Syllabus: Interference and diffraction – introduction – interference in thin film by reflection – Newton's rings – Fraunhofer diffraction due to single slit, double slit and diffraction grating

Interference

1. Introduction

In the 17th century, the properties of light were explained by Sir Isaac Newton and Christian Huygens. Sir Isaac Newton was explained the properties of light by introducing Corpuscular theory in 1675. It explains reflection, refraction, and dispersion properties of light. It fails to explain interference, diffraction, polarization, photo electric effect, and double refraction.

In 1679, Christian Huygens proposed the wave theory of light. According to Huygens wave theory, each point on the wave front is to be considered as a source of secondary wavelets. It explains reflection, refraction, dispersion, double refraction, diffraction, interference, and polarization properties of light. It fails to explain, photo electric effect, black body radiation etc,

2. Interference of light

The best evidence for the wave nature of light is interference phenomenon. This was experimentally demonstrated by Thomas Young in 180, through double slit experiment. Due to interference, we will observe many observations in our day today life, such as multiple colours on soap bubbles as well as on oil film when viewed under sun light. Interference concept is explained on the basis of superposition of wave's concept.

When two light waves superimpose, then the resultant amplitude or intensity in the region of superposition is different than the amplitude of individual waves.

Definition:-

The modification in the distribution of intensity in the region of superposition is known as interference.

In case of interference pattern we observe two cases

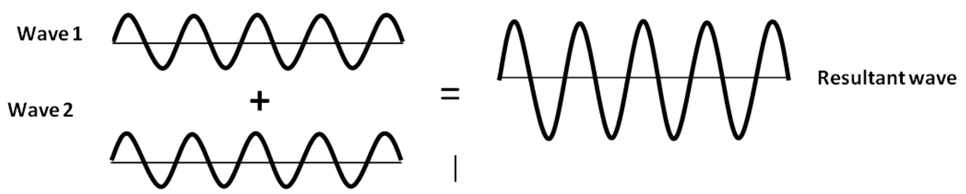
- Constructive interference
- Destructive interference

Constructive interference

- The waves are reaching at a point are in phase constructive interference occurs
- In constructive interference, the resultant amplitude is always equal to the sum of the amplitudes of two individual waves.
- *Condition*

The path difference between the two waves is equal to the integral multiple of wave length (λ) the constructive interference occurs.

$path\ diference = n\lambda$ Where n = 0, 1, 2, 3, 4 …



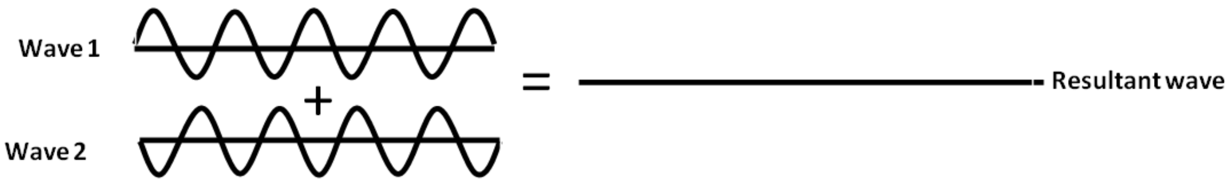
Destructive interference

- The waves are reaching at a point are in out of phase destructive interference occurs
- In Destructive interference, the resultant amplitude is always equal to the difference of the amplitudes of two individual waves.
- Condition

The path difference between the two waves is equal to the odd integral multiple of $\lambda/2$ destructive interference occurs

$$path\ diference = \frac{(2n-1)\lambda}{2} \qquad \text{Where } n = 1, 2, 3, 4 \ \dots \ or$$

$$path\ diference = \frac{(2n+1)\lambda}{2} \qquad \text{Where } n = 0, 1, 2, 3, 4 \ \dots$$



3. Types of interference:-

For the formation of interference pattern, two coherent light sources are required. To get two coherent sources form a single light source, two techniques are used. They are

1. Division of wave front
2. Division of amplitude

Division of wave front

The wave front from a single light source is divided into two parts using the phenomenon of reflection, refraction, or diffraction. Young's double slit experiment is belongs to this class of interference.

Division of amplitude

The amplitude of a single light beam is divided into two parts by parallel reflection or refraction. Newton's ring experiment, Michelson's interferometer is belongs to this class of interference.

