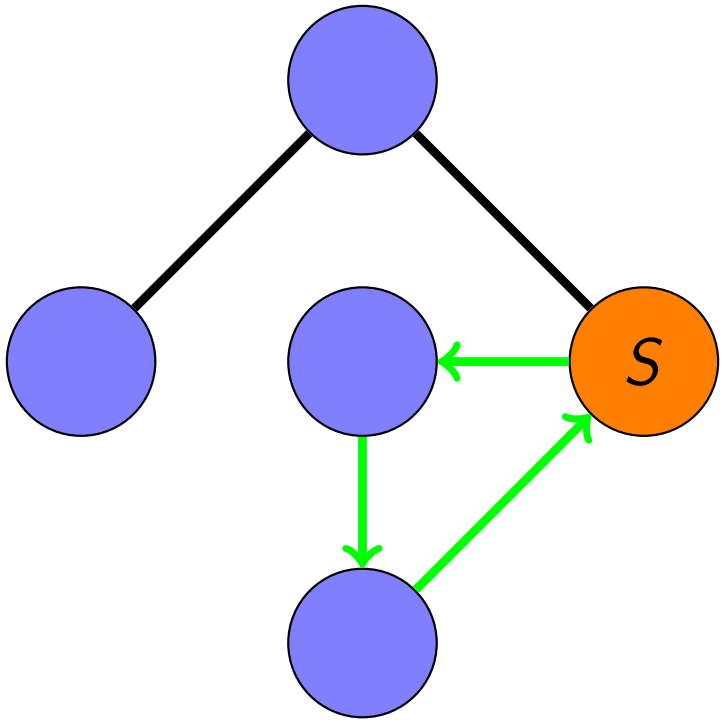
Finding Eulerian Path continue...



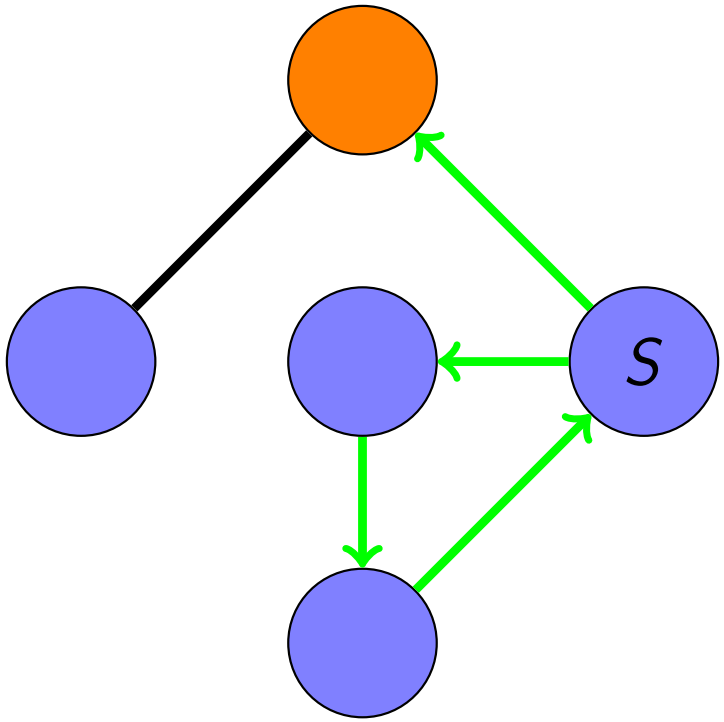
Finding Eulerian Path continue...



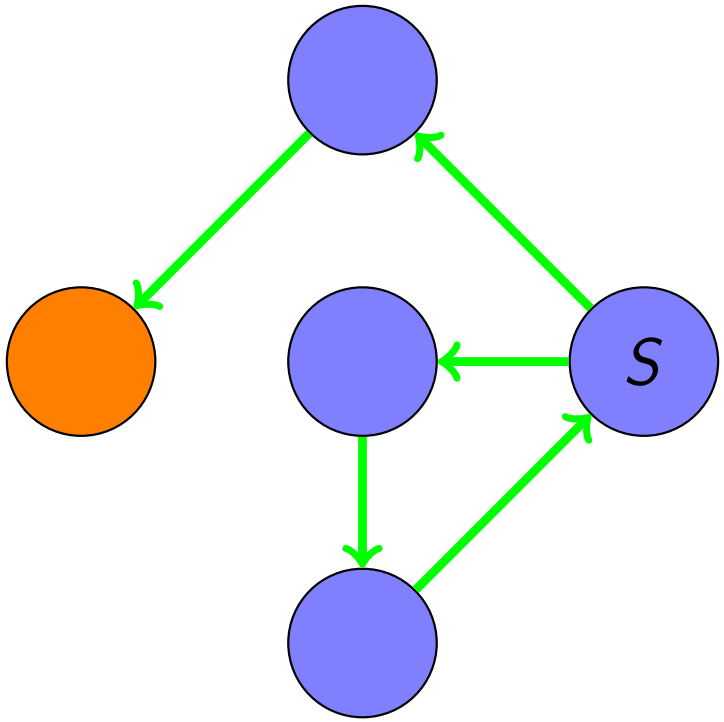
Finding Eulerian Path continue...



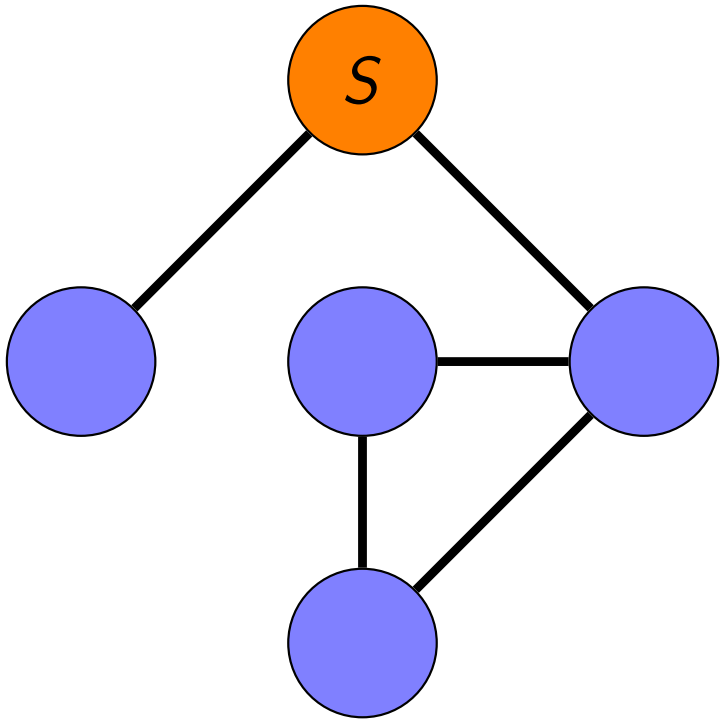
Finding Eulerian Path continue...



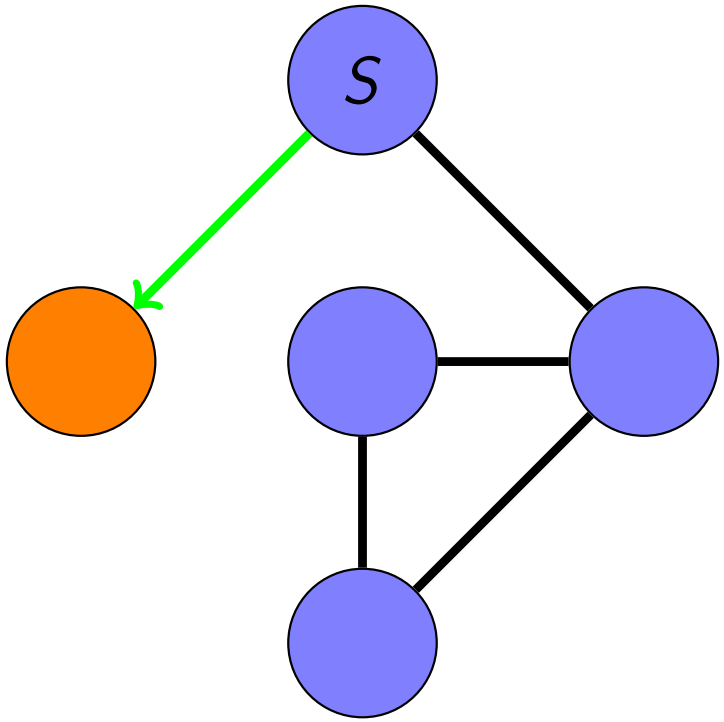
Finding Eulerian Path continue...



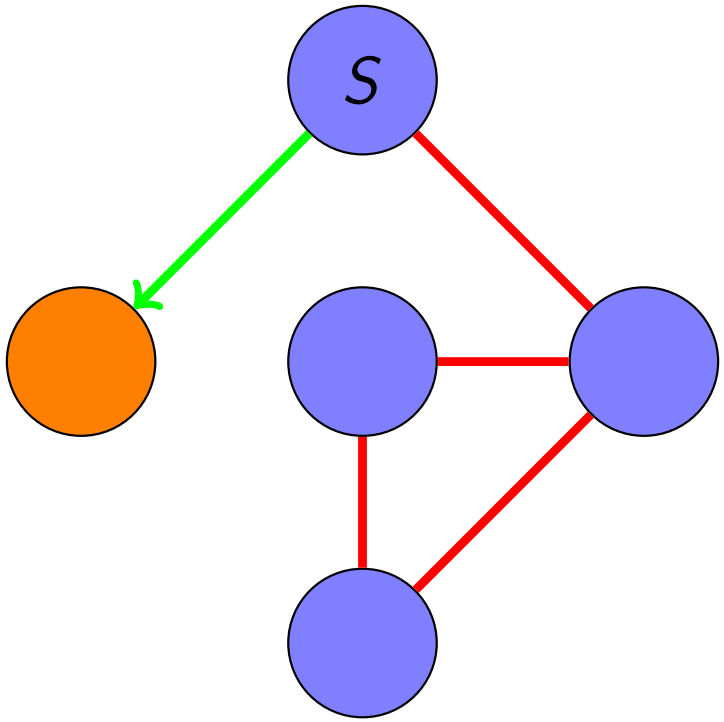
Trapped!



