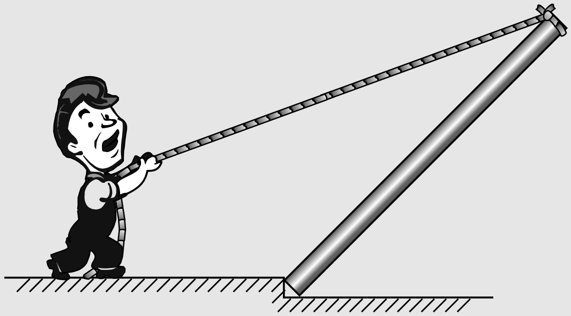
3

EQUILIBRIUM OF SYSTEM OF COPLANAR FORCES



3.1 Introduction

In the previous chapter, we have discussed the different types of force system (i.e., *concurrent*, *parallel* and *general*) which were easily identified and their resultants were calculated. In this chapter, we shall discuss the *equilibrium analysis of engineering problem* for which we must originate the force system.

Here we shall introduce the *free body diagram* which is perhaps the most important physical concept in this text. This is always the initial step in solving a problem and often the most critical step. The matter of this chapter is very important for dealing with the subject. Here we shall discuss many basic concepts, such as *two-force body*, *three-force body*, *Lami's theorem*, *types of supports*, *types of loads*, *analysis of simple body*, *analysis of composite bodies*, *frames,* etc.

3.2 Equilibrant

A *force*, which is *equal, opposite* and *collinear to the resultant of a concurrent force system* is known as the *equilibrant of the concurrent force system*.

Equilibrant is the force which, when applied to a body acted by the concurrent force system, keeps the body in equilibrium.

A *single force* which brings the *system to equilibrium*, thus equilibrant is *equal in magnitude*, *opposite in direction* and *collinear to resultant force*.

The force that cancels the effect of the force system acting on the body is also known as *equilibrant*.

3.3 Free Body Diagram (FBD)

*The **Free Body Diagram (FBD)** is a sketch of the body showing all active and reactive forces that acts on it after removing all supports with consideration of geometrical angles and distance given.*

To investigate the equilibrium of a body, we remove the supports and replace them by the reactions which they exert on the body.

The first step in equilibrium analysis is to identify all the forces that act on the body, which is represented by a free body diagram. Therefore, the free body diagram is the most important step in the solution of problems in mechanics.

Importance of FBD

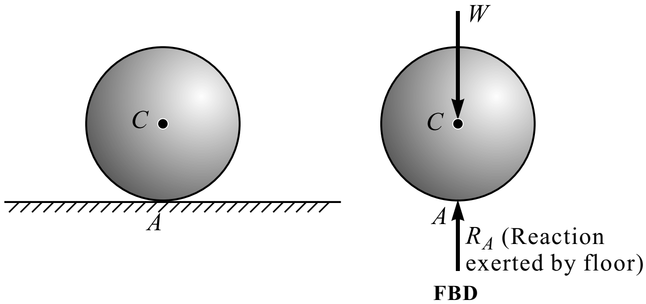
1. The sketch of FBD is the key step that translates a physical problem into a form that can be analysed mathematically.
2. The FBD is the sketch of a body, a portion of a body or two or more connected bodies completely isolated or free from all other bodies, showing the force exerted by all other bodies on the one being considered.
3. FBD represents all active (applied) forces and reactive (reactions) forces. Forces acting on the body that are not provided by the supports are called *active force* (weight of the body and applied forces). *Reactive forces* are those that are exerted on a body by the supports to which it is attached.
4. FBD helps in identifying known and unknown forces acting on a body.
5. FBD helps in identifying which type of force system is acting on the body so by applying appropriate condition of equilibrium, the required unknowns are calculated.

Procedure for Drawing a FBD

1. Draw a neat sketch of the body assuming that all supports are removed.
2. FBD may consist of an entire assembled structure or any combination or part of it.
3. Show all the relevant dimensions and angles on the sketch.
4. Show all the active forces on corresponding point of application and insert their magnitude and direction, if known.
5. Show all the reactive forces due to each support.
6. The FBD should be legible and neatly drawn, and of sufficient size, to show dimensions, since this may be needed in computation of moments of forces.
7. If the sense of reaction is unknown, it should be assumed. The solution will determine the correct sense. A positive result indicates that the assumed sense is correct, whereas a negative result means the assumed sense is incorrect, so the correct sense is opposite to the assumed sense.
8. Use principle of transmissibility wherever convenient.

Example 1

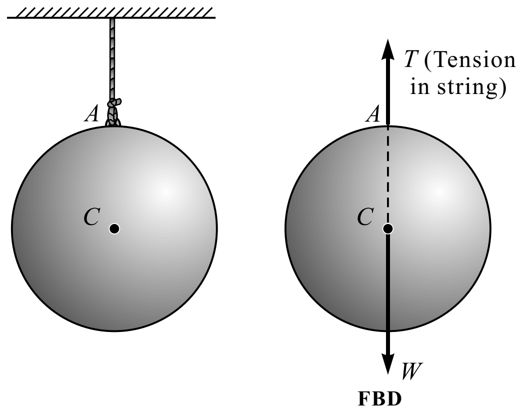
A sphere having weight W is resting on the horizontal floor. It is free to move along the horizontal plane but cannot move vertically downward. A sphere exerts a vertical push against the horizontal surface at the point of contact A. As per Newton's third law, action and reaction are equal and opposite. In FBD, we remove the supporting horizontal plane and replace it by reactive force R_A. Weight W is the active force.



Note : In this chapter, all contact surface are assumed to be a frictionless. Therefore no FBD is represented with frictional force.

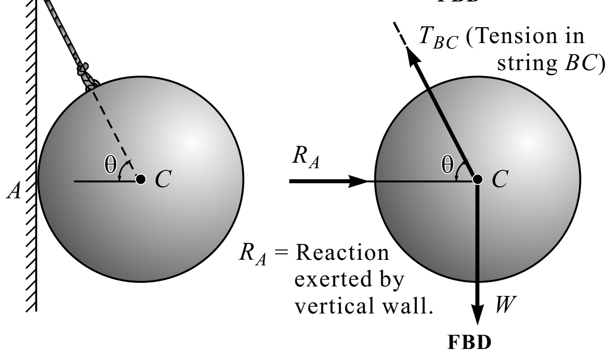
Example 2

A sphere having weight W is freely suspended by string connected to ceiling. Here sphere can be swing as pendulum but cannot move vertically down as attached by string, thus the sphere exerts a downward pull on the end of the supporting string. In FBD, we remove the string and replace it by reactive force, i.e., the tension T in the string. Weight W is the active force.



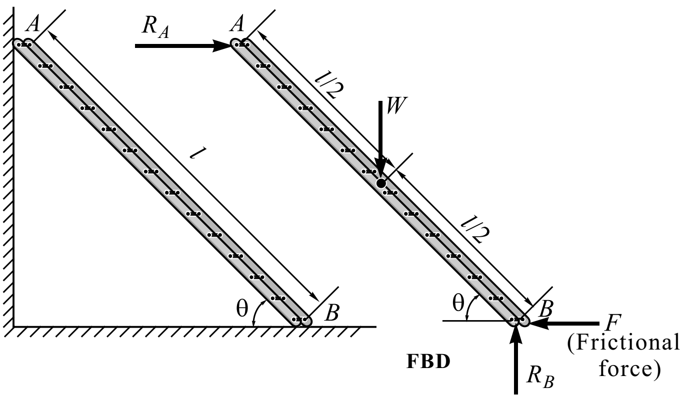
Example 3

A sphere having weight W is suspended by a string but rest against the vertical wall. Here the sphere is constrained to move downward by string and towards left due to vertical wall. The sphere not only pulls down on string BC but also pushes to the left against the vertical wall at A. In FBD, we remove the string and vertical wall and replace by tension T and reaction R_A. Weight W is the active force.



Example 4

Ladder having weight W is resting against the rough horizontal floor and smooth vertical wall. R_A is reactive force exerted by vertical wall and R_B is the reactive force exerted by horizontal floor on ladder. F is the reactive frictional force between horizontal floor and ladder, weight W is the active force of ladder.

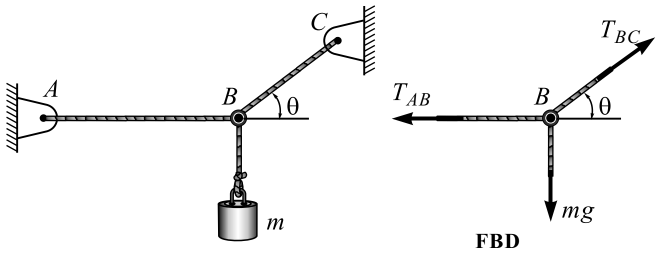


Example 5

A block of mass m kg is suspended by ropes, as shown in the figure.

$T_{AB} \Rightarrow$ Tension in rope AB

$T_{BC} \Rightarrow$ Tension in rope BC

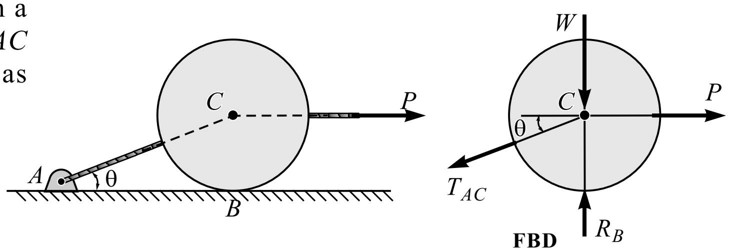


Example 6

A cylinder of weight W supported on a smooth horizontal plane by a cord AC and pulled by applied force P as shown in the figure.

$T_{AC} \Rightarrow$ Tension in cord AC

$R_B \Rightarrow$ Reaction exerted by the floor on cylinder at B

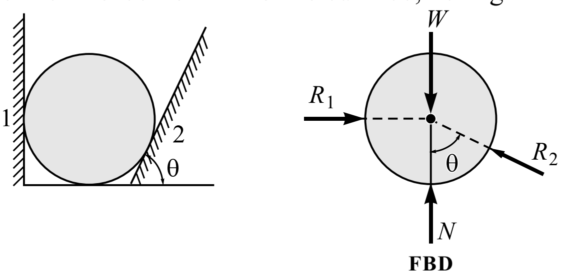


Note : String, rope, cord, cable, wire, thread, chain always experiences tension which is shown by drawing an arrow away from joint or body in FBD.

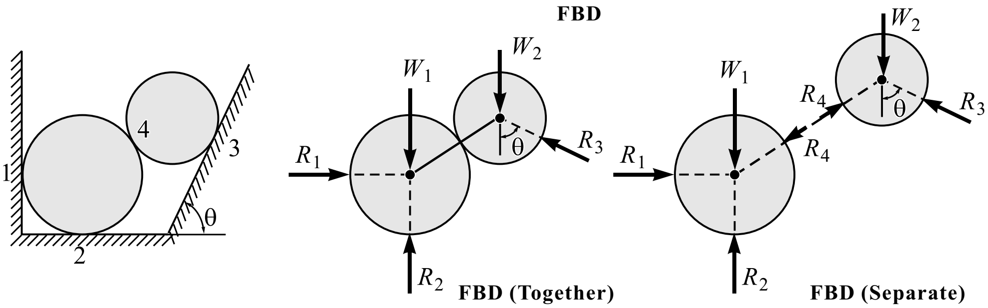
Example 7

Smooth Surface Contact : When a body is in contact with a smooth (frictionless) surface at only one point, the reaction is a force normal to the surface, acting at the point of contact.

(i)



(ii)



3.4 Types of Supports

While drawing FBD the most important step to master is the determination of the support reactions. The structure in the field may be various types such as beam, truss, frame, levers ladder, etc. They are supported with specific arrangements.

Generally, the support offers reactions. Different types of supports and their reactions are classified as follows.

1. **Roller Support :** A roller support is equivalent to a frictionless surface. It can only exert a force that is perpendicular to the supporting surface. The magnitude of the force is then the only unknown force introduced in a FBD when the support is removed. The roller support is free to roll along the surface on which it rests. Since the linear displacement in normal direction to surface of roller is restricted, it offers a reaction in normal direction to surface of roller. For example, a sliding door slides smoothly with the help of a roller support, whereas a conveyer belt can move smoothly on a roller support.

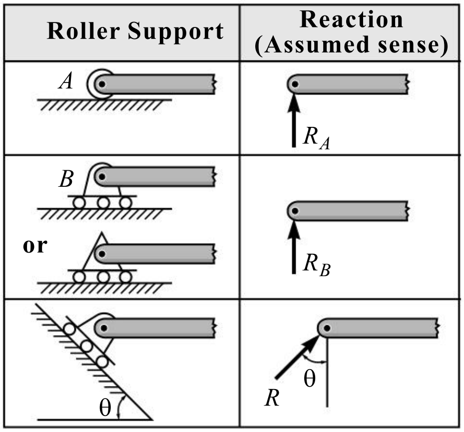


Fig. 3.4-(1)

2. **Hinge (Pin) Support :** The hinge support allows free rotation about the pin end but it does not allows linear displacement of that end. Since linear displacements are restricted in horizontal and vertical directions, the reaction offered at hinge support (say R_A at θ) is resolved into two components, i.e., H_A and V_A. The direction of these two components are uncertain. Therefore, they are initially assumed in FBD

   A pin is a cylinder that is slightly smaller than the hole into which it is inserted [refer to Fig. 3.4(3)-i]. Neglecting friction, the pin can only exert a force that is normal to the contact surface (say at point *A*) shown as R_A [refer to Fig. 3.4(3)-ii]. *A* pin support thus introduces two unknown, the magnitude of R_A and the angle θ that specifies the direction of R_A. This reaction R_A at θ can be resolved into two components, i.e., horizontal components (H_A) and vertical component (V_A). One should identify the figures given in Fig. 3.4(3)-iv as pin support.

   For example, opening and closing of a door is possible by hinges. Same principle applies in the opening and closing of a suitcase or a laptop too.

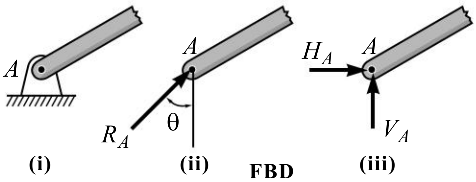


Fig. 3.4(2)

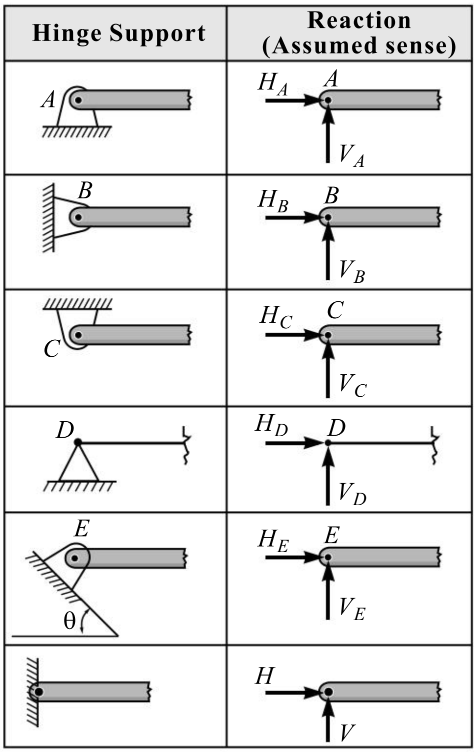
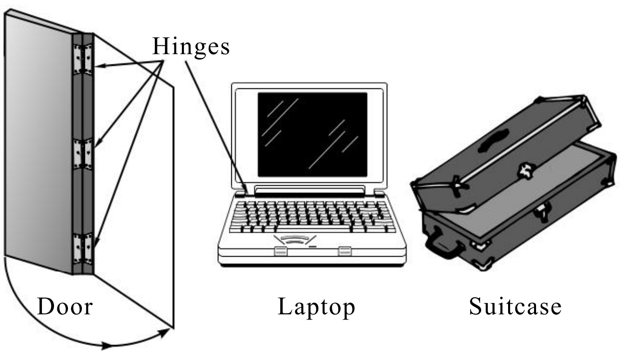


Fig. 3.4(3)-iv



3. **Fixed (Built in) Support :** When the end of a beam is fixed (built in) then that support is said to be fixed support. Fixed support neither allows linear displacement nor rotation of a beam. Due to these restrictions, the components reaction offered at fixed supports are horizontal component H_A, vertical component V_A and couple component M_A. These components are shown in assumed direction. Refer to Fig. 3.4(3).

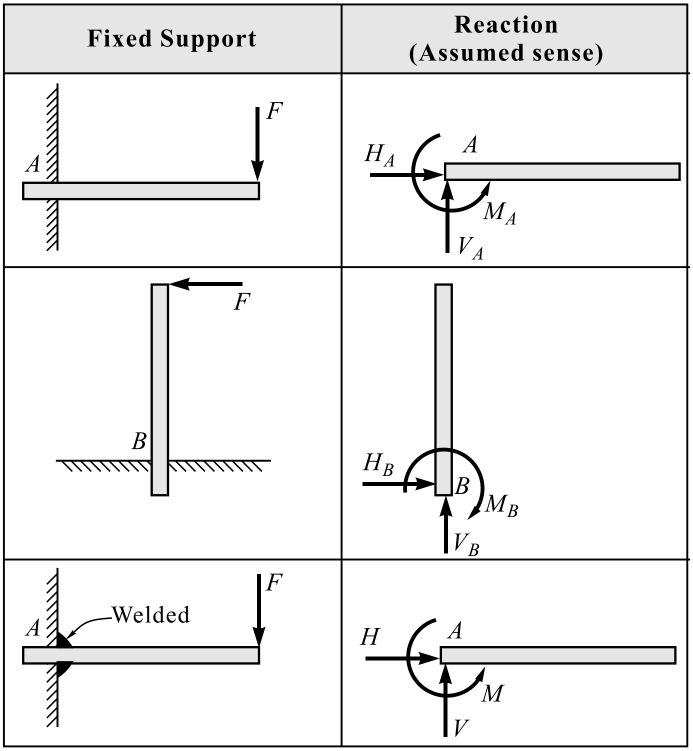


Fig. 3.4(3)

4. **Freely Sliding Guide :** Collar or slider free to move along smooth guides can support force normal to guide only.

   For example, a slider is free to move along a horizontal slot, whereas a collar is free to move along a vertical rod (guide).

   Refer to Fig. 3.4(4).

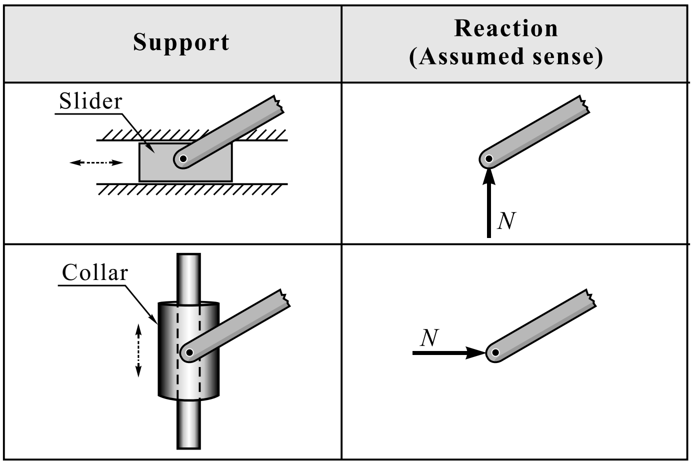


Fig. 3.4(4)

5. **Gravitational Attraction :** The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts towards the Earth through the centre of mass G.

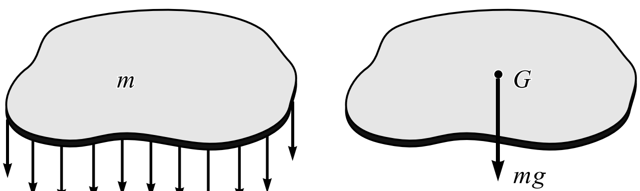


Fig. 3.4(5)

6. **Spring Force :** Spring force is given by the relation $F = kx$ where k is the spring constant and x is the deformation of the spring. Deformation may be due to tension if spring is stretched and compression, if compressed.

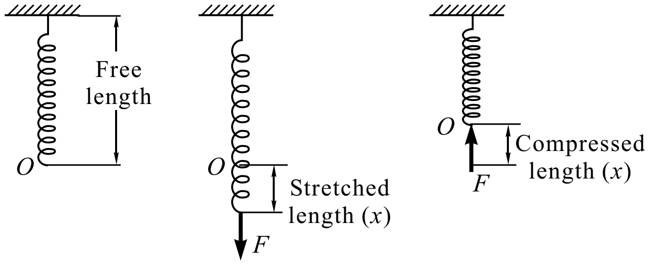


Fig. 3.4(6)

7. **Inextensible String, Cable, Belt Rope, Cord, Chain or Wire :** The force developed in rope is always a tension away from the body in the direction of rope. When one end of a rope is connected to a body, then the rope is not to be considered as a part of the system and it is replaced by tension in FBD as shown in Fig. 3.4(7).

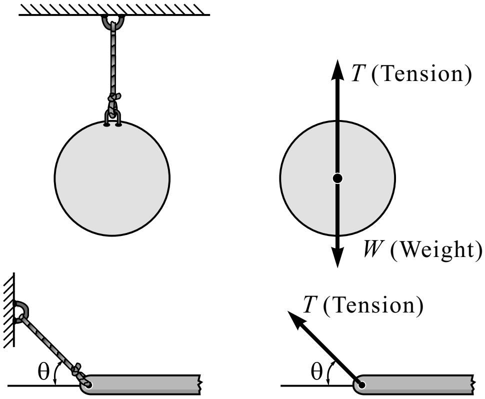


Fig. 3.4(7)

8. **Rope and Frictionless Pulley Arrangement :** When a rope is passing over a frictionless pulley, then the tension on both sides of the rope is same as shown in Fig. 3.4(8)-i, ii and iii.

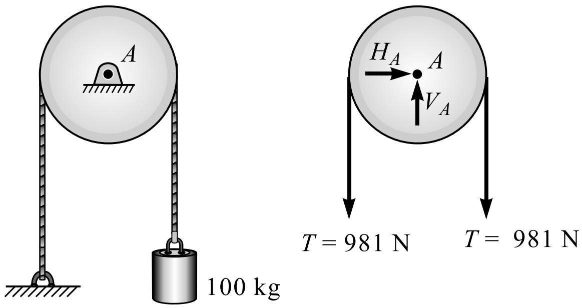


Fig. 3.4(8)-i

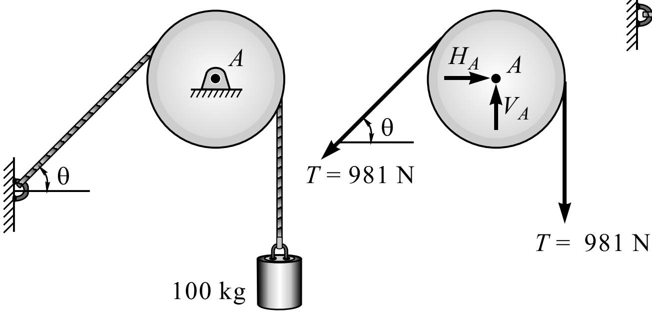


Fig. 3.4(8)-ii

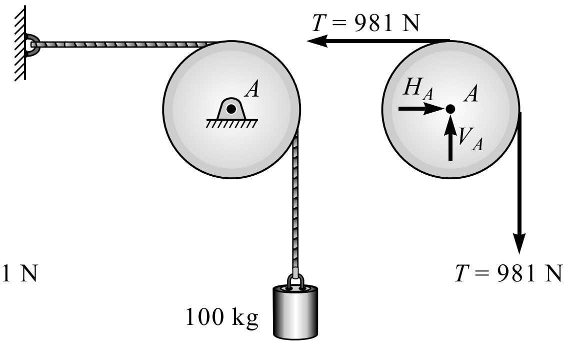


Fig. 3.4(8)-iii

9. Transfer of Tension of Rope on Frictionless Pulley from Circumference to the Centre of Pulley : For frictionless pulley, tension on both sides of rope is equal. Therefore, in FBD of pulley, the tension on two sides can be shown at the centre of pulley.