4. Conditions for interference

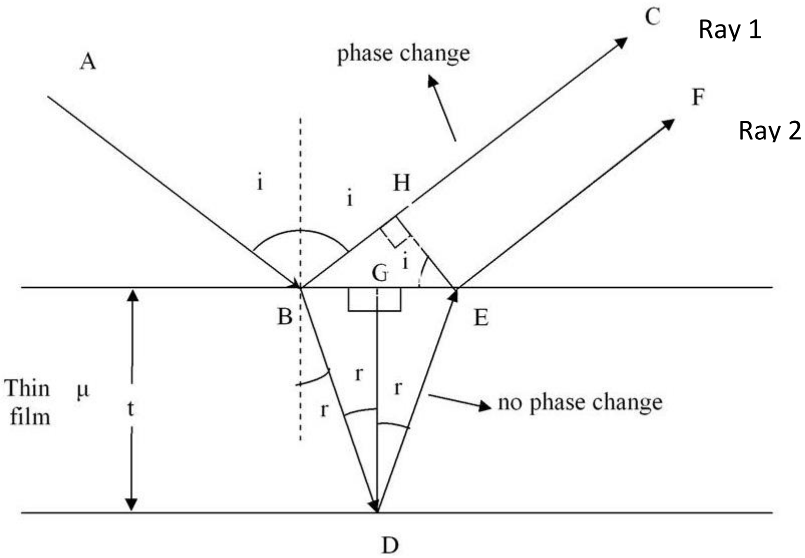
1) Two light sources of emitting light waves should be coherent.
2) Two sources must emit continuous light waves of same wavelengths or frequency.
3) The separation between the two sources should be small.
4) The distance between the two sources and the screen should be large.
5) To view interference fringes, the background should be dark.
6) The amplitude of light waves should be equal or nearly equal.
7) The sources should be narrow.
8) The sources should be monochromatic.

5. Interference in thin films by reflection

Principle:-

The formation of colours in thin films can explained as due to the phenomenon of interference. In this example, the formation of interference pattern is by the division of amplitude.

Consider a thin film of uniform thickness *'t'* and refractive index 'μ'. Let a monochromatic light ray AB is incident is on the upper surface of the film at point 'A' with an angle *'i'*. The incidence light ray AB is divided into two light rays ray 1 (BC) and ray 2 (EF) by the division of amplitude principle. These two light rays BC and EF are parallel and superimpose and produce interference. The intensity of interference fringe depends up on the path difference between the ray 1 and ray 2.



The path difference between the light rays (1) and (2) is

$$path\ difference = \mu(BD + DE)\ in\ film - BH\ in\ air \quad (1)$$

From ΔBDG $\cos r = \frac{DG}{BD} = \frac{t}{BD} \Rightarrow BD = \frac{t}{\cos r}$

Similarly from ΔDEG $\cos r = \frac{DG}{DE} = \frac{t}{DE} \Rightarrow DE = \frac{t}{\cos r}$

$$\therefore BD = DE = \frac{t}{\cos r} \quad (2)$$

From ΔBEH $\sin i = \frac{BH}{BE} = \frac{BH}{BG+GE}$

$\therefore BH = (BG + GE).\sin i$

From $\Delta BDG\ and\ \Delta DEG$ $BG = GE = t\tan r$

$BH = (2t\tan r).\sin i$

From Snell's law at point B

$$\sin i = \mu \sin r$$

$$\therefore BH = 2\mu t \tan r . \sin r \quad (3)$$

Substituting the equations (2) and (3) in equation (1), we get

$$Path\ difference = \frac{2\mu t}{\cos r} - 2\mu t \tan r . \sin r$$

$$= \frac{2\mu t}{\cos r} - 2\mu t . \frac{\sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r}(1 - \sin^2 r)$$

$$= \frac{2\mu t}{\cos r} \cos^2 r$$

$$= 2\mu t \cos r$$

At point B the light ray (1) is reflected at the surface of thin film (denser medium). So the light ray (1) undergoes a phase change π or an additional path difference λ/2.

Total path difference $= 2\mu t \cos r - \frac{\lambda}{2}$

Constructive interference (or Bright fridge)

General condition; $pathdifference = n\lambda$

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$2\mu t \cos r = n\lambda + \frac{\lambda}{2}$$

$$2\mu t \cos r = \frac{(2n+1)\lambda}{2}$$

Destructive interference (or Dark fridge)