Trapped! continue...



Trapped! continue...

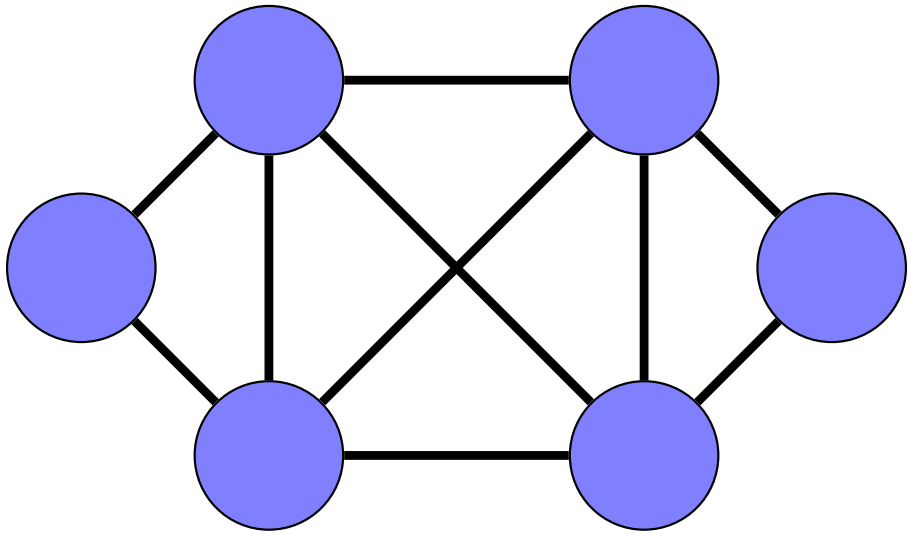


Eulerian Circuit Recap

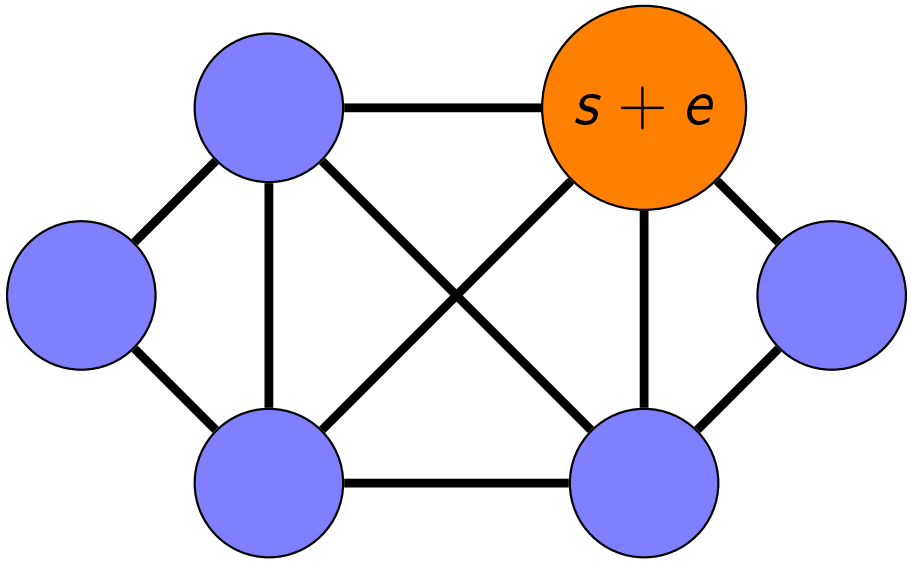
Definition

An **Eulerian Circuit** is an euler path which starts and ends on the same vertex .

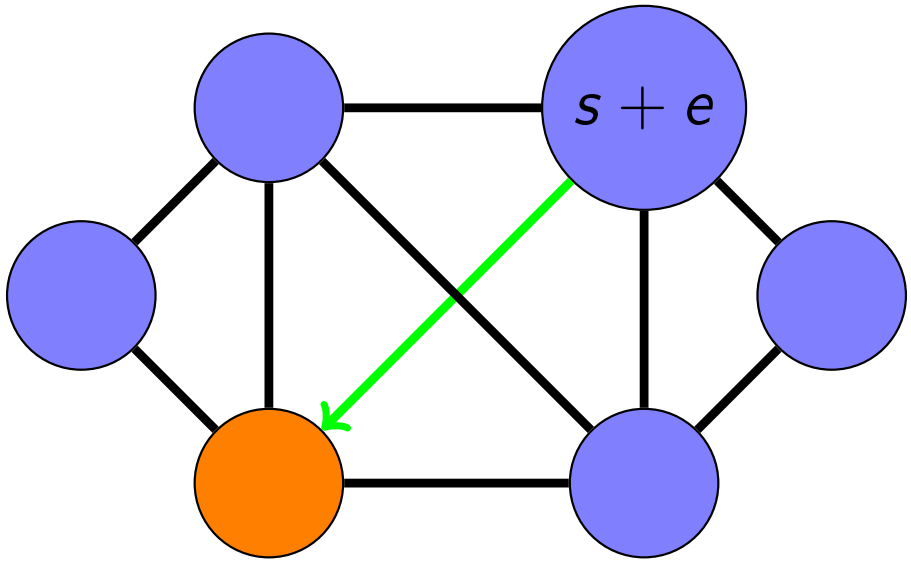
Find Euler Circuit



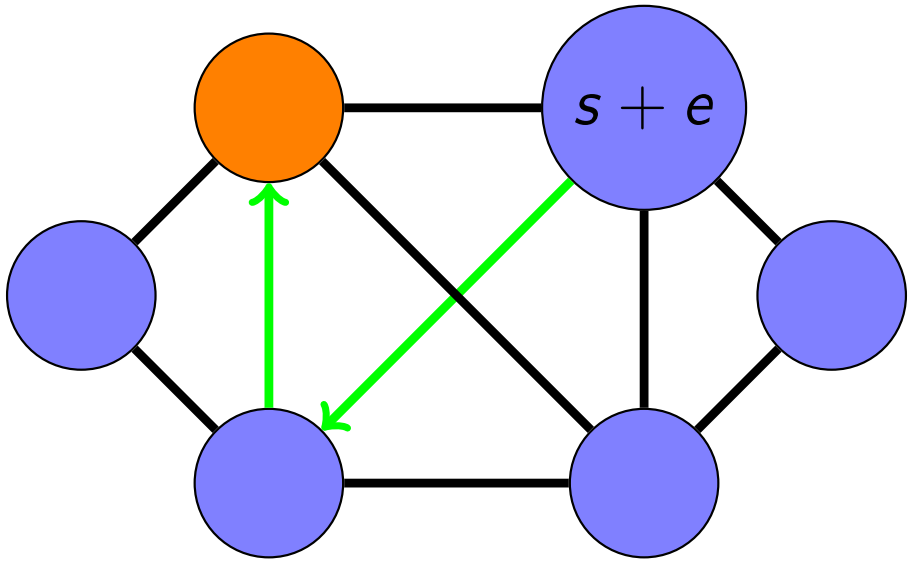
Finding Euler Circuit Continue...



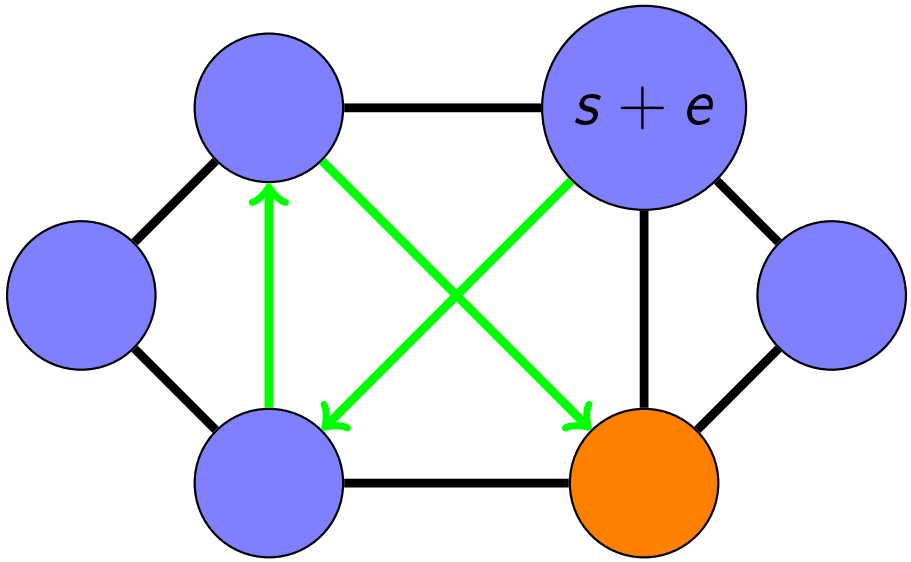
Finding Euler Circuit continue...



