Module 1 Unit 1

THIN FILM INTERFERENCE – NUMERICAL PROBLEMS

1. F A parallel beam of light of wavelength 6000 Å is incident on a plain transparent film of R. I. 1.5. If the angle of refraction is 28°. Find thickness of the film if it appears bright in the reflected light. Assume n = 1.
2. A soap film of R. I. 1.33 and thickness 1.5×10^{-5} cm is illuminated by light incident at 30°. Light reflected from it shows a dark band in the 2^{nd} order. Calculate wavelength corresponding the dark band.
3. A parallel beam of light falls normally on an oil film of R. I. 1.25. Complete destructive interference is observed for wavelengths 5000 Å and 6000 Å and for no wavelength in-between. Find the thickness of the oil.
4. White light falls normally on a soap film (μ = 1.33) of thickness 3800 Å. Which wavelength/s within the visible spectrum (4000 – 7000 Å) will be intensified in the reflected light?
5. A parallel beam light falls normally on an oil film of R. I. 1.2 having uniform thickness which is spread on water (R. I. 1.33). Brightness is obtained for wavelengths 5000 Å and 7500 Å and for no wavelength in-between. Find thickness of the oil film.
6. White light is incident on a soap film of R. I. 1.25 at 50°. Find minimum thickness of the film required if it appears bright yellow (λ = 5893 A) in the transmitted light.
7. A soap film of R. I. 1.33 and thickness 0.11 µm is illuminated by light incident at 30°. The reflected light shows a bright band corresponding to a wavelength of 5×10^{-5} cm. Calculate order of interference of the band.
8. A parallel beam of light is incident at an angle of 30° on a film of refractive index 1.5. Find the colours from the visible spectrum (4000 – 7000 Å) which will be intensified in transmitted light? Thickness of the film is 5×10^{-5} cm.
9. A soap film is illuminated by monochromatic light of wavelength 6000 7000 Å incident at certain angle. The film appears bright in reflected light. If its thickness is 1.5×10^{-5} cm and refractive index is 1.33, find the angle of incidence for minimum thickness. At what angle would it exhibit destructive interference?
10. White light is incident on a thin film of oil of R. I. 1.4 and thickness 0.0045 cm deposited on a glass plate of R. I. 1.52. The reflected light shows red colour of a particular wavelength at an angle of incidence of ~~9.94°~~ 9.8° in a certain order and again at 5.74° in the next higher order. Determine wavelength of the red colour getting reflected.
11. Calculate the wavelength which would be cut-off from reflection due to a film of thickness 1 micron and refractive index 1.28.
12. Can a thin film of MgF_2 of R. I. 1.22 act as an antireflection film if deposited on glass of R. I. 1.52? If yes, determine the minimum thickness required to cut-off reflection due to wavelength 5500 Å.
13. Determine the thickness of thin coating required for which, it will act as anti-transmitting film for the wavelength of 5000 Å. Given R. I. of the film = 1.28. ~~A film of magnesium fluoride of thickness 0.1 micron and R.I 1.22 is deposited on a glass plate. Calculate the wavelength which would be cut-off on reflection.~~
14. In above example, calculate the wavelength which will be highly transmitted by the same film.
15. A binocular has two-layer antireflection coating. The outer coating is MgF_2 (μ = 1.38) and the inner coating is ZrO_2 (μ = 2.10) to reduce reflections due to 6600 Å and 5700 Å respectively. Determine thickness of each coating required.

Module 1 Unit 1

THIN FILM INTERFERENCE – FORMULA SHEET

Parameter	Formula	
1. Interference conditions		
a) Rarer-denser-rarer **OR** Denser-rarer-denser	Maxima	Minima
Reflected light:	$2\mu t\cos r = \left(n - \frac{1}{2}\right)\lambda$	$2\mu t\cos r = n\lambda$
Transmitted light:	$2\mu t\cos r = n\lambda$	$2\mu t\cos r = \left(n - \frac{1}{2}\right)\lambda$
b) Rarer-intermediate-denser	Maxima	Minima
Reflected light (only):	$2\mu t\cos r = n\lambda$	$2\mu t\cos r = \left(n - \frac{1}{2}\right)\lambda$
2. Anti-reflecting film **OR** Highly transmitting film	$t = \frac{\lambda}{4\mu_f}$	
3. Anti-transmitting film **OR** Highly reflecting film	$t = \frac{\lambda}{2\mu_f}$	