General condition: $pathdifference = (2n+1)\frac{\lambda}{2}$

$$2\mu t \cos r - \frac{\lambda}{2} = \frac{(2n-1)\lambda}{2}$$

$$2\mu t \cos r = n\lambda$$

6. Newton's rings

Principle:-

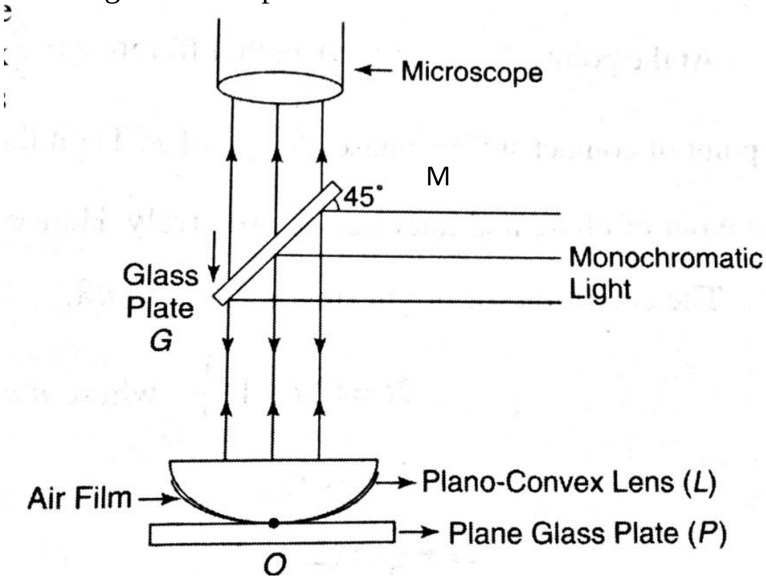
The formation of Newton's rings can explained as due to the phenomenon of interference. In this example, the formation of interference pattern is obtained by the division of amplitude.

Experimental arrangement

Experimental arrangement

- ✓ The experimental arrangement of Newton's rings is shown in figure.
- ✓ The Plano -convex lens (L) of large radius of curvature is placed with its convex surface on the glass plate (P). The Plano convex lens touches the glass plate at O.
- ✓ A monochromatic light is allowed to fall normally on the lens with the help of glass plate M kept at 45^0 to the incident monochromatic light.
- ✓ A part of light is reflected by the curved surface of the lens 'L' and a part of light is transmitted is partly reflected back by the upper surface of the plane glass plate P.

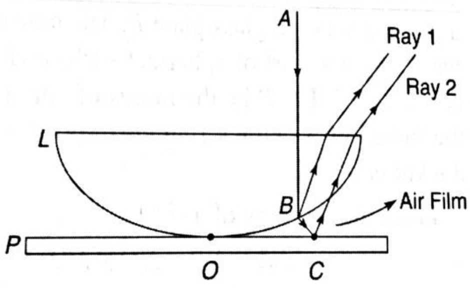
✓ These reflected rays interfere and give rise to an interference pattern in the form of circular fringes. These rings are seen through microscope.



O

Explanation of Newton's rings

Newton's rings are formed due to the interference between the light rays reflected from the lower surface of the lens and the upper surface of the glass plate (or top and bottom surfaces of the air film).



Let a vertical light ray AB be partially reflected from the curved surface of Plano convex lens with out phase change and partially transmitted light ray BC is again reflected at C on the glass plate with additional phase change of π or path difference λ/2.

The path difference between the two rays is

$$2\mu t\cos r+\frac{\lambda}{2}$$

For air film $\mu=1$ and for normal incidence $r=0$, so

The path difference $=2t+\frac{\lambda}{2}$

At the point of contact $t=0$, path difference is $\frac{\lambda}{2}$ i.e., the reflected and incidence light are out of phase and destructive interference occur. So the center fringe is always dark.

<u>Constructive interference (or Bright fridge)</u>

General condition: $pathdifference=n\lambda$

$$2t+\frac{\lambda}{2}=n\lambda$$

$$2t=(2n-1)\frac{\lambda}{2}$$

Where $n=0,1,2,\dots\dots\dots..$

<u>Destructive interference (or Dark fridge)</u>

General condition: $pathdifference=(2n+1)\frac{\lambda}{2}$

$$2t+\frac{\lambda}{2}=(2n+1)\frac{\lambda}{2}$$

$$2t=n\,\lambda$$

Where $n=0,1,2,\dots\dots\dots..$

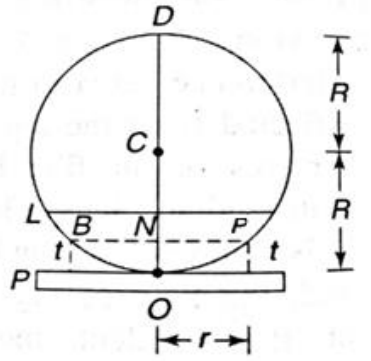
<u>**Theory of Newton's rings**</u>

To find the diameters of a dark and bright fringes construct a circle with the radius of curvature R of a lens L. Let us choose a point P at a distance 'r' from the center of lens and t be the thickness of air film at point p.