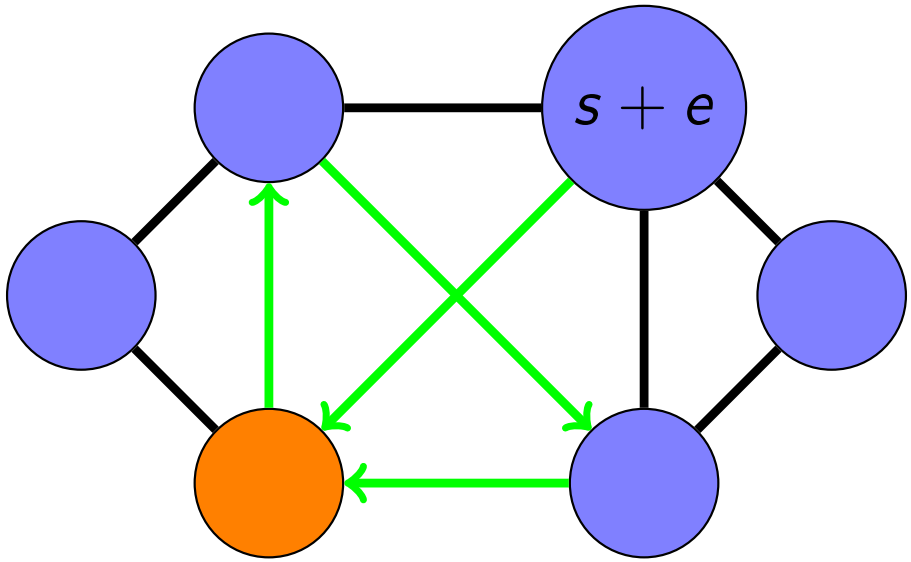
Finding Euler Circuit continue...



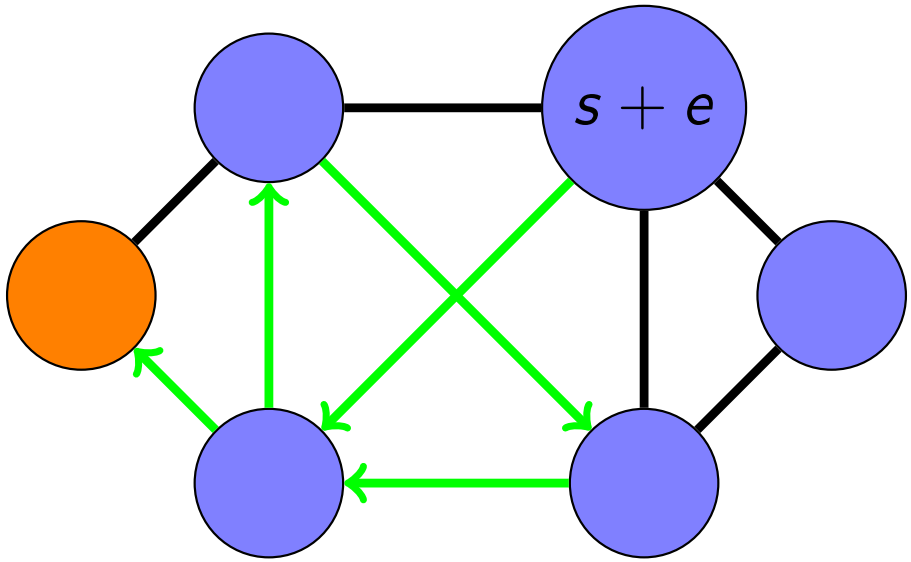
Finding Euler Circuit continue...



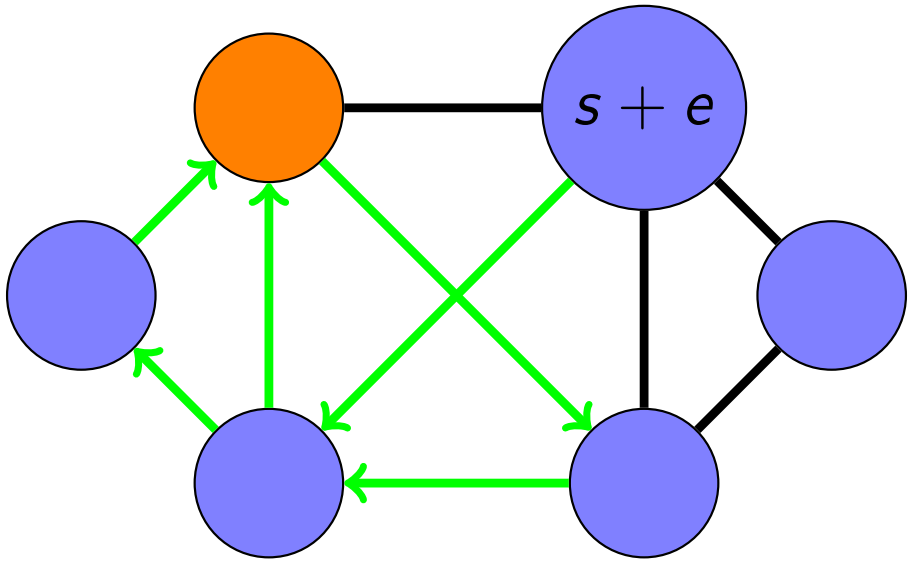
Finding Euler Circuit continue...



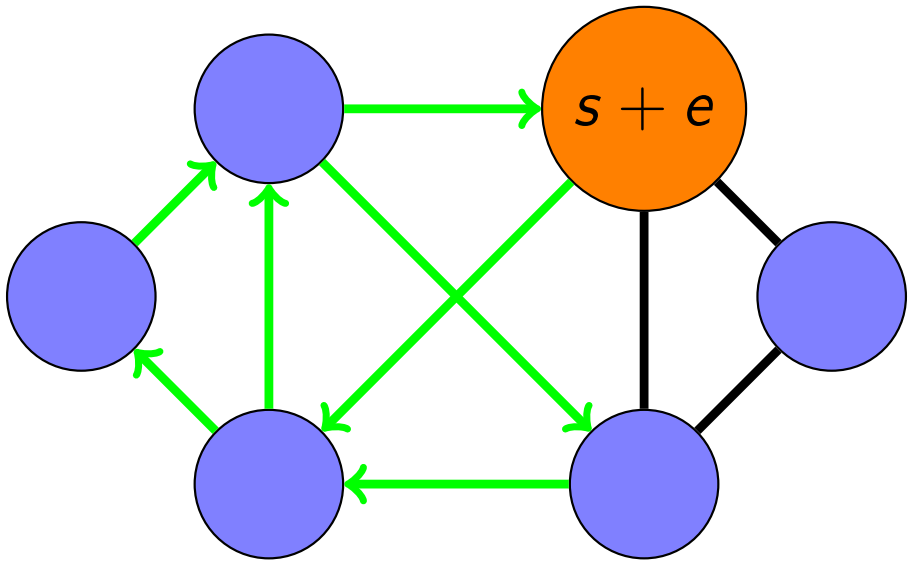
Finding Euler Circuit continue...



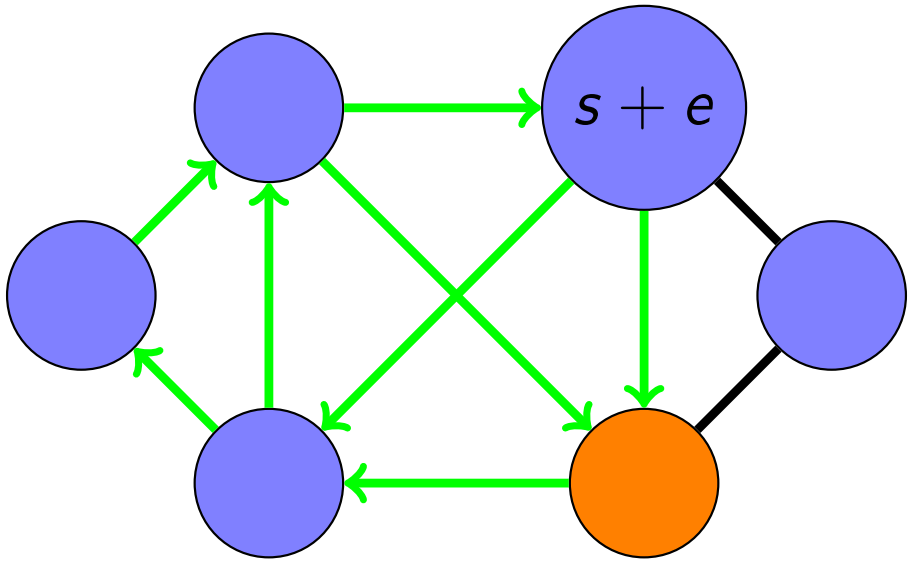
Finding Euler Circuit continue...



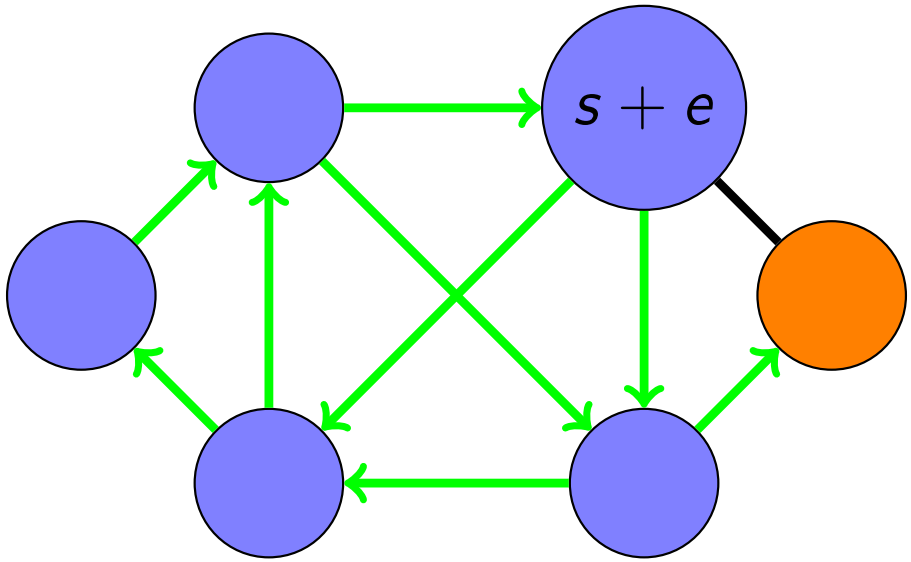
Finding Euler Circuit continue...