Soved numerical problems on thin film interference

① $\lambda = 6000\,\mathring{A} = 6\times10^{-7}\,m$, $\mu = 1.5$, $r = 28^\circ$, $n = 1$

Formula: $2\mu t\cos r = (n-\frac{1}{2})\lambda$ — formula 1(a) maxima in reflected

$$\therefore t = \frac{(n-\frac{1}{2})\lambda}{2\mu\cos r}$$

$$= \frac{(1-\frac{1}{2})\times 6\times10^{-7}}{2\times1.5\times\cos(28^\circ)}$$

$$= 1.13\times10^{-7}\,m$$

(2) $\mu = 1.33$, $t = 1.5\times10^{-5}$ cm, $i = 30^\circ$, $n = 2$

Formula : $2\mu t\cos r = n\lambda$ — formula 1(a) minima in reflected

$$\therefore \lambda = \frac{2\mu t\cos r}{n}$$

$$\mu = \frac{\sin i}{\sin r} \Rightarrow \sin r = \frac{\sin i}{\mu} \Rightarrow \cos r = \sqrt{1-\sin^2 r} = \sqrt{1-\frac{\sin^2 i}{\mu^2}}$$

$$\therefore \lambda = \frac{2t\cdot\mu\cdot\sqrt{1-\frac{\sin^2 i}{\mu^2}}}{n} = \frac{2t\sqrt{\mu^2-\sin^2 i}}{n}$$

$$\boxed{\mu\cos r = \sqrt{\mu^2-\sin^2 i}}$$

$$\therefore \lambda = \frac{2\times1.5\times10^{-5}\times\sqrt{1.33^2-\sin^2(30)}}{2}$$

$$= 1.85\times10^{-5} \text{ cm}$$

(3) $\mu = 1.25$, $\lambda_1 = 5000\,Å = 5 \times 10^{-7}\,m$, $\lambda_2 = 6000\,Å = 6 \times 10^{-7}\,m$

Formula: $2\mu t \cos r = n\lambda$ — assuming reflected light condition for min

two or more λ's then n is also not unique

since min occur in consecutive order

$$\therefore\ 2\mu t \cos r = n\lambda_1 = (n-1)\lambda_2 \quad \text{as } \lambda_2 > \lambda_1$$

$$= (n+1)\lambda_1 = n\lambda_2 \quad -\,\text{"}\,-$$

$$\therefore\ n\lambda_1 = (n-1)\lambda_2 = n\lambda_2 - \lambda_2$$

$$\Rightarrow\ n = \frac{\lambda_2}{\lambda_2 - \lambda_1} = \frac{6000}{6000 - 5000}$$

$$\therefore\ n = 6$$

sub. it back

$$\boxed{\text{for normal incidence } \cos r = 1}$$

$$2\mu t \cos r = n\lambda_1$$

for normal incidence, $i = 0 \Rightarrow r = 0$

$\therefore \cos r = 1$

$$\therefore\ t = \frac{n\lambda_1}{2\mu} = \frac{6 \times 5 \times 10^{-7}}{2 \times 1.25}$$

$$t = 12 \times 10^{-7}\,m$$

(4) $\mu = 1.33$, $t = 3800\,\text{Å}$, n: various as λ's are many

visible light spectrum : 4000 Å – 7000 Å

Formula : $2\mu t \cos r = (n - \frac{1}{2})\lambda$

For normal incidence $\cos r = 1$

$$\therefore \lambda = \frac{2\mu t}{n - \frac{1}{2}}$$

For $n = 1$, $\lambda = \frac{2\mu t}{1 - \frac{1}{2}} = 4\mu t$

$= 4 \times 1.33 \times 3800$

$= 20216\,\text{Å}$ — outside

For $n = 2$, $\lambda = \frac{4\mu t}{3} = \frac{20216}{3}$

$= 6738.67\,\text{Å}$ – within

For $n = 3$, $\lambda = \frac{4\mu t}{5} = \frac{20216}{5}$

$= 4043.2\,\text{Å}$ – within

$n = 4$, $\lambda = \frac{4\mu t}{7} = \frac{20216}{7}$

outside

(5) $\mu = \mu_{oil} = 1 \cdot 2$, $\lambda_1 = 5000 \, \mathring{A} = 5 \times 10^{-7} \, m$, $\lambda_2 = 7500 \, \mathring{A} = 7 \cdot 5 \times 10^{-7} \, m$

