Quantum Image Processing - FRQI Image Representations

Quantum image representation (QIR) is a necessary part of quantum image processing (QIP) and plays an important role in quantum information processing. Research of QIP is mainly divided into two aspects: quantum image representations and quantum image processing algorithms. Among them, the quantum image representation, as the basis of image processing, specifies the representation of images in a quantum computer. The quantum image representation model plays an important role as the basis of quantum image processing. There have been many research results on quantum image representation models, here I describe a flexible representation for quantum images (FRQI). These quantum image representations encode colour pixels as well as positions in different ways, making them somewhat different in terms of image processing applications and algorithmic complexity.

Flexible representation for quantum images (FRQI)

A Flexible Representation of Quantum Images (FRQI) is proposed to provide a representation for images on quantum computers in the form of a normalized state which captures information about colours and their corresponding positions in the images. A constructive polynomial preparation for the FRQI state from an initial state, an algorithm for quantum image compression (QIC), and processing operations for quantum images are combined to build the whole process for quantum image processing on FRQI. The simulation experiments on FRQI include storing, retrieving of images and a detection of a line in binary images by applying quantum Fourier transform as a processing operation. The goal of the Flexible Representation of Quantum Images (FRQI) is to provide a quantum representation of images that allows an efficient encoding of the classical data into a quantum state and the subsequent use of operators for image processing operations. In this case, encoding the classical image into a quantum state requires a polynomial number of simple gates.

Here I propose, a flexible representation of quantum images (FRQI) which captures information about colours and their corresponding positions in an image into a normalized quantum state.

The FRQI State

The quantum state representing the image is:

$$|I(\theta)\rangle = \frac{1}{2^n}\sum_{i=0}^{2^{2n}-1}(\cos\theta_i\,|0\rangle + \sin\theta_i\,|1\rangle)\otimes|i\rangle \qquad (1.1)$$

$$\theta_i \in \left[0, \frac{\pi}{2}\right], i = 0, 1, \cdots, 2^{2n}-1 \qquad (1.2)$$

The FRQI state is a normalized state as from equation (1.1) we see that $\|I(\theta)\| = 1$ and is made of two parts:

- color information encoding: $\cos\theta_i\,|0\rangle + \sin\theta_i\,|1\rangle$
- associated pixel position encoding: $|i\rangle$

A 2×2 image is given below, with corresponding θ angles (colour encoding) and associated kets (position encoding) :

$\theta_0, \|00\rangle$	$\theta_1, \|01\rangle$
$\theta_2, \|10\rangle$	$\theta_3, \|11\rangle$

And the equivalent quantum state is

$$
\begin{aligned}
|I\rangle = \frac{1}{2}[\quad & (\cos\theta_0 |0\rangle + \sin\theta_0 |1\rangle) \otimes |00\rangle \\
& + (\cos\theta_1 |0\rangle + \sin\theta_1 |1\rangle) \otimes |01\rangle \\
& + (\cos\theta_2 |0\rangle + \sin\theta_2 |1\rangle) \otimes |10\rangle \\
& + (\cos\theta_3 |0\rangle + \sin\theta_3 |1\rangle) \otimes |11\rangle \ ]
\end{aligned}
\tag{1.3}
$$

Building the FRQI State:

From an initialized state $|0\rangle^{\otimes 2n+1}$ to the FRQI state is in full superposition, except for the last qubit which encoded the colour. $H^{\otimes 2n}$ being the tensor product of $2n$ Hadamard operations, intermediate state is

$$|H\rangle = \frac{1}{2^n}|0\rangle \otimes \sum_{i=0}^{2^{2n}-1} |i\rangle = \mathcal{H}\left(|0\rangle^{\otimes 2n+1}\right) \tag{2.1}$$

As demonstrated in [1] there exist a unitary transformation $\mathcal{P} = \mathcal{RH}$ transforming the initial state $|0\rangle^{\otimes 2n+1}$ into the FRQI $I(\theta)$ state and

$$\mathcal{R}|H\rangle = \left(\prod_{i=0}^{2^{2n}-1} R_i\right)|H\rangle = |I(\theta)\rangle \tag{2.2}$$