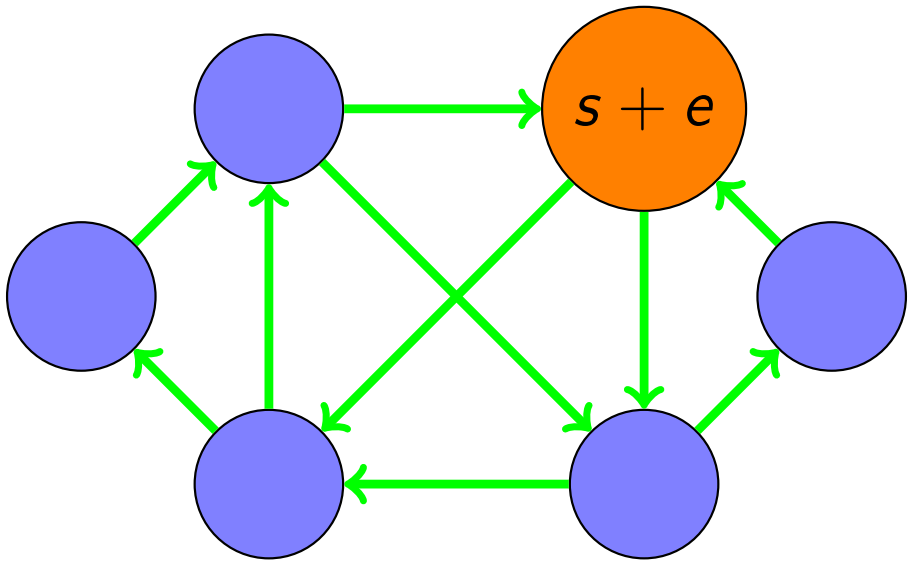
Finding Euler Circuit continue...



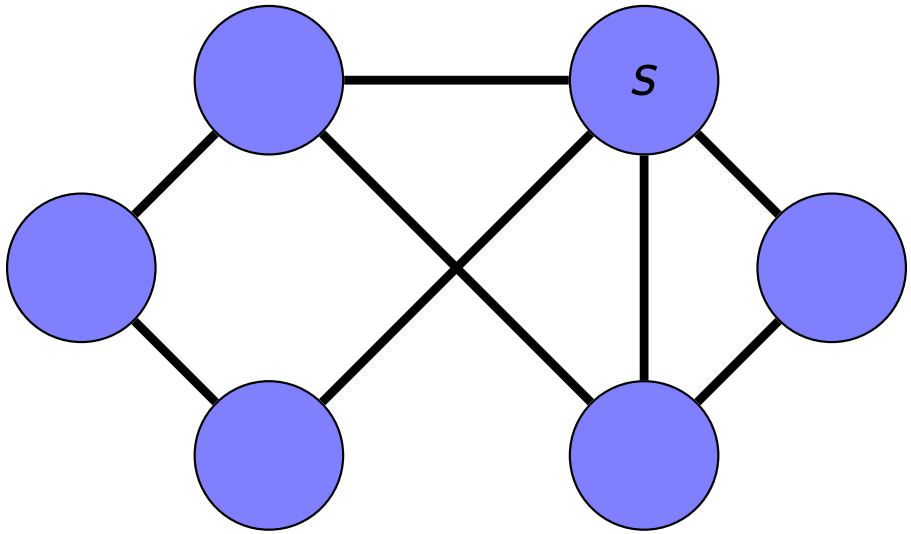
Finding Euler Circuit continue...



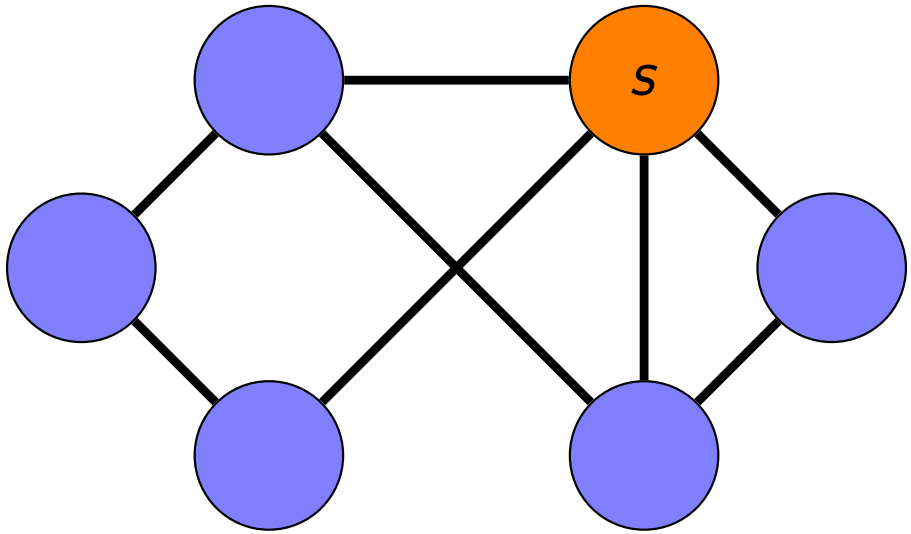
Finding Euler Circuit continue...



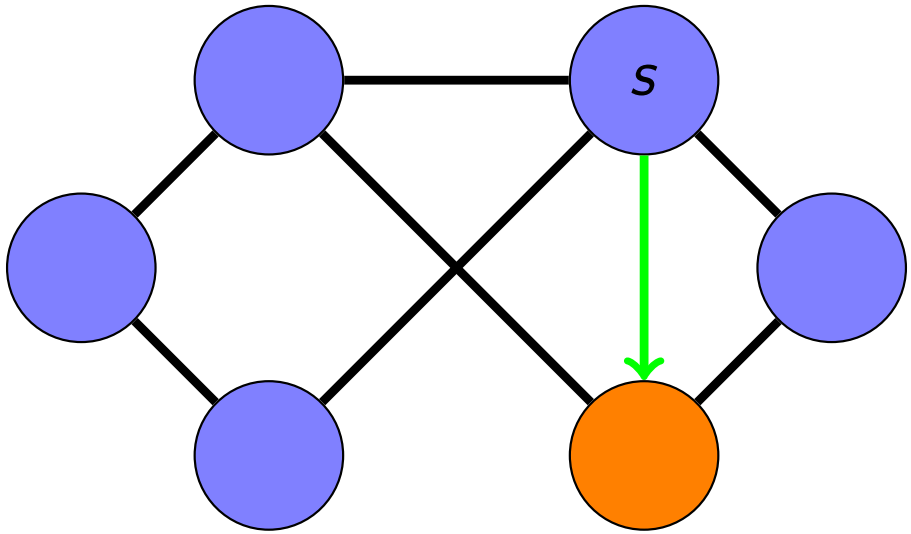
Circling the Unknown



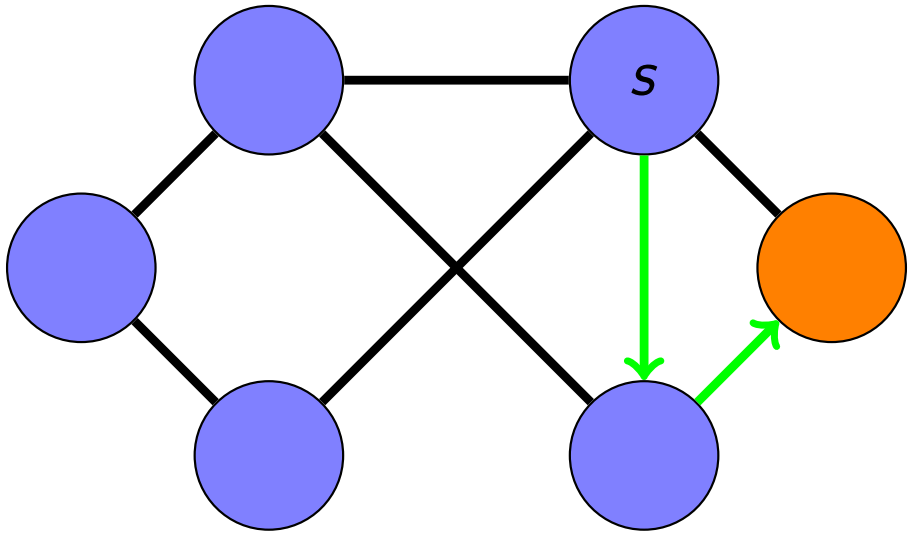
Circling the Unknown continue...



