Optimization Project Report

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Problem Statement:

As per a CMO survey, marketing budgets are generally 11% of the total company budget. However, it can be seen that a marketing budget often results in either little to no improvement in effectiveness of marketing, or a mass improvement. One main reason for this could be how the marketing budget is allocated, and how much each company invests in advertisement.

This study aims to find the optimal allocation of the company's marketing budget. Our company has been approved a budget of $10M for the year, and we are focusing on how to distribute it. However, there are certain restrictions to the budget that are as follow:

Requirements:

1. The amount invested in print and TV should be no more than the amount spent on Facebook and Email. Surprisingly, email seems to be a great channel for reaching real people.
2. The total amount used in social media (Facebook, LinkedIn, Instagram, Snapchat, and Twitter) should be at least twice of SEO and AdWords.
3. For each platform, the amount invested should be no more than $3M.

Approach:

Consulting Firm I:

Our department has assisted in the help of an outside consulting firm, who has suggested the following spread of the budget:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	3.1%	4.9%	2.4%	3.9%	1.6%	2.4%	4.6%	2.6%	3.3%	4.4%

We first had to create an objective function for the ROI to be maximized. Using Gurobi, we used the following code:

```
[ ]  # gurobi
     marketingModel = gp.Model()

     marketingModX = marketingModel.addMVar(10)
     marketingModCon = marketingModel.addMConstrs(A, marketingModX, sense, b)
     marketingModel.setMObjective(None,obj,0,sense=gp.GRB.MAXIMIZE)

     marketingModel.Params.OutputFlag = 0
     marketingModel.optimize()
```

```
[ ]  df = pd.DataFrame(columns = roi_data.columns.values.tolist() + ['Optimal'])
     df.loc['First Consulting Firm'] = marketingModel.x + [marketingModel.objval]
```

Through running a linear program, we found that the optimal budget allocation is as follows:

The budget allocation for each medium and the optimal ROI is

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	Optimal
First Consulting Firm	0.0	3000000.0	0.0	1000000.0	0.0	0.0	3000000.0	0.0	0.0	3000000.0	456000.0

We can see that there is a higher importance on TV, AdWords, Instagram, and Email as they have proved to have a higher effect than the other channels provided. The highest amount allocated belongs to TV, Instagram, and Email, with each channel having $3 Million to use for advertising.

Consulting Firm II:

There can often be doubts about optimized solutions. We cannot be entirely sure if this is the best way to allocate the budget efficiently. To ensure that this budget allocation method is optimal and will be successful, we decided to compare this allocation to other consulting firms. The second consulting firm provided the ROI estimates in the table that follows:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	4.9%	2.3%	2.4%	3.9%	4.4%	4.6%	2.6%	1.9%	3.7%	2.6%

We then repeated the same operations using the ROI estimates from the second firm. The constraints remained the same.

```
[ ]  # gurobi
     marketingModel = gp.Model()

     marketingModX = marketingModel.addMVar(10)
     marketingModCon = marketingModel.addMConstrs(A, marketingModX, sense, b)
     marketingModel.setMObjective(None,obj,0,sense=gp.GRB.MAXIMIZE)

     marketingModel.Params.OutputFlag = 0
     marketingModel.optimize()
```

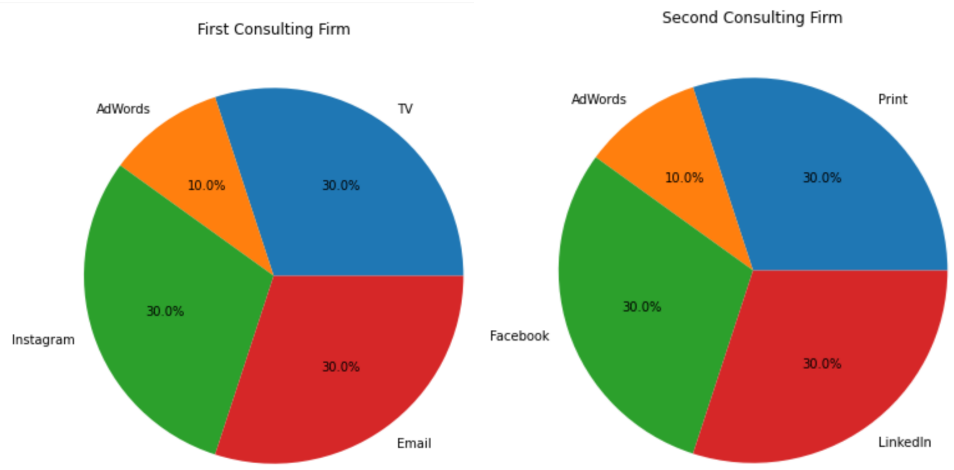
```
[ ]  df.loc['Second Consulting Firm'] = marketingModel.x + [marketingModel.objval]
```

After repeating the linear program for the ROI estimates, we found the budget allocation according to the second consulting firm and compiled them onto one table that follows:

The budget allocation for each medium and the optimal ROI as per the first consulting firm and second consulting firm is

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	Optimal
First Consulting Firm	0.0	3000000.0	0.0	1000000.0	0.0	0.0	3000000.0	0.0	0.0	3000000.0	456000.0
Second Consulting Firm	3000000.0	0.0	0.0	1000000.0	3000000.0	3000000.0	0.0	0.0	0.0	0.0	456000.0

As we can see from the table, the allocation varies noticeably. While the amount to each channel is approximately the same, the main channels used changed. We then decided to compare the two to see how they differed. The following two graphs show how each consulting firm allocated the budget.



