AUDIO LABS

A Method to Analyze the Spatial Response of Informed Spatial Filters

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1. Introduction

Motivation

- Informed spatial filters (ISFs) [1] are designed to capture multiple sound sources with a desired, arbitrary spatial response at each TF instant while attenuating undesired signal components.
- The desired spatial response function is different from the directivity pattern of the spatial filter.
- A comparative analysis of the obtained and the desired spatial response has not been performed yet.

Aim

- Propose a method to analyse the obtained spatial response and compare it to the desired response.
- Gain insight into the obtained spatial response at the output of the spatial filter.
- Analyse the effect of DOA estimation errors on the obtained spatial response, and provide an objective motivation for the need of robustness against DOA estimation errors.

2. Informed Spatial Filter: Review

- Consider a uniform linear array (ULA) of M microphones located at $\mathbf{d}_{1\ldots M}$. The microphone signals are given by

$$\mathbf{y}(n,k) = \underbrace{\sum_{l=1}^{L} \mathbf{x}_l(n,k)}_{\mathbf{x}(n,k)} + \mathbf{x}_\mathrm{n}(n,k)$$

- Assuming signal components are mutually uncorrelated

$$\boldsymbol{\Phi}_\mathbf{y}(n,k) = \mathrm{E}\{\mathbf{y}(n,k)\mathbf{y}^\mathrm{H}(n,k)\} = \boldsymbol{\Phi}_\mathbf{x}(n,k) + \boldsymbol{\Phi}_\mathrm{n}(n,k)$$

- Desired signal

$$Z(n,k) = \sum_{l=1}^{L} G(\theta_l,k) X_l(n,k,\mathbf{d}_1)$$

- Estimate of the desired signal

$$\widehat{Z}(n,k) = \mathbf{w}^\mathrm{H}(n,k)\mathbf{y}(n,k)$$

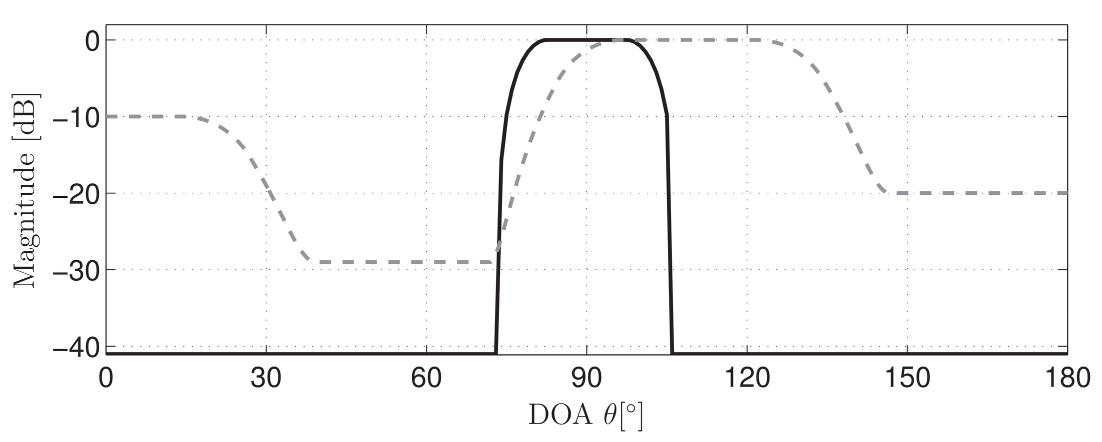
Definitions

- Microphone signal of l-th plane wave
  $\mathbf{x}_l(n,k) = [X_l(n,k,\mathbf{d}_1)\ldots X_l(n,k,\mathbf{d}_M)]^T$
- Sound pressure of l-th plane wave
  $\mathbf{x}_l(n,k) = \mathbf{a}(\theta_l,k)X_l(n,k,\mathbf{d}_1)$
- m-th element of steering vector
  $a_m(\theta_l,k) = \exp\{-j\kappa r_m \cos\theta_l(n,k)\}$
- Noise PSD matrix
  $\boldsymbol{\Phi}_\mathrm{n}(n,k) = \phi_\mathrm{n}(k)\,\mathbf{I}$
- Direction dependent gain
  $G(\theta_l,k)$
- Filter weights
  $\mathbf{w}(n,k)$
- Propagation vectors of L sources
  $\mathbf{A} = [\mathbf{a}(\theta_1,k),\ldots,\mathbf{a}(\theta_L,k)]$