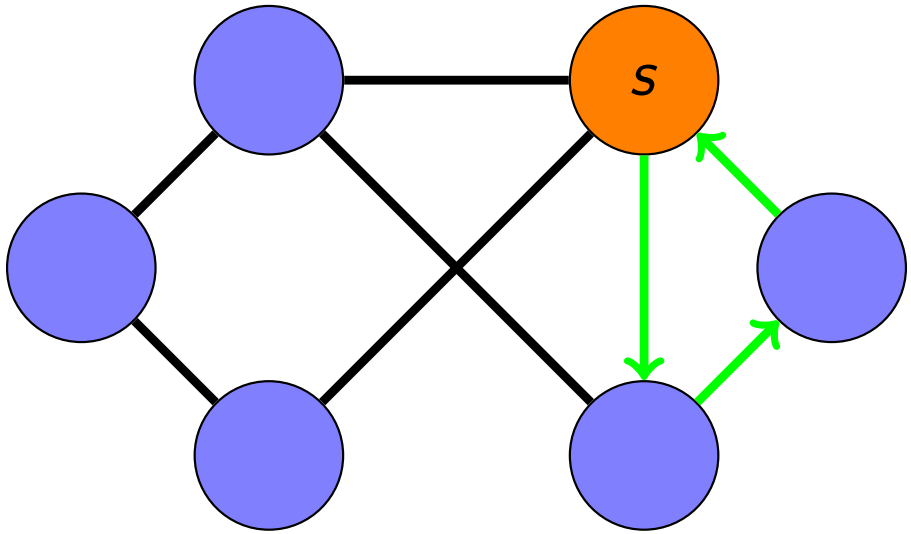
Circling the Unknown continue...



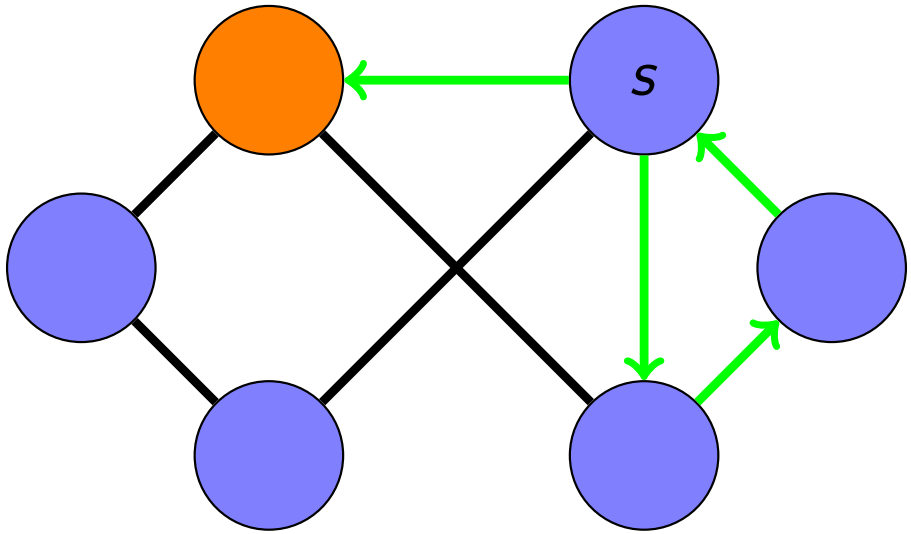
Circling the Unknown continue...



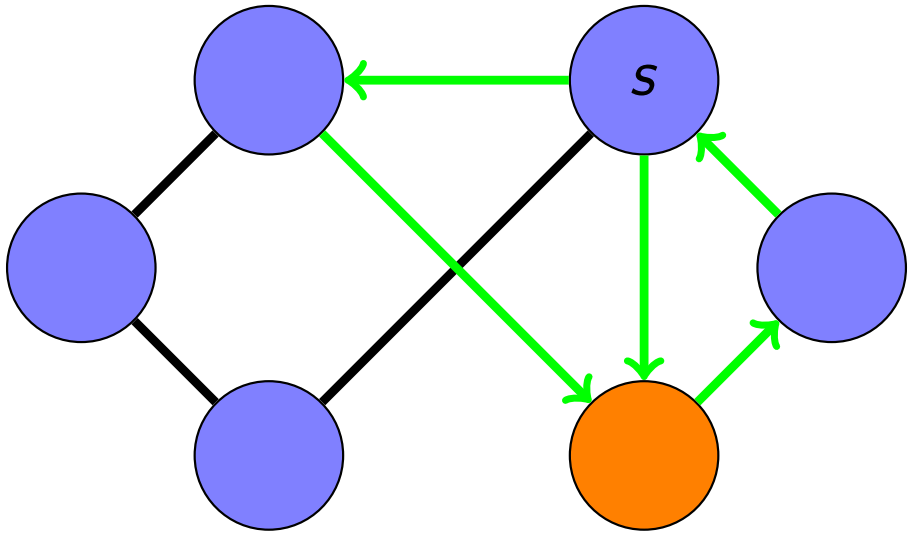
Circling the Unknown continue...



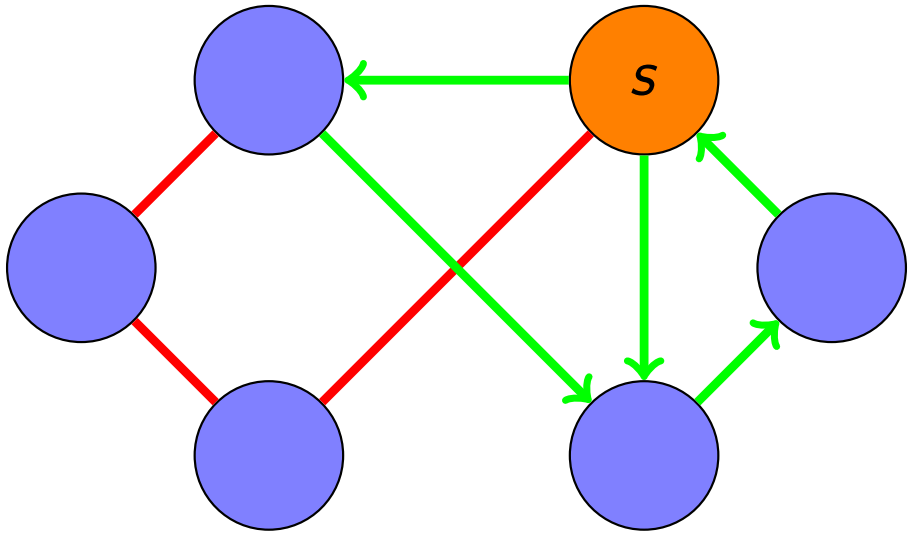
Circling the Unknown continue...



Circling the Unknown continue...



Circling the Unknown continue...



What conditions are required for a valid Eulerian Circuit?

Undirected Graph	Every vertex has an even degree
Directed Graph	Every vertex has equal indegree and outdegree

Hamiltonian Graph

Definition

An **Hamiltonian Graph** is a circuit that traverses every **vertex** of a graph exactly once and returns to the starting point.

Differnece with Euler Circuit

Euler circuit covers all **Edges**

Conditions for Hamiltonian Graphs

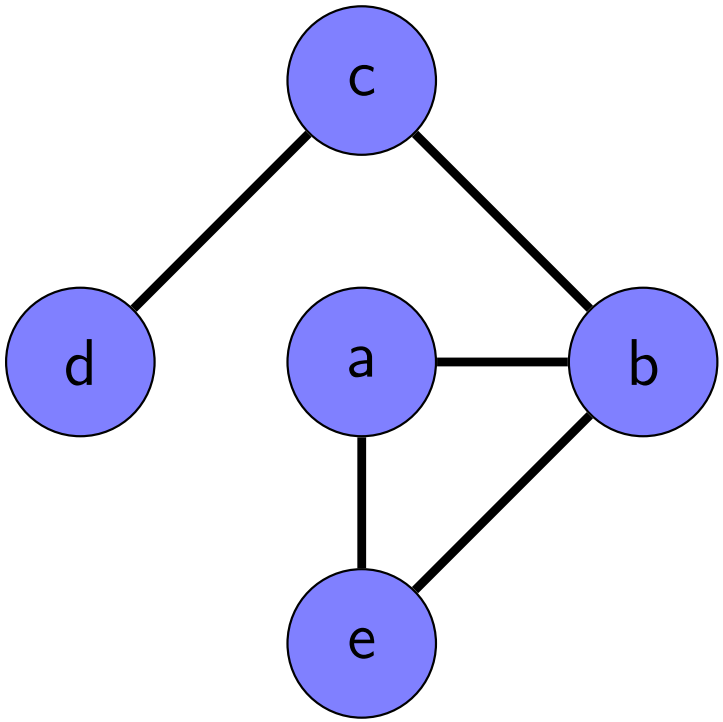
Conditions

1. **Connectedness:** The graph must be connected.
2. The Graph must have a **Hamiltonian Path** and a **Hamiltonian Cycle**

Important Note

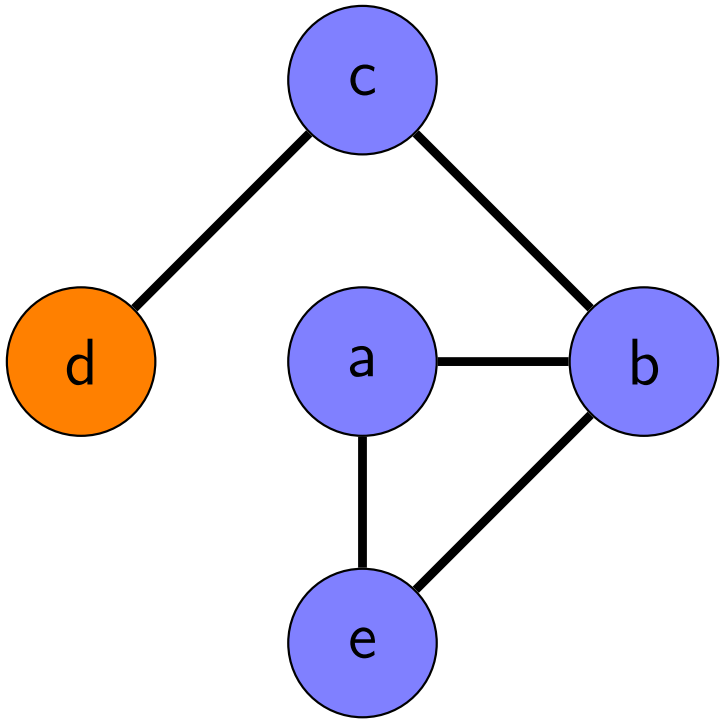
Hamiltonian Graph is a special kind of graph, not all graph have them

How to find Hamiltonian graph????