Assuming the first ROI data is correct, if we were to use the second allocation, the objective relative to the optimal objective would lower by: 204000.0.

Assuming the second ROI data is correct, if we were to use the first allocation, the objective relative to the optimal objective would lower by: 192000.0.

```
CASE_1= df.loc['First Consulting Firm','Optimal'] - np.array(roi_data.loc['ROI'].tolist()) @ np.array(df.iloc[1,:-1].tolist())
print("Assuming the first ROI data is correct, if we were to use the second allocation, the objective relative to the optimal objective would lower by: " + str(CASE_1))
```

```
Assuming the first ROI data is correct, if we were to use the second allocation, the objective relative to the optimal objective would lower by: 204000.0
```

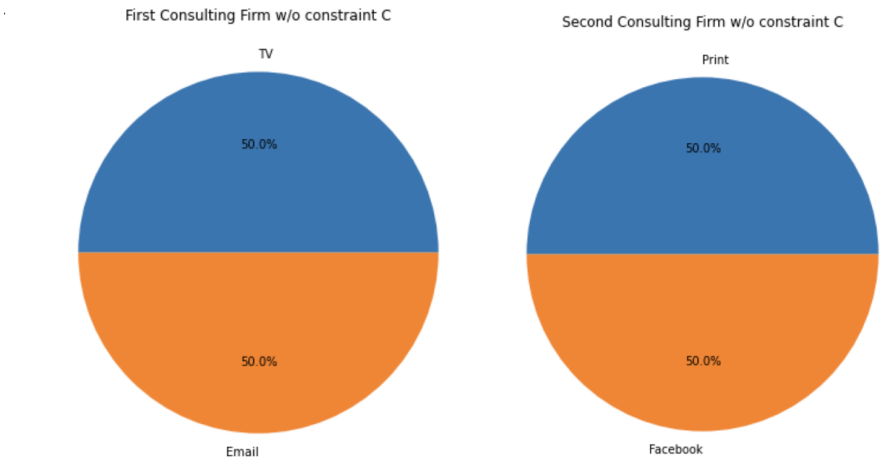
```
CASE_2= df.loc['Second Consulting Firm','Optimal'] - np.array(roi_data.loc['Second Firms ROI Estimate'].tolist()) @ np.array(df.iloc[0,:-1].tolist())
print("Assuming the second ROI data is correct, if we were to use the first allocation, the objective relative to the optimal objective would lower by: " + str(CASE_2))
```

```
Assuming the second ROI data is correct, if we were to use the first allocation, the objective relative to the optimal objective would lower by: 192000.0
```

The third constraint listed below has proved to be useful because if it were to be ignored as seen above, there would be a very uneven distribution among the channels. All of the budget would be put into two channels - TV & Email for the First and Print & Facebook for the Second Consulting Firm - which can be very risky because if they were to fail it would cause a huge loss and would result in low effectiveness.

```
The budget allocation for each medium and the optimal ROI as per the first consulting firm and second consulting firm if third constraint was ingnored is
```

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	Optimal
First Consulting Firm	0.0	5000000.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5000000.0	465000.0
Second Consulting Firm	5000000.0	0.0	0.0	0.0	5000000.0	0.0	0.0	0.0	0.0	0.0	465000.0



We then ran a sensitivity analysis to see the upper and lower bounds at which the optimal solution remains the same. We can see the results below:

```
Below table shows how much each advertising medium's ROI could increase/decrease while the optimal allocation from step (3) stays the same.
```

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Lower Bound	-inf	0.039	-inf	0.033	-inf	-inf	0.039	-inf	-inf	0.029
Upper Bound	0.049	0.062	0.039	0.046	0.029	0.039	inf	0.039	0.039	inf

The results of our upper and lower bound analysis essentially tell us by how much each advertising medium's ROI could change and still result in the same optimal allocation that we had found previously. One important thing to note here is that some of the values are positive or negative infinity. This implies that no matter how much we change in that direction (lower or upper), the optimal allocation for that medium will not change. Again, for the values that are not infinity, that is informing us of exactly how we could slightly tweak the ROI without impacting the optimal allocation that we had reported earlier.