From the property of a circle $NP.NB = NO.ND$

$$r.r = t.(2R - t)$$

$$r^2 = 2Rt - t^2$$



If t is small t^2 is negligible.

$$r^2 = 2Rt$$

$$t = \frac{r^2}{2R}$$

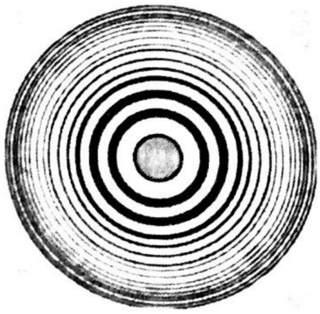
Bright rings

For bright ring, the condition is $2t = (2n-1)\frac{\lambda}{2}$

$$\frac{2\,r^2}{2R} = (2n-1)\frac{\lambda}{2}$$

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

By replacing r by D/2, the diameter of the bright ring is



$$\frac{D^2}{4} = \frac{(2n-1)\lambda R}{2}$$

$$D^2 = 2(2n-1)\lambda R$$

$$D = \sqrt{2(2n-1)\lambda R}$$

$$D = \sqrt{(2n-1)}\sqrt{2\lambda R}$$

$$D \propto \sqrt{(2n-1)}$$

$$D \propto \sqrt{odd\ natural\ number}$$

Dark rings

For dark rings, the condition is

$$2t = n\,\lambda$$

$$\frac{2\,r^2}{2R} = n\,\lambda$$

$$r^2 = n\,\lambda R$$

By replacing r by D/2, the diameter of the dark ring is

$$\frac{D^2}{4} = n\,\lambda R$$

$$D = \sqrt{4n\,\lambda R}$$

$$D = 2\sqrt{n\,\lambda R}$$

$$D \propto \sqrt{n}$$

$$D \propto \sqrt{natural\ number}$$

Note: suppose a liquid is taken in between the lens and glass plate having refractive index μ, then the diameter of the dark n^{th} dark ring can be written as

$$D = \frac{\sqrt{4n\,\lambda R}}{\mu}$$

7. Determination of wave length of sodium light using Newton's rings

By forming Newton's rings and measuring the radii of the rings formed, we can calculate the wavelength of the light used if the radius of curvature of the lens is known. Let R be the radius of curvature of the lens and λ is the wavelength of the light used.

So the diameter of the m^{th} dark ring can be written as

$$D_m{}^2 = 4\,m\,\lambda\,R \qquad (1)$$

Similarly the diameter of the nth dark ring is

$D_n{}^2 = 4\, n\, \lambda\, R$ (2)

Subtracting equation (1) from (2) we get

$$D_n{}^2 - D_m{}^2 = 4\, n\lambda\, R - 4\, m\, \lambda\, R$$

$$D_n{}^2 - D_m{}^2 = 4\,(n - m)\, \lambda\, R$$

$$\lambda = \frac{D_n{}^2 - D_m{}^2}{4(n - m)R}$$

Using the above relation wavelength can be calculated

8. Determination of refractive index of a liquid using Newton's rings

By forming Newton's rings and measuring the diameter of the rings formed, we can calculate the refractive index of the liquid.

In air film, the diameters of the mth and nth dark rings are D_m and D_n are measured with the help of travelling microscope.

The diameter of the nth dark ring is

$D_n{}^2 = 4\, n\, \lambda\, R$ (1)

The diameter of the mth dark ring is

$D_m{}^2 = 4\, m\lambda\, R$ (2)

Subtracting equation (1) from (2) we get

$D_n{}^2 - D_m{}^2 = 4\,(n - m)\, \lambda\, R$ (3)

The Newton's rings setup is taken in a liquid. Now the air film is replaced by liquid film. In liquid film, the diameters of the same nth and mth dark rings are D'_n and D'_m are measured with the help of travelling microscope.

$D'_n{}^2 = \frac{4\, n\, \lambda\, R}{\mu}$

And $D'_m{}^2 = \frac{4\, m\, \lambda\, R}{\mu}$

So $D'_n{}^2 - D'_m{}^2 = \frac{4\,(n-m)\, \lambda\, R}{\mu}$ (4)

Dividing equation (3) by (4)

$$\frac{D_n{}^2 - D_m{}^2}{D'_n{}^2 - D'_m{}^2} = \frac{4\,(n - m)\, \lambda\, R}{\frac{4\,(n - m)\, \lambda\, R}{\mu}}$$

$$\frac{D_n{}^2 - D_m{}^2}{D'_n{}^2 - D'_m{}^2} = \mu$$

Using the above relation μ can be calculated.

Diffraction

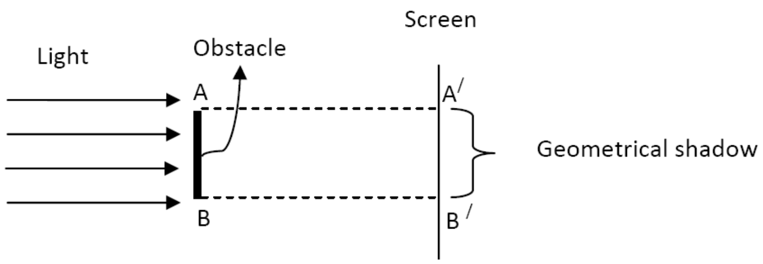
1. Introduction

The wave nature of light is first confirmed by the phenomenon of interference. Further it is confirmed by the phenomenon of diffraction. The word 'diffraction' is derived from the Latin word diffractus which means break to piece. When the light waves encounter an obstacle, they bend round the edges of the obstacle. The bending is predominant when the size of the obstacle is comparable with the wavelength of light. The bending of light waves around the edges of an obstacle is diffraction. It was first observed by Gremaldy.