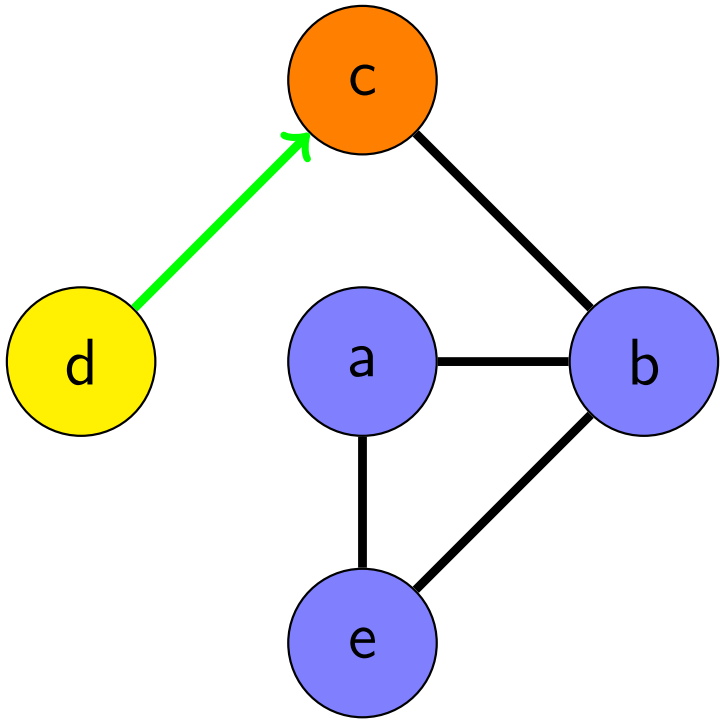


- Search for Hamiltonian Path

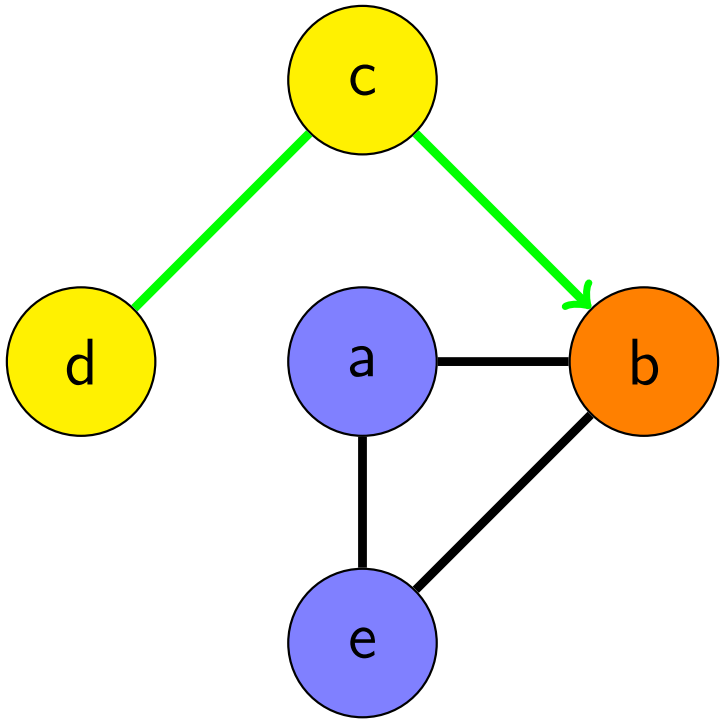
Searching Continued....



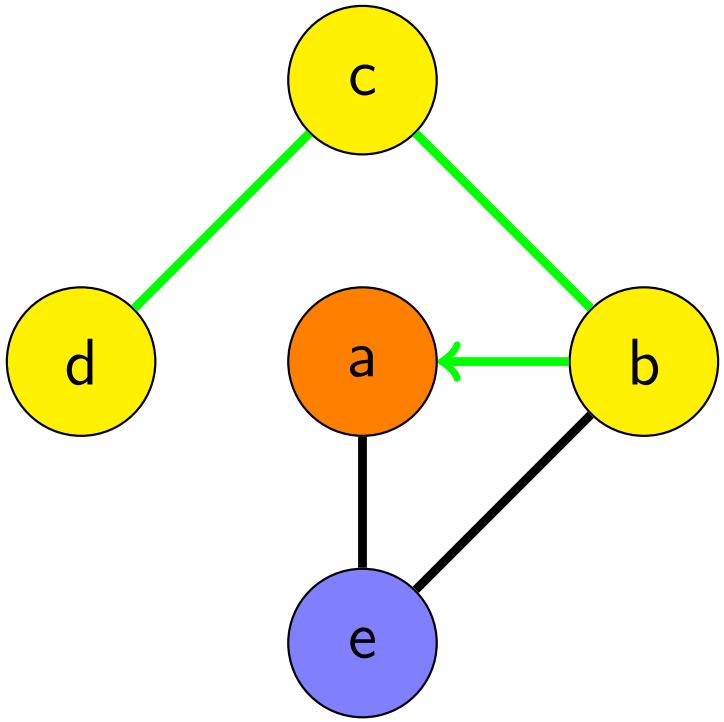
Searching Continued....



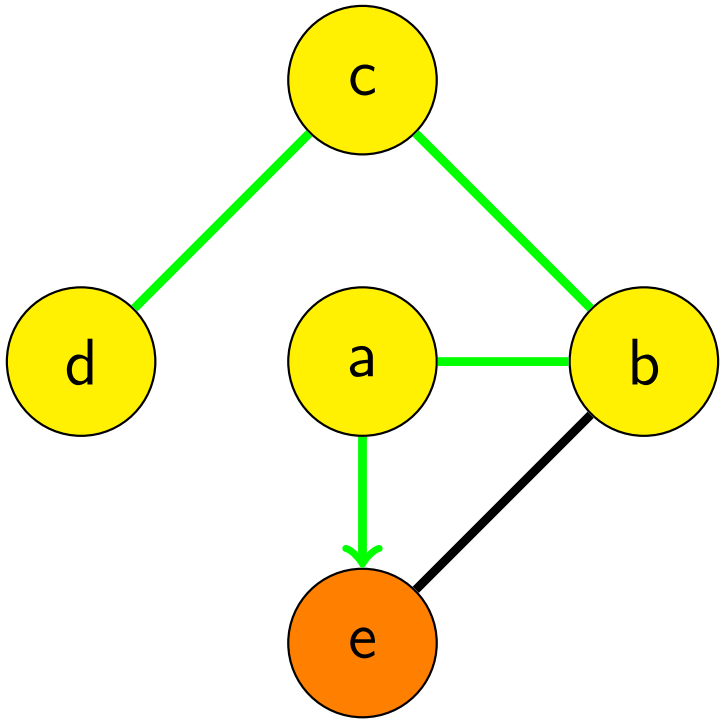
Searching Continued....



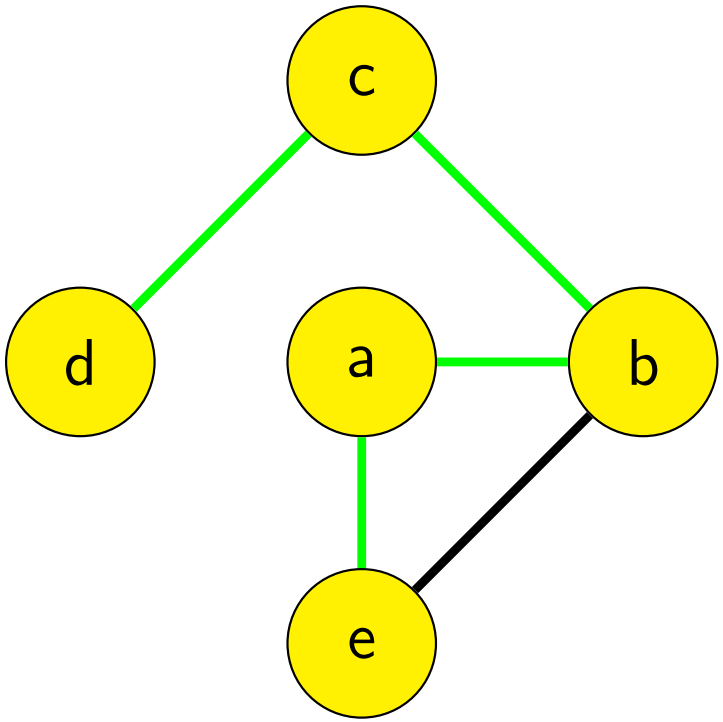
Searching Continued....



Searching Continued....

