LAA Homework 6 (Report)

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Expression Used in Classification

The key expression used in the digit classification model is:

$$\rho_n(z) := \min \|D_n x - z\|$$

Where:

- $\rho_n(z)$ is the residual norm for digit n.
- D_n is the matrix whose columns are the principal components (obtained via Singular Value Decomposition) of the images corresponding to digit n. Although we have named D_n as $array_n$.
- x is the vector of coefficients that best approximates the test image n in the subspace formed by the columns of D_n.
- z is the flattened vector representing the test image.

Justification for Using This Expression

Dimensionality Reduction

The matrix D_n, formed by the first few singular vectors, represents a lower-dimensional space that captures the most variance (or most critical information) of the training images for the digit n. This reduction in dimensionality accelerates computations and reduces the effects of noise and overfitting.

Efficiency

Instead of using the full matrix of image data for n, using D_n focuses on the most significant aspects of the images, making the computation more efficient without a substantial loss of information. The computation involves fewer dimensions, hence it's computationally less expensive and faster.

Optimal Approximation in Subspace

By minimizing the norm $\|D_n x - z\|$, the model seeks the point in the subspace spanned by D_n that is closest to the test image z. This is equivalent to finding the best approximation of z within the subspace defined by the principal components of D_n, which are expected to be the most representative features of that digit.

Robustness to Variance

Since D_n captures the primary modes of variation among the images of digit n, the approach is robust against variations within the class represented by n. It effectively handles different handwriting styles and distortions common in handwritten digit recognition tasks.

Quantitative Comparison Across Classes

By calculating $\rho_n(z)$ for each digit n and selecting the n with the minimum residual, the model quantitatively compares how closely z resembles each digit class. This method ensures that the classification is based on the most objective criterion available in the context of the data's principal components.