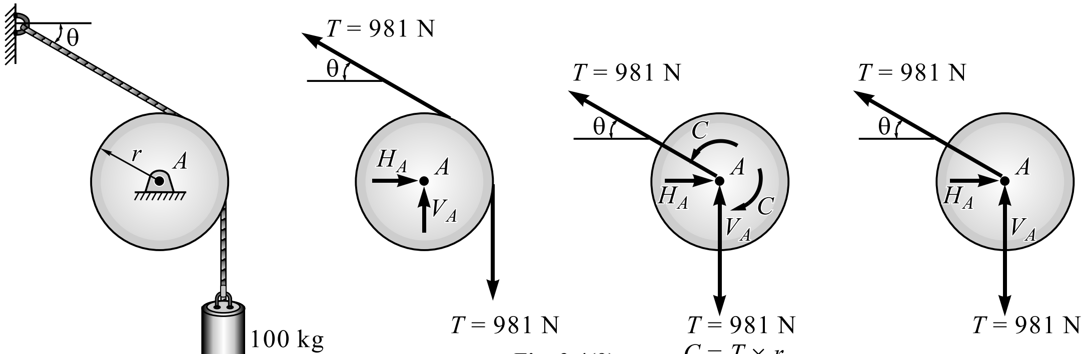


Fig. 3.4(9)

Reason **:** We know that the force can be transferred from one point to the other on the same rigid body by moving at the new point with parallel line of action and adding a couple whose magnitude is equal to the moment of force about new transferred point, here it is centre A. Since both the forces (tension of rope) are tangential to the circle, they are at perpendicular distance, equal to radius of pulley. So the two forces of the same magnitude when transferred to centre A, they also carry the couple of equal magnitude $C = T \times r$ but opposite in sense. Hence they cancel each other.

10. Straight Rod Supported by Knife Edge (Fulcrum) :

A straight rod having weight W rests against a horizontal smooth surface and knife edge support at B. It is an exceptional case where the reaction exerted by knife edge is normal to straight rod.

In Fig. 3.4(10), A is the smooth surface and we know reaction R_A will be perpendicular to the smooth surface. Here our emphasis is on the knife edge support which is at B and it exerts the normal reaction R_B perpendicular to the straight rod.

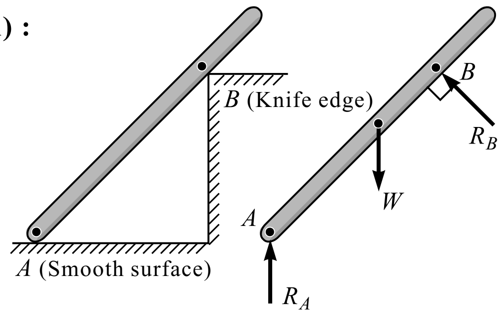


Fig. 3.4(10)

11. Rigid Body Supported by Knife Edge : A cylinder having weight W resting against a rectangular block is pulled by force P which is just enough to roll the cylinder over the rectangular block. Since the cylinder is just about to roll over the rectangular block, the reaction at contact B will become zero. The cylinder is subjected to two active forces, i.e., self-weight W and applied force P and one reactive force R due to knife edge support by rectangular block. We know by 'Three-force principle', three non-parallel forces must form concurrent force system for equilibrium condition. Therefore W, P and R must pass through the point of concurrence, as shown in Fig. 3.4(11).

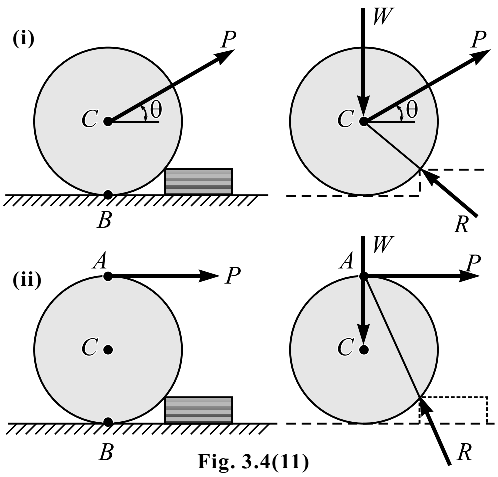


Fig. 3.4(11)

3.5 Lami's Theorem

If three concurrent coplanar forces acting on a body having same nature (i.e., pulling or pushing) are in equilibrium, then each force is proportional to the sine of angle included between the other two forces.

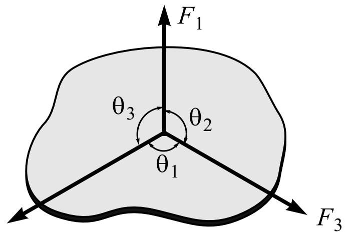


Fig. 3.5-i

By Lami's theorem, we have

$$\frac{F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta_2} = \frac{F_3}{\sin \theta_3}$$

Proof

By three-force triangle theorem, we get a closed triangle. Refer to Fig. 3.5-ii.

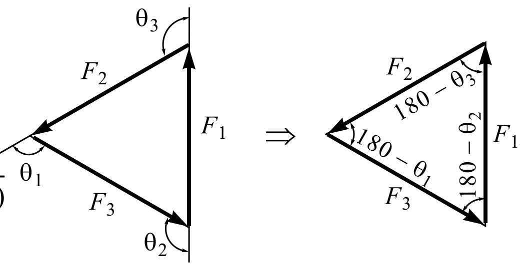


Fig. 3.5-ii

By sine rule, we have

$$\frac{F_1}{\sin (180^o - \theta_1)} = \frac{F_2}{\sin (180^o - \theta_2)} = \frac{F_2}{\sin (180^o - \theta_2)}$$

$$\therefore \quad \frac{F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta_2} = \frac{F_3}{\sin \theta_3}$$

Hence proved.

Limitations of Lami's Theorem

1. It is applicable to three non-parallel coplanar concurrent forces only.
2. Nature of three forces must be same (i.e., pulling or pushing)

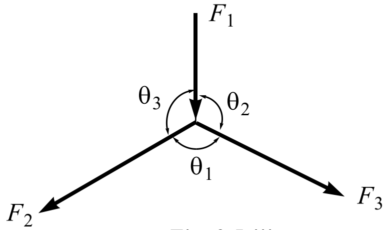


Fig. 3.5-iii

In Fig. 3.5-iii, F_2 and F_3 are pulling forces while F_1 is a pushing force. By principle of transmissibility, one can transmit F_1 on the other side of point of concurrency to make three forces of the same nature (i.e., pulling) and can apply Lami's theorem with due consideration of geometrical change in angles.

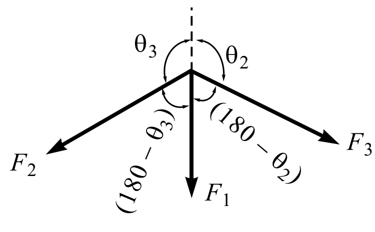


Fig. 3.5-iv

Easy Alternative Vector Approach

Since force is a vector quantity, negative sign (if placed), indicates opposite sense. So, one can prefer to change the force F_1 by introducing negative sign and then applying Lami's theorem as shown in Fig. 3.5-v.

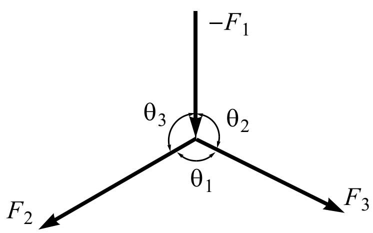


Fig. 3.5-v

$$\frac{-F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta_2} = \frac{F_3}{\sin \theta_3}$$

3.6 Solved Problems

Problem 1

Find the tension in each rope in Fig. 3.1(a).

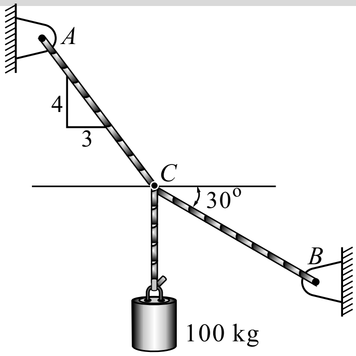


Fig. 3.1(a)

Solution

(i) Consider the FBD of Point C.

(ii) By Lami's theorem,

$$\frac{981}{\sin 156.87^\circ} = \frac{T_{AC}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 143.13^\circ}$$

$T_{AC} = 2162.76$ N

$T_{BC} = 1498.41$ N ***Ans.***

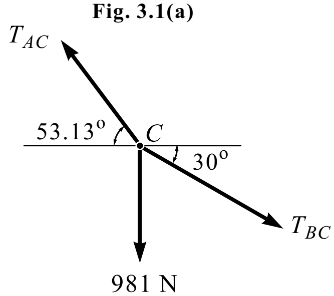


Fig. 3.1(b) : FBD

Problem 2

Block P 5 kg and block Q of mass m kg are suspended through the chord which is in the equilibrium position, as shown in Fig. 3.2(a). Determine the mass of block Q.

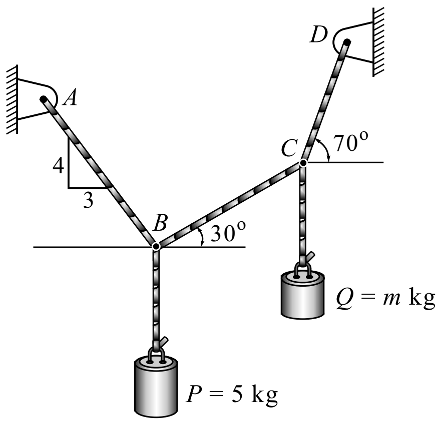


Fig. 3.2(a)

Solution

(i) Consider the FBD of Point B.

(ii) By Lami's theorem,

$$\frac{5 \times 9.81}{\sin 96.87^\circ} = \frac{T_{AB}}{\sin 120^\circ} = \frac{T_{BC}}{\sin 143.13^\circ}$$

$\therefore T_{AB} = 42.79$ N

$T_{BC} = 29.64$ N

(iii) Consider the FBD of Point C.

(iv) By Lami's theorem,

$$\frac{m \times 9.81}{\sin 140^\circ} = \frac{29.64}{\sin 160^\circ}$$

$\therefore m = 5.678$ kg ***Ans.***

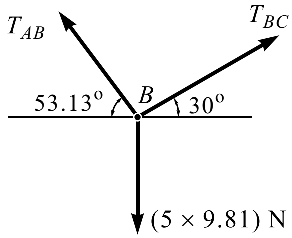


Fig. 3.2(b) : FBD of *B*

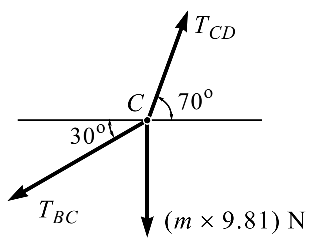


Fig. 3.2(c) : FBD of *C*

Problem 3

Find force transmitted by cable BC as shown in Fig. 3.3(a); E is a frictionless pulley, while B and D are weightless rings.

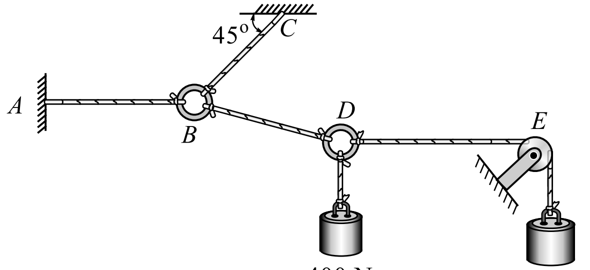


Fig. 3.3(a)

Solution

Let T_{BA} be the tension in the string BA

and T_{BC} be the tension in the string BC.

(i) Consider the FBD of portion *BD*.

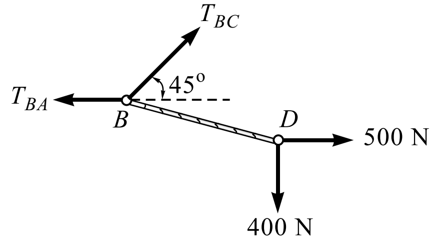


Fig. 3.3(b) : FBD of String BD

From equilibrium condition,

$$\Sigma F_y = 0$$

$$T_{BC} \sin 45^\circ = 400$$

$$\therefore T_{BC} = 565.69 \text{ N} \; (\measuredangle 45^\circ) \quad \textbf{\textit{Ans.}}$$

Problem 4

A circular roller of weight 1000 N and radius 20 cm hangs by a tie rod AB = 40 cm and rests against a smooth vertical wall at C as shown in Fig. 3.4(a). Determine the tension in the rod and reaction at point C.

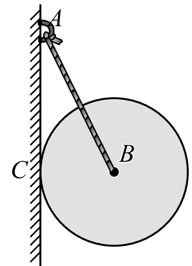


Fig. 3.4(a)

Solution

(i) Draw the FBD of the roller

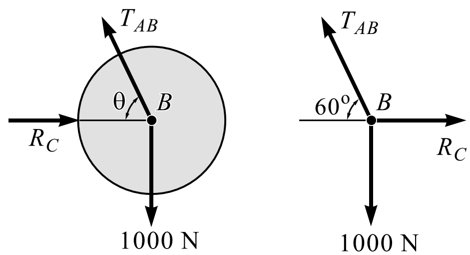


Fig. 3.4(b)

$$\cos \theta = \frac{20}{40}$$

$$\therefore \theta = 60^\circ$$

(ii) By Lami's theorem, we have

$$\frac{1000}{\sin 120^\circ} = \frac{T_{AB}}{\sin 90^\circ} = \frac{R_C}{\sin 150^\circ}$$

$$\therefore T_{AB} = 1154.7 \text{ N} \; (60^\circ \nwarrow) \quad \textbf{\textit{Ans.}}$$

$$\therefore R_C = 577.35 \text{ N} \; (\rightarrow) \quad \textbf{\textit{Ans.}}$$

Problem 5

A roller of weight W = 1000 N rests on a smooth inclined plane. It is kept from rolling down the plane by string AC as shown in Fig. 3.5(a). Find the tension in the string and reaction at the point of contact D.

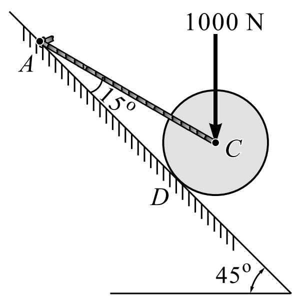


Fig. 3.5(a)

Solution

(i) Draw the FBD of the roller.

(ii) By Lami's theorem,

$$\frac{1000}{\sin 75^\circ} = \frac{R_D}{\sin 60^\circ} = \frac{-T_{AC}}{\sin 225^\circ}$$

$\therefore R_D = 896.58 \text{ N}$ $(\measuredangle 45^\circ)$ ***Ans.***

$\therefore T_{AC} = 732 \text{ N}$ ***Ans.***

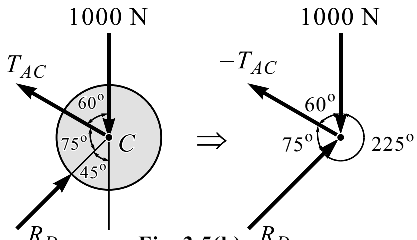


Fig. 3.5(b)

Problem 6

A cylinder of mass 50 kg is resting on a smooth surface, which is inclined at 30° and 60° to horizontal as shown in Fig. 3.6(a). Determine the reaction at contact A and B.

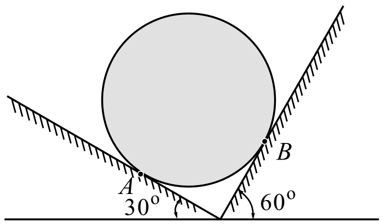


Fig. 3.6(a)

Solution

(i) Consider the FBD of the cylinder.

(ii) By Lami's theorem, we have

$$\frac{50 \times 9.81}{\sin 90^\circ} = \frac{R_A}{\sin 120^\circ} = \frac{R_B}{\sin 150^\circ}$$

$R_A = 424.79 \text{ N}$

$R_B = 245.25 \text{ N}$ ***Ans.***

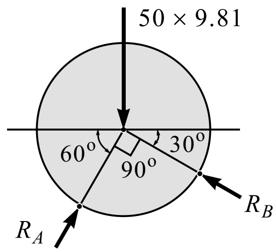


Fig. 3.6(b) : FBD of Cylinder

Problem 7

A 30 kg collar may slide on frictionless vertical rod and is connected to a 34 kg counter weight. Find the value of h for which the system is in equilibrium.

Refer to Fig. 3.7(a) for details.

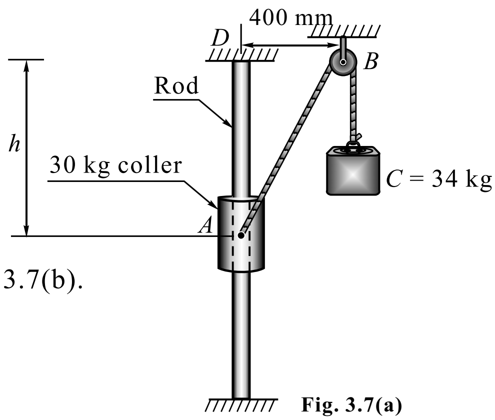


Fig. 3.7(a)

Solution

(i) Consider the FBD of the collar shown in Fig. 3.7(b).

(ii) By Lami's theorem,

$$\frac{T}{\sin 90^\circ} = \frac{30 \times 9.81}{\sin (180^\circ - \theta)}$$

$$\sin \theta = \frac{30 \times 9.81}{34 \times 9.81} \times \sin 90^\circ$$

$\therefore \theta = 61.93^\circ$

(iii) Refer to Fig. 3.7(c).

$$\tan 61.93^\circ = \frac{h}{400}$$

$h = 750 \text{ mm}$ ***Ans.***

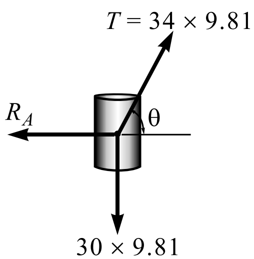


Fig. 3.7(b) : FBD of Collar

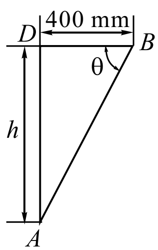


Fig. 3.7(c)

Problem 8

Determine the force P applied at 30^o to the horizontal just necessary to start a roller having radius 50 cm over an obstruction 12 cm high, if the roller is of mass 100 kg, shown in Fig. 3.8(a). Also find the magnitude and direction of P when it is minimum.

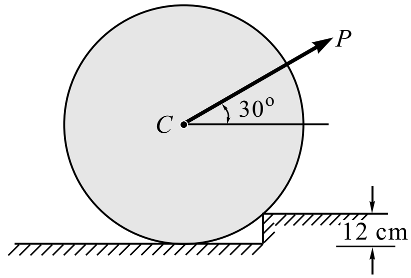


Fig. 3.8(a)

Solution

(i) Consider the FBD of the roller

As per the given condition, P is just sufficient to start the roller. At this instant, the roller will not have any pressure on the horizontal surface. Therefore, the surface will not offer any reaction. We can identify that this body is subjected to three forces, viz., 100×9.81, P and R. Since $100 \times 9.81$and P are passing through C therefore the third force, i.e., the reaction R must also pass through same point C, as per three-force principle.

Let us draw concurrent forces for simplicity, refer to Fig. 3.8(b).

$$\sin\theta = \frac{38}{50} \quad \therefore \theta = 49.46^o$$

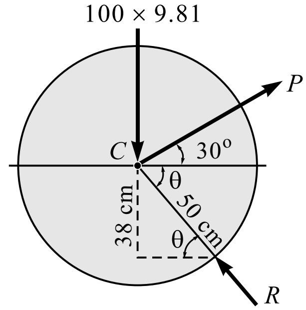


Fig. 3.8(b) : FBD of Roller

(ii) Before applying Lami's theorem, we should overcome its limitation, i.e., nature of three forces must be the same. Here P is directed away from point C. Since we know that force is a vector quantity, by placing negative sign we can satisfy the Lami's theorem requirement. Now, nature of three forces is pushing towards point C.

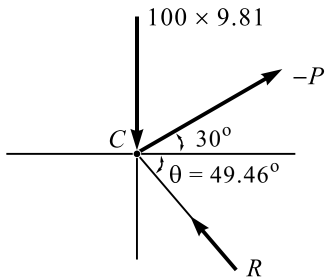


Fig. 3.8(c)

(iii) By Lami's theorem,

$$\frac{981}{\sin 79.46^o} = \frac{-P}{\sin 220.54^o}$$

$$P = \frac{-981 \times \sin 220.54^o}{\sin 79.46^o}$$

$$\therefore P = 648.57 \text{ N} \quad \textbf{\textit{Ans.}}$$

(iv) Method I : To find P_{min}

By Lami's theorem,

$$\frac{-P_{min}}{\sin 220.54^o} = \frac{981}{\sin(\alpha + 49.46^o)}$$

$$P_{min} = \frac{-981 \times \sin 220.54^o}{\sin(\alpha + 49.46^o)} \quad \dots (I)$$

For P_{min} the denominator should be maximum, i.e.,

$$\sin(\alpha + 49.46^o) = 1$$

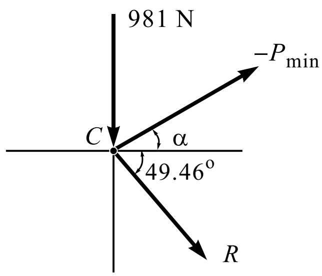


Fig. 3.8(d)

$\therefore \ \alpha + 49.46 = 90 \qquad \therefore \ \alpha = 40.54^{\circ}$

From Eq. (I)

$$P_{min} = \frac{-981 \times \sin 220.54^{\circ}}{\sin (40.54^{\circ} + 49.46^{\circ})}$$

$\therefore \ P_{min} = 637.63$ N ($\measuredangle \alpha$) ***Ans.***

(iv) Method II : To find P_{min}

To start the roller over the obstruction, P should balance the anticlockwise moment of W about point A with an equal clockwise moment.

The maximum distance between the point A and line of action of P is AC. Therefore, to create a given moment about A, the force P will be minimum when it acts at right angle to AC as shown in Fig. 3.8(e). Then the P_{min} will make an angle $\alpha = 40.54^{\circ}$.

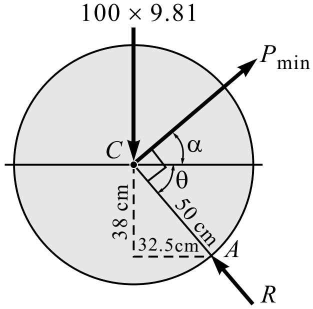


Fig. 3.8(e) : FBD of Roller

$\alpha + \theta = 90^{\circ}$; $\alpha + 49.49^{\circ} = 90^{\circ} \therefore \ \alpha = 40.54^{\circ}$

$\Sigma M_A = 0$

$100 \times 9.81 \times 32.5 - P_{min} \times 50 = 0$

$\therefore \ P_{min} = 637.63$ N ($\measuredangle \alpha$) ***Ans.***

Problem 9

Two identical rollers each of mass 50 kg are supported by an inclined plane and a vertical wall as shown in Fig. 3.9(a). Assuming smooth surfaces, find the reactions induced at the point of support A, B and C.

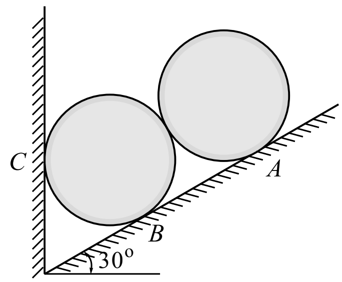


Fig. 3.9(a)

Solution

(i) Consider FBD of both rollers together and let R be the radius of rollers.