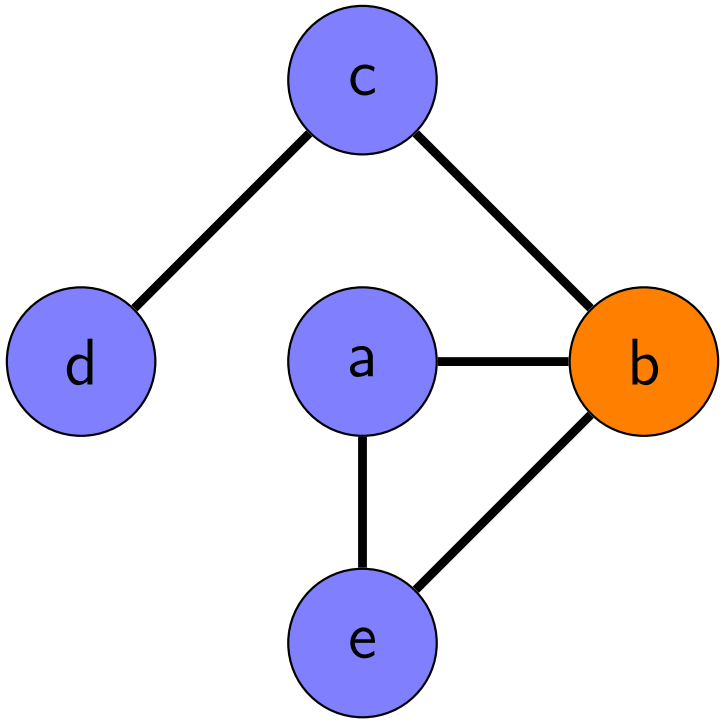


Searching Continued....

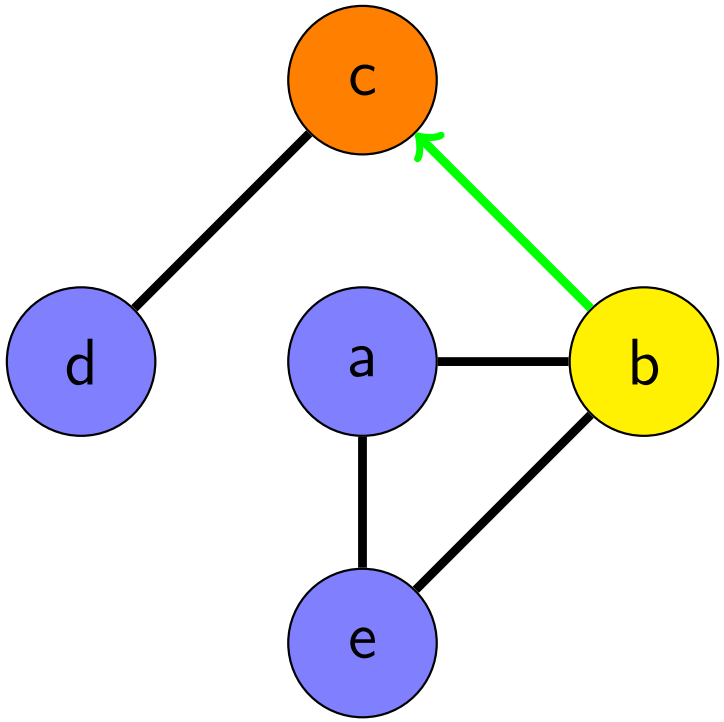


- Ureka! Path is found
- But what about Cycle????

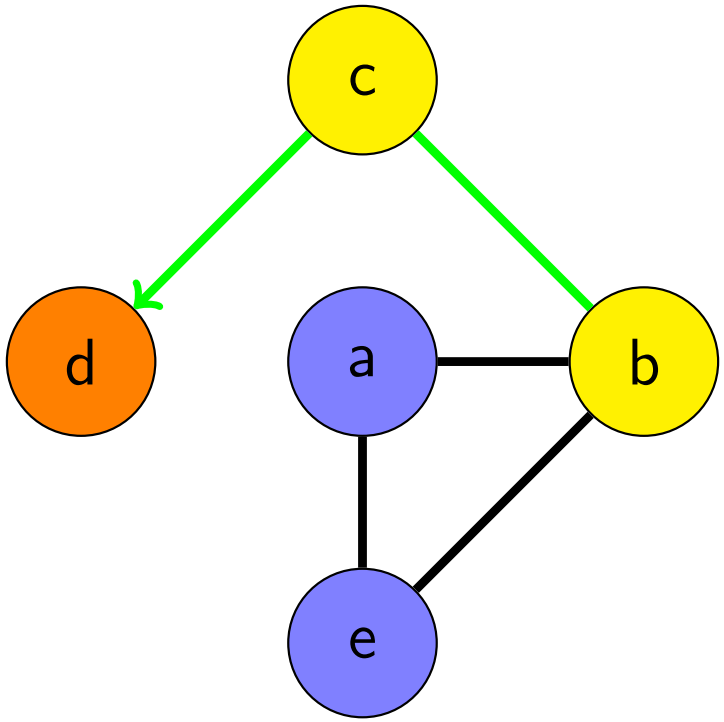
Be choosy in Life!!!!



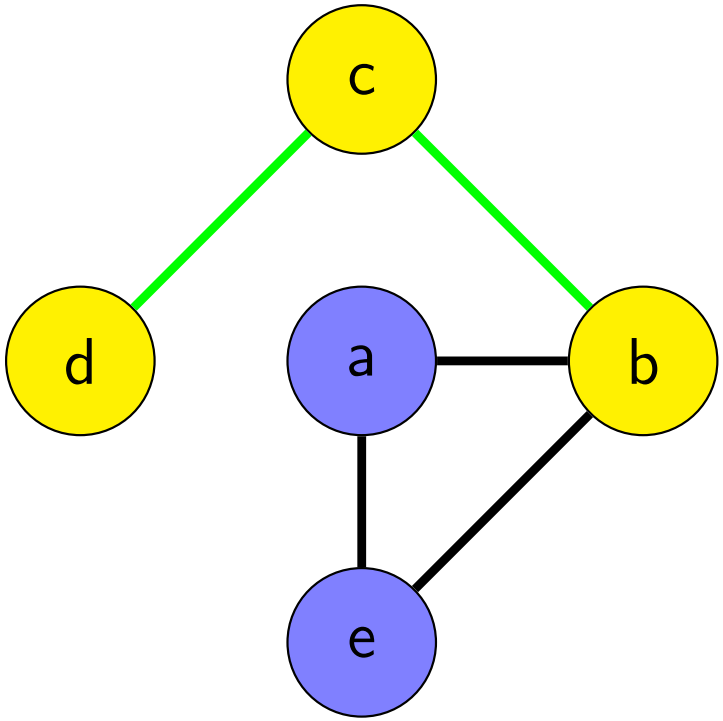
Keep Going....



Keep Going....

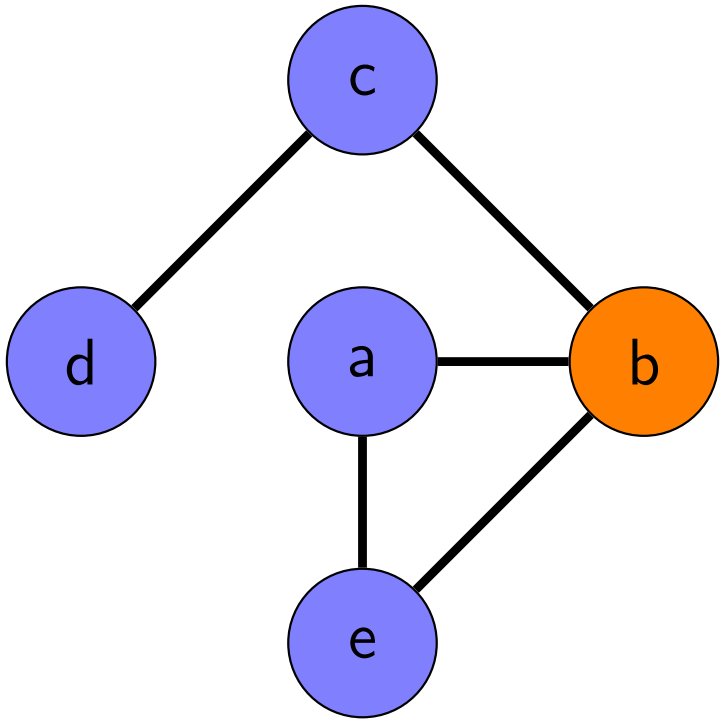


Keep Going....

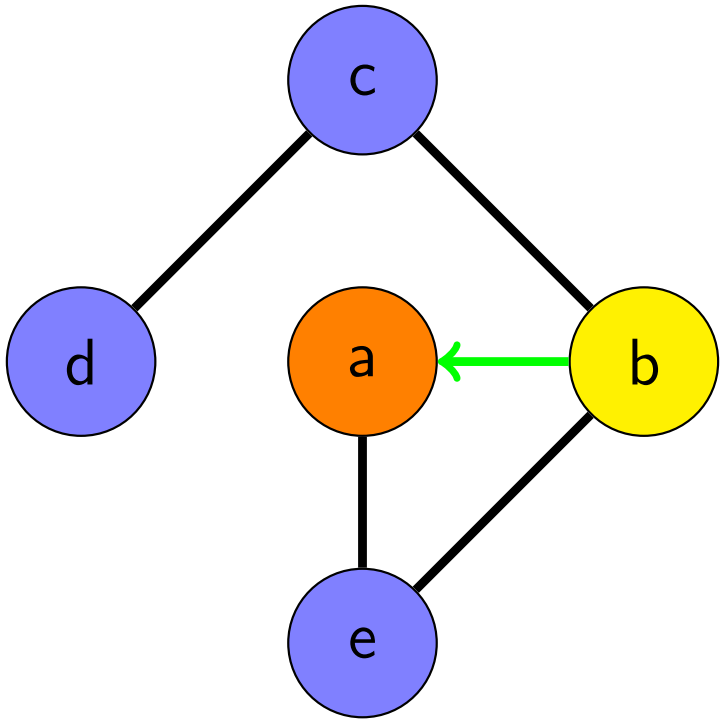


- Oh No!!