Formula : $2\mu t \cos r = n\lambda$ — assuming reflected light from oil-water formula 1(b) maxima in reflected

for normal incidence $\cos r = 1$

$\therefore 2\mu t = (n+1)\lambda_1 = n\lambda_2$

since these are in consecutive orders

$\therefore (n+1)\lambda_1 = n\lambda_2$

$$\therefore n = \frac{\lambda_1}{\lambda_2 - \lambda_1}$$

$$= \frac{5000}{7500 - 5000}$$

$n = 2$

sub back

$2\mu t = n\lambda_2$

$$\therefore t = \frac{n\lambda_2}{2\mu}$$

$$= \frac{2 \times 7 \cdot 5 \times 10^{-7}}{2 \times 1 \cdot 2}$$

$t = 6 \cdot 25 \times 10^{-7} \, m$

(6) $\mu = 1.25$, $i = 50°$, $\lambda = 5893\,\text{Å} = 5.893 \times 10^{-7}\,m$

For minimum thickness, take $n = 1$

Formula : $2\mu t\cos r = n\lambda$

$$\therefore \quad t = \frac{n\lambda}{2\mu\cos r}$$

$$\mu\cos r = \sqrt{\mu^2 - \sin^2 i}$$

$$\therefore \quad t = \frac{n\lambda}{2\sqrt{\mu^2 - \sin^2 i}} = \frac{1 \times 5.893 \times 10^{-7}}{2 \times \sqrt{1.25^2 - \sin^2(50)}}$$

$$= 2.98 \times 10^{-7}\,m$$

7) $\mu = 1.33$, $t = 0.11\ \mu m = 1.1 \times 10^{-7} m$, $i = 30°$, $\lambda = 5 \times 10^{-5} cm = 5 \times 10^{-7} m$

Formula: $2\mu t \cos r = (n - \frac{1}{2})\lambda$

$$\therefore 2t\sqrt{\mu^2 - \sin^2 i} = (n - \tfrac{1}{2})\lambda$$

$$\therefore (n - \tfrac{1}{2}) = \frac{2t\sqrt{\mu^2 - \sin^2 i}}{\lambda}$$

$$\therefore n = \frac{1}{2} + \frac{2t\sqrt{\mu^2 - \sin^2 i}}{\lambda}$$

$$= \frac{1}{2} + \frac{2 \times 1.1 \times 10^{-7} \times \sqrt{1.33^2 - \sin^2(30)}}{5 \times 10^{-7} m}$$

$$= \frac{1}{2} + 0.54$$

$$= 1.04 \approx 1$$

(8) $i = 30^\circ$, $\mu = 1.5$, $t = 5\times10^{-5}$ cm $= 5000$ Å, n various

visible light spectrum : 4000 Å – 7000 Å

Formula : $2\mu t\cos r = n\lambda$

$$\therefore 2t\sqrt{\mu^2 - \sin^2 i} = n\lambda$$

$$\therefore \lambda = \frac{2t\sqrt{\mu^2-\sin^2 i}}{n}$$

For $n = 1$, $\lambda = \dfrac{2\times5\times10^{-5}\times\sqrt{1.5^2 - \sin^2(30)}}{1}$

$= 1.414\times10^{-4}$ cm

$= 14140\times10^{-8}$ cm

$= 14140$ Å outside

For $n = 2$

$\lambda = \dfrac{14140}{2}$

$= 7071$ Å outside

For $n = 3$

$\lambda = \dfrac{14140}{3}$

$= 4714$ Å within

For $n = 4$

$\lambda = \dfrac{14140}{4}$

$= 3535$ Å outside

(9) $\lambda \doteq 7000\,\mathring{A}$ (used corrected value) $= 7\times10^{-7}\,m$,
$t = 1.5\times10^{-5}\,cm = 1.5\times10^{-7}\,m$, $\mu = 1.33$