(ii) $\Sigma M_O = 0$

$R_A \times 2R - 50 \times 9.81 \cos 30^{\circ} \times 2R = 0$

$R_A = 424.79$ N (60°) ***Ans.***

(iii) $\Sigma F_y = 0$

$R_B \cos 30^{\circ} + R_A \cos 30^{\circ} - 50 \times 9.81 - 50 \times 9.81 = 0$

$R_B = 707.97$ N (60°) ***Ans.***

(iv) $\Sigma F_x = 0$

$R_C - R_A \sin 30^{\circ} - R_B \sin 30^{\circ} = 0$

$R_C = 424.79 \sin 30^{\circ} + 707.97 \sin 30^{\circ}$

$R_C = 566.38$ N ($\rightarrow$) ***Ans.***

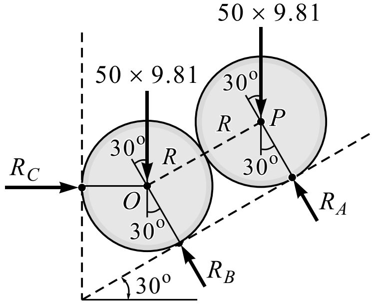


Fig. 3.9(b)

Problem 10

A uniform wheel of 60 cm diameter and weighing 1000 N rest against a rectangular block 15 cm high lying on a horizontal plane as shown in Fig. 3.10(a). It is to be pulled over the block by a horizontal force P applied to the end of a string wound round the circumference of the wheel. Find force P when the wheel is just about to roll over the block.

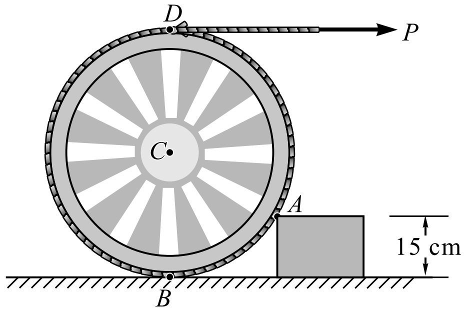


Fig. 3.10(a)

Solution

Since the wheel is just about to roll over the rectangular block, the reaction at contact B will become zero. The wheel is subjected to two active forces, i.e., self-weight 1000 N and horizontal applied force P and one reactive force due to knife edge support by rectangular block. Therefore, by three force principle, all three forces must be concurrent and should pass through point D.

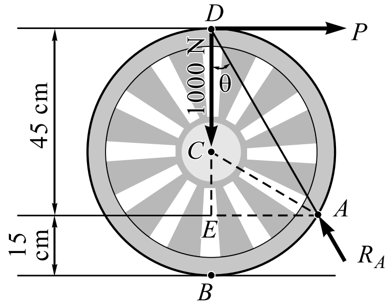


Fig. 3.10(b)

(i) Draw the FBD of wheel corresponding to above discussion. Refer to Fig. 3.10(b).

In $\Delta\, ACE$

$$AE = \sqrt{AC^2 - CE^2} = \sqrt{30^2 - 15^2}$$

$$AE = 25.98 \text{ cm}$$

$$\therefore \tan\theta = \frac{AE}{DE} = \frac{25.98}{45} \quad \therefore \theta = 30^\circ$$

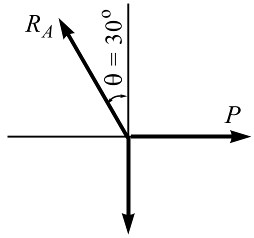


Fig. 3.10(c)

(ii) By Lami's theorem, we have

$$\frac{1000}{\sin 120^\circ} = \frac{P}{\sin 150^\circ}$$

$\therefore P = 577.35$ N ***Ans.***

Problem 11

Two cylinders each of diameter 100 mm and each weighing 200 N are placed as shown in Fig. 3.11(a). Assuming that all the contact surfaces are smooth, find the reactions at A, B and C.

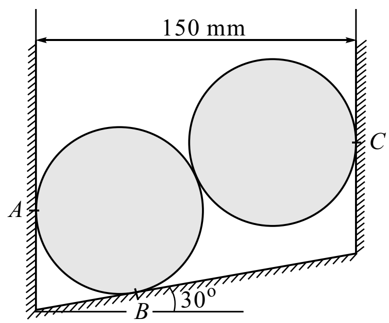


Fig. 3.11(a)

Solution

Note : Assuming the base line inclined at 30° to horizontal.

(i) Consider the FBD of both the rollers together as shown in Fig. 3.11(b).

(ii) From the FBD of lower cylinder.

$$\Sigma F_y = 0$$

$$R_B \cos 30^\circ - 200 - R \sin 60^\circ = 0$$

$$R_B = 461.89 \text{ N } \left(60^\circ \nwarrow\right) \quad \textbf{\textit{Ans.}}$$

$$\Sigma F_x = 0$$

$$R_A - R\cos 60^\circ - R_B \sin 30^\circ = 0$$

$$R_A = 346.42 \text{ N } (\rightarrow) \quad \textbf{\textit{Ans.}}$$

(iii) From the FBD of upper cylinder.

By Lami's theorem,

$$\frac{200}{\sin 120^\circ} = \frac{R_C}{\sin 150^\circ} = \frac{R}{\sin 90^\circ}$$

$$R_C = 115.47 \text{ N } (\leftarrow)$$

$$R = 230.94 \text{ N } \left(\nearrow 60^\circ\right) \quad \textbf{\textit{Ans.}}$$

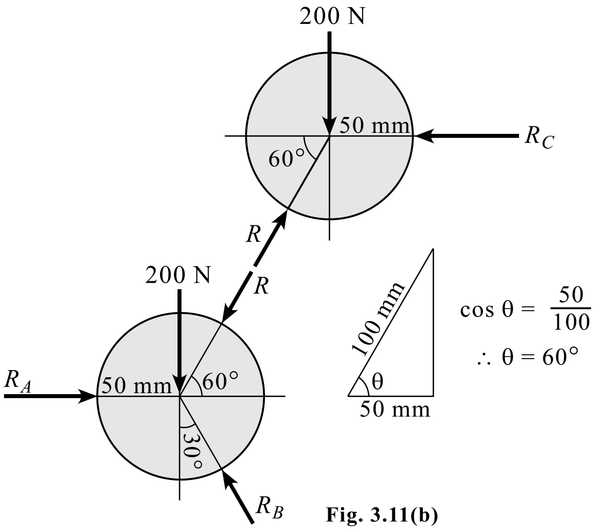


Fig. 3.11(b)

Problem 12

Two spheres, A and B, are resting in a smooth through as shown in Fig. 3.12(a). Draw the free body diagrams of A and B showing all the forces acting on them, both in magnitude and direction. Radius of spheres A and B are 250 mm and 200 mm, respectively.

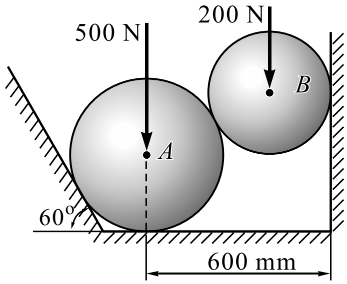


Fig. 3.12(a)

Solution

(i) From Fig. 3.12(b). AB = 450 mm and AC = 400 mm

$$\cos\theta = \frac{AC}{AB} = \frac{400}{450} \qquad \therefore\ \theta = 27.27^\circ$$

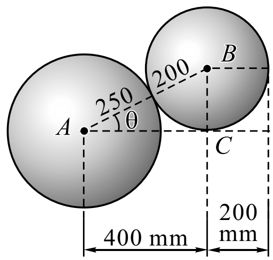


Fig. 3.12(b)

(ii) Consider the FBD of Sphere *B* [Fig. 3.12(c)]

By Lami's theorem,

$$\frac{200}{\sin 152.73^\circ} = \frac{R_1}{\sin 117.27^\circ} = \frac{R_2}{\sin 90^\circ}$$

$$\therefore\ R_1 = 388 \text{ N } (\leftarrow) \text{ and}$$

$$R_2 = 436.51 \text{ N } \left(\nearrow 27.27^\circ\right) \quad \textbf{\textit{Ans.}}$$

(iii) Consider the FBD of Sphere *A* [Fig. 3.12(d)]

$$\Sigma F_x = 0$$

$$R_4 \cos 30^\circ - 436.51 \cos 27.27^\circ = 0$$

$$R_4 = 448 \text{ N } \left(\nearrow 30^\circ\right) \quad \textbf{\textit{Ans.}}$$

$$\Sigma F_y = 0$$

$$-500 + R_3 - 436.51 \sin 27.27^\circ + 448 \sin 30^\circ = 0$$

$$R_3 = 476 \text{ N } (\uparrow) \quad \textbf{\textit{Ans.}}$$

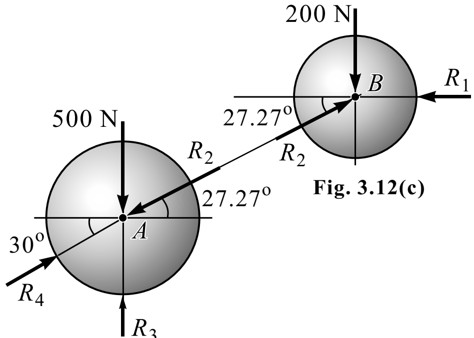


Fig. 3.12(c)

Fig. 3.12(d)

Problem 13

Two spheres A and B of weight 1000 N and 750 N, respectively are kept as shown in the Fig. 3.13(a). Determine the reactions at all contact points 1, 2, 3 and 4. Radius of A = 400 mm and Radius of B = 300 mm.

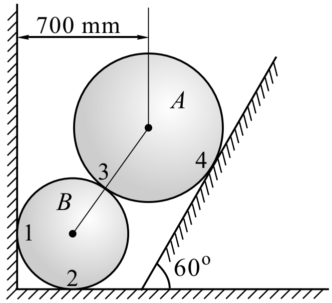


Fig. 3.13(a)

Solution

(i) Consider the FBD of Sphere *A* [Fig. 3.13(b)]

By Lami's theorem, we have

$$\frac{1000}{\sin(180-30-55.15)^\circ} = \frac{R_3}{\sin(90+30)^\circ} = \frac{R_4}{\sin(90+55.15)^\circ}$$

$R_3 = 869.14$ N $(\angle 55.15^\circ)$

$R_4 = 573.48$ N (30°) ***Ans.***

(ii) Consider the FBD of Sphere *B*

$\Sigma F_x = 0$

$R_1 - R_3 \cos 55.15^\circ = 0$

$R_1 = 496.65$ N $(\rightarrow)$ ***Ans.***

$\Sigma F_y = 0$

$R_2 - 750 - R_3 \sin 55.15^\circ = 0$

$R_2 = 1463.26$ N $(\uparrow)$ ***Ans.***

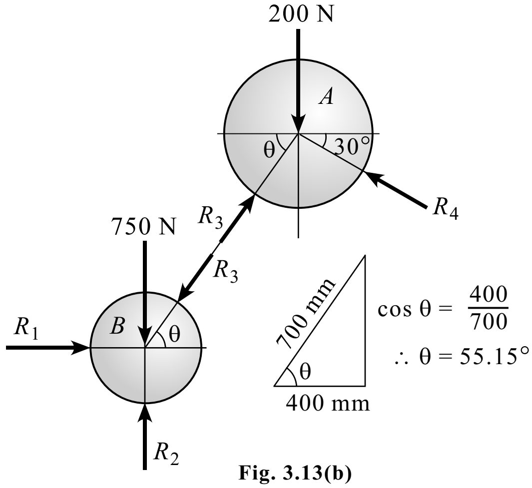


Fig. 3.13(b)

Problem 14

A right-circular cylinder of diameter 40 cm, open at both ends, rests on a smooth horizontal plane. Inside the cylinder, there are two sphere having weights and radii as given [Fig. 3.14(a)]. Find the minimum weight of the cylinder for which it will not tip over.

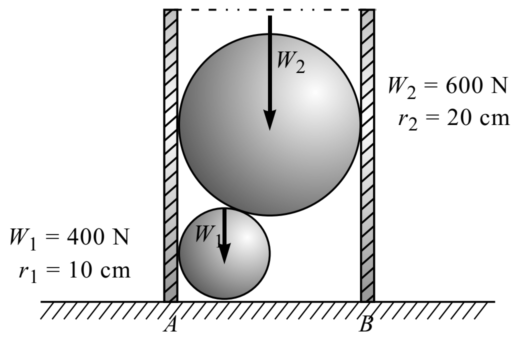


Fig. 3.14(a)

Solution

(i) Consider the FBD of both the spheres together

Let P and Q be the centre points of the spheres.

$PQ = 30$ cm

$30^2 = 10^2 + h^2$

$h = 28.28$ cm

$$\Sigma M_P = 0$$

$$R_2 \times h - W_2 \times 10 = 0$$

$$R_2 = \frac{600 \times 10}{28.28}$$

$$\therefore R_2 = 212.16 \text{ N } (\leftarrow) \quad \textit{Ans.}$$

$$\Sigma F_x = 0$$

$$R_1 - R_2 = 0$$

$$\therefore R_1 = 212.16 \text{ N } (\rightarrow) \quad \textit{Ans.}$$

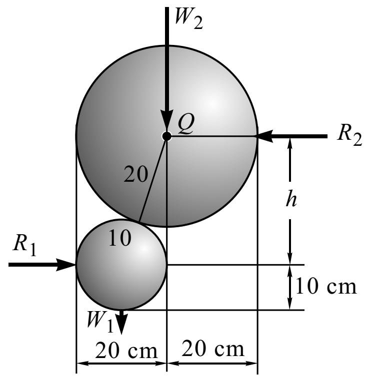


Fig. 3.14(b) : FBD of Both the Spheres Together

(ii) Consider the FBD of the cylinder

If the weight of the cylinder is negligible, the cylinder will tip about point B.

So, to avoid tipping $W_{minimum}$ is required.

$$\Sigma M_B = 0$$

$$W \times 20 + 212.16 \times 10 - 212.16 \times 38.28 = 0$$

$$W = 300 \text{ N } (\downarrow) \quad \textit{Ans.}$$

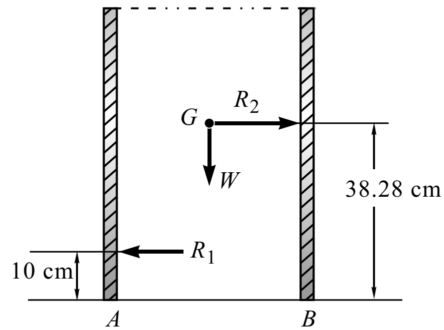
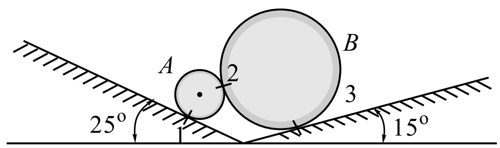


Fig. 3.14(c) : FBD of Cylinder

Problem 15

Determine the reactions at points of contact 1, 2 and 3. Assume smooth surfaces.



$W_A = 1$ kg $r_A = 1$ cm,

$W_B = 4$ kg $r_B = 4$ cm.

Fig. 3.15(a)

Solution

(i) Consider the FBD of both the cylinders together

$$\Sigma F_x = 0$$

$$R_1 \cos 65^\circ - R_3 \cos 75^\circ = 0$$

$$\therefore R_1 = 0.6124 \, R_3$$

$$\Sigma F_y = 0$$

$$R_1 \sin 65^\circ + R_3 \sin 75^\circ + 1 \times 9.81 + 4 \times 9.81 = 0$$

$$0.6124 \, R_3 \sin 65^\circ + R_3 \sin 75^\circ + 5 \times 9.81 = 0$$

$$R_3 = 32.216 \text{ N } \left(\measuredangle 75^\circ\right) \text{ and } R_1 = 19.729 \text{ N } \left(\measuredangle 65^\circ\right) \quad \textit{Ans.}$$

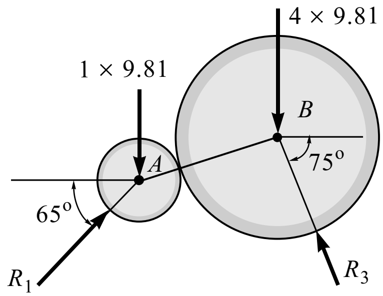


Fig. 3.15(b) : FBD of A and B Together as a Single Body

(ii) Consider the FBD of 1 kg cylinder A

$$\Sigma F_y = 0$$

$$19.729 \sin 65^\circ - 1 \times 9.81 - R_2 \sin \alpha = 0$$

$$R_2 \sin \alpha = 8.071 \qquad \text{......(I)}$$

$$\Sigma F_x = 0$$

$$19.729 \cos 65^\circ - R_2 \cos \alpha = 0$$

$$R_2 \cos \alpha = 8.338 \qquad \text{......(II)}$$

Dividing Eq.(I) by Eq. (II), we get

$$\alpha = 44.07^\circ$$

From Eq. (I), we get

$$R_2 = 11.604 \text{ N } \left(\measuredangle 44.07^\circ\right)$$ ***Ans.***

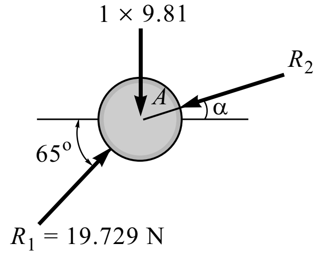


Fig. 3.15(c) : FBD of *A*

Problem 16

Three identical tubes of weights 8 kN each are placed as shown in Fig. 3.16(a). Determine the forces exerted by the tubes on the smooth walls and floor.

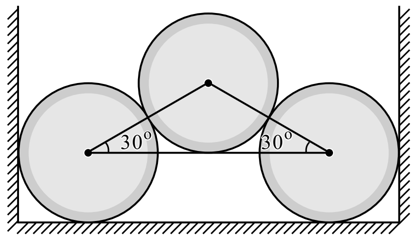


Fig. 3.16(a)

Solution

(i) Consider the FBD of upper tube shown in Fig. 3.16(b).

Since tubes are identical and placed symmetrically, reactions R at contact will be same.

(ii) By Lami's theorem

$$\frac{8}{\sin 120^\circ} = \frac{R}{\sin 120^\circ} \quad \therefore R = 8 \text{ kN}$$ ***Ans.***

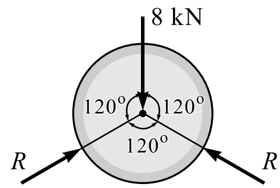


Fig. 3.16(b)

(iii) Consider the FBD of any lower tube (say left) Fig. 3.16(c).

(iv) By Lami's theorem,

$$\frac{8}{\sin 90^\circ} = \frac{R_W}{\sin 120^\circ} = \frac{R_F}{\sin 150^\circ}$$

(v) $R_W = 6.928$ kN (Force exerted by the tubes on the smooth wall) ***Ans.***

$R_F = 4$ kN (Force exerted by the tubes on the smooth floor) ***Ans.***

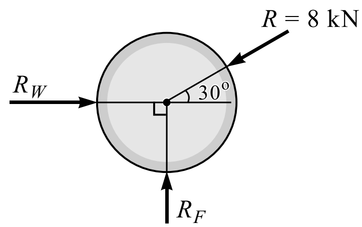


Fig. 3.16(c)

Problem 17

Two smooth circular cylinder is of weight $W = 500$ N each and radius $r = 150$ mm are connected at their centre by a string of length $l = 400$ mm and rest upon a horizontal plane supporting above them a third cylinder of weight 1000 N and radius $r = 150$ mm as shown in Fig. 3.17(a). Find the tension in the string and pressure at the points of contact D and E.

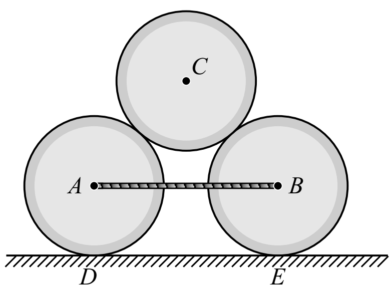


Fig. 3.17(a)

Solution

(i) Consider the $\Delta\ ABC$ and simplify its geometric length and angle, as shown in Fig. 3.17(b).

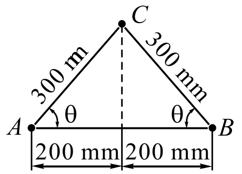


Fig. 3.17(b)

$$\cos\theta = \frac{200}{300} \qquad \therefore\ \theta = 48.19^\circ$$

(ii) Draw the FBD of the upper cylinder C [Fig. 3.20(c)].

(iii) By Lami's theorem

$$\frac{1000}{\sin 83.62^\circ} = \frac{R}{\sin 138.19^\circ}$$

$\therefore\ R = 670.82$ N ***Ans.***

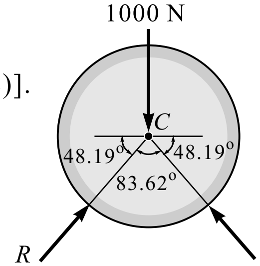


Fig. 3.17(c)

(iv) Draw the FBD of the lower cylinder A as shown in Fig. 3.17(d).

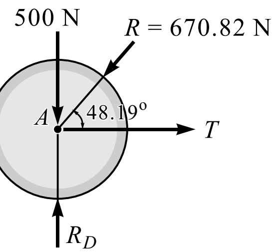


Fig. 3.17(d)

$$\Sigma F_x = 0$$

$$T - 670.82 \cos 48.19^\circ = 0$$

$T = 447.21$ N ***Ans.***

$$\Sigma F_y = 0$$

$$R_D - 500 - 670.82 \sin 48.19^\circ = 0$$

$$R_D = 1000 \text{ N}$$

$R_D = R_E = 1000$ N ***Ans.*** (Since the loading is symmetric, therefore, reaction will be equal.)

Problem 18

Three cylinders are piled up in a rectangular channel, as shown in Fig. 3.18(a). Determine the reactions at point 6 between the cylinder A and the vertical wall of the channel.

(Cylinder A : radius = 4 cm, m = 15 kg,
Cylinder B : radius = 6 cm, m = 40 kg,
Cylinder C : radius = 5 cm, m = 20 kg).

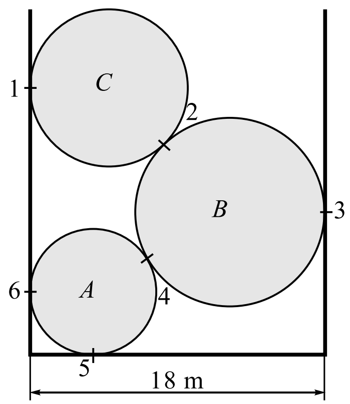


Fig. 3.18(a)

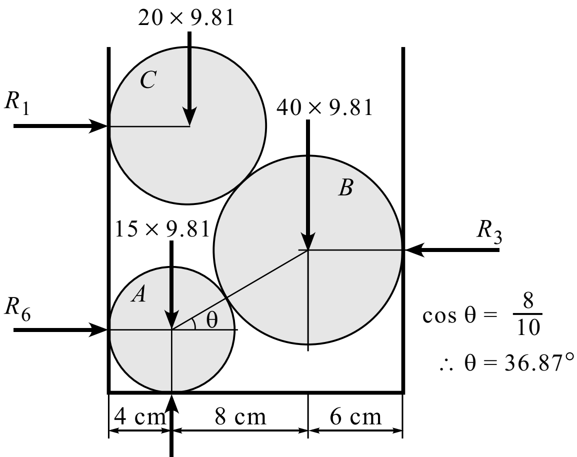


Fig. 3.18(b)

Solution

(i) Consider FBD of entire system, as shown in Fig. 3.18(b).