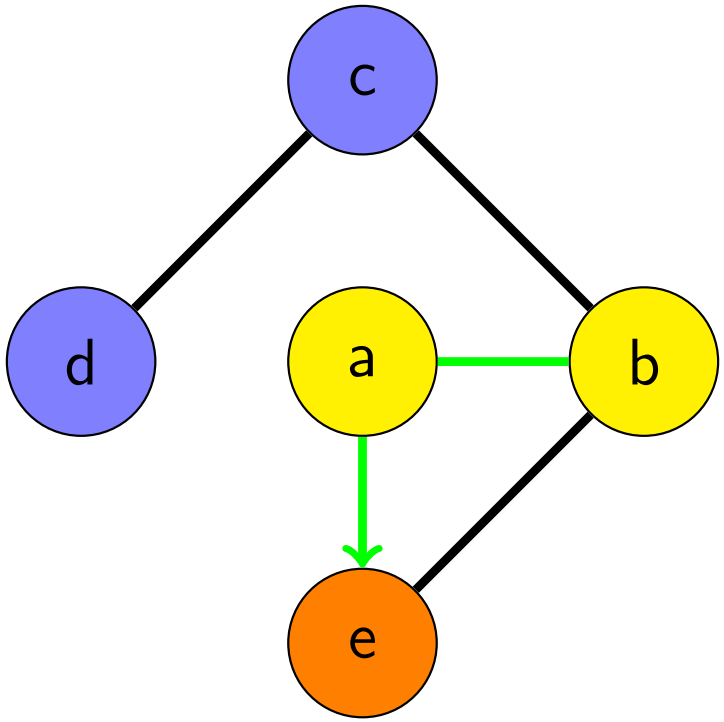
Let's Try another way!!!



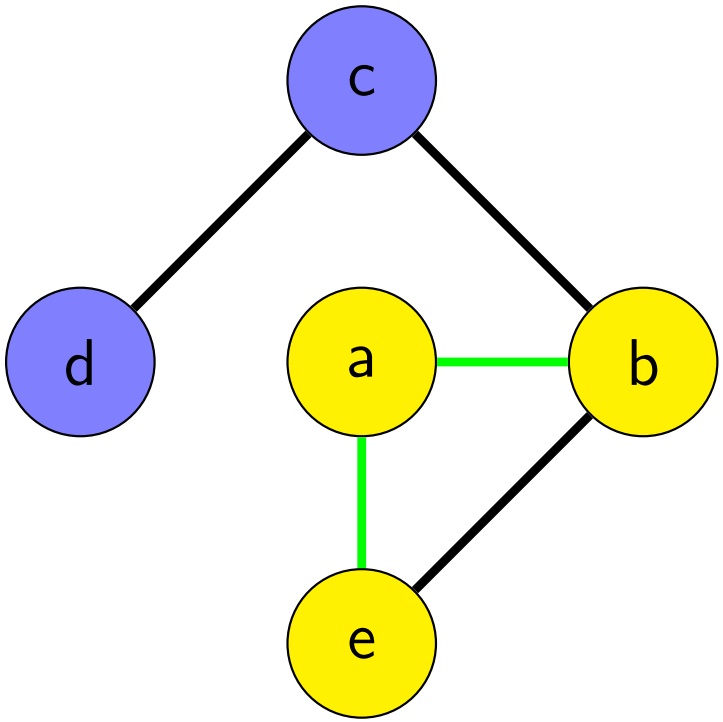
Trying....



Trying....

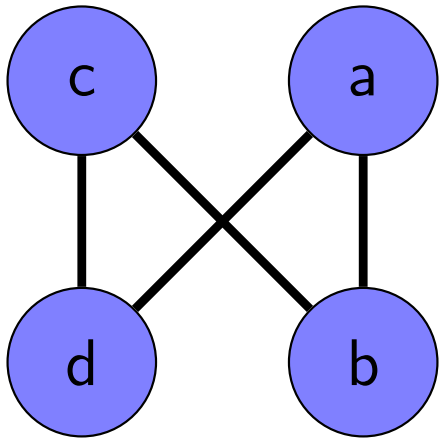


Found???

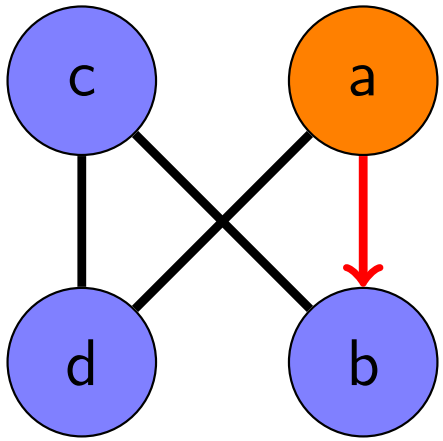


- Sad! Better luck Next time!!!

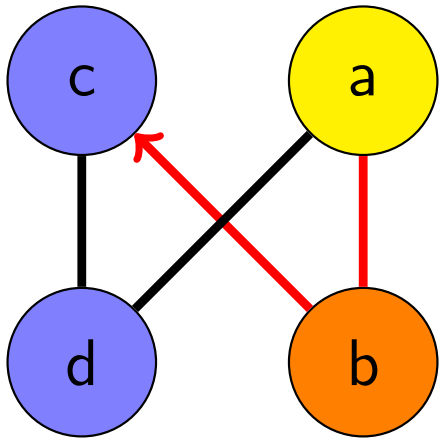
Can we find this time??



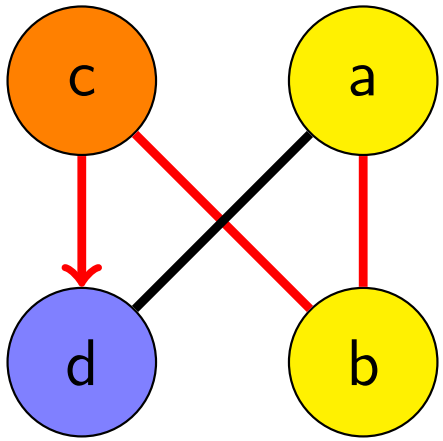
Hoping.....



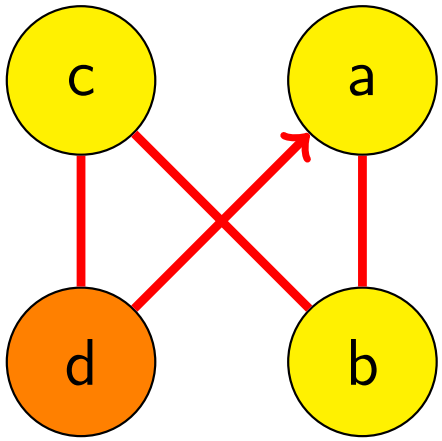
Hoping....



Hoping....

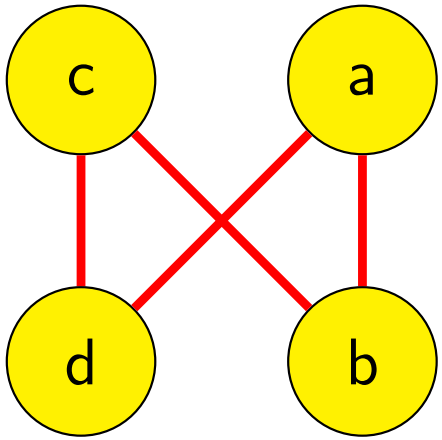


Almost....



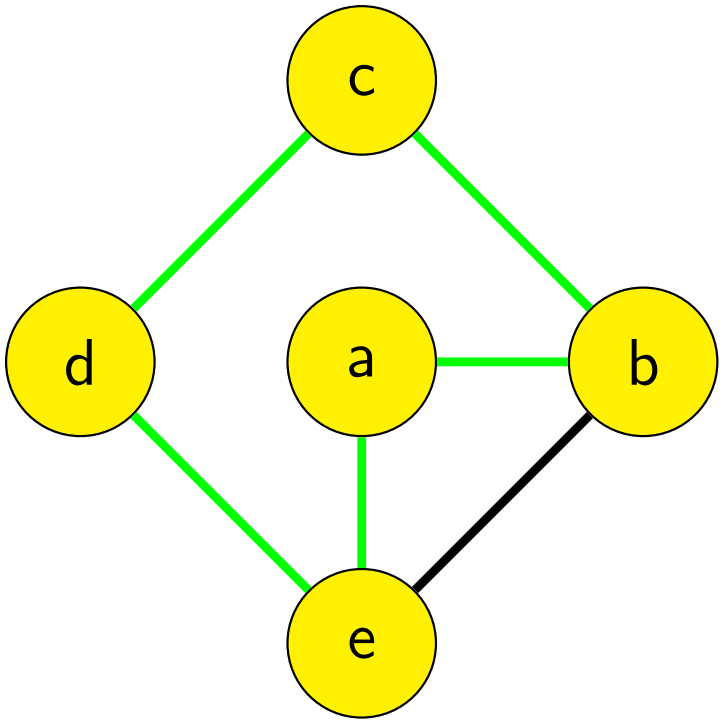
- Find a path

Happy now??



- Finally find Hamiltonian Graph!!!!

Let's get back to First example



- Is it now a Hamiltonian Graph?

Applications

- Travelling Salesman Problem (TSP)
- Circuit Design:
- Network Routing
- Vehicle Routing
- And many more....