2. Diffraction

When the light falls on the obstacle whose size is comparable with the wavelength of light then the light bends around the obstacle and enters in the geometrical shadow. This bending of light is called diffraction.

When the light is incident on an obstacle AB, their corresponding shadow is completely dark on the screen. Suppose the width of the slit is comparable to the wavelength of light, then the shadow consists of bright and dark fringes. These fringes are formed due to the superposition of bended waves around the corners of an obstacle. The amount of bending always depends on the size of the obstacle and wavelength of light used.



3. Types of diffraction

The diffraction phenomena are classified into two ways

I. Fresnel diffraction
II. Fraunhofer diffraction.

Fresnel diffraction:-

In this diffraction the source and screen are separated at finite distance. To study this diffraction lenses are not used because the source and screen separated at finite distance. This diffraction can be studied in the direction of propagation of light. In this diffraction the incidence wave front must be spherical of cylindrical.

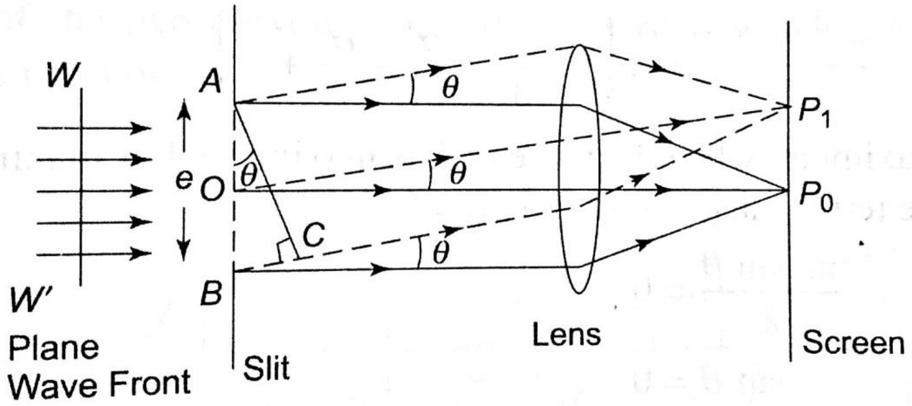
Fraunhofer diffraction:-

In this diffraction the source and screen are separated at infinite distance. To study this diffraction lenses are used because the source and screen separated at infinite distance. This diffraction can be studied in any direction. In this diffraction the incidence wave front must be plane.

4. Fraunhofer single slit diffraction:-

Let us consider a slit AB of width 'e'.
Let a plane wave front ww' of monochromatic light of wavelength λ is incident on the slit AB.
According to Huygens principle, every point on the wave front is a source of secondary wavelets. The wavelets spread out to the right in all directions.

The secondary wavelets which are travelling normal to the slit are brought to focus at point P_0 on the screen by using the lens.
These secondary wavelets have no path difference. Hence at point P_0 the intensity is maxima and is known as central maximum. The secondary wavelets travelling at an angle θ with the normal are focused at point P_1.



Intensity at point P_1 depends up on the path difference between the wavelets A and B reaching to point P_1. To find the path difference, a perpendicular AC is drawn to B from A.
The path difference between the wavelets from A and B in the direction of θ is

$$path\ difference = BC = AB\ sin\,\theta$$
$$= e\ sin\,\theta$$
$$phase\ difference = \frac{2\pi}{\lambda}(path\ difference)$$
$$= \frac{2\pi(e\sin\theta)}{\lambda}$$

Let the width of the slit is divided into 'n' equal parts and the amplitude of the wave front each part is 'a'. Then the phase difference between any two successive waves from these parts would be

$$\frac{1}{n}(phase\ difference) = \frac{1}{n}\left(\frac{2\pi e\sin\theta}{\lambda}\right) = d$$

Using the vector addition method, the resultant amplitude R is

$$R = \frac{a\sin\frac{nd}{2}}{\sin\frac{d}{2}}$$

$$R = A\frac{\sin\alpha}{\alpha} \qquad \because na = A\ and\ \alpha = \frac{\pi e\sin\theta}{\lambda}$$

Therefore resultant intensity $I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$

Principal maximum:-

The resultant amplitude R can be written as

$$R = \frac{A}{\alpha}\left(\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \cdots\right)$$
$$= \frac{A\alpha}{\alpha}\left(1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \cdots\right)$$
$$= A\left(1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \cdots\right)$$

In the above expression for $\alpha = 0$ values the resultant amplitude is maximum

$$R = A$$

Then $$I_{max} = R^2 = A^2$$

$$\alpha = \frac{\pi e \sin\theta}{\lambda} = 0$$
$$\sin\theta = 0$$
$$\theta = 0$$

For $\theta = 0$ and $\alpha = 0$ value the resultant intensity is maximum at P_0 and is known as principal maximum.

