# Width Parameter

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## 1 Band Function

Band Function:

$$E^2\mathrm{N(E)} = \mathrm{N}_0 \begin{cases} E^{\alpha+2}\exp\left(-\frac{E}{E_0}\right) & \text{for } E < (\alpha-\beta)E_0 \\ \left[(\alpha-\beta)E_0\right]^{(\alpha-\beta)} E^{\beta+2}\exp(\beta-\alpha) & \text{for } E > (\alpha-\beta)E_0 \end{cases}$$

$where$ ( $\alpha-\beta)E_0$ is the break energy.

## 2 Expression for FWHM

To find the FWHM, we set the expressions of both power laws in the Band function equal to half of the peak value of $E^2N(E)$ which is obtained by substituting E in the low energy part with $E_p = (\alpha+2)E_0$.

For the energy corresponding to the FWHM, falling on the low energy part of the spectrum, we considered the case of $E > (\alpha - \beta)E_0$

$$E^{\alpha+2} exp(-\frac{E}{E_0}) = \frac{[(\alpha+2)E_0]^{\alpha+2} e^{-(\alpha+2)}}{2} \tag{1}$$

Now, let a $= \alpha + 2$

$$E^{a} exp(-\frac{E}{E_0}) = \frac{a^a \cdot e^{-a}}{2} \tag{2}$$

Taking natural log on both sides.

$$a \cdot ln(\frac{E}{E_0}) - \frac{E}{E_0} = a \cdot ln(\frac{a \cdot e^{-1}}{2}) \tag{3}$$

$$\tag{4}$$

On dividing both sides by a we get

$$ln(\frac{E}{E_0}) - \frac{E}{a \cdot E_0} = ln(\frac{a \cdot e^{-1}}{2^{1/a}}) \tag{5}$$

$$\tag{6}$$

Taking exponent on both sides

$$\frac{E}{E_0} \cdot exp(-\frac{E}{aE_0}) = \frac{a}{e \cdot 2^{1/a}} \tag{7}$$

Dividing both sides by -a

$$-\frac{E}{a \cdot E_0} exp(-\frac{E}{a \cdot E_0}) = -\frac{1}{e \cdot 2^{1/a}} \tag{8}$$

$$\tag{9}$$

By applying Lambert W function on both sides and using the fact that $W(x \cdot e^x) = x$ we get

$$-\frac{E}{a \cdot E_0} = W(-\frac{a}{e \cdot 2^{1/a}}) \tag{10}$$

Therefore, the energy corresponding to FWHM falling on the lower energy part of the spectrum is given by

$$E_1 = -a \cdot E_0 \cdot W(-\frac{1}{e \cdot 2^{1/a}}) \tag{11}$$

For the energy corresponding to the FWHM, falling on the high energy part of the spectrum, we considered the case of $E > (\alpha - \beta)E_0$

$$[(\alpha - \beta)E_0]^{\alpha-\beta} \cdot E^{\beta+2} \cdot e^{(\beta-2)} = \frac{(aE_0)^a \cdot e^{-a}}{2} \tag{12}$$

After cancelling the common terms on both sides we get

$$E^{\beta+2} = \frac{(aE_0)^a \cdot e^{-(\alpha+2)}}{2 \cdot [(\alpha - \beta)E_0]^{\alpha-\beta} \cdot e^{(\beta-2)}} \tag{13}$$

$$E^{\beta+2} = \frac{a^a \cdot E_0^{\beta+2} \cdot e^{-(\beta+2)}}{2 \cdot (\alpha - \beta)^{\alpha-\beta}} \tag{14}$$

After taking log on both sides and rearranging the terms we get

$$log(E_2) = log(\frac{a^{\frac{a}{\beta+2}} E_0 \cdot e^{-1}}{2^{\frac{1}{\beta+2}} \cdot (\alpha - \beta)^{\frac{\alpha-\beta}{\beta+2}}}) \tag{15}$$

Where $E_2$ is the energy corresponding to FWHM falling on the high energy part of the spectrum.

Now, FWHM is defined as $W' = log(E_2) - log(E_1)$ (We're taking FWHM as W' because we have already used W for Lambert W function)

$$W' = log(\frac{a^{\frac{a}{\beta+2}} E_0 \cdot e^{-1}}{2^{\frac{1}{\beta+2}} \cdot (\alpha - \beta)^{\frac{\alpha-\beta}{\beta+2}}}) - log(-a \cdot E_0 \cdot W(-\frac{1}{e \cdot 2^{1/a}})) \tag{16}$$

By using the fact that $log(a) - log(b) = log(a/b)$, cancelling out $E_0$ and simplifying the power of a we get

$$W' = log(\frac{-a^{\frac{\alpha-\beta}{\beta+2}} \cdot e^{-1}}{2^{\frac{1}{\beta+2}} \cdot (\alpha - \beta)^{\frac{\alpha-\beta}{\beta+2}} \cdot W(-\frac{1}{e \cdot 2^{1/a}})}) \tag{17}$$

Therefore, we can see that W' depends only on alpha and beta.

Our next steps are to simplify the expression and derive $\beta$ in terms of $W'$ and $\alpha$. We will then develop a local model for XSPEC to fit time-resolved spectra of single-peaked GRBs with high fluence, allowing us to study the evolution of the width parameter.