Informed LCMV filter

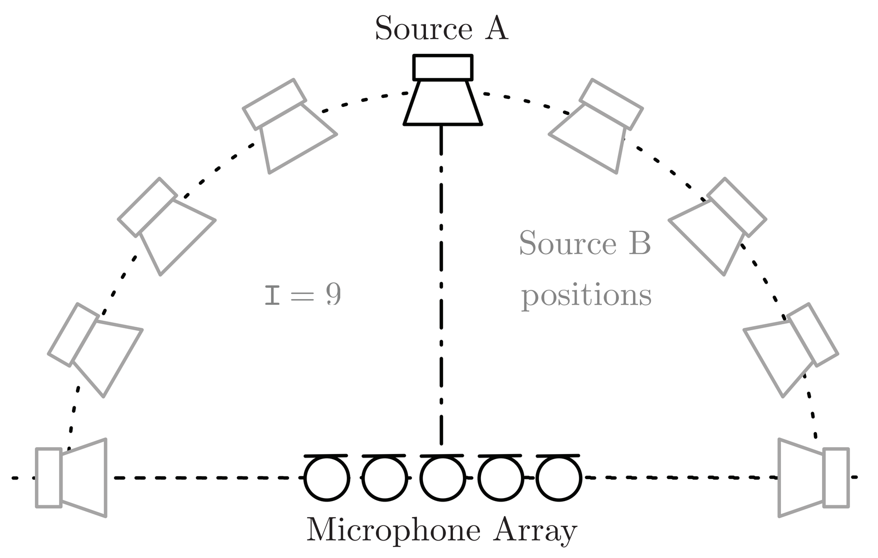
- Formulation: $\mathbf{w}(n,k) = \arg\min_{\mathbf{w}} \mathbf{w}^\mathrm{H}\boldsymbol{\Phi}_\mathrm{n}(n,k)\mathbf{w}$ s.t. $\mathbf{w}^\mathrm{H}(n,k)\mathbf{a}(\theta_l,k) = G(\theta_l,k)$
- Solution: $\mathbf{w}(n,k) = \boldsymbol{\Phi}_\mathrm{n}^{-1}\mathbf{A}\left[\mathbf{A}^\mathrm{H}\boldsymbol{\Phi}_\mathrm{n}^{-1}\mathbf{A}\right]^{-1}\mathbf{g}$ with $\mathbf{g} = [G(\theta_1,k),\ldots,G(\theta_L,k)]^\mathrm{H}$
- The direction dependent gain $G(\theta_l,k)$ corresponds to the value of an arbitrary spatial response function $\mathrm{g}(\theta,k)$, evaluated at the DOA of the l-th plane wave.

3. Desired Spatial Response



- The spatial response is an arbitrary, user-defined function that can be potentially complex valued and frequency dependent.
- The design of the desired spatial response function is also dependent on the application.

4. Proposed Analysis Method



Setup

- Assumption: L = 2
- Sample complete DOA range at I discrete points.
- Source A is kept static at broadside
- Source B is moved through the whole DOA range, placing it at the Idiscrete points.

Average Directional Array Gain

$$\mathcal{A}_{\mathrm{d},i} = \frac{1}{\mathrm{card}(\mathcal{T}_\mathrm{i})}\sum_{(n,k)\in\mathcal{T}_\mathrm{i}} \frac{G_\mathrm{A}(n,k)}{G_{\mathrm{B,i}}(n,k)}$$

$$\mathcal{T}_\mathrm{i} = \{(n,k) : |X_\mathrm{A}(n,k,\mathbf{d}_1)| \geq \epsilon_\mathrm{A} \wedge |X_{\mathrm{B,i}}(n,k,\mathbf{d}_1)| \geq \epsilon_\mathrm{B}\}$$

$$G_\mathrm{A}(n,k) = \frac{|\tilde{X}_\mathrm{A}(n,k)|^2}{|X_\mathrm{A}(n,k,\mathbf{d}_1)|^2}$$

$$G_{\mathrm{B,i}}(n,k) = \frac{|\tilde{X}_{\mathrm{B,i}}(n,k)|^2}{|X_{\mathrm{B,i}}(n,k,\mathbf{d}_1)|^2}$$

- **Proposed method can also be used for L =1 or L > 2.**
- **By varying the input signal-to-interference ratio (iSIR) the proposed method can be used to identify the critical iSIR where the single plane wave signal model is violated.**

5. Simulation Experiment

Simulation Setup

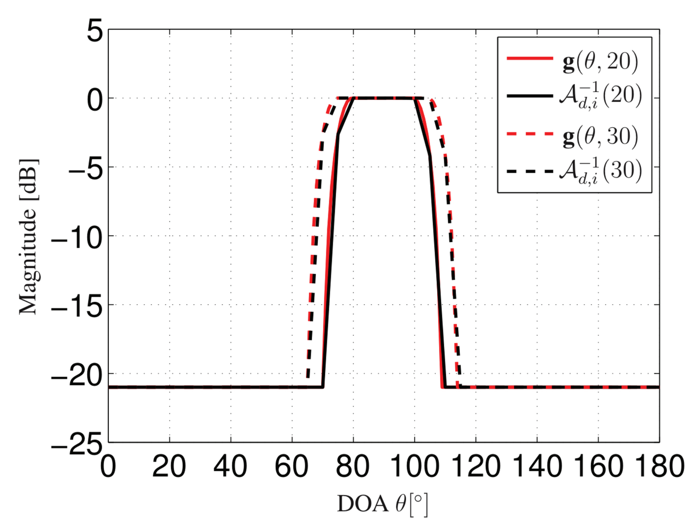
- ULA with M=5
- Source B placed at I = 37 discrete points
- Anechoic environment
- Source-microphone distance: 1.8 m