$\Sigma F_y = 0$

$R_5 - 20 \times 9.81 - 40 \times 9.81 - 15 \times 9.81 = 0$

$R_5 = 735.75$ N ***Ans.***

(ii) Consider the FBD of cylinder *A* [Refer to Fig. 3.18(c)].

$\Sigma F_y = 0$

$735.75 - 15 \times 9.81 - R_4 \sin 36.87^o = 0$

$R_4 = 981$ N ***Ans.***

$\Sigma F_x = 0$

$R_6 - R_4 \cos 36.87^o = 0$

$R_6 = 784.8$ N $(\rightarrow)$ ***Ans.***

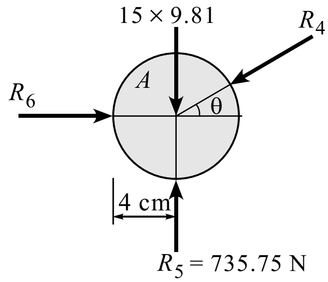


Fig. 3.18(c)

Problem 19

Three identical spheres *P*, *Q*, *R* of weight *W* are arranged on smooth inclined surface, as shown in Fig. 3.19(a). Determine the angle α which will prevent the arrangement from collapsing.

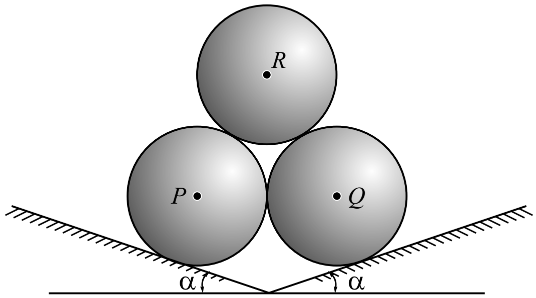


Fig. 3.19(a)

Solution

(i) Consider the FBD of upper sphere *R*

Since spheres are identical, therefore, due to symmetry, reaction at contact point will be same (*R*).

Δ *PQR* is forming equilateral triangle. Thus, included angle between reactions *R* is 60°.

By Lami's theorem,

$$\frac{W}{\sin 60^o} = \frac{R}{\sin 150^o} \quad \therefore R = 0.577\, W$$

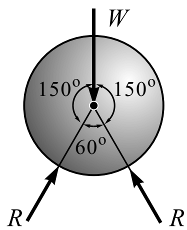


Fig. 3.19(b) : FBD of Sphere *R*

(ii) Consider the FBD of any one lower sphere (say *P*)

The reaction at contact between two lower spheres will be zero because at the required angle α the arrangement is about to collapse.

$\therefore R_2 = 0$

$\Sigma F_x = 0$

$R_1 \sin \alpha = 0.577\, W \cos 60^o$ ......(I)

$\Sigma F_y = 0$

$R_1 \cos \alpha = W + 0.577\, W \sin 60^o$

Dividing Eq. (I) by Eq. (II)

$$\tan \alpha = \frac{0.577\, W \cos 60^o}{W + 0.577\, W \sin 60^o} \quad \therefore \alpha = 10.89^o \quad \textbf{\textit{Ans.}}$$

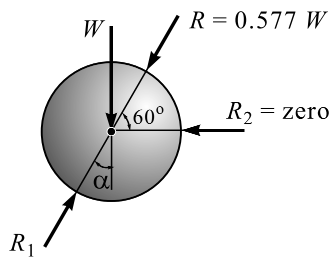


Fig. 3.19(c) : FBD of Sphere *P*

Problem 20

A mass raises a 10 kg joist of length 4 m by pulling on a rope. Find the tension in the rope and reaction at A. Refer Fig. 3.20(a).

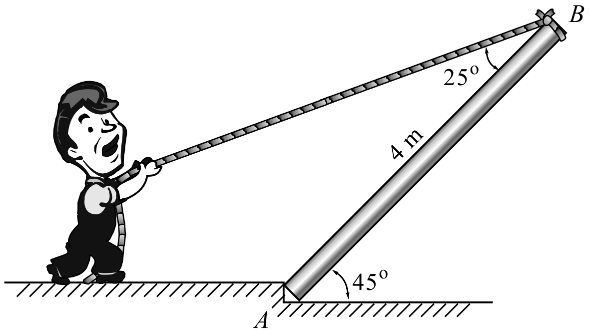


Fig. 3.20(a)

Solution

Method I

(i) Consider the FBD of the joist

By three-force principle in equilibrium R_A, T and 10×9.81 N must pass through a common point say D.

In $\Delta\, BCD$, by sine rule

$$\frac{CD}{\sin 25^\circ} = \frac{2}{\sin 110^\circ}$$

$$CD = 0.9 \text{ m}$$

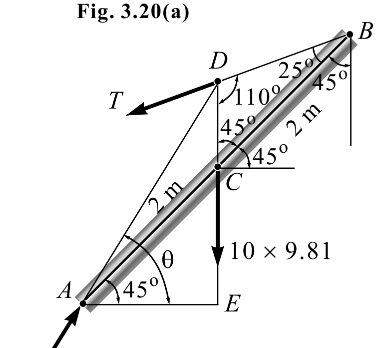


Fig. 3.20(b) : FBD of Joist

(ii) In $\Delta\, AEC$,

$$AE = CE = \sqrt{2}$$

$$\tan\theta = \frac{DE}{AE} = \frac{CD + CE}{AE}$$

$$\tan\theta = \frac{0.9 + \sqrt{2}}{\sqrt{2}} \qquad \therefore\ \theta = 58.57^\circ$$

(iii) Considering three concurrent forces at point D

By Lami's theorem, we have

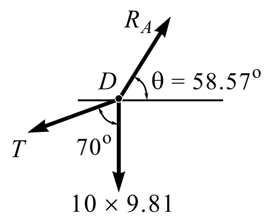


Fig. 3.20(c)

$$\frac{98.1}{\sin 141.43^\circ} = \frac{T}{\sin 148.57^\circ} = \frac{R_A}{\sin 70^\circ}$$

$$T = 82.05 \text{ N}$$

$R_A = 147.86$ N $\left(\measuredangle\theta = 58.57^\circ\right)$ ***Ans.***

Method II

(i) $\Sigma M_A = 0$

$$T \sin 25^\circ \times 4 - 10 \times 9.81 \times 2 \cos 45^\circ = 0 \ \therefore\ T = 82.07 \text{ N}$$

(ii) $\Sigma F_x = 0$

$$H_A - T\cos 20^\circ = 0 \ \therefore\ H_A = 77.12 \text{ N}$$

(iii) $\Sigma F_y = 0$

$$V_A - 10 \times 9.81 - T \sin 20^\circ = 0 \ \therefore\ V_A = 126.17 \text{ N}$$

(iv) $\tan\theta = \dfrac{V_A}{H_A} = \dfrac{126.17}{77.12} \quad \therefore\ \theta = 58.57^\circ$

(v) $R_A = \sqrt{H_A^2 + V_A^2} = \sqrt{77.12^2 + 126.17^2}$

$R_A = 147.87$ N $\left(\measuredangle\theta = 58.57\right)$ ***Ans.***

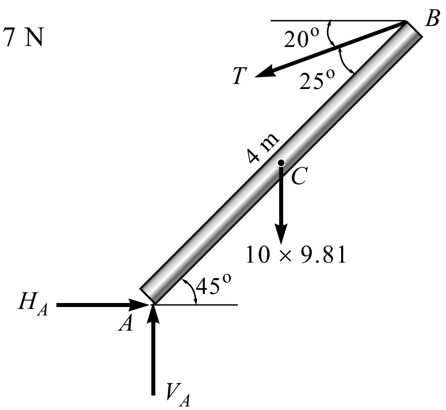


Fig. 3.20(d) : FBD of Joist

Problem 21

A lever AB is hinged at C and attached to a control cable at A, as shown in Fig. 3.21(a). If the lever is subjected to a 75 N vertical force at B, determine the tension in the cable and reaction at C.

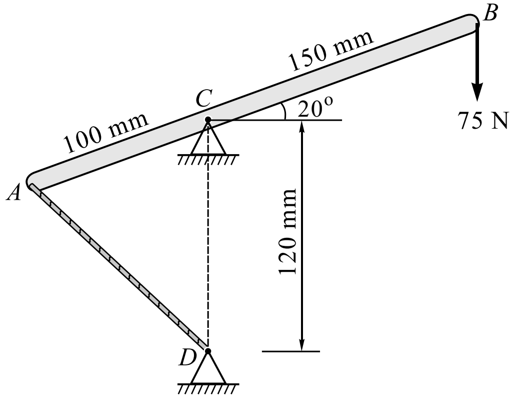


Fig. 3.21(a)

Solution

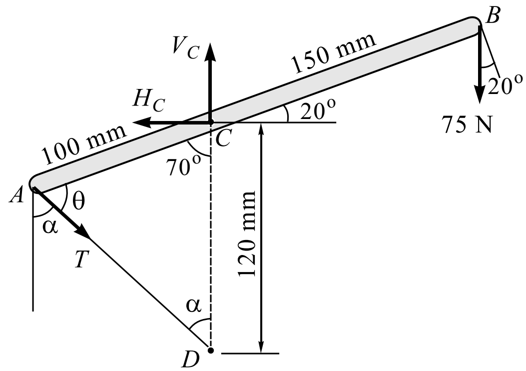


Fig. 3.21(b)

(i) Consider the FBD of Lever AB in Fig. 3.21(b).

(ii) In ΔACD, by cosine rule

$$AD = \sqrt{100^2 + 120^2 - 2 \times 100 \times 120 \cos 70^\circ}$$

$$AD = 127.25 \text{ mm}$$

By sine rule,

$$\frac{127.25}{\sin 70^\circ} = \frac{120}{\sin \theta} \qquad \therefore \theta = 62.39^\circ$$

$$\alpha = 180 - 70 - \theta \qquad \therefore \alpha = 47.61^\circ$$

(iii) $\Sigma M_C = 0$

$$T \sin 62.39^\circ \times 100 - 75 \cos 20^\circ \times 150 = 0$$

$$T = 119.3 \text{ N}$$

(iv) $\Sigma F_x = 0$

$$-H_C + 119.3 \sin 47.61^\circ = 0$$

$H_C = 88.11$ N (←) ***Ans.***

(v) $\Sigma F_y = 0$

$$V_C - 75 - 119.3 \cos 47.61^\circ = 0$$

$V_C = 155.43$ N (↑) ***Ans.***

Problem 22

Two cylinders, having weight W_A = 2000 N and W_B = 1000 N are resting on smooth inclined planes having inclination 60° and 45° with the horizontal respectively, as shown in Fig. 3.22(a). They are connected by a weightless bar *AB* with hinge connections. The bar *AB* makes 15° angle with the horizontal. Find the magnitude of the force *P* required to hold the system in equilibrium.

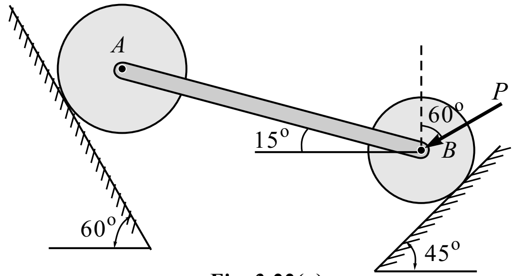


Fig. 3.22(a)

Solution

(i) In a given system of rigid bodies (two cylinders and one bar), the bar *AB* is connected at its extreme ends by frictionless pin. So, we can identify bar *AB* is a two-force member which can be indicated by F_{AB}.

(ii) Consider the FBD of cylinder W_A = 2000 [Fig. 3.22(b)].

By Lami's theorem,

$$\frac{2000}{\sin 135^\circ} = \frac{F_{AB}}{\sin 120^\circ} \quad \therefore \ F_{AB} = 2449.49 \text{ N}$$

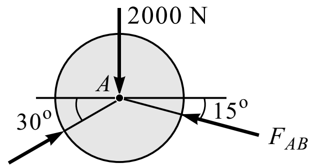


Fig. 3.22(b)

(iii) Consider the FBD of cylinder W_B = 1000 N [Fig. 3.22(c)].

$$\Sigma F_x = 0$$

$$2449.49 \cos 15^\circ - R \cos 45^\circ - P \sin 60^\circ = 0$$

$$R \cos 45^\circ = 2449.49 \cos 15^\circ - P \sin 60^\circ \ \ldots \text{(I)}$$

$$\Sigma F_y = 0$$

$$R \cos 45^\circ - P \cos 60^\circ - 2449.49 \sin 15^\circ - 1000 = 0$$

$$R \cos 45^\circ = P \cos 60^\circ + 2449.49 \sin 15^\circ + 1000 \ldots \text{(II)}$$

Now, solving Eqs. (I) and (II), we get

P = 535.9 N ***Ans.***

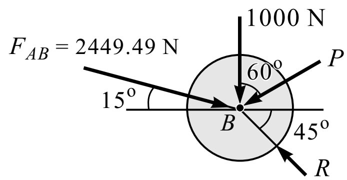


Fig. 3.22(c)

Problem 23

A uniform rod *AB* of length 3*R* and weight *W* rests inside a hemispherical bowl of radius *R* as shown in Fig. 3.23(a). Neglecting friction, determine angle θ corresponding to equilibrium.

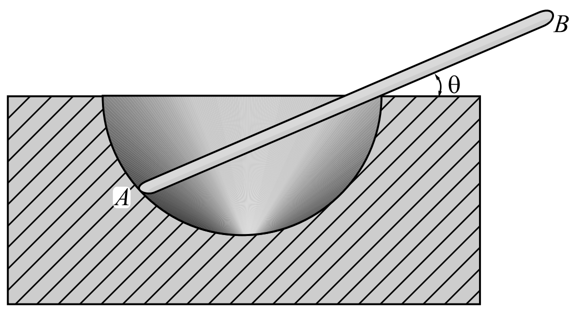


Fig. 3.23(a)

Solution

(i) Refer to the FBD of rod *AB* as shown in Fig. 3.23(b).

The one active force is weight of rod W acting vertically down through the centre of gravity of rod AB and two reactive forces reaction R_A acting along the normal to hemisphere at A and passing through centre O and reaction R_D acting along the normal to the rod AB at D (knife edge support).

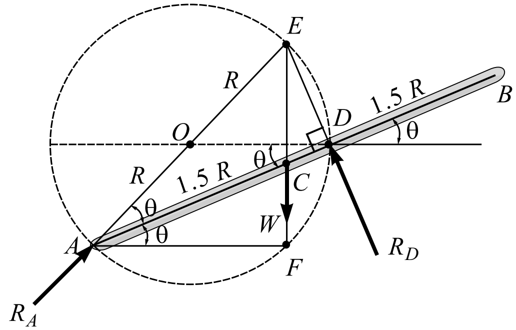


Fig. 3.23(b) : FBD of Rod *AB*

(ii) By three-force principle in equilibrium these three forces must pass through a common point (say E). This point E must lie on the circle as shown because $\angle ADE = 90^o$ (Angle subtended by a diameter at any point lying on the circumference of the circle is a right angle).

(iii) By the geometry of the figure

$\angle\theta = \angle ODA = \angle OAD = \angle DAF$

In $\Delta\, EAF$ and $\Delta\, CAF$

$EA = 2\,R$ and $CA = 1.5\,R$

$\angle EAF = 2\theta$ and $\angle CAF = \theta$

$AF = 2\,R \cos 2\theta$ and $AF = 1.5\,R \cos\theta$

(iv) $2\,R\cos 2\theta = 1.5\,R \cos\theta$

$2\cos 2\theta = 1.5\cos\theta$

$2(2\cos^2\theta - 1) = 1.5\cos\theta$

$4\cos^2\theta - 1.5\cos\theta - 2 = 0$

Solving the quadratic equation, we get $\theta = 23.2^o$ ***Ans.***

3.7 Types of Beam

In a structure, horizontal member which takes transverse load in addition to other loading is called ***beam***. Transverse load means *load perpendicular to the length of the beam*.

In engineering structures like bridges, beam is one of the important structural member. In trusses and frames, pin-joined members take only tensile or compressive load. Beam is capable to take all types of load, i.e., transverse load, tensile load, compressive load, twisting load, etc.

Further beams may carry different types of transverse load such as point load, uniformly distributed load, uniformly varying load, etc.

Classification of Beams : Beams are classified depending upon the type of support, as shown below.

1. **Simply Supported Beam :** As the name indicates, it is the simplest of all beams which is supported by a hinge at one end and a roller at the other end.

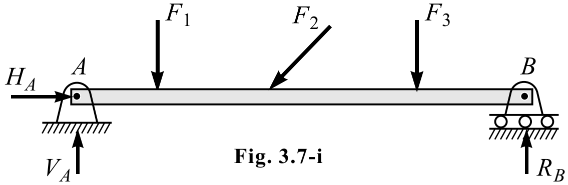


Fig. 3.7-i

2. **Simply Supported Beam with Overhang:** Here, one end or both the ends of simply supported beam is projected beyond the supports, which means that the portion of beam extends beyond the hinge and roller supports.

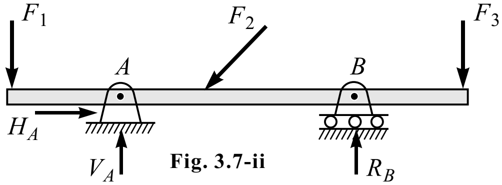


Fig. 3.7-ii

3. **Cantilever Beam :** A beam which is fixed at one end and free at the other end is called a *cantilever beam*. The fixed end is also known as built-in support. The common example is wall bracket, projected balconies, etc. One end of the beam is cast in concrete and is nailed, bolted, riveted or welded. The fixed end does not allow horizontal linear movement, vertical linear movement or rotational movement.

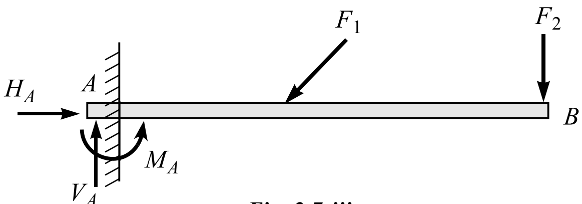


Fig. 3.7-iii

4. **Continuous Beam :** A beam which has more than two support is said to be a *continuous beam*. The extreme left and right supports are the end supports of the beam. Two intermediate supports are shown. Such beams are also called *statically indeterminate beams* because the reactions cannot be obtained by the equation of equilibrium.

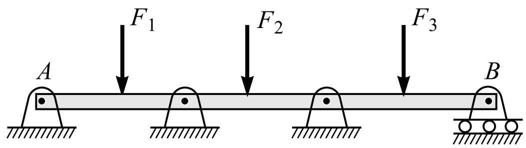


Fig. 3.7-iv

5. **Beams Linked with Internal Hinges :** Here two or more beams are connected to each other by pin joint and continuous beam is formed. Such a joint are called *internal hinges.* Internal hinges allow us to draw FBD of beam at its joint, if required.

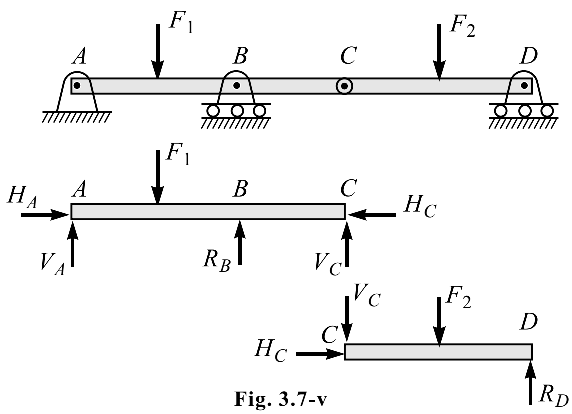


Fig. 3.7-v

3.8 Types of Load

There are two types of load : Point load and distributed load.

1. **Point Load :** If the whole intensity of load is assumed to be concentrated at a point then it is known as *point load*.

   Refer to Fig. 3.8.

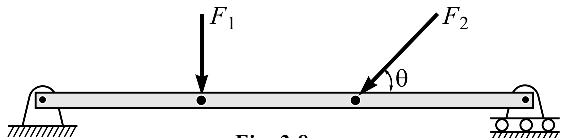


Fig. 3.8

2. **Distributed Load :** The concept of a centroid of an area may be used to solve problem dealing with a beam supporting a distributed load. This load may consist of the weight of materials supported directly or indirectly by the beam or it may be caused by wind or hydraulic pressure. The distributed load may be represented by *plotting the load intensity W supported per unit length*. The load intensity is expressed in N/m or kN/m.

(a) *Uniformly Distributed Load (UDL)* : If the whole intensity of load is distributed uniformly along the length of loading then it is called Uniformly Distributed Load (UDL). For example a truck loaded with sand of equal height and slab of a building flooring.

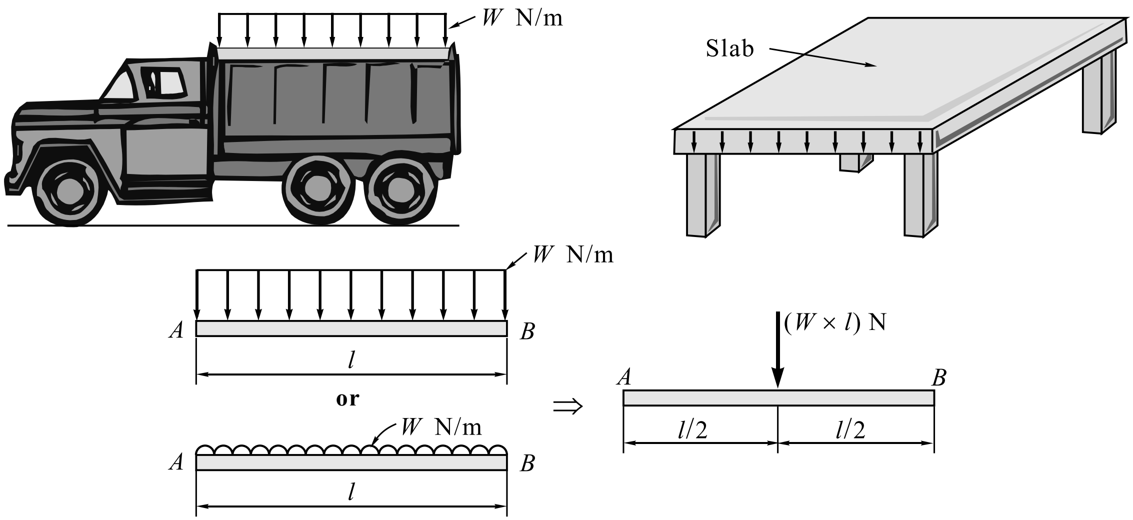
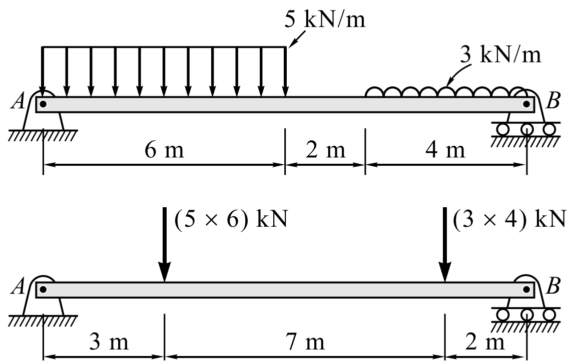


Fig. 3.8-i

Example

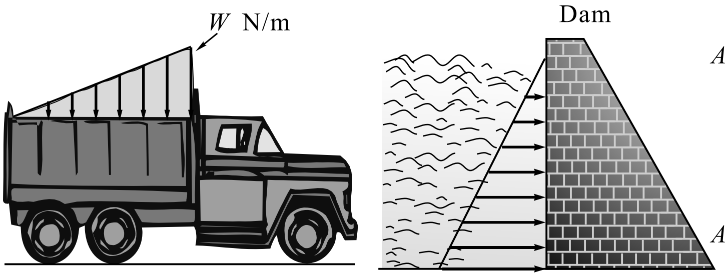
A uniformly distributed load on a beam can be replaced by a concentrated point load. The magnitude of this equivalent point load is equal to the area under loading diagram and it acts through the centroid. The area under the loading diagram is calculated by multiplying the load intensity with length of loading.