Formula : $2\mu t\cos r = (n-\frac{1}{2})\lambda$

$$\therefore\ 2t\sqrt{\mu^2-\sin^2 i} = (n-\tfrac{1}{2})\lambda$$

let $n=1$ then $2t\sqrt{\mu^2-\sin^2 i} = \frac{\lambda}{2}$

$$\therefore\ \sqrt{\mu^2-\sin^2 i} = \frac{\lambda}{4t}$$

[destructive
$\sqrt{\mu^2-\sin^2 i} = \frac{\lambda}{2t}$]

$$\therefore\ \mu^2-\sin^2 i = \frac{\lambda^2}{16t^2}$$

$$\therefore\ \sin^2 i = \mu^2 - \frac{\lambda^2}{16t^2}$$

$$\therefore\ \sin i = \sqrt{\mu^2 - \frac{\lambda^2}{16t^2}}$$

For minima

$2\mu t\cos r = n\lambda$

$i = \sin^{-1}\left[\sqrt{\mu^2 - \frac{n^2\lambda^2}{4t^2}}\right]$

$\downarrow$

> 1

no value of i is possible for destructive interference

$i = \sin^{-1}\left[\sqrt{\mu^2 - \frac{\lambda^2}{16t^2}}\right]$

$= \sin^{-1}\left[\sqrt{1.33^2 - \frac{(7\times10^{-7})^2}{16\times(1.5\times10^{-7})^2}}\right]$

$= \sin^{-1}\left[\sqrt{1.33^2 - \frac{49}{16\times2.25}}\right]$

$= 24.06^\circ$

(10) $\mu = 1.4$, $t = 0.0045\ cm = 4.5\times10^{-3}\ cm$, $i_1 = 9.8^\circ$, $i_2 = 5.74^\circ$

Formula : $2\mu t\cos r = (n-\frac{1}{2})\lambda$ here n's will be different for different i's

$\therefore 2t\sqrt{\mu^2-\sin^2 i} = (n-\frac{1}{2})\lambda$ use n & (n+1) or (n-1) & n

here, for $\sin i_1$, use n
for $\sin i_2$ use n+1 $\Big\}$ as $\sin i_2 < \sin i_1$

$\therefore 2t\sqrt{\mu^2-\sin^2 i_1} = (n-\frac{1}{2})\lambda$ — (1)

and $2t\sqrt{\mu^2-\sin^2 i_2} = \left[(n+1)-\frac{1}{2}\right]\lambda_1$

$= \left[(n-\frac{1}{2})+1\right]\lambda$

$= (n-\frac{1}{2})\lambda + \lambda$

$\therefore 2t\sqrt{\mu^2-\sin^2 i_2} = 2t\sqrt{\mu^2-\sin^2 i_1} + \lambda$

$\therefore \lambda = 2t \times \left[\sqrt{\mu^2-\sin^2 i_2} - \sqrt{\mu^2-\sin^2 i_1}\right]$

$\therefore \lambda = 2\times 4.5\times10^{-3} \left[\sqrt{1.4^2-\sin^2(5.74)} - \sqrt{1.4^2-\sin^2(9.8)}\right]$

$\therefore \lambda = 6.1275\times10^{-5}\ cm$
$= 6127.5\ \mathring{A}$

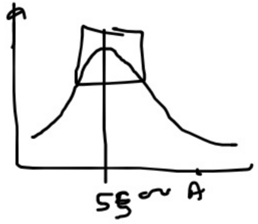
(11) $t = 1\mu m$, $\mu_f = 1.28$

Formula : $t = \dfrac{\lambda}{4\mu_f}$ — formula 2 of formula sheet

$\therefore \lambda = 4\mu_f \cdot t = 4 \times 1.28 \times 1 = 5.12 \mu m = 5120\ nm = 51200\ \mathring{A}$

