Ratio & Proportion

General Terms and Rules:

1. **Ratio:**

   The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as a : b.

   In the ratio a : b, we call a as the first term or **antecedent** and b, the second term or **consequent**.

   Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

   Eg. 4 : 5 = 8 : 10 = 12 : 15. Also, 4 : 6 = 2 : 3.

2. **Proportion:**

   The equality of two ratios is called proportion.

   If $a : b = c : d$, we write $a : b :: c : d$ and we say that a, b, c, d are in proportion.

   Here a and d are called **extremes**, while b and c are called **mean terms**.

   Product of means = Product of extremes.

   Thus, $a : b :: c : d \Leftrightarrow$ **ad = bc**

3. **Fourth Proportional:**

   If $a : b = c : d$, then d is called the fourth proportional to a, b, c.

4. **Third Proportional:**

   $a : b = c : d$, then c is called the third proportion to a and b.

5. **Mean Proportional:**

   Mean proportional between a and b is $\sqrt{ab}$.

6. **Comparison of Ratios:**

   $$(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$$

7. **Compounded Ratio:**

   The compounded ratio of the ratios: $(a : b)$, $(c : d)$, $(e : f)$ is $(ace : bdf)$.

8. **Duplicate Ratios:**

Duplicate ratio of $(a : b)$ is $(a^2 : b^2)$.

Sub-duplicate ratio of $(a : b)$ is $(a^{1/2} : b^{1/2})$.

Triplicate ratio of $(a : b)$ is $(a^3 : b^3)$.

Sub-triplicate ratio of $(a : b)$ is $(a^{1/3} : b^{1/3})$.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-c} = \frac{c+d}{c-d}$ [Componendo and Dividendo]

9. **Variations:**

If x is directly proportional to y, we write,

$$x \propto y \Rightarrow x = ky \text{ for some constant } k$$

If x is indirectly proportional to y, we write,

$$x \propto \frac{1}{y} \Rightarrow x = \frac{k}{y} \Rightarrow xy = k, \text{ for some constant } k$$