Refer to the adjacent figure.



(ii) *Uniformly Varying Load (UVL)* : If the whole intensity of load is distributed uniformly at varying rate along the length of loading then, it is known as Uniformly Varying Load (UVL). For example a truck loaded with sand, hydraulic pressure varies linearly with the depth.

Refer to Fig. 3.8-ii.



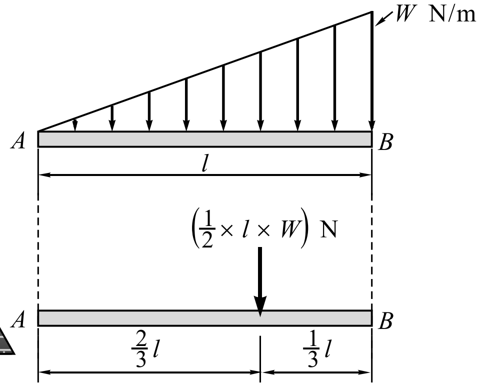
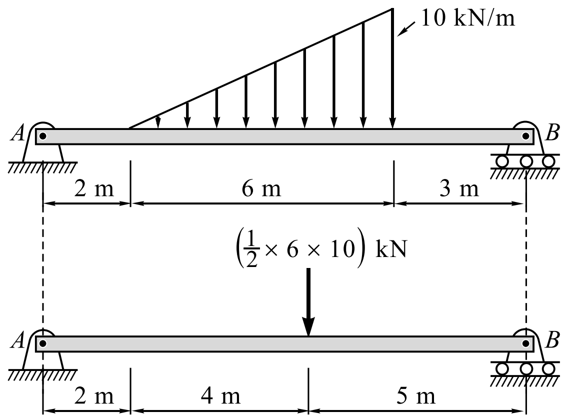


Fig. 3.8-ii

Example

A uniformly varying load on a beam can be replaced by a concentrated point load. The magnitude of this equivalent point load is equal to the area under loading diagram and it acts through the centroid. Generally, UVL is represented by right angled triangle. Area under loading diagram is the area of triangle, i.e.,

$$\frac{1}{2} \times \text{Length of loading} \times \text{Load intensity}$$



(iii) *UDL and UVL Combined*

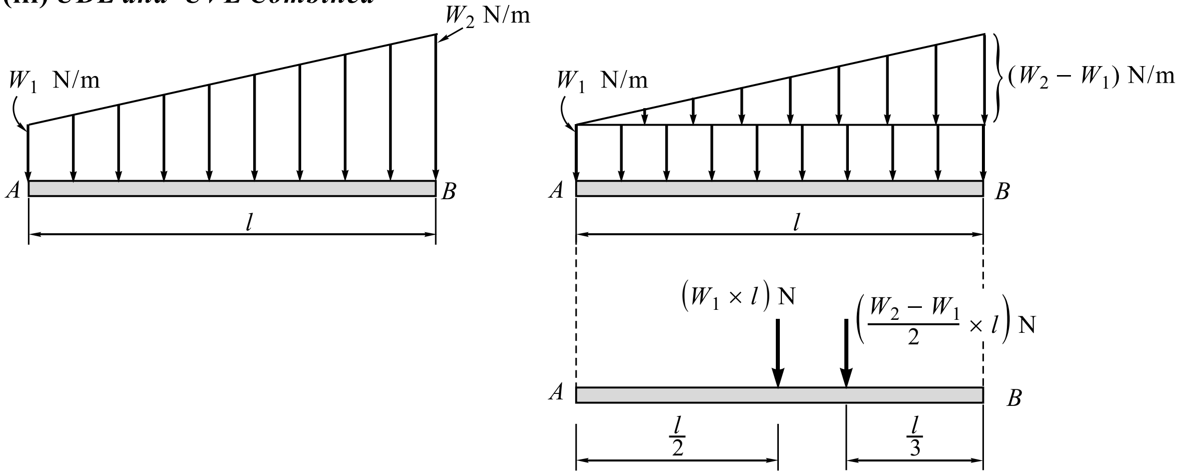
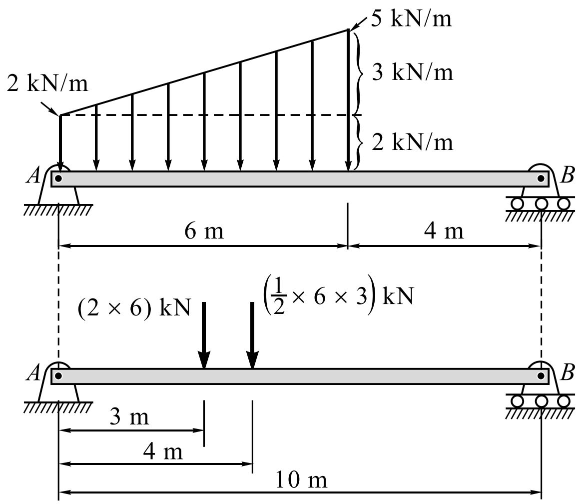


Fig. 3.8-iii

Example



(iv) ***Varying Load with Some Relation*** **:** The varying load is given by some relation, say parabolic nature. It can be replaced by concentrated point load. The magnitude of the equivalent point load is *equal to the area under loading diagram and it acts through the centroid*.

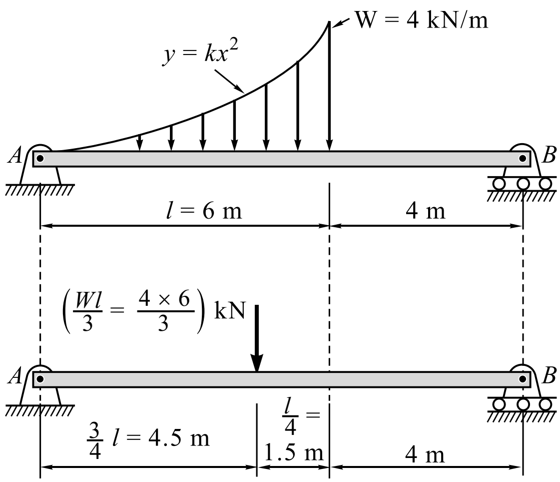


Fig. 3.8-iv

(v) ***Couple***

Example

Though on beam, couples C_1 = 5 kN-m and C_2 = 7 kN-m are shown at specific positions but we know that couple is a free vector, so it can act anywhere along the beam *AB*. In other words, distance of couples C_1 and C_2 from point *A* (given 2 m and 6 m) respectively, has no significance as far as position is concerned. As per the requirement of solution say ΣM_A, we can consider given couples C_1 = 5 kN-m (↻) and C_2 = 7 kN-m (↺). Refer to Fig. 3.8-v.

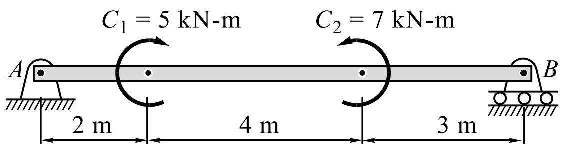


Fig. 3.8-v

3.9 Solved Problems on Support Reactions of Beams

Problem 24

Calculate the support reactions for the beam shown in Fig. 3.24(a).

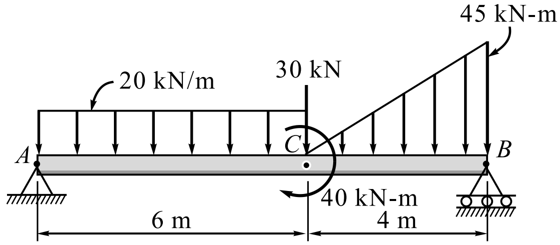


Fig. 3.24(a)

Solution

(i) Consider the FBD of Beam AB [Fig. 3.24(b)]

(ii) $\Sigma M_A = 0$

$-120 \times 3 - 30 \times 6 - 40 - 90 \times 8.67 + R_B \times 10 = 0$

$R_B = 136.03$ kN (↑)

(iii) $\Sigma F_x = 0$

$H_A = 0$

(∵ there is no horizontal force acting)

(iv) $\Sigma F_y = 0$

$V_A - 120 - 30 - 90 + 136.03 = 0$

$V_A = 103.97$ kN (↑) ***Ans.***

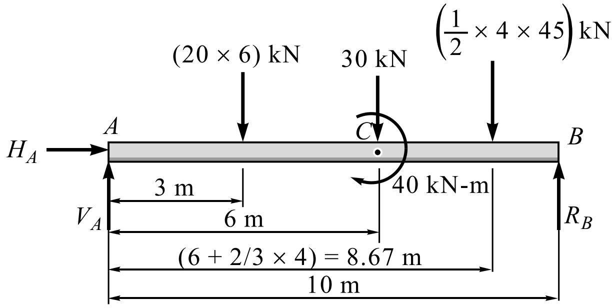


Fig. 3.24(b)

Problem 25

Find the support reactions at A and B for the beam loaded as shown in Fig. 3.25(a).

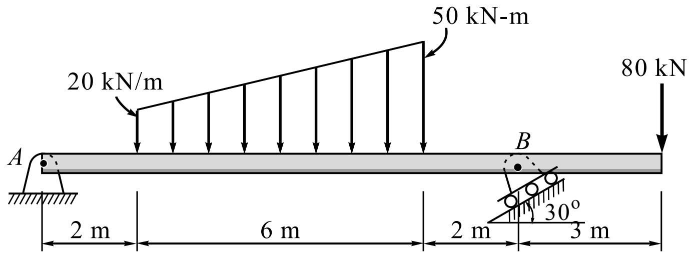


Fig. 3.25(a)

Solution

(i) Consider the FBD of Beam AB [Fig. 3.25(b)].

(ii) $\Sigma M_A = 0$

$R_B \sin 60^\circ \times 10 - 120 \times 5 - 90 \times 6 - 80 \times 13 = 0$

$R_B = 251.73$ kN (60°)

(iii) $\Sigma F_x = 0$

$H_A - 251.73 \cos 60^\circ = 0$

$H_A = 125.87$ kN (→)

(iv) $\Sigma F_y = 0$

$V_A - 120 - 90 + 251.73 \sin 60^\circ - 80 = 0$

$V_A = 72$ kN (↑) ***Ans.***

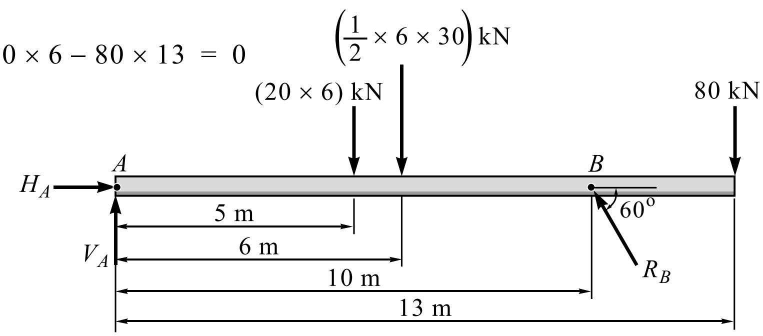


Fig. 3.25(b)

Problem 26

Find analytically the support reaction at B and the load P, for the beam shown in Fig. 3.26(a), if the reaction of support A is zero.

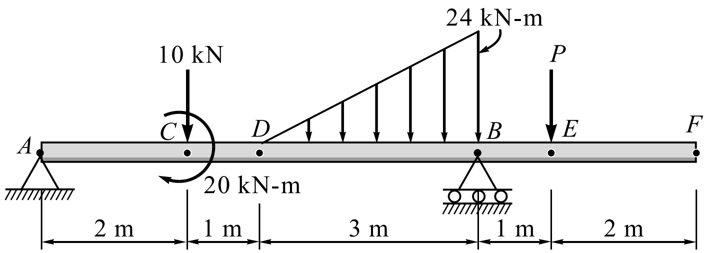


Fig. 3.26(a)

Solution

(i) Consider the FBD of Beam AF

(ii) $\Sigma F_y = 0$

$V_A + R_B - 10 - 36 - P = 0 \quad (V_A = 0 \text{ given})$

$R_B - P = 46 \qquad \ldots\text{(I)}$

(iii) $\Sigma M_A = 0$

$R_B \times 6 - 10 \times 2 - 20 - 36 \times 5 - P \times 7 = 0$

$6\,R_B - 7\,P = 220 \qquad \ldots\text{(II)}$

(iv) Solving Eqs. (I) and (II)

$R_B = 102 \text{ kN } (\uparrow)$ ***Ans.***

(v) From Eq. (I)

$P = 102 - 46$

$P = 56 \text{ kN } (\downarrow)$ ***Ans.***

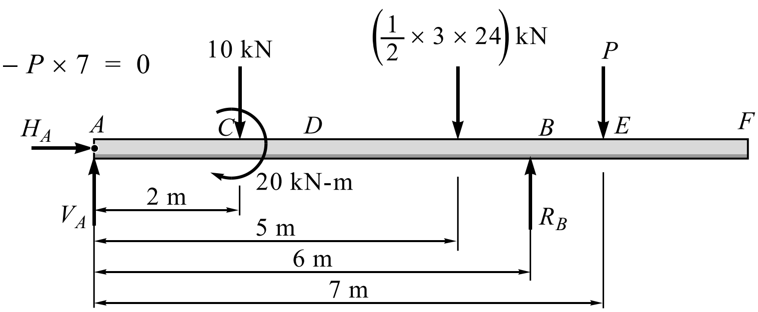


Fig. 3.26(b) : F.B.D of Beam *AF*

Problem 27

Find the support reactions at A and F for the given Fig. 3.27(a).

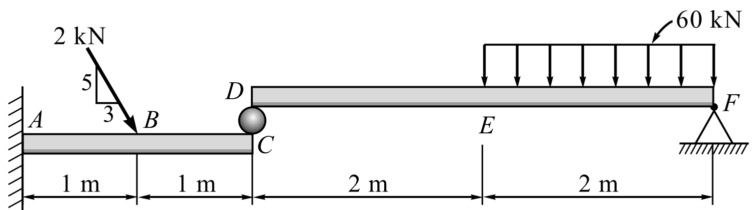


Fig. 3.27(a)

Solution

(i) **Consider the FBD of Beam *DF*** [Fig. 3.27(b)]

$\Sigma M_F = 0$

$120 \times 1 - R_D \times 4 = 0 \quad \therefore \; R_D = 30 \text{ kN}$

$\Sigma F_x = 0 \qquad \therefore \; H_F = 0$

$\Sigma F_y = 0$

$R_D + V_F - 120 = 0$

$V_F = 90 \text{ kN } (\uparrow)$ ***Ans.***

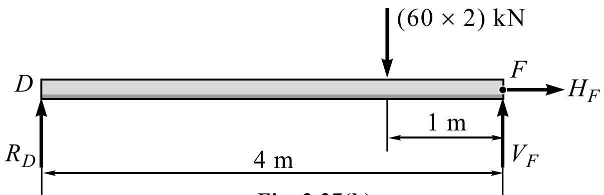


Fig. 3.27(b)

(ii) **Consider the FBD of Beam *AC*** [Fig. 3.27(c)]

$\Sigma M_A = 0$

$M_A - 2 \sin 59.04^\circ \times 1 - 30 \times 2 = 0$

$M_A = 61.72 \text{ kN-m } (\circlearrowleft)$

$\Sigma F_x = 0$

$2 \cos 59.04^\circ - H_A = 0$

$H_A = 1.03 \text{ kN } (\leftarrow)$

$\Sigma F_y = 0$

$V_A - 2 \sin 59.04^\circ - 30 = 0$

$V_A = 31.72 \text{ kN } (\uparrow)$ ***Ans.***

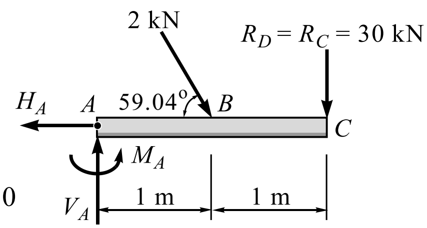


Fig. 3.27(c)

Problem 28

Two beams AB and CD are arranged as shown in Fig. 3.38(a). Find the support reactions at D.

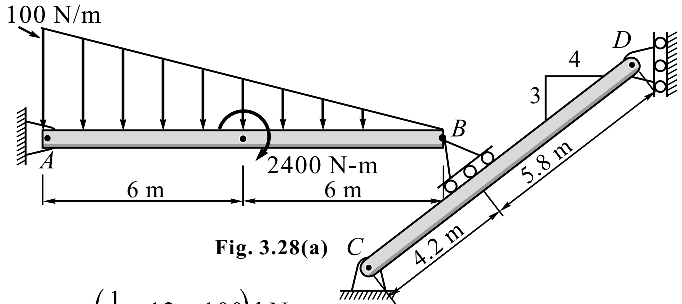


Fig. 3.28(a)

Solution

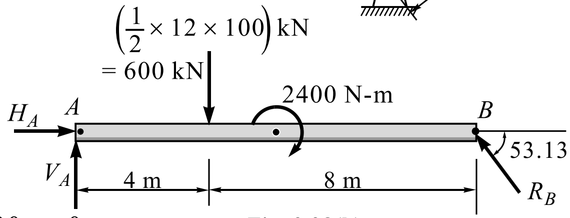


Fig. 3.28(b)

(i) Consider the FBD of Beam *AB*

$$\Sigma M_A = 0$$

$$R_B \sin 53.13^\circ \times 12 - 600 \times 4 - 2400 = 0$$

$R_B = 500$ N ***Ans.***

(ii) Consider the FBD of Beam *CD*

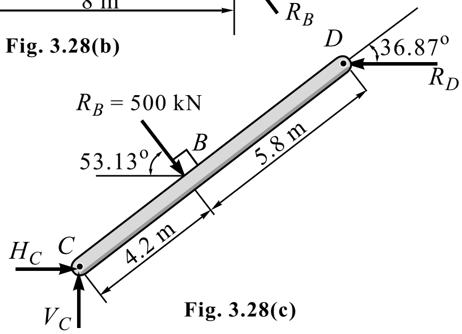


Fig. 3.28(c)

$$\Sigma M_C = 0$$

$$R_D \sin 36.87^\circ \times 10 - 500 \times 4.2 = 0$$

$R_D = 350$ N $(\leftarrow)$ ***Ans.***

Problem 29

Determine the intensity of distributed load W at the end C of the beam ABC as shown in Fig. 3.29(a), for which the reaction at C is zero. Also calculate the reaction at B.

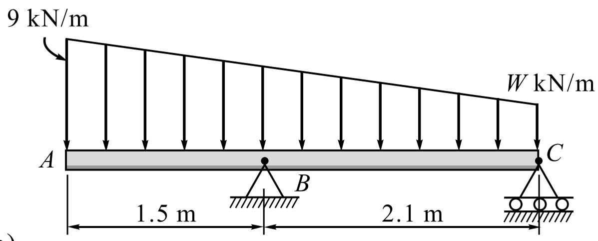


Fig. 3.29(a)

Solution

(i) Consider the FBD of beam ABC with equivalent point load shown in Fig. 3.29(b).

(ii) $\Sigma M_B = 0$

$$\frac{1}{2} \times 3.6 \times (9 - W) \times 0.3 + R_C \times 0 - W \times 3.6 \times 0.3 = 0$$

$$4.86 - 0.54W - 1.08W = 0$$

$$1.62\,W = 4.86 \quad \therefore W = 3 \text{ kN}$$

(iii) $\Sigma F_x = 0 \quad \therefore H_B = 0$

(iv) $\Sigma F_y = 0$

$$V_B - \frac{1}{2} \times 3.6 \times (9 - W) - W \times 3.6 + 0 = 0$$

$$V_B = 10.8 + 10.8$$

$\therefore V_B = 21.6$ kN $(\uparrow)$ ***Ans.***

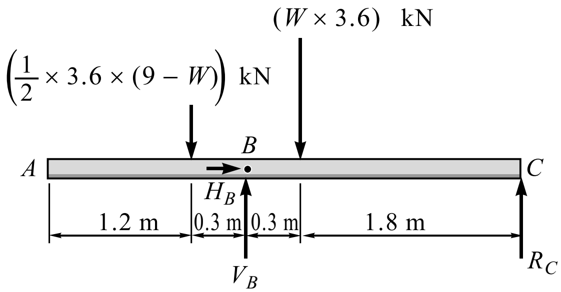


Fig. 3.29(b) : F.B.D of Beam *ABC*

Problem 30

Bars AB and CD are rigidly connected by welding at D as shown in Fig. 3.30(a). Bar AB weighs 5 kN/m whereas weight of bar CD is negligible.

Determine the support reactions.

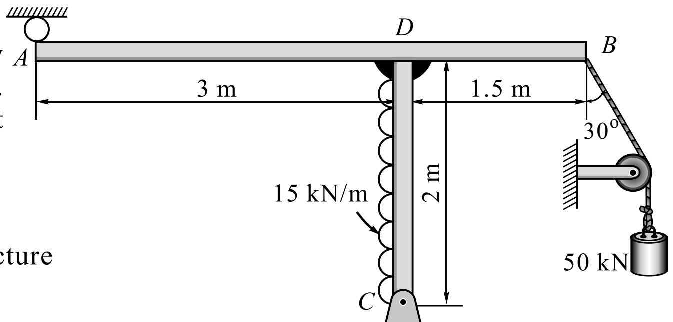


Fig. 3.30(a)

Solution

(i) Consider the FBD of the whole structure since it is a single rigid body.

(ii) $\Sigma M_C = 0$

$$R_A \times 3 + (4.5 \times 5) \times 0.75 - 50 \cos 30^\circ \times 1.5$$

$$- 50 \sin 30^\circ \times 2 - (15 \times 2) \times 1 = 0$$

$$R_A = 42.69 \text{ kN } (\downarrow) \quad \textit{Ans.}$$

(iii) $\Sigma F_x = 0$

$$(15 \times 2) + 50 \sin 30^\circ - H_C = 0$$

$$H_C = 55 \text{ kN } (\leftarrow) \quad \textit{Ans.}$$

(iv) $\Sigma F_y = 0$

$$V_C - R_A - (4.5 \times 5) - 50 \cos 30^\circ = 0$$

$$V_C = 108.49 \text{ kN } (\uparrow) \quad \textit{Ans.}$$

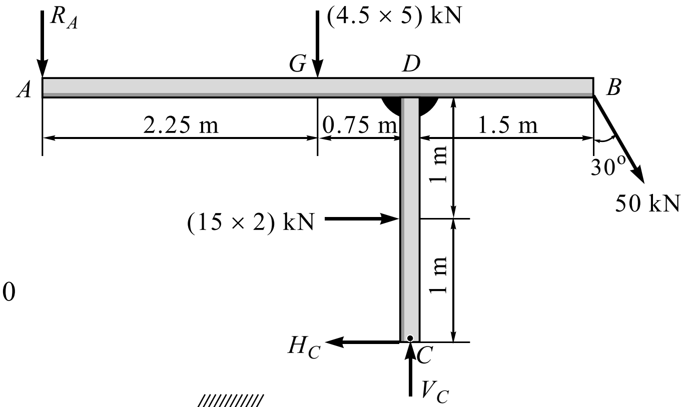


Fig. 3.30(b)

Problem 31

A single rigid bar ABC of 'L' shape is loaded and supported, as shown Fig. 3.31(a). Find the support reactions.