The R_i operations are controlled rotations matrices defined by:

$$R_i = \left(I \otimes \sum_{j=0, j\neq i}^{2^{2n}-1} |j\rangle\langle j|\right) + R_y(2\theta_i) \otimes |i\rangle\langle i| \tag{2.3}$$

Where $R_y(2\theta_i)$ are the standard rotation matrices:

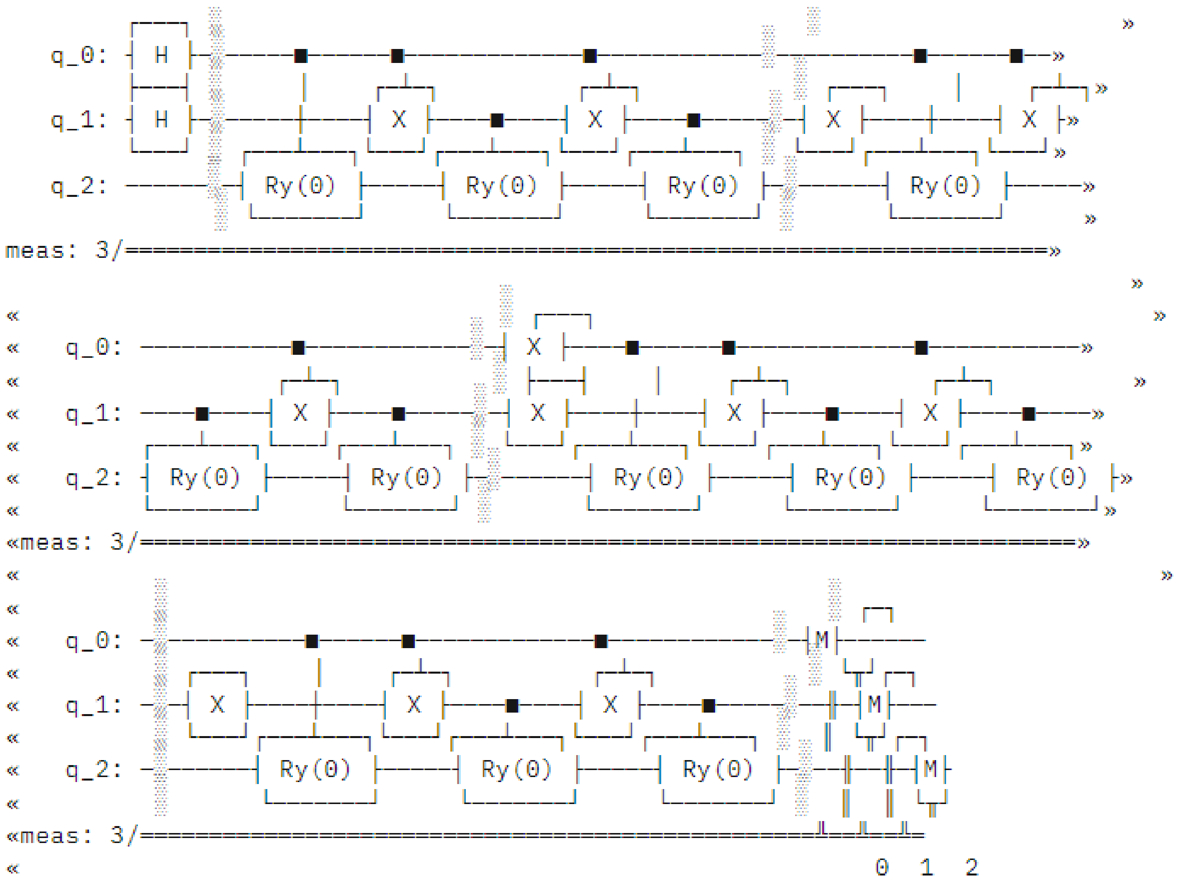
$$R_y(2\theta_i) = \begin{pmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{pmatrix} \tag{2.4}$$

The controlled rotations can be implemented via the generalized $C^{2n}(R_y(2\theta_i))$ which can be broken down into standard rotations and $CNOT$ gates. For instance, if the case for $n=1$, 4 pixels (i.e. a 2×2 image)