Minimum intensity positions

I Will be minimum when $\sin\alpha = 0$

$$\alpha = \pm m\pi \qquad m = 1,2,3,4,5 \ldots\ldots$$
$$\alpha = \frac{\pi e \sin\theta}{\lambda} = \pm m\pi$$
$$\pi e \sin\theta = \pm m\lambda$$

So we obtain the minimum intensity positions on either side of the principal maxima for all $\alpha = \pm m\pi$ values.

Secondary maximum

In between these minima secondary maxima positions are located. This can be obtained by differentiating the expression of I w.r.t α and equation to zero

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left(A^2\left[\frac{\sin\alpha}{\alpha}\right]^2\right) = 0$$

$$A^2\frac{2\sin\alpha}{\alpha}\frac{d}{d\alpha}\left(\frac{\sin\alpha}{\alpha}\right) = 0$$

$$A^2\frac{2\sin\alpha}{\alpha}\cdot\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0$$

In the above expression α can never equal to zero, so

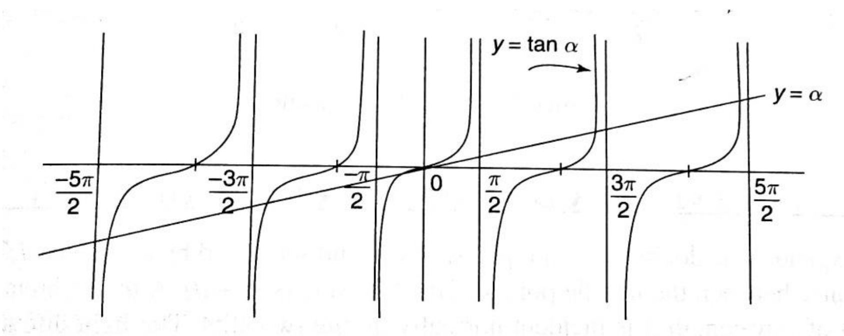
Either $\sin\alpha = 0$ or $\alpha\cos\alpha - \sin\alpha = 0$

$\sin\alpha = 0$ Gives the positions of minima

The condition for getting the secondary maxima is

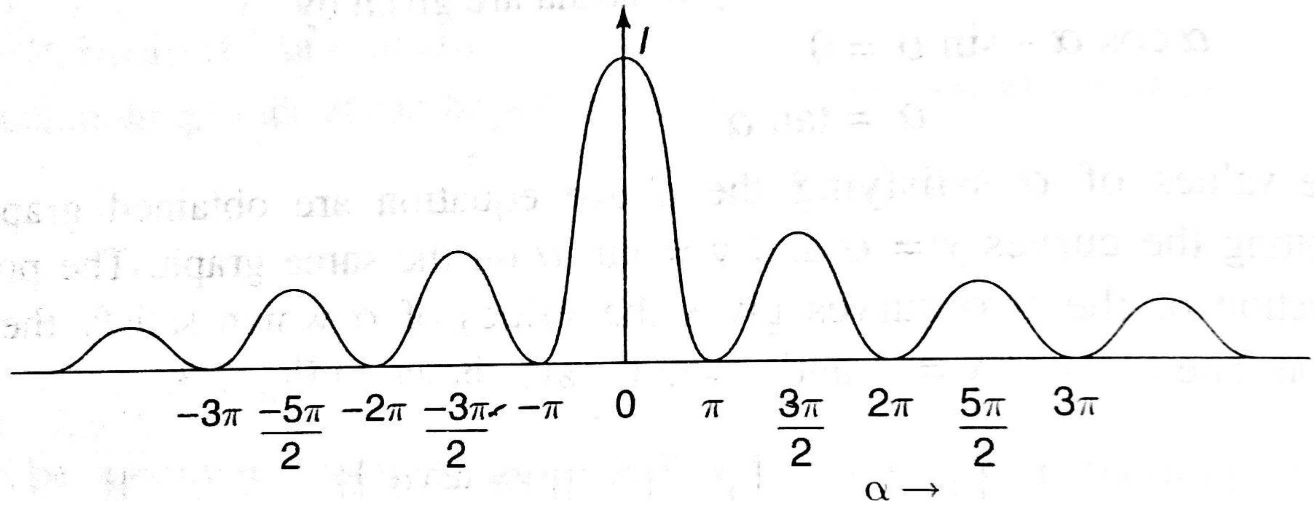
$$\alpha\cos\alpha - \sin\alpha = 0$$
$$\alpha\cos\alpha = \sin\alpha$$
$$\alpha = \tan\alpha$$

The values of α satisfying the above equation are obtained graphically by plotting the curves $Y = \alpha$ and $Y = \tan\alpha$ on the same graph. The plots of $Y = \alpha$ and $Y = \tan\alpha$ is shown in figure.