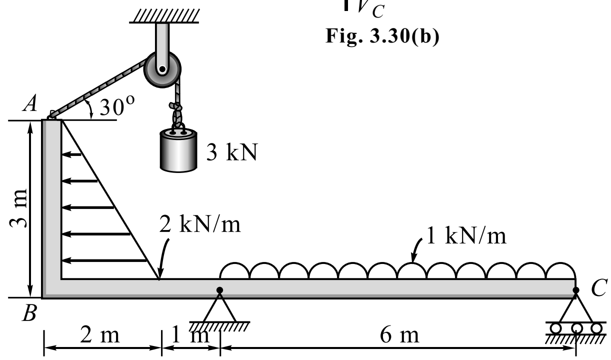


Fig. 3.31(a)

Solution

(i) Consider the FBD of Beam ABC.

(ii) $\Sigma M_D = 0$

$$R_C \times 6 - 3 \cos 30^\circ \times 3 - 3 \sin 30^\circ \times 3$$

$$- 6 \times 3 + \left(\frac{1}{2} \times 3 \times 2\right) \times 1 = 0$$

$$R_C = 4.55 \text{ kN } (\uparrow) \qquad \textit{Ans.}$$

(iii) $\Sigma F_x = 0$

$$3 \cos 30^\circ - \left(\frac{1}{2} \times 3 \times 2\right) + H_D = 0$$

$$H_D = 0.4 \text{ kN } (\rightarrow) \qquad \textit{Ans.}$$

(iv) $\Sigma F_y = 0$

$$V_D + 3 \sin 30^\circ + R_C - 6 = 0$$

$$V_D = -0.05 \text{ (Wrong assumed direction)}$$

$$V_D = 0.05 \text{ kN } (\downarrow) \quad \textit{Ans.}$$

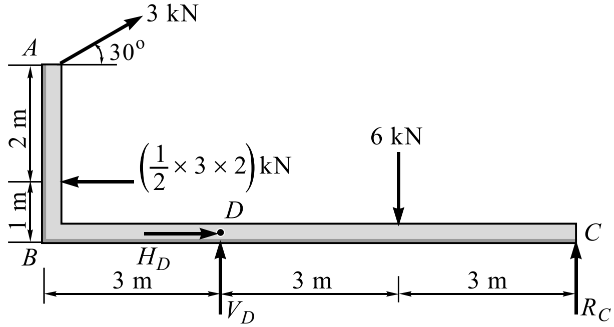


Fig. 3.31(b)

Problem 32

Find the support reactions of the beam shown in Fig. 3.32(a). *E* is internal hinge.

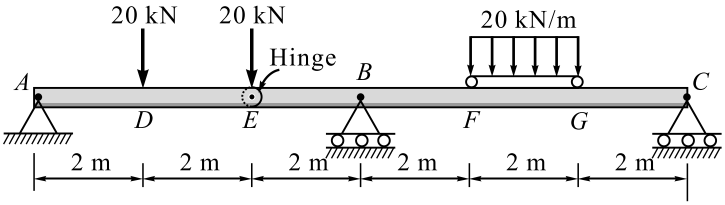


Fig. 3.32(a)

Solution

(i) Consider the FBD of Beam *AE*

$\Sigma M_E = 0$

$20 \times 2 - V_A \times 4 = 0$

$V_A = 10 \text{ kN } (\uparrow)$ ***Ans.***

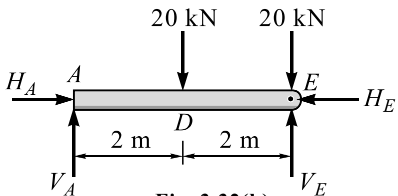


Fig. 3.32(b)

$\Sigma F_y = 0$

$V_A + V_E - 20 - 20 = 0$

$V_E = 30 \text{ kN } (\uparrow)$ ***Ans.***

Since there is a horizontal or inclined external force acting, therefore, horizontal component of reaction will be zero.

$H_A = H_E = 0$

(ii) Method I

Consider the FBD of Beam *EC*

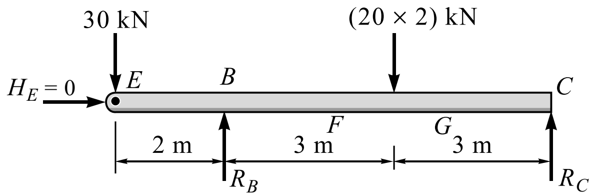


Fig. 3.32(c)

$\Sigma M_B = 0$

$30 \times 2 - 20 \times 2 \times 3 + R_C \times 6 = 0$

$R_C = 10 \text{ kN } (\uparrow)$ ***Ans.***

$\Sigma F_y = 0$

$R_B + R_C - 30 - 20 - 20 = 0$

$R_B = 60 \text{ kN } (\uparrow)$ ***Ans.***

(iii) Method II

Consider the FBD of Beam *AC*

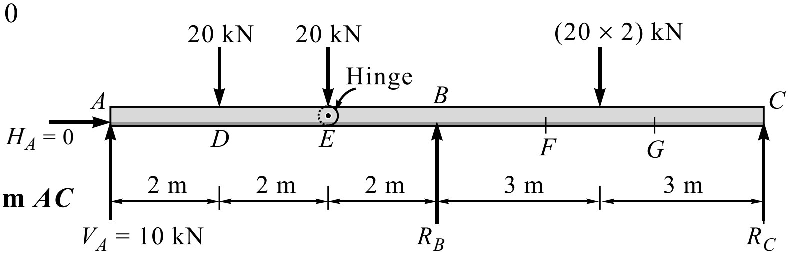


Fig. 3.32(d)

$\Sigma M_B = 0$

$R_C \times 6 - 20 \times 2 \times 3 + 20 \times 2 + 20 \times 4 - 10 \times 6 = 0$

$R_C = 10 \text{ kN } (\uparrow)$ ***Ans.***

$\Sigma F_y = 0$

$10 + R_B + 10 - 20 - 20 - 20 \times 2 = 0$

$R_B = 60 \text{ kN } (\uparrow)$ ***Ans.***

Exercises

[I] Problems

1. An electric light weighing 15 N hangs from a point C by the two strings AC and BC as shown in Fig. 3.E1. AC is inclined at 60° to the horizontal and BC at 45° to the vertical as shown. Using the Lami's theorem find the forces in the strings AC and BC.

   [**Ans.** $T_{AC} = 10.98$ N and $T_{BC} = 7.76$ N.]

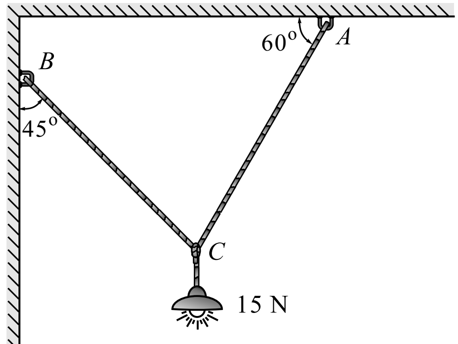


Fig. 3.E1

2. A force P is applied at O to the strings AOB as shown in Fig. 3.E2. If the tension in each string is 50 N, find the magnitude and direction of force P for equilibrium conditions.

   [**Ans.** $\theta = 7.5^\circ$ and $P = 60.88$ N.]

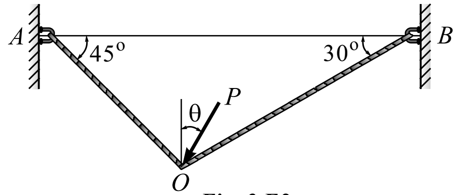


Fig. 3.E2

3. A smooth sphere of mass 2 kg is supported by a chain as shown in Fig. 3.E3. The length of chain AB is equal to the radius of the sphere. Draw free body diagram of each element and find the tension in the chain and reaction of the wall.

   [**Ans.** $T_{AB} = 22.6$ N and $R_C = 11.33$ N.]

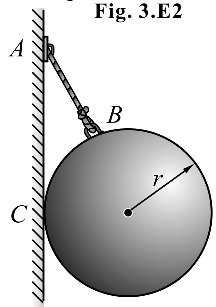


Fig. 3.E3

4. A smooth sphere of weight 500 N rests in a V shaped groove, whose sides are inclined at 25° and 65° to the horizontal, as shown in Fig. 3.E4. Find the reactions at A and B.

   [**Ans.** $R_A = 453.15$ N ($\measuredangle 65^\circ$) and
   $R_B = 211.31$ N (25°) .]

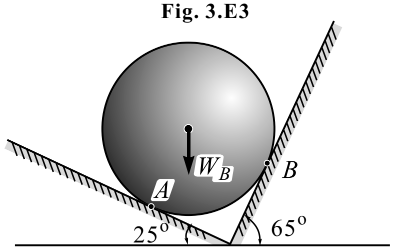


Fig. 3.E4

5. Determine the horizontal distance to which a 1 m long in extensible string holding weight of 500 N can be pulled before the string breaks. The string can withstand the maximum pull of 1000 N, as shown in Fig. 3.E5. Determine also the required force F.

   [**Ans.** $F = 866$ N and $x = 0.86$ m.]

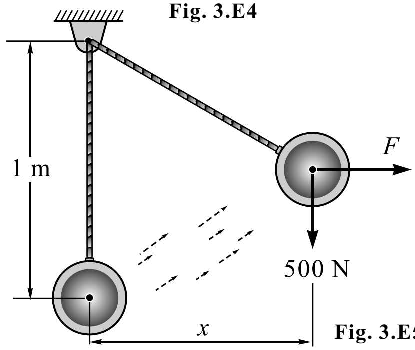


Fig. 3.E5

6. A bar *AB* of weight 1 kN is hinged to a vertical wall at *A* and supported by a cable *BD* as shown in Fig. 3.E6. Find the tension in the cable and the magnitude and direction of reaction at the hinge.

[***Ans.*** $T = 0.866$ kN and
$R_A = 0.5$ kN $\angle\theta = 30^\circ$.]

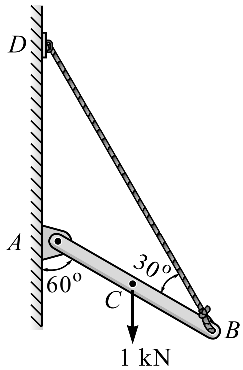


Fig. 3.E6

7. A string *ACB* of length l carries a small pulley *C* from which a load *W* is suspended as shown in Fig. 3.E7. Find the position of equilibrium as defined by the angle α.

Given : $d = \dfrac{l}{2}$, $h = \dfrac{l}{4}$.

[***Ans.*** $\alpha = 60^\circ$]

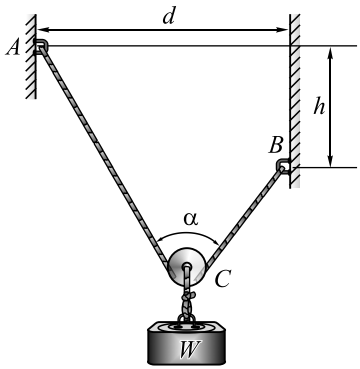


Fig. 3.E7

8. A system of connected flexible cables shown in Fig. 3.E8 is supporting two vertical forces 200 N and 250 N at points *B* and *D*. Determine the forces in various segments of the cable.

[***Ans.*** $T_{DE} = 224.14$ N, $T_{BD} = 183.01$ N
$T_{BC} = 336.6$ N and $T_{AB} = 326.79$ N.]

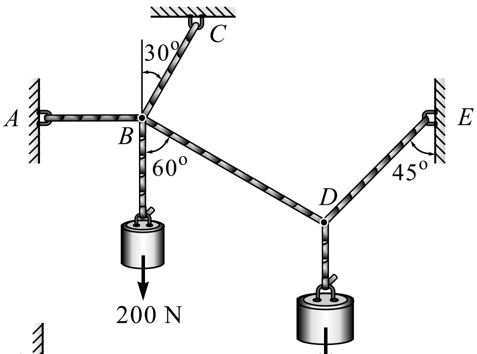


Fig. 3.E8

9. Two equal loads of 2500 N are supported by a flexible string *ABCD* at points *B* and *C* as shown in Fig. 3.E9. Find the tension in the portion *AB*, *BC*, *CD* of the string.

[***Ans.*** $T_{AB} = 4330$ N, $T_{BC} = 2500$ N
and $T_{CD} = 2500$ N.]

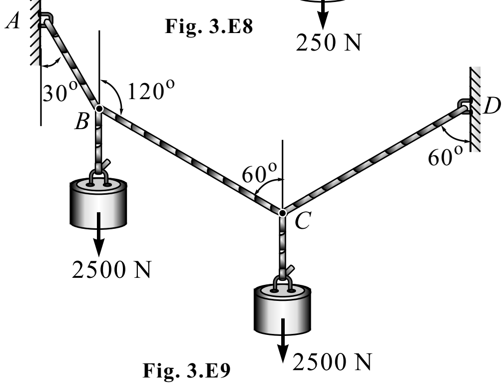


Fig. 3.E9

10. Two cables are tied together at C and loaded as shown in Fig. 3.E10. Determine the tensions in AC and BC.

[**Ans.** $T_{AC} = 326$ N and $T_{BC} = 368$ N.]

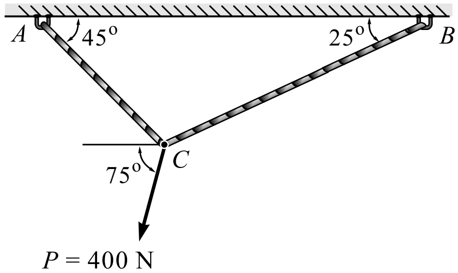


Fig. 3.E10

11. Two smooth spheres of weight 100 N and radius 250 mm each are in equilibrium in a horizontal channel of width 870 mm as shown in Fig. 3.E11. Find the reactions at the surfaces of contact A, B, C, D, assuming all the surfaces to be smooth.

[**Ans.** $R_A = 133.3$ N ($\rightarrow$), $R_B = 200$ N ($\uparrow$),
$R_C = 133.3$ N ($\leftarrow$) and $R_D = 166.6$ N.]

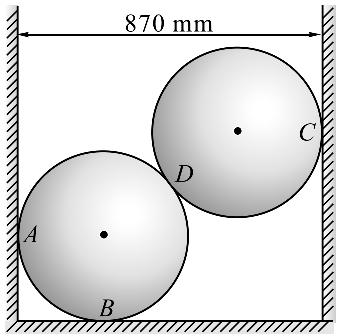


Fig. 3.E11

12. Two smooth cylinder with diameters 250 mm and 400 mm respectively are kept in a groove with slanting surfaces, making angles 60° and 30° respectively as shown in Fig. 3.E12. Determine the reactions at contact points A, B, and C.

[**Ans.** $R_B = 297.37$ N, $R_B = 1125.85$ N
and $R_A = 352.69$ N.]

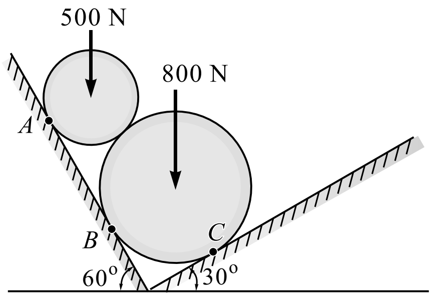


Fig. 3.E12

13. Two cylinders P and Q are in a channel as shown in Fig. 3.E13. The cylinder P has a diameter of 100 mm and weight 200 N and Q has 180 mm and 500 N. Determine the reaction at all the contact surfaces.

[**Ans.** $R_1 = 134.2$ N ($\leftarrow$), $R_2 = 240.8$ N,
$R_3 = 154.9$ N ($\measuredangle 30°$) and $R_4 = 622.5$ N ($\uparrow$).]

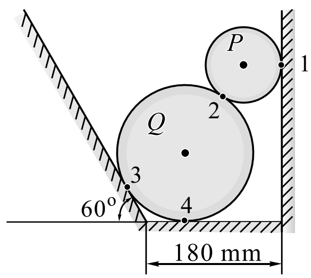


Fig. 3.E13

14. Two identical rollers each of weight 500 N are kept in a right-angle frame ABC having negligible weight as shown in Fig. 3.E14. Assuming smooth surfaces, find the reactions induced at the points P, Q, R and S. Also find the reactions at B and C.

Ans. $R_P = 500$ N, $R_Q = R_S = 433$ N,
$R_R = 250$ N, $R_C = 246.41$ N (60°),
$H_B = 123.21$ N ($\rightarrow$) and $V_B = 786.6$ N ($\uparrow$).

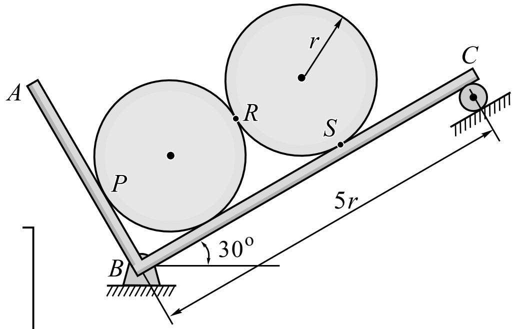


Fig. 3.E14

15. A light bar is suspended from a cable BE and supports a 200 N block at C. The extremities A and D of the bar are in contact with frictionless vertical walls, as shown in Fig. 3.E15. Determine the tension in cable BE and the reactions at A and D.

Ans. $T = 200$ N, $R_D = 75$ N ($\leftarrow$) and $R_A = 75$ N ($\rightarrow$).

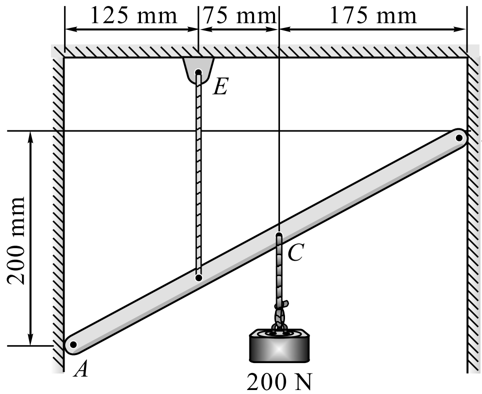


Fig. 3.E15

16. Neglecting friction, determine the tension in cable ABD and the reaction at support C, as shown in Fig. 3.E16.

Ans. $T = 80$ N,
$H_C = 80$ N ($\rightarrow$) and
$V_{BC} = 40$ N ($\uparrow$).

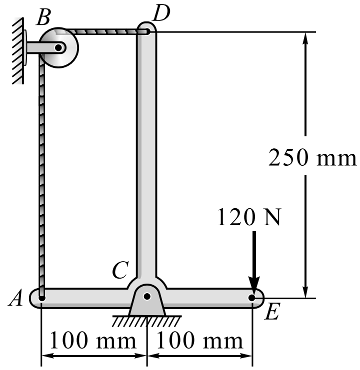


Fig. 3.E16

17. Determine the tension in cable BC, as shown in Fig. 3.E17. Neglect the self-weight of AB.

Ans. $V_A = 5850$ N,
$H_A = 4070.3$ N,
and $T = 4700$ N.

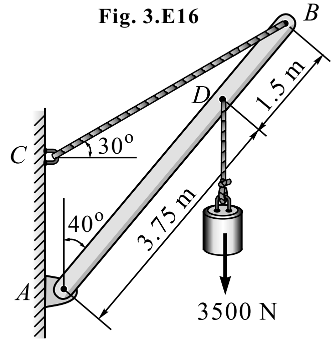


Fig. 3.E17

18. Determine the force P applied at 45° to the horizontal just necessary to start a roller 100 cm diameter over an obstruction 25 cm high, if the roller weighs 1000 N, as shown in Fig. 3.E18. Also find the magnitude and direction of P when it is minimum.

Ans. (a) $P = 866.48$ N and
(b) $P_{\text{min}} = 866$ N at 60° to horizontal.

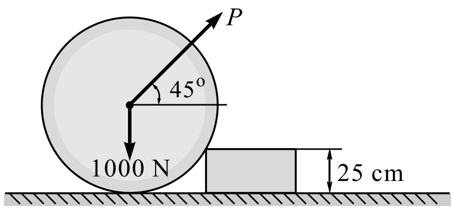


Fig. 3.E18

19. A roller of radius 400 mm, weighing 4 kN is to be pulled over a rectangular block of height 200 mm as shown in Fig. 3.E19, by a horizontal force applied at the end of a string wound round the circumference of the roller. Find the magnitude of the horizontal force P and the reaction at B, which will just turn the roller over the corner of the rectangular block. Also, determine the least force and its line of action at the roller centre, for turning the roller over the rectangular block.

Ans. (a) $P = 2.31$ kN and
(b) P_{min} at roller centre $= 3.46$ kN.

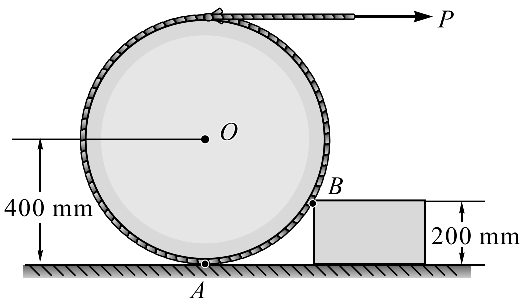


Fig. 3.E19

20. Determine the magnitude and direction of the smallest force P required to start to wheel over the block as shown in Fig. 3.E20.

Ans. $P = 9.47$ kN and
$\theta = 71.4°$.

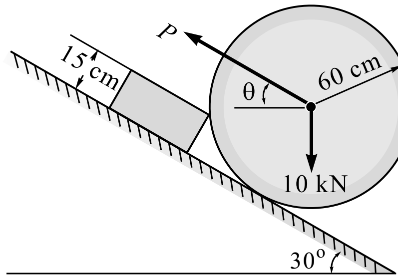


Fig. 3.E20

21. A vertical pole is anchored in a cement foundation. Three wires are attached to the pole as shown in Fig. 3.E21. If the reaction at point A consists of the reactions as shown, find the tensions in the wires.

Ans. $T_1 = 8104.6$ N,
$T_2 = 6784.3$ N and
$T_3 = 4444.4$ N.

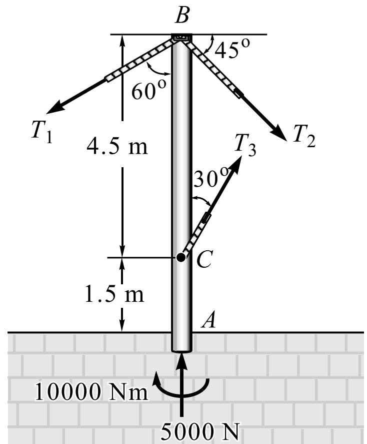


Fig. 3.E21

22. The spanner shown in Fig. 3.E22 is used to rotate a shaft. A pin fits in a hole at A, while a flat frictionless surface rests against the shaft at B. If the moment about C of the force exerted on the shaft at A is to be 87 N-m, find (a) the force P which should be exerted on the spanner at D and (b) the corresponding value of the force exerted on the spanner at B.

[***Ans.*** $P = 240$ N and $R_B = 1768$ N $(\rightarrow)$.]

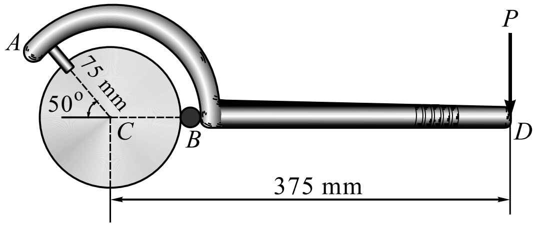


Fig. 3.E22

23. A smooth pipe rests against the wall at the points of contact A, B and, C, as shown in Fig. 3.E23. Determine the reactions at these points needed to support the vertical force of 200 N. Neglect the pipe's thickness in the calculation.

[***Ans.*** $R_C = 284$ N,
$R_B = 53.1$ N and
$R_A = 115.5$ N.]