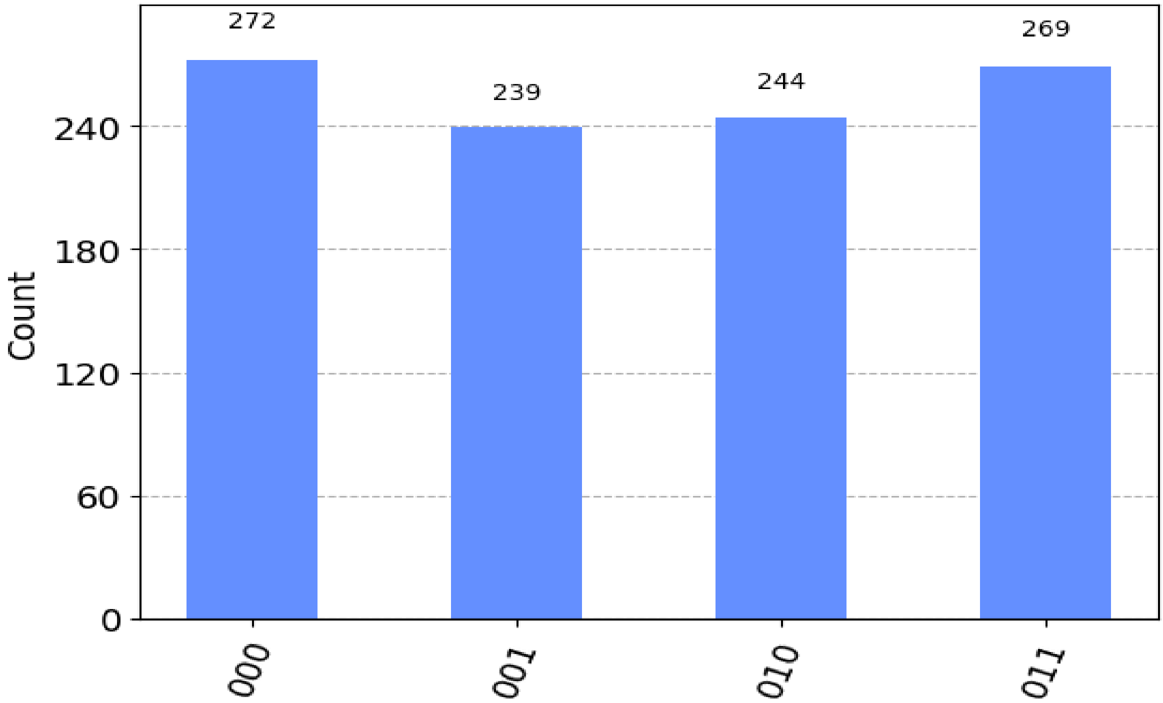
Implementation and Measurement: 2×2 Image with Greyscale Values

Barriers are used for added clarity on the different blocks associated with individual pixels. Greyscale images (i.e. the L component of a LRGB image), which means only one value is of interest for the colour encoding: the intensity is used. In other words, all angles θ_i equal to 0 means that all the pixels are black, if all θ_i values are equal to $\pi/2$then all the pixels are white, and so on.