DOA estimates with error

$$\hat{\theta}_\mathrm{A}(n,k) = \theta_\mathrm{A} + \Delta\theta_\mathrm{A}(n,k),$$
$$\hat{\theta}_{\mathrm{B,i}}(n,k) = \theta_{\mathrm{B,i}} + \Delta\theta_{\mathrm{B,i}}(n,k)$$

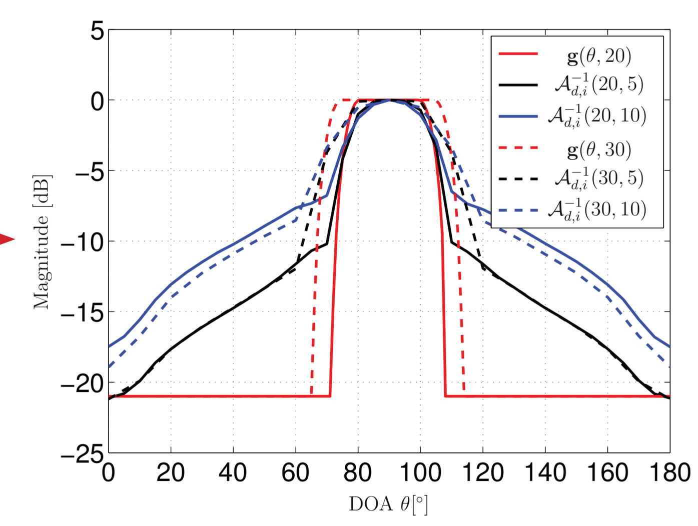
Error variance

$$\sigma^2_{\mathrm{DOA}} = \mathrm{E}\{\Delta\theta^2\}$$



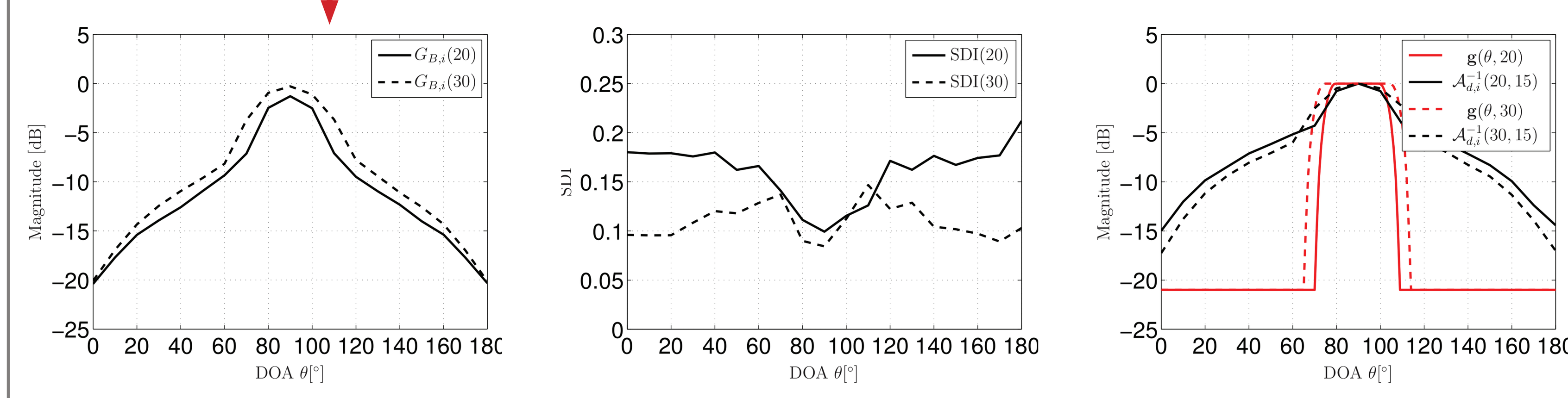
Obtained vs Desired : Known DOAs

- DOA error variance: 5° and 10°
- Passband widths: 20° and 30°



Obtained vs Desired : DOA errors + Different passband widths

- DOA error variance : 15°
- Passband widths: 20° and 30°



Array Gain: Source B | SDI | Obtained vs Desired

6. Conclusions

- A method to analyze the ability of ISF to obtain an arbitrary, user-defined, desired spatial response at the output of the filter was presented.
- With perfect knowledge of the source DOAs the desired spatial response can be obtained at the output of the ISF.
- Through the analysis it was shown that robustness against DOA estimation errors is important and essential for the ISF framework.

[1] O. Thiergart, M. Taseska, and E.A.P. Habets, "An informed parametric spatial filter based on instantaneous direction-of-arrival estimates," IEEE Trans Acoust., Speech, Signal Process., vol. 22, no. 12, pp.2182–2196, Dec 2014.