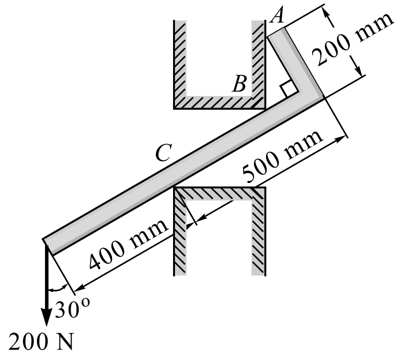


Fig. 3.E23

24. Two rollers of weights 50 N and 100 N are connected by a flexible string AB. The rollers rest on two mutually perpendicular surfaces DE and EF as shown in Fig. 3.E24. Find the tension in the string and the angle θ that it makes with the horizontal when the system is in equilibrium.

[***Ans.*** $\theta = 10.9^{\circ}$ and $T = 66.15$ N.]

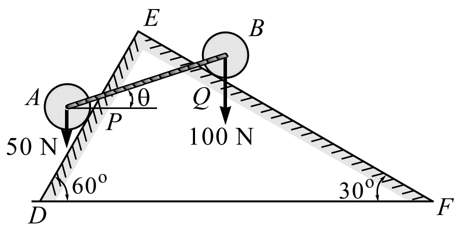


Fig. 3.E24

25. A vertical post PQ of a crane is pivoted at P and supported by a guide Q. Find the reactions at P and Q due to the loads acting, as shown in Fig. 3.E25.

[***Ans.*** $R_Q = 4000$ N and $R_P = 5656.8$ N.]

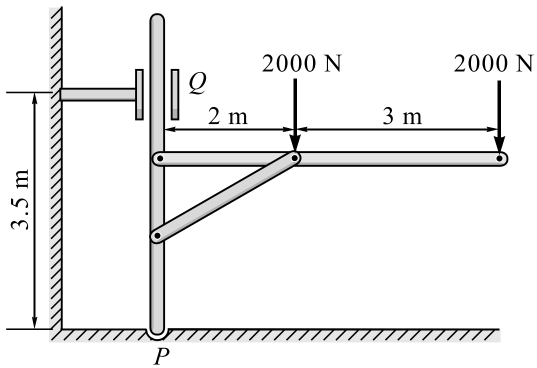


Fig. 3.E25

26. *A* and *B* are identical smooth cylinders having masses of 100 kg each as shown in Fig. 3.E26. Determine the maximum force *P* which can be applied without causing *A* to leave the floor.

[**Ans.** $P = 1699$ N.]

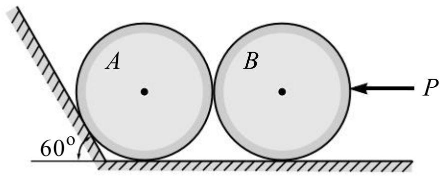


Fig. 3.E26

27. A man weighing 75 N stands on the middle rung of a 25 N ladder resting on a smooth floor and against a wall as shown in Fig. 3.E27. The ladder is prevented from slipping by a string *OD*. Find the tension in the string and reactions at *A* and *B*.

[**Ans.** $R_A = 120.26$ N, $R_B = 35.13$ N and $T = 40.56$ N.]

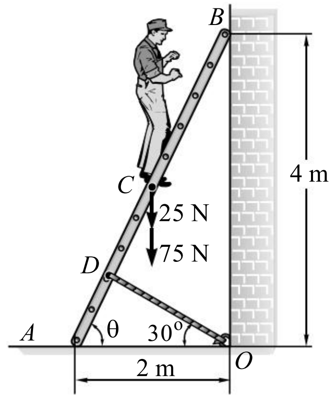


Fig. 3.E27

28. The frame *BCD* as shown in Fig. 3.E28 supports a 600 N cylinder. The frame is hinged at *D*. Determine the reactions at *A*, *B*, *C* and *D*.

[**Ans.** $R_A = 200$ N, $R_B = 600$ N, $R_C = 200$ N and $R_D = 632.46$ N.]

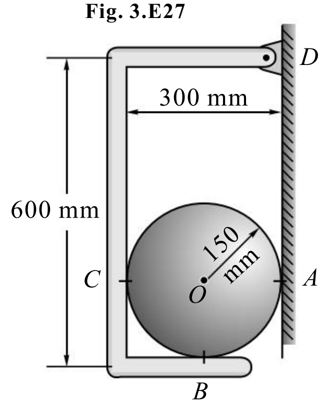


Fig. 3.E28

29. A square block of wood of mass *M* is hinged at *A* and rests on a roller at *B* as shown in Fig. 3.E29. It is pulled by means of a string attached at *D* and inclined at an angle 30° with the horizontal. Determine the force *P* required to be applied to string to just lift the block off the roller.

[**Ans.** $P = 0.366\, Mg$]

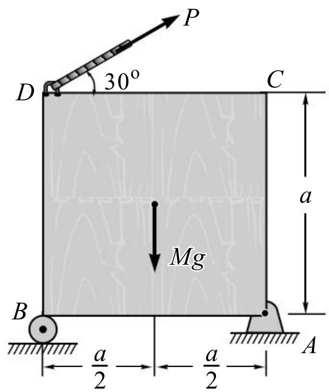


Fig. 3.E29

30. Figure 3.E30 shows several identical smooth rollers of weight W each stacked on an inclined plane. Determine (a) the maximum number of rollers which will lie in a single row as shown and (b) all forces acting on roller A under condition (a).

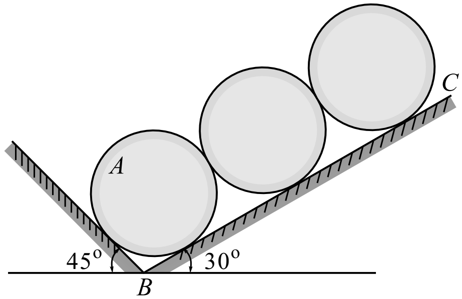


Fig. 3.E30

[***Ans.*** (a) 6 Nos
(b) $3.11\,W$ (θ) $\theta = 45^\circ$,
$0.062\,W$ (θ) $\theta = 60^\circ$,
$2.5\,W$ (θ) $\theta = 30^\circ$ and W (↓).]

31. Two prismatic bar AB and CD are welded together in the form of a rigid T and are suspended in a vertical plane as shown in the Fig. 3.E31. Determine the angle θ that the bar will make with the vertical when a load of 100 N is applied at the end D. Two bars are identical and each weighing 50 N.

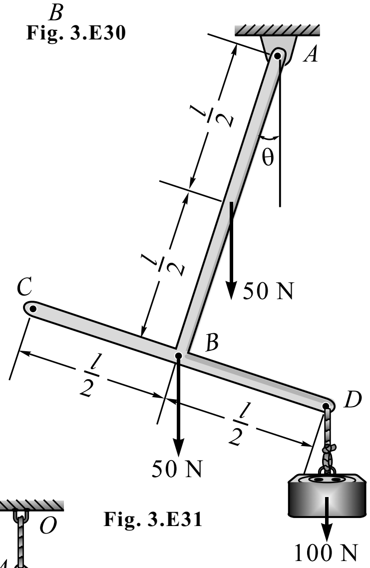


Fig. 3.E31

[***Ans.*** $\theta = 15.86^\circ$]

32. Two bars AB and BC of length 1 m and 2 m and weights 100 N and 200 N respectively, are rigidly joined at B and suspended by a string AO as shown in Fig. 3.E32. Find the inclination θ of the bar BC to the horizontal when the system is in equilibrium.

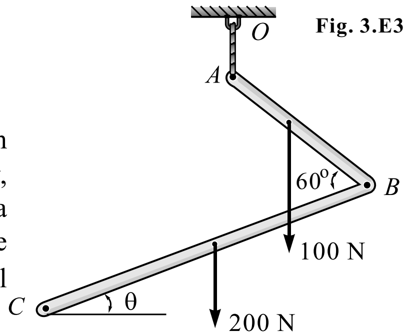


Fig. 3.E32

[***Ans.*** $\theta = 19.1^\circ$]

33. A pulley of 1 m radius, supporting a load of 500 N, is mounted at B on a horizontal beam as shown in Fig. 3.E33. If the beam weighs 200 N and pulley weighs 50 N, find the hinge force at C.

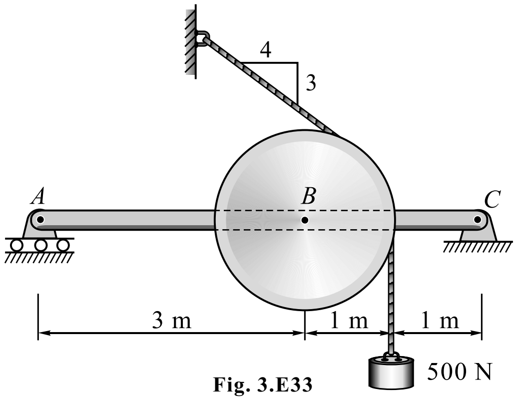


Fig. 3.E33

[***Ans.*** $R_C = 472$ N $\angle 32^\circ$]

34. Determine the reactions at all the supports of the beam/ structure shown in Fig. 3.E34.

Ans. $V_A = 3.6$ kN $(\uparrow)$,
$V_D = 10.4$ kN $(\uparrow)$ and $H_A = 0$.

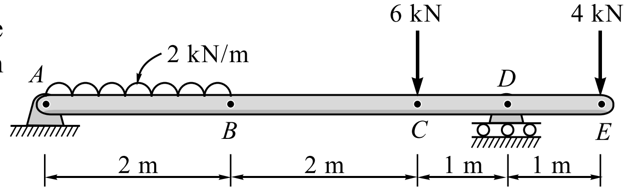


Fig. 3.E34

35. Determine the reactions at all the supports of the beam/ structure shown in Fig. 3.E35.

Ans. $R_B = 44.17$ kN (60°),
$V_A = 36.75$ kN $(\uparrow)$ and
$H_A = 22.1$ kN $(\rightarrow)$.

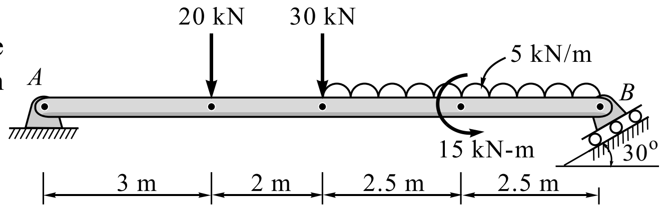


Fig. 3.E35

36. Determine the reactions at all the supports of the beam shown in Fig. 3.E36.

Ans. $H_A = 0$,
$V_A = 10.56$ kN $(\uparrow)$ and
$R_B = 15.44$ kN $(\uparrow)$.

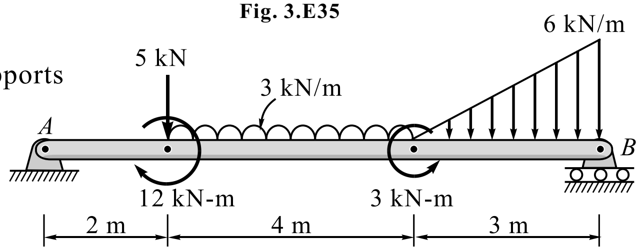


Fig. 3.E36

37. Determine the reactions at all the supports of the beam shown in Fig. 3.E37.

Ans. $H_A = 8.66$ kN $(\rightarrow)$,
$V_A = 8.79$ kN $(\uparrow)$ and
$V_B = 9.21$ kN $(\uparrow)$.

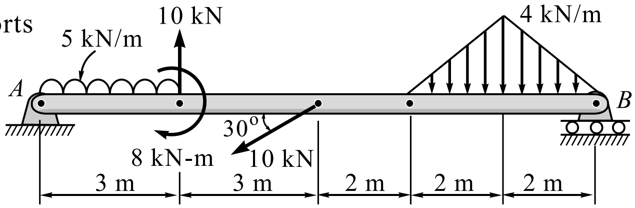


Fig. 3.E37

38. Determine the reactions at all the supports of the beam shown in Fig. 3.E38.

Ans. $H_A = 10$ kN $(\leftarrow)$,
$V_A = 127.32$ kN $(\uparrow)$ and
$M_A = 694.6$ kNm $(\circlearrowleft)$.

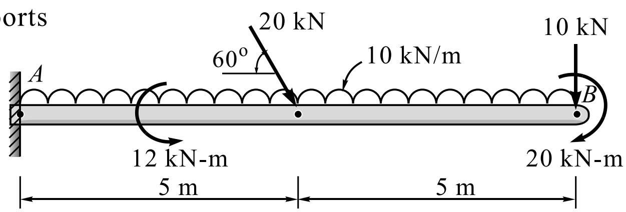


Fig. 3.E38

39. Calculate the reactions at A and B for the beam subjected to two linearly distributed loads as shown in Fig. 3.E39.

Ans. $H_A = 0$,
$V_A = 21.1$ kN $(\uparrow)$ and
$V_B = 20.9$ kN $(\uparrow)$.

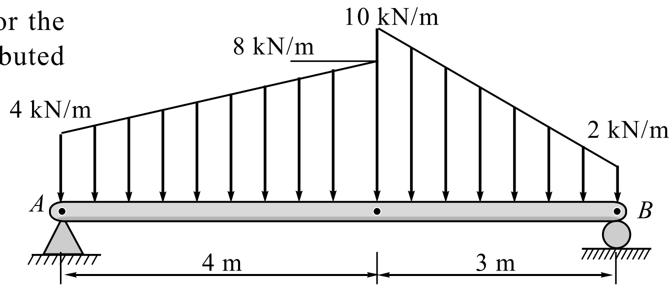


Fig. 3.E39

40. The beams *AB* and *CF* are arranged as shown in Fig. 3.E40. Determine the reactions at *A*, *C* and *D* due to the forces acting on the beam.

Ans. H_A = 28.28 kN (←),
V_A = 9.43 kN (↓),
R_D = 40.92 kN (↑),
H_C = 10 kN (→) and
V_C = 34.12 kN (↓).

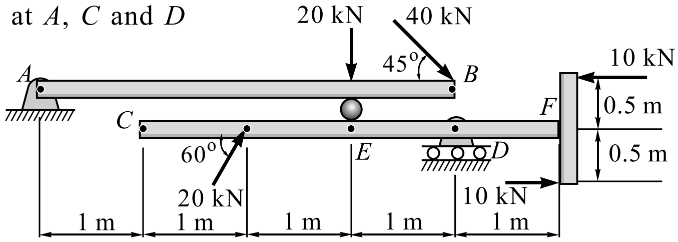


Fig. 3.E40

41. A beam weighs 4 kN/m is loaded and supported as shown in the Fig. 3.E41. Find the support reactions.

Ans. V_A = 70.81 kN (↑),
H_A = 5 kN (→) and
R_A = 72.15 kN (↓).

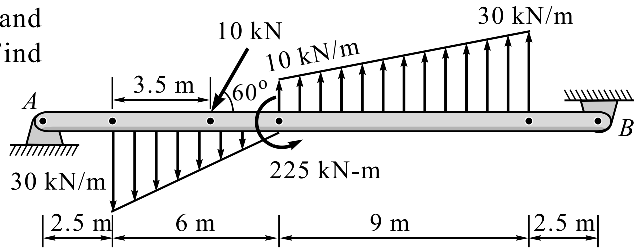


Fig. 3.E41

[II] Review Questions

1. What are the conditions of equilibrium for concurrent, parallel and general force system ?
2. Describe FBD and its importance in the analysis of problems.
3. Explain the types of supports and indicate the unknown reactions they offer.
4. What are the different types of loads ?
5. How do you identify the two-force member in a structure ?
6. Explain three-force member principle.
7. State and prove Lami's theorem.

[III] Fill in the Blanks

1. If a system is in equilibrium and acted by two forces, then these two forces must be ________ in magnitude, ________ in direction and collinear in action.
2. If three concurrent force system is in equilibrium, then the resultant of two forces should be ________ and _______ to the third force.
3. Full form of UDL is ___________________ and that of UVL is ___________________.
4. Fixed support is also named as ________ support.
5. Tensile force of a straight member in FBD is represented by drawing an arrow ________ from joint or body.

8

FRICTION

8.1 Introduction

In most of the equilibrium problems that we have analysed up to this point, surfaces of contact have been assumed to be frictionless. The concept of a frictionless surface is, of course, an idealisation. All real surfaces have some roughness.

When a body moves or tends to move over another body, a force opposing the motion develops at the contact surfaces. Whenever a tendency exists for one contacting surfaces to slide along another surface, tangential force is generated between contacting surface. This force which opposes the movement or tendency of movement is called a ***frictional force*** *or simply* ***friction***.

When two bodies in contact are in motion then the frictional force is opposite to the relative motions.

Cause of Friction : Friction is due to *the resistance offered to motion by minutely projecting portions at the contact surfaces*. These microscopic projections get interlocked. To a small extent, the material of two bodies in contact also produces resistance to motion due to intra-molecular force of attraction, i.e., adhesive properties.

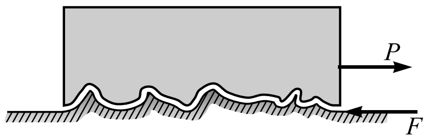


Fig. 8.1-i : Magnified Microscopic View of Rough Surface

8.2 Types of Friction

Dry Friction : Dry friction develops when *the unlubricated surface of two solids are in contact under a condition of sliding or a tendency to slide*. A frictional force tangent to the surfaces of contact is developed both during the interval leading up to impending slippage and while slippage takes place. The direction of the frictional force always opposes the relative motion or impending motion. This type of friction is also known as *Coulomb friction*.

Fluid Friction : Fluid friction is developed when *adjacent layers in a fluid (liquid or gas) are moving at different velocities.* This motion gives rise to frictional forces between fluid elements and these forces depend on the relative velocity between layers. Fluid frictions depends not only on the velocity gradients within the fluid but also on viscosity of fluid, which is a measure of its resistance to shearing action between fluid layers. Fluid friction is treated in the study of fluid mechanics and is beyond the scope of this text. So we are going to deal with dry friction only.

8.3 Mechanism of Friction

Consider a block of weight W resting on a horizontal surface as shown in Fig. 8.3-i. The contacting surface possesses a certain amount of roughness. Let P be the horizontal force applied which will vary continuously from zero to a value sufficient to just move the block and then to maintain the motion. The free body diagram of the block shows active forces (i.e., Applied force P and weight of block W) and reactive forces (i.e., normal reaction N and tangential frictional force F).

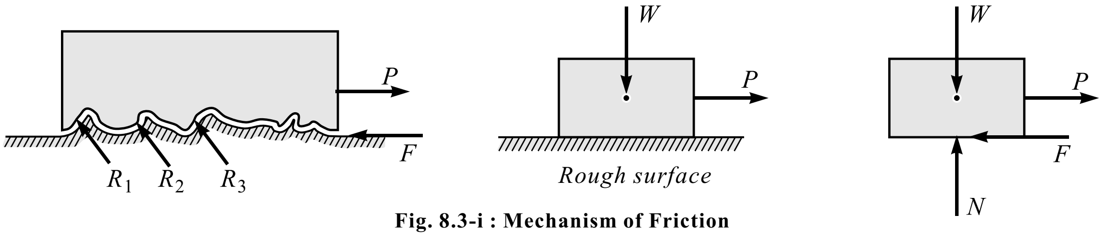


Fig. 8.3-i : Mechanism of Friction

Frictional force F has the remarkable property of adjusting itself in magnitude equal to the applied force P till the limiting equilibrium condition.

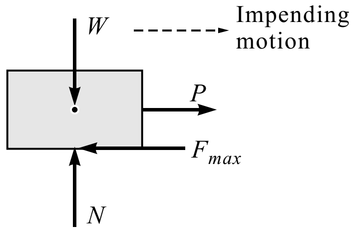
Limiting Equilibrium Condition : As applied force P increases, the frictional force F is equal in magnitude and opposite in direction. However, there is a limit beyond which the magnitude of the frictional force cannot be increased. If the applied force is more than this maximum frictional force, there will be movement of one body over the other. Once the body begins to move, there is decrease in frictional force F from maximum value observed under static condition. The frictional force between the two surface when the body is in motion is called *kinetic* or *dynamic friction* F_K.

Limiting Frictional Force (F_{max}) : It is the maximum frictional force developed at the surface when the block is at the verge of motion (impending motion).

Coefficient of Friction : By experimental evidence it is proved that limiting frictional force is directly proportional to normal reaction.

$$F_{max} \propto N$$

$$F_{max} = \mu_S N \quad \Rightarrow \quad \mu_S = \frac{F_{max}}{N} \qquad \text{...(8.1)}$$

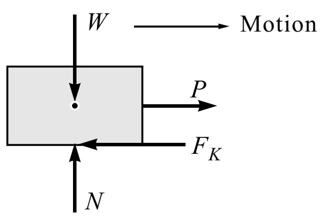


Coefficient of Static Friction : The ratio of limiting frictional force (F_{max}) and normal reaction (N) is a constant. This constant is called the *coefficient of static friction* (μ_S).

If a body is in motion, we have

$$F_K \propto N$$

$$F_K = \mu_K N \quad \Rightarrow \quad \mu_K = \frac{F_K}{N} \qquad \text{...(8.2)}$$



Coefficient of Kinetic Friction : The ratio of kinetic frictional force (F_K) and normal reaction (N) is a constant. This constant is known as *coefficient of kinetic friction* (μ_K).

Kinetic friction is always less than limiting friction.

8.4 Laws of Friction

1. The frictional force is always tangential to the contact surface and acts in a direction opposite to that in which the body tends to move.
2. The magnitude of frictional force is self-adjusting to the applied force till the limiting frictional force is reached and at the limiting frictional force the body will have impending motion.
3. Limiting frictional force F_{max} is directly proportional to normal reactions (i.e., $F_{max} = \mu_S N$).
4. For a body in motion, kinetic frictional force F_K developed is less than that of limiting frictional force F_{max} and the relation $F_K = \mu_K N$ is applicable.
5. Frictional force depends upon the roughness of the surface and the material in contact.
6. Frictional force is independent of the area of contact between the two surfaces.
7. Frictional force is independent of the speed of the body.
8. Coefficient of static friction μ_S is always greater than the coefficient of kinetic friction μ_K.

Angle of Friction : It is the angle made by the resultant of the limiting frictional force F_{max} and the normal reaction N with the normal reactions.

Consider the block with weight W and applied force P.

When the block is at the verge of motion, limiting frictional force F_{max} will act in opposite direction of applied force and normal reaction N will act perpendicular to surface, as shown in Fig. 8.4-i. We can replace the F_{max} and N by resultant reaction R which acts at an angle ϕ to the normal reaction.

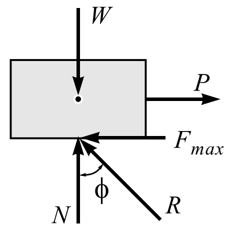


Fig. 8.4-i

This angle ϕ is called as the *angle of friction*.

From Fig. 8.4-ii, we have

$$F_{max} = R \sin \phi$$

$$\mu_S N = R \sin \phi \qquad \text{... (I)} \qquad (\because F_{max} = \mu_S N)$$

$$N = R \cos \phi \qquad \text{... (II)}$$

Dividing Eq. (I) by Eq. (II), we get

$$\tan \phi = \mu_S \text{ or } \phi = \tan^{-1} \mu_S \qquad \text{...(8.3)}$$

Angle of Repose : It is the minimum angle of inclination of a plane with the horizontal at which the body so kept will just begin to slide down on it without the application of any external force (due to self-weight).

Consider the block with weight W resting on an inclined plane, which makes an angle θ with horizontal as shown in Fig. 8.4-ii. When θ is small the block will rest on the plane. If θ is increased gradually a slope is reached at which the block is about to start sliding. This angle θ is called the *angle of repose*.