$\boldsymbol{\theta_i=\pi/2}$, all pixels at maximum intensity



Measurement and Image Retrieval



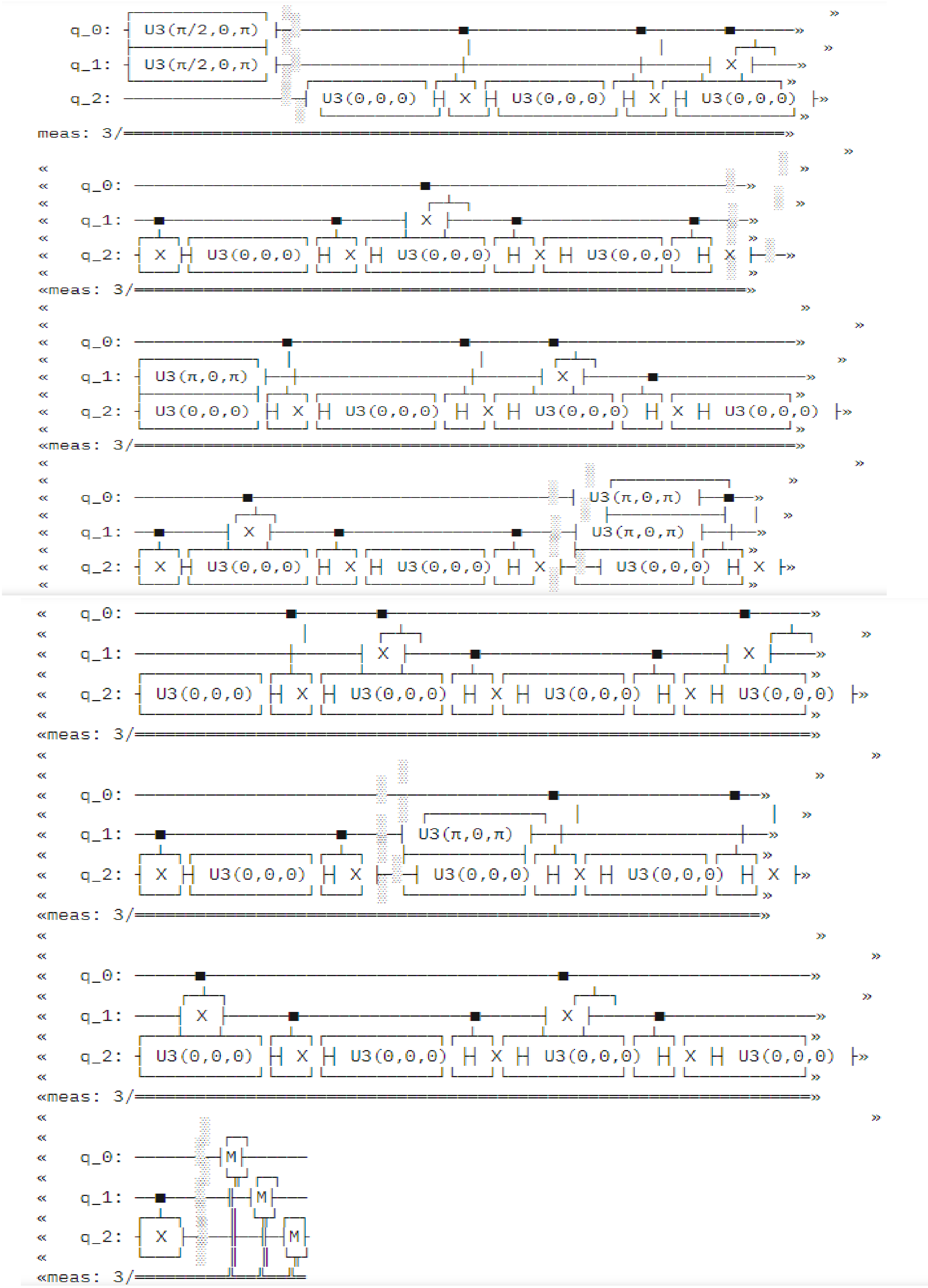
Circuit Analysis and Running on a Real Device

As the only difference between the circuits is the rotation angle θ, the depth and number of gates needed for this class of circuits (2×2 images):

Circuit Analysis

With $\theta_i = \pi/2$ (maximum intensity for all pixels).