(12) $\mu_f = 1.22$, $\mu_g = 1.52$, $\lambda = 5500\ \mathring{A} \longrightarrow$ green component

Check if $\mu_f \overset{?}{=} \sqrt{\mu_g} \Rightarrow \sqrt{1.52} = 1.23 \simeq \mu_f$



$\therefore$ MgF_2 can act as A-R film on glass

Formula : $t = \dfrac{\lambda}{4\mu_f} = \dfrac{5500}{4 \times 1.22} = 1145.8\ \mathring{A}$

↓

xe

(13) $\lambda = 5000\ \text{Å}$, $\mu_f = 1.28$

Formula: $t = \dfrac{\lambda}{2\mu_f}$ — formula 3 of formula sheet

$$= \frac{5000}{2 \times 1.28} = \underline{1953\ \text{Å}}$$

— it is highly reflecting film (anti-transmitting)

(14) For highly transmitting film (i.e. anti-reflecting film)

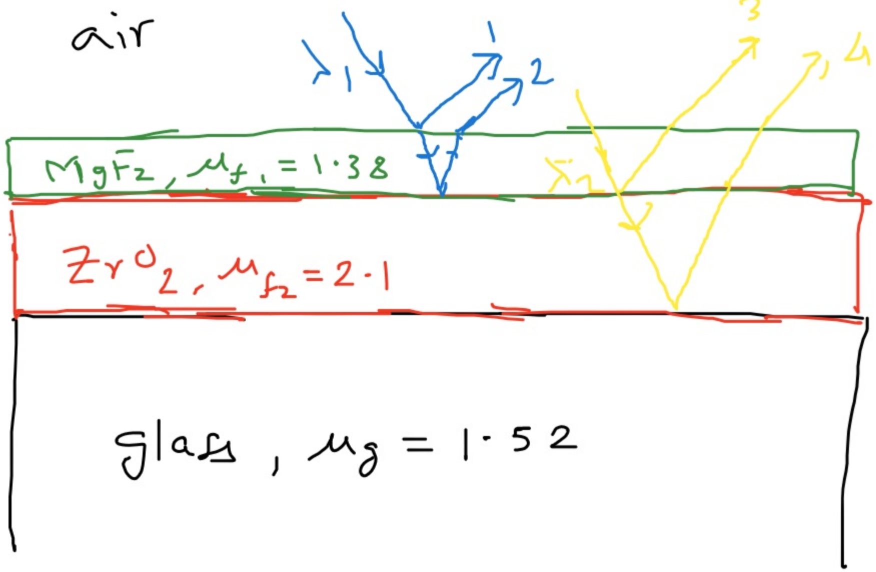
Formula: $t = \dfrac{\lambda}{4\mu_f}$, use t obtained above

$$t = 1953\ \text{Å}$$

$$\therefore \lambda = 4\mu_f \cdot t = 4 \times 1.28 \times 1953\ \text{Å}$$

$$\simeq 10000\ \text{Å}$$

(15) $\lambda_1 = 6600\,\mathring{A}$ (for MgF_2), $\mu_{f_1} = 1.38$
$\lambda_2 = 5700\,\mathring{A}$ (for ZrO_2), $\mu_{f_2} = 2.1$
glass has $\mu_g = 1.52$



condition for minima for light reflected from MgF_2

$$2\mu t_1 \cos r = (n - \tfrac{1}{2})\lambda_1$$

let $\cos r = 1$, $n = 1$

$$\Rightarrow t_1 = \frac{\lambda_1}{4\mu_{f_1}} =$$

---

condition for minima for light reflected from ZrO_2

$$2\mu t_2 \cos r = n\lambda_2$$

let $\cos r = 1$, $n = 1$

for ZrO_2, $t_2 = \frac{\lambda_2}{2\mu f_2}$

MgF_2

$$t_1 = \frac{\lambda_1}{4\mu f_1}$$

$$= \frac{6600}{4 \times 1.38}$$

$$= 1195.6\ \text{Å}$$

ZrO_2

$$t_2 = \frac{\lambda_2}{2\mu f_2}$$

$$= \frac{5700}{2 \times 2.1}$$

$$= 1357.1\ \text{Å}$$