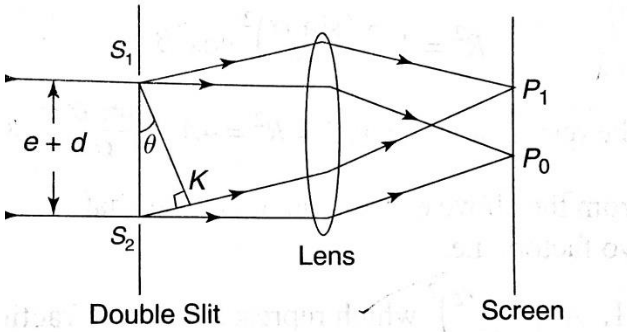
In the graph the two curves intersecting curves gives the values of satisfying of α satisfying the above equation. From the graph intersecting points are $\alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$...........

From the above concepts the intensity distribution curve verses α is shown in figure.



5. Fraunhofer double slit diffraction

- Let $S_1$and S_2 be the two slits equal width e and separated by a distance d.
- The distance between the two slits is (e + d).
- A monochromatic light of wavelength λ is incident on the two slits.
- The diffracted light from these slits is focused on the screen by using a lens.
- The diffraction of a two slits is a combination of diffraction and interference.
- When the plane wave front is incident on the two slits, the secondary wavelets from these slits travel in all directions.
- The wavelets travelling in the direction of incident light is focused at P_0. The wavelets travelling at an angle θ with the incident light are focused at point P_1.
- From Fraunhofer single slit experiment, the resultant amplitude is $R = A\frac{\sin\alpha}{\alpha}$
- So the amplitude of each secondary wavelet travelling with an angle θ can be taken as $A\frac{\sin\alpha}{\alpha}$. These two wavelets interference and meet at point P_1 on the screen.



To find out the path difference between the two wavelets, let us draw a normal s_1k to the wavelet S_2.

$$Path\ difference\ = S_2\,k$$

From $\Delta S_1\, S_2 k$ $\sin\theta = \frac{S_2\,k}{S_1 S_2} = \frac{S_2\,k}{(e+d)}$

$$S_2\,k = (e + d\,)\sin\theta$$

$$phase\ difference = \frac{2\pi}{\lambda}(path\ difference)$$

$$phase\ difference = \frac{2\pi}{\lambda}(e + d\,)\sin\theta = \delta$$

By using vector addition method, we can calculate the resultant amplitude at point P_1 by taking the resultant amplitudes of the two slits $S_1$and S_2 as sides of the triangle. The third side gives resultant amplitude.

$$AC^2 = AB^2{+}BC^2 + 2(AB)(BC)\cos\delta$$

$$R^2 = \left(A\frac{\sin\alpha}{\alpha}\right)^2 + \left(A\frac{\sin\alpha}{\alpha}\right)^2 + 2\left(A\frac{\sin\alpha}{\alpha}\right)\left(A\frac{\sin\alpha}{\alpha}\right)\cos\delta$$

$$R^2 = 4\left(A\frac{\sin\alpha}{\alpha}\right)^2 \cos^2{}_\beta$$

$$where\ \alpha = \frac{\pi e\sin\theta}{\lambda}\ and\ \beta = \frac{\pi(e+d)\sin\theta}{\lambda}$$

$$R^2 = 4\left(A\frac{\sin\alpha}{\alpha}\right)^2 \cos^2\beta$$

The resultant intensity $I = R^2 = 4\left(A\frac{\sin\alpha}{\alpha}\right)^2 \cos^2\beta$

From the above equation, it is clear that the resultant intensity is a product of two factors i.e.,

1. $\left(A\frac{\sin\alpha}{\alpha}\right)^2$ Represents the diffraction pattern due to a single slit.
2. $\cos^2\beta$ Represents the interference pattern due to wavelets from double slit.

<u>Diffraction effect</u>

The diffraction pattern consists of central principal maxima for $\alpha = 0$ value.

The secondary maxima of decreasing intensity are present on either side of the central maxima for $\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$ values.

Between the secondary maxima the minima values are present for $\alpha = \pm\pi, \pm2\pi, \pm\cdots$ values.

In diffraction pattern the variation of I w.r.t α as shown in figure (a)

<u>Interference effect</u>

In interference pattern the variation of $\cos^2\beta$ w.r.t β as shown in figure (b)

$\cos^2\beta$ Represent the interference pattern.