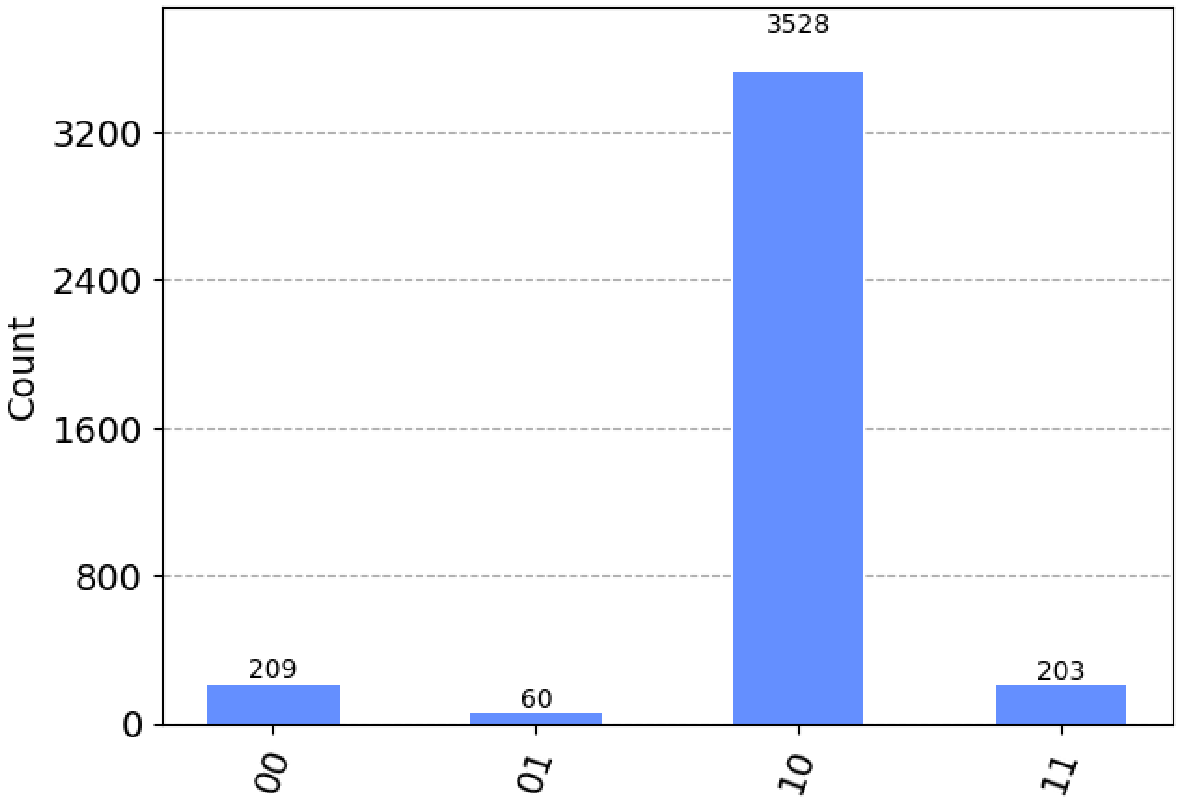
Depth : 23
Operations: OrderedDict([('cry', 12), ('cx', 8), ('barrier', 5), ('x', 4), ('measure', 3), ('h', 2)])



Depth : 50
Operations: OrderedDict([('cx', 32), ('u3', 30), ('barrier', 5), ('measure', 3)])

Run on a Real Device

By simulating on real device : ibmq_quito



Conclusions :

To establish a foundation for quantum image processing operations using unitary operators and the polynomial preparation process, a proposal has been made for a flexible representation of quantum images (FRQI). The FRQI is capable of capturing image colours and their corresponding positions in a quantum state, and a quantum circuit using Hadamard gates and controlled rotation gates is suggested for the transform. The positions in an image can be grouped based on their colour, and the QIC algorithm can be used on this information to reduce the number of simple gates required for FRQI preparation. QIC is based on minimizing Boolean equations derived from the binary strings encoding positions in the same colour groups. Quantum image processing operators based on unitary transforms are addressed on FRQI and divided into three categories, depending on whether they deal with colours, colours at specific positions, or a combination of colours and positions. Experiments were conducted on FRQI, including storing and retrieving quantum images, compression ratios of the QIC algorithm, and the application of QFT as an image processing operation. The results indicate that the FRQI can serve as a fundamental tool for representing and processing images on quantum computers, and the QIC demonstrates the efficiency of FRQI in both theory and practice, as evidenced by the results of PPT and measurements of identical quantum states.

References

1. Le, P.Q., Dong, F. & Hirota, K. A flexible representation of quantum images for polynomial preparation, image compression, and processing operations. *Quantum Inf Process* **10**, 63–84 (2011).
2. Su, J., Guo, X., Liu, C. *et al.* An improved novel quantum image representation and its experimental test on IBM quantum experience. *Sci Rep* **11**, 13879 (2021).
3. https://qiskit.org/textbook/ch-applications/image-processing-frqi-neqr.html