We also had been given permission to reinvest half of the return from the previous month, as long as we kept the three original constraints in place. To account for this, we reran our optimization model with an updated amount for the budget in each month, corresponding to the previous total budget, plus half of the previous month's return. Our results are summarized in the table below, and the following code shows how each month's allocation is calculated using gurobi and how the budget is updated.

	month	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
0	January	3000000.00	0.0	0.0	1333333.33	0.00	0.00	2666666.67	0.0	0.00	3000000.00
1	February	3000000.00	0.0	0.0	2395500.00	3000000.00	0.00	0.00	0.0	1791000.00	0.00
2	March	0.00	0.0	0.0	3000000.00	0.00	3000000.00	1389648.00	0.0	3000000.00	0.00
3	April	0.00	0.0	0.0	3000000.00	0.00	3000000.00	3000000.00	0.0	1596856.49	0.00
4	May	1804099.90	0.0	0.0	0.00	0.00	0.00	3000000.00	0.0	3000000.00	3000000.00
5	June	3000000.00	0.0	0.0	0.00	0.00	0.00	3000000.00	0.0	2020171.65	3000000.00
6	July	0.00	0.0	0.0	3000000.00	2247554.91	0.00	3000000.00	0.0	3000000.00	0.00
7	August	3000000.00	0.0	0.0	1827294.08	0.00	654588.16	0.00	0.0	3000000.00	3000000.00
8	September	1362932.65	0.0	0.0	3000000.00	0.00	3000000.00	0.00	0.0	3000000.00	1362932.65
9	October	0.00	0.0	0.0	3000000.00	0.00	3000000.00	3000000.00	0.0	0.00	2955475.27
10	November	3000000.00	0.0	0.0	2056420.96	0.00	1112841.91	3000000.00	0.0	0.00	3000000.00
11	December	3000000.00	3000000.0	0.0	427950.65	3000000.00	0.00	0.00	0.0	0.00	3000000.00

```
for index, row in roi_month.iterrows():
    roi_list = [row[1]/100,row[2]/100,row[3]/100,row[4]/100,row[5]/100,row[6]/100,row[7]/100,row[8]/100,row[9]/100,row[10]/100]
    obj = np.array(roi_list) # objective vector

    A = np.zeros((3,10)) # initialize constraint matrix
    A[0,:] = [1,1,1,1,1,1,1,1,1,1] # 10M
    A[1,:] = [1,1,0,0,-1,0,0,0,0,-1] # Constraint A
    A[2,:] = [0,0,-2,-2,1,1,1,1,1,0] # Constraint B

    b = np.array([spend,0,0])
    sense = np.array(['<','<','>']) # all constraints are less than or equal constraints

    marketingModel = gp.Model()
    marketingModX = marketingModel.addMVar(10, ub = np.array([3000000,3000000,3000000,3000000,3000000,3000000,3000000,3000000,3000000,3000000]))
    marketingModCon = marketingModel.addMConstrs(A, marketingModX, sense, b)
    marketingModel.setMObjective(None,obj,0,sense=gp.GRB.MAXIMIZE)
    marketingModel.Params.OutputFlag = 0
    marketingModel.optimize()

    spend = spend + (marketingModel.objVal*0.5) # reinvest half of the return
```

As we can see from our results, if we are allowed to reinvest our previous returns, our optimal allocation of funds continuously changes. Some interesting things to note are the prevalence of

AdWords, Print, and some social media platforms, and the lack of prevalence of TV, SEO, Facebook, and Snapchat.

Finally, we checked to ensure that we had a stable budget, meaning that the monthly allocation for a particular platform does not exceed a change of $1 million, from one month to the next. We quickly calculated the monthly differences in budget allocation, summarized in the table below.

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
0	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
1	0.00	0.0	0.0	1062166.67	3000000.00	0.00	-2666666.67	0.0	1791000.00	-3000000.00
2	-3000000.00	0.0	0.0	604500.00	-3000000.00	3000000.00	1389648.00	0.0	1209000.00	0.00
3	0.00	0.0	0.0	0.00	0.00	0.00	1610352.00	0.0	-1403143.51	0.00
4	1804099.90	0.0	0.0	-3000000.00	0.00	-3000000.00	0.00	0.0	1403143.51	3000000.00
5	1195900.10	0.0	0.0	0.00	0.00	0.00	0.00	0.0	-979828.35	0.00
6	-3000000.00	0.0	0.0	3000000.00	2247554.91	0.00	0.00	0.0	979828.35	-3000000.00
7	3000000.00	0.0	0.0	-1172705.92	-2247554.91	654588.16	-3000000.00	0.0	0.00	3000000.00
8	-1637067.35	0.0	0.0	1172705.92	0.00	2345411.84	0.00	0.0	0.00	-1637067.35
9	-1362932.65	0.0	0.0	0.00	0.00	0.00	3000000.00	0.0	-3000000.00	1592542.62
10	3000000.00	0.0	0.0	-943579.04	0.00	-1887158.09	0.00	0.0	0.00	44524.73
11	0.00	3000000.0	0.0	-1628470.31	3000000.00	-1112841.91	-3000000.00	0.0	0.00	0.00

What we found was that we did not have a stable budget, as there are multiple instances where the change from month to month is greater than $1 million. To change the model so that there would be a stable budget, we would have to add a variable for each month in the code that would contain the optimal allocation for the previous month. Then, we would have to add a constraint that checks whether or not each month of the current year's optimal allocation is within $1 million of the previous month's budget. In terms of the business, we would recommend having a stable budget, as it would allow the business to better plan out the monthly financial advertising contributions.

This concludes our study into our firm’s marketing budget optimization.