Interference maximum will occur for $\cos^2\beta = 1$

$$\beta = \pm m\pi \qquad where\ m = 0, 1, 2, 3, 4 \ldots\ldots\ldots$$

$$\beta = \pm\pi, \pm2\pi, \pm3\pi, \pm4\pi \ldots\ldots\ldots$$

$$\frac{\pi(e+d)\sin\theta}{\lambda} = \pm m\pi$$

$$(e+d)\sin\theta = m\lambda$$

Interference minima will occur for $\cos^2\beta = 0$

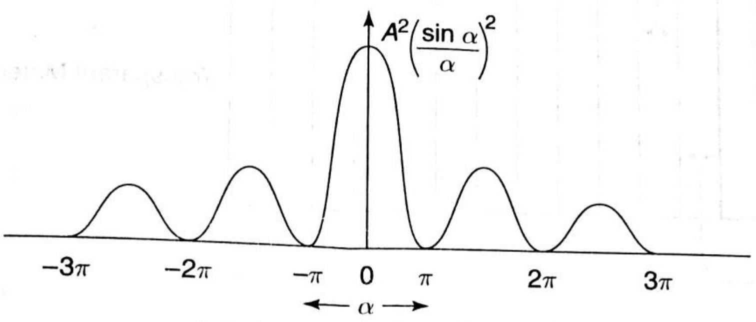
$$\beta = \pm(2m+1)\,{}^{\pi}/_{2}$$

$$\frac{\pi(e+d)\sin\theta}{\lambda} == \pm(2m+1)\,{}^{\pi}/_{2}$$

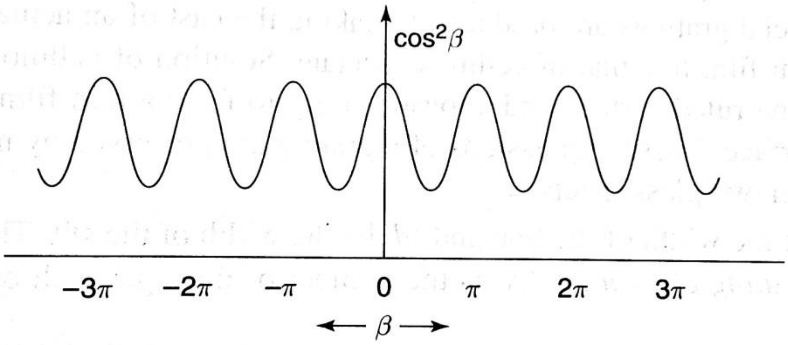
$$(e+d)\sin\theta = \pm(2m+1)\,{}^{\lambda}/_{2}$$

Intensity distribution:-

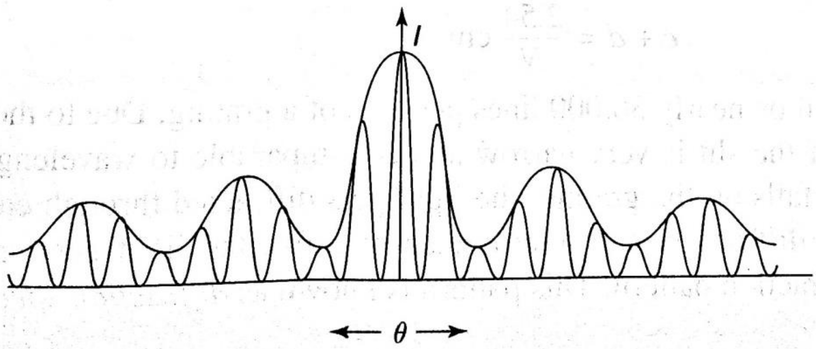
Figure (a), (b) and (c) represents the intensity variations of due to diffraction, interference and both effects respectively. From figure (c) it is clear that the resultant minima are not equal to zero.



(a)



(b)



(c)

6. Diffraction grating

A set of large number of parallel slits of same width and separated by opaque spaces is known as diffraction grating. Fraunhofer used the first grating consisting of a large number of parallel wires placed side by side very closely at regular separation. Now the gratings are constructed by ruling the equidistance parallel lines on a transparent material such as glass with fine diamond point. The ruled lines are opaque to light while the space between the two lines is transparent to light and act as a slit.

Let ‘e’ be the width of line and‘d’ be the width of the slit. Then (e + d) is known as grating element. If N is the number of lines per inch on the grating then

$N(e + d) = 1\ ich = 2.54\ cm$

$(e + d) = \frac{2.54\ cm}{N}$

Commercial gratings are produced by taking the cost of actual grating on a transparent film like that of cellulose acetate. Solution of cellulose acetate is poured on a ruled surface and allowed to dry to form a thin film, detachable from the surface. This film of grating is kept between the two glass plates.