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Comment on the Anisotropy of the Primeval Fireball*

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This note presents an exact theory for the anisotropy of the primeval fireball as a function of our motion through the radiation.

An important application of the primeval fireball is that one should be able to detect our motion relative to this sea of blackbody radiation. In the Princeton isotropy experiment[1] an upper limit to this velocity was obtained using the fact that, in first order, the fractional perturbation to the radiation temperature in a given direction ought to be equal to the component of our velocity in that direction, divided by the velocity of light. Apparently this effect is not well understood. Recently Condon and Harwit[2] derived a formula for the front-back asymmetry in the radiation flux. An erratum[2] corrects an error (due to neglect of aberration on the detector solid angle) which gave the wrong sign and magnitude at the wavelength of the Princeton experiment. Our purpose in this note is to present the full derivation of this very important effect, taking account of all angles of detector orientation, and all orders in v/c. The result is simply a variation of the blackbody radiation temperature with orientation and velocity.

Consider two observers immersed in the blackbody radiation: O is at rest relative to the radiation, and he sees strictly isotropic blackbody radiation, and O' is moving with speed v along the X axis of O. The moving observer carries with him a detector with collecting area A', with its normal at angle θ' to the axis. The detector responds to radiation incident normal to its surface within the solid angle $d\Omega'$, and in the bandwidth $d\nu'$. We introduce the area $A_0 \equiv A'/|\cos\theta'|$, perpendicular to v, and through which the photons have passed to get to A'. This is convenient because A_0 has the same magnitude and orientation in both frames of reference.

Let $n'(\nu', \theta')$ be the photon density measured in units of photons $\mathrm{cm}^{-3}\ \mathrm{sr}^{-1}\ \mathrm{Hz}^{-1}$ as seen by observer O'. Then in time dt' observer O' finds that his detector reponds to

$$d\mathfrak{N} = n'(\nu',\theta')d\nu' d\Omega' A_0 |\cos\theta'| c dt' \tag{1}$$

photons. In the same interval, observer O sees the same number of counts, but expresses it as

$$d\mathfrak{N} = n(\nu)d\nu d\Omega A_0 |v + c\cos\theta| dt, \tag{2}$$

where $A_0|v+c\cos\theta|dt$ is the volume swept out by the $d\mathfrak{N}$ photons.

The quantities needed to relate $n'(\nu',\theta')$ and $n(\nu)$ transform as follows[3]:

$$dt' = dt(1-v^2/c^2)^{1/2}, \tag{3}$$

$$\cos\theta' = (\cos\theta + v/c)[1+(v/c)\cos\theta]^{-1}, \tag{4}$$

$$d\Omega' = d\varphi' d\cos\theta' = d\Omega(1-v^2/c^2)[1+(v/c)\cos\theta]^{-2}, \tag{5}$$

$$\nu' = \nu[1+(v/c)\cos\theta][1-(v/c)^2]^{1/2}. \tag{6}$$

Using Eqs. (1)–(6), we have

$$n'(\nu',\theta') = n(\nu)(\nu'/\nu)^2. \tag{7}$$

We express this result in terms of the effective thermodynamic temperature $\mathcal{T}'$ by means of the Planck formula,

$$n'(\nu',\theta') = 2\nu'^2 c^{-3}[\exp(h\nu'/k\mathcal{T}') - 1]^{-1}. \tag{8}$$

It is evident from Eqs. (6)–(8) that

$$\mathcal{T}'(\theta') = \mathcal{T}(1-v^2/c^2)^{1/2}[1-(v/c)\cos\theta']^{-1}, \tag{9}$$

where $\mathcal{T}$ is the temperature of the radiation in the frame O. The Lorentz transformation thus changes the radiation temperature; but the observer O' looking in the fixed direction θ' still would map out a blackbody spectrum. Evidently, at any wavelength of observation the temperature $\mathcal{T}'$ is greatest in the direction of v, so the detected power is greatest in the direction of v.

Equation (7) is a special relativity effect, but the general Liouville theorem[4] also applies here, and gives the same answer.

To first order in v/c, a moving radiometer pointed at direction θ relative to v measures blackbody radiation of temperature $\mathcal{T}[1+(v/c)\cos\theta]$. This is the formula used in the Princeton isotropy experiment.[1]

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