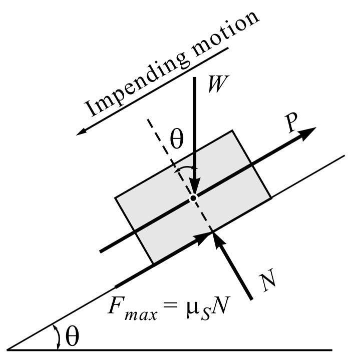


Fig. 8.4-ii

For limiting equilibrium condition, we have

$$\Sigma F_x = 0$$

$$\mu_S N - W \sin\theta = 0$$

$$W \sin\theta = \mu N \qquad \text{... (I)}$$

$$\Sigma F_y = 0$$

$$N - W\cos\theta = 0$$

$$W\cos\theta = N \qquad \text{... (II)}$$

Dividing Eq. (I) by Eq. (II), we get

$$\tan\theta = \mu_S \qquad \text{...(8.4)}$$

In previous discussion, we had $\tan\phi = \mu_S$, which shows

Angle of friction ϕ = Angle of repose θ

The above relation also shows that the angle of repose is independent of the weight of the body and it depends on μ.

Cone of Friction : When the applied force P is just sufficient to produce the impending motion of given body, angle of friction ϕ is obtained which is the angle made by resultant of limiting friction force and normal reaction with normal reaction as shown in Fig. 8.4-iii. If the direction of applied force P is gradually changed through 360°, the resultant R generates a right circular cone with semi vertex angle equal to ϕ. This is called the *cone of friction*.

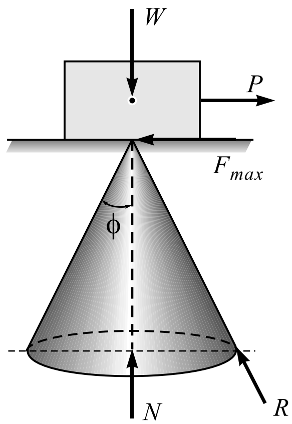


Fig. 8.4-iii

8.5 Types of Friction Problems

The above discussion can be represented by a graph with applied force P v/s frictional force F as shown in Fig. 8.5-i.

Referring to the graph we recognize three distinct types of friction problems. Here, we have static friction, limiting friction and kinetic friction.

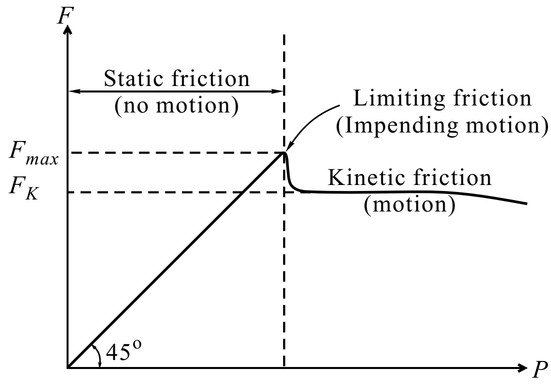


Fig. 8.5-i

1. **Static Friction :** If there is neither the condition of impending motion nor that of motion then to determine the actual force, we first assume static equilibrium and take F as a frictional force required to maintain the equilibrium condition.

Here, we have three possibilities

(i) $F < F_{max}$ $\Rightarrow$ Body is in static equilibrium condition which means body is purely at rest.

(ii) $F = F_{max}$ $\Rightarrow$ Body is in limiting equilibrium condition which means impending motion and hence $F = F_{max} = \mu_S N$ is valid equation.

(iii) $F > F_{max}$ $\Rightarrow$ Body is in motion which means $F = F_K = \mu_K N$ is valid equation (this condition is impossible, since the surfaces cannot support more force than the maximum frictional force. Therefore, the assumption of equilibrium is invalid, the motion occurs).

2. **Limiting Friction :** The condition of impending motion is known to exist. Here a body which is in equilibrium is on the verge of slipping which means the body is in limiting equilibrium condition.

   $F_{max} = \mu_S N$ is valid equation.

3. **Kinetic Friction :** The condition of relative motion is known to exist between the contacting surfaces. So, the body is in motion.

   Kinetic friction takes place $F_K = \mu_K N$ is valid equation.

8.6 Applications of Friction

1. The running vehicle is controlled by applying brake to its tyre because of friction. The vehicle moves with better grip and does not slip due to appropriate friction between the tyre and the road.
2. One can walk comfortably on the floor because of proper gripping between floor and the sole of the shoes. It is difficult to walk on oily or soapy floor.
3. Belt and pulley arrangement permits loading and unloading of load effectively because of suitable friction.
4. Lift moves smoothly without slipping due to proper rope and pulley friction combination.
5. A simple lifting machine like screw jack functions effectively based on principle of wedge friction.

8.7 Solved Problems

8.7.1 Body Placed on Horizontal Plane

Problem 1

Determine the frictional force developed on the block shown in Fig. 8.1 when

(i) $P = 40$ N,

(ii) $P = 80$ N. Coefficient of static friction between the block and floor is $\mu_S = 0.3$ and $\mu_K = 0.25$ and

(iii) Also find the value of P when the block is about to move.

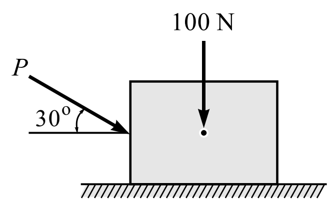


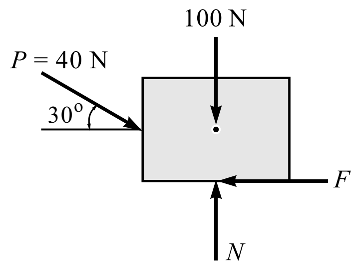
Fig. 8.1

Solution

(i) When *P* = 40 N

Consider the FBD of the block

Let F be the frictional force required to maintain the static equilibrium condition.



FBD of Block

$\Sigma F_y = 0$

$N - 100 - 40 \sin 30^\circ = 0$

$N = 120$ N

$\Sigma F_x = 0$

$40 \cos 30^\circ - F = 0$

$F = 34.64$ N

For limiting equilibrium condition, we have

$F_{max} = \mu_S \times N = 0.3 \times 120$

$F_{max} = 36$ N

Since $F < F_{max}$ therefore block is in static equilibrium condition.

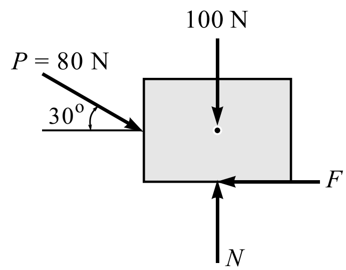
Therefore, actual frictional force is $F = 34.64$ N ***Ans.***

Here block is not moving.

(ii) When *P* = 80 N

Consider the FBD of the block

Let F be the frictional force required to maintain the static equilibrium condition.



FBD of Block

$\Sigma F_y = 0$

$N - 100 - 80 \sin 30^\circ = 0$

$N = 140$ N

$\Sigma F_x = 0$

$80 \cos 30^\circ - F = 0$

$F = 69.28$ N

For limiting equilibrium condition, we have

$F_{max} = \mu_S \times N = 0.3 \times 140$

$F_{max} = 42$ N

$\because F > F_{max}$

$\therefore$ The block is in motion and kinetic friction is considered.

$\therefore$ $F_K = \mu_K N$ is applicable and $F_{max} = \mu_S N$ is not applicable.

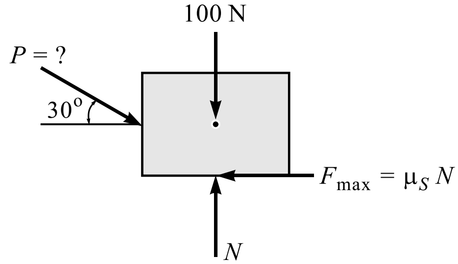
$F_K = 0.25 \times 140$

$F_K = 35$ N

$\therefore$ Actual frictional force acting at surface is $F_K = 35$ N and block is in motion. ***Ans.***

(iii) Find P = ?

For limiting equilibrium condition, consider the FBD of the block.



FBD of Block

$$\Sigma F_y = 0$$

$$N - 100 - P \sin 30^\circ = 0$$

$$N = 100 + P \sin 30^\circ \qquad \ldots \text{(I)}$$

$$\Sigma F_x = 0$$

$$P \cos 30^\circ - \mu_S N = 0$$

From Eq. (I)

$$P \cos 30^\circ - 0.3\,(100 + P \sin 30^\circ) = 0$$

$$P \cos 30^\circ - 0.3\,P \sin 30^\circ = 0.3 \times 100$$

$$P = 41.9 \text{ N}$$

When $P = 41.9$ N the block is about to move (Impending motion). ***Ans.***

Problem 2

A wooden block of mass 40 kg is on rough inclined plane as shown in Fig. 8.2. Find the frictional force at surface in contact if $\mu_S = 0.4$ and $\mu_K = 0.35$.

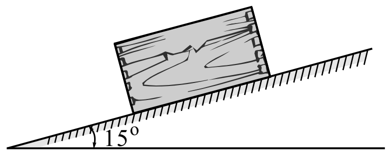


Fig. 8.2

Solution

$$\tan \phi = \mu_S$$

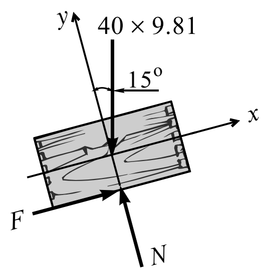
$$\therefore \ \phi = 21.8^\circ$$

We know angle of friction is equal to angle of repose for limiting equilibrium condition where self-weight of block is just sufficient to slide down without any external force acting on it.

In the above case, inclination of surface at an angle 15° is less than angle of friction $\phi = 21.8^\circ$. Therefore, the block will be in static equilibrium condition (i.e., stationary).

(i) Consider the FBD of the block

Let F be the frictional force required to maintain the static equilibrium condition.



FBD of Block

$$\Sigma F_x = 0$$

$$F - 40 \times 9.81 \sin 15^\circ = 0$$

$$F = 101.56 \text{ N} \quad \textbf{\textit{Ans.}}$$

Problem 3

For Problem 2, what is the external force required to be applied parallel to the inclined plane in downward direction for impending motion ?

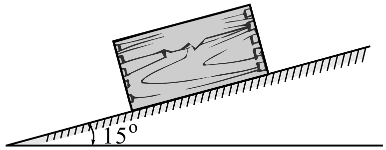


Fig. 8.3

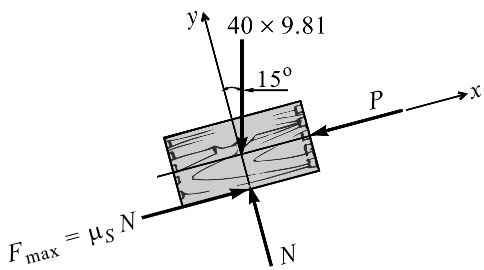
Solution

Impending motion means limiting equilibrium condition is applicable, i.e.,

$$F_{max} = \mu N$$

(i) Consider the FBD of the block

Let P be the force required to be applied to develop impending motion.



FBD of Block

$$\Sigma F_y = 0$$

$$N - (40 \times 9.81 \cos 15^{o}) = 0$$

$$N = 379.03 \text{ N}$$

$$\Sigma F_x = 0$$

$$\mu_S N - P - (40 \times 9.81 \sin 15^{o}) = 0$$

$$P = 0.4 \times 379.03 - 40 \times 9.81 \sin 15^{o}$$

$$P = 50.05 \text{ N} \left(15^{o}\swarrow\right)$$ ***Ans.***

Problem 4

A support block is acted upon by two forces as shown in Fig. 8.4, knowing that the coefficient of friction between the block and incline are $\mu_S = 0.35$, $\mu_K = 0.25$, determine the force P required

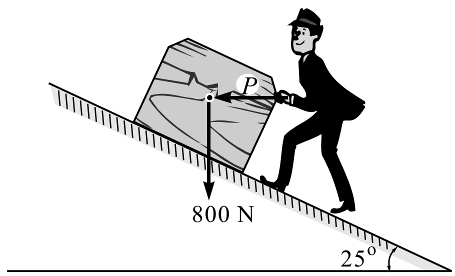


Fig. 8.4

(i) to start the block moving up the plane,

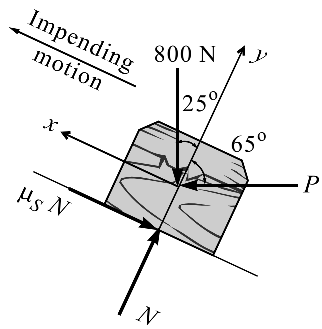
(ii) to keep it moving up, and

(iii) to prevent it from sliding down.

Solution

(i) Force *P* required to start the block moving up the plane

Consider the FBD of the block



FBD of Block

$$\Sigma F_y = 0$$

$$N - 800 \cos 25^{o} - P \cos 65^{o} = 0$$

$$N = 800 \cos 25^{o} + P \cos 65^{o}$$

$$\Sigma F_x = 0$$

$$P \sin 65^\circ - \mu_S N - 800 \sin 25^\circ = 0$$

$$P \sin 65^\circ - 0.35(800 \cos 25^\circ + P \cos 65^\circ) - 800 \sin 25^\circ = 0$$

$$P \sin 65^\circ - 0.35 \times P \cos 65^\circ - 0.35 \times 800 \cos 25^\circ - 800 \sin 25^\circ = 0$$

$$P (\sin 65^\circ - 0.35 \cos 65^\circ) = 0.35 \times 800 \cos 25^\circ + 800 \sin 25^\circ$$

$P = 780.42$ N ***Ans.***

(ii) Force *P* required to keep it moving up

Consider the FBD of the block

$$\Sigma F_y = 0$$

$$N - 800 \cos 25^\circ - P \cos 65^\circ = 0$$

$$N = 800 \cos 25^\circ + P \cos 65^\circ$$

$$\Sigma F_x = 0$$

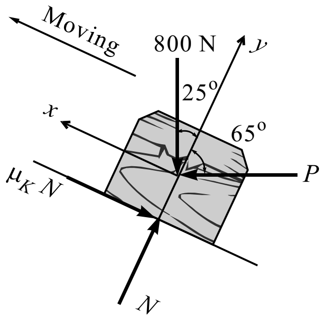
$$P \sin 65^\circ - \mu_K N - 800 \sin 25^\circ = 0$$

$$P \sin 65^\circ - 0.25(800 \cos 25^\circ + P \cos 65^\circ) - 800 \sin 25^\circ = 0$$

$$P \sin 65^\circ - 0.25 \times P \cos 65^\circ - 0.25 \times 800 \cos 25^\circ - 800 \sin 25^\circ = 0$$

$$P (\sin 65^\circ - 0.25 \cos 65^\circ) = 0.25 \times 800 \cos 25^\circ + 800 \sin 25^\circ$$

$P = 648.67$ N ***Ans.***



FBD of Block

(iii) Force *P* required to prevent it from sliding down

Consider the FBD of the block

$$\Sigma F_y = 0$$

$$N - 800 \cos 25^\circ - P \cos 65^\circ = 0$$

$$N = 800 \cos 25^\circ + P \cos 65^\circ$$

$$\Sigma F_x = 0$$

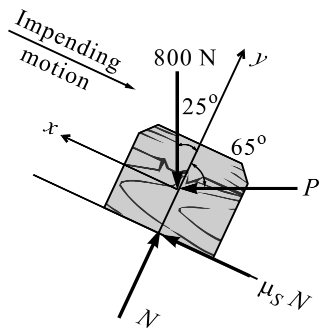
$$\mu_S N + P \sin 65^\circ - 800 \sin 25^\circ = 0$$

$$0.35(800 \cos 25^\circ + P \cos 65^\circ) + P \sin 65^\circ - 800 \sin 25^\circ = 0$$

$$0.35 \times 800 \cos 25^\circ - 0.35 \times P \cos 65^\circ + P \sin 65^\circ - 800 \sin 25^\circ = 0$$

$$P (0.35 \cos 65^\circ + \sin 65^\circ) = 800 \cos 25^\circ - 0.35 \times 800 \sin 25^\circ$$

$P = 80$ N ***Ans.***



FBD of Block

Problem 5

Two blocks W_1 and W_2 , resting on two inclined planes, are connected by a horizontal bar AB, as shown in Fig. 8.5. If W_1 equals 1000 N, determine the maximum value of W_2 for which the equilibrium can exist. The angle of limiting friction is 20^o at all rubbing faces.

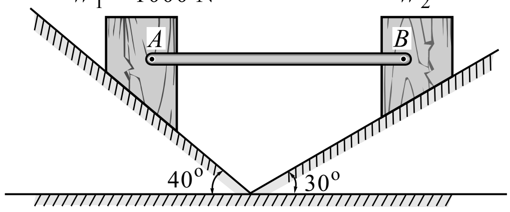


Fig. 8.5

Solution

For maximum weight of the block B in limiting equilibrium condition, tendency of block B will be to impend upwards.

$\therefore$ Impending motion of block A will be downward.

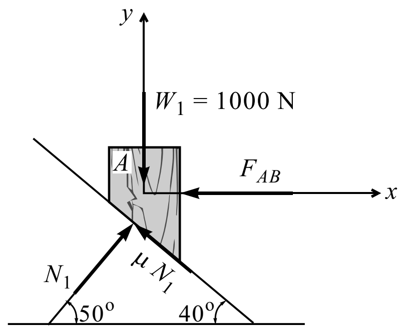
(i) Consider the FBD of Block *A*

$$\Sigma F_y = 0$$

$$N_1 \sin 50^o + \mu N_1 \sin 40^o - 1000 = 0$$

$$N_1 = \frac{1000}{\sin 50^o + \tan 20^o \sin 40^o}$$

$$N_1 = 1000 \text{ N}$$



FBD of Block *A*

$$\Sigma F_x = 0$$

$$N_1 \cos 50^o - \mu N_1 \cos 40^o - F_{AB} = 0$$

$$F_{AB} = 1000 \cos 50^o - \tan 20^o \times 1000 \cos 40^o$$

$$F_{AB} = 363.97 \text{ N}$$

(ii) Consider the FBD of Block *B*

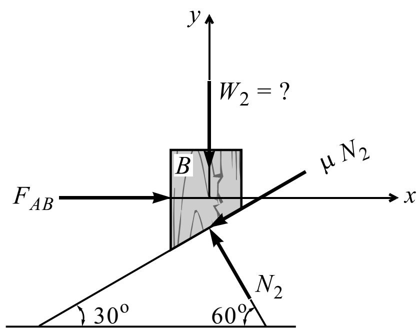
$$\Sigma F_x = 0$$

$$F_{AB} - \mu N_2 \cos 30^o - N_2 \cos 60^o = 0$$

$$363.97 - \tan 20^o N_2 \cos 30^o - N_2 \cos 60^o = 0$$

$$N_2 = \frac{363.97}{(\cos 60^o + \tan 20^o \cos 30^o)}$$

$$N_2 = 446.48 \text{ N}$$



FBD of Block *B*

$$\Sigma F_y = 0$$

$$N_2 \sin 60^o - \mu N_2 \sin 30^o - W_2 = 0$$

$$W_2 = 446.48 \sin 60^o - \tan 20^o \times 446.48 \sin 30^o$$

$$W_{2(\max)} = 305.41 \text{ N} \quad \textbf{\textit{Ans.}}$$

Problem 6

Two blocks W_1 and W_2 which are connected by a horizontal bar AB are supported on rough planes as shown in Fig. 8.6. The coefficient of friction for the block A = 0.4. The angle of friction for the block B is 20^o. Find the smallest weight W_1 of the block A for which the equilibrium can exist, if W_2 = 2250 N.

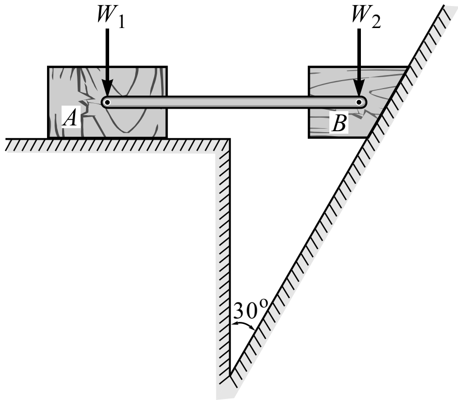


Fig. 8.6

Solution

(i) Consider the FBD of Block *B*

$\Sigma F_y = 0$

$N_1 \sin 30^o + \mu_1 N_1 \sin 60^o - 2250 = 0$

$N_1 (\sin 30^o + \tan 20^o \sin 60^o) = 2250$

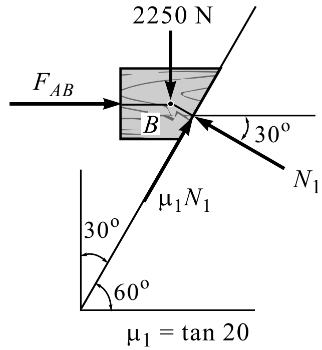
$N_1 = 2760.03$ N

$\Sigma F_x = 0$

$F_{AB} + \mu_1 N_1 \cos 60^o - N_1 \cos 30^o = 0$

$F_{AB} = 2760.03 \cos 30^o - \tan 20^o \times 2760.03 \cos 60^o$

$F_{AB} = 1887.97$ N



FBD of Block *B*

(ii) Consider the FBD of Block *A*

$\Sigma F_x = 0$

$\mu_2 N_2 - F_{AB} = 0$

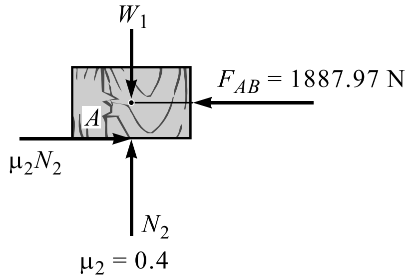
$0.4 N_2 = 1887.97$

$N_2 = 4719.93$ N

$\Sigma F_y = 0$

$N_2 - W_1 = 0$

$\therefore W_1 = 4719.93$ N ***Ans.***



FBD of Block *A*

Problem 7

Two blocks A = 100 N and B = W are connected by a rod at their ends by frictionless hinges as shown in Fig. 8.7. Find the weight of block B (W) required for limiting equilibrium of the system if coefficient of friction at all sliding surface is 0.3.

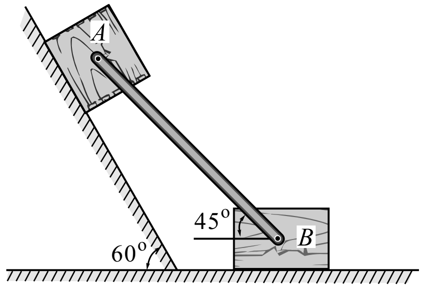
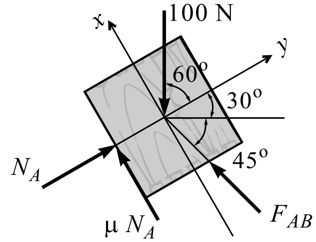


Fig. 8.7

Solution

(i) Consider the FBD of Block *A*



FBD of Block *A*

$$\Sigma F_y = 0$$

$$N_A - 100 \cos 60^o - F_{AB} \cos 75^o = 0$$

$$N_A = 100 \cos 60^o + F_{AB} \cos 75^o$$

$$\Sigma F_x = 0$$

$$\mu N_A + F_{AB} \sin 75^o - 100 \sin 60^o = 0$$

$$0.3(100 \cos 60^o + F_{AB} \cos 75^o) + F_{AB} \sin 75^o - 100 \sin 60^o = 0$$

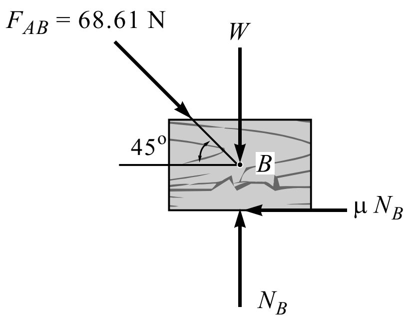
$$(0.3 \times F_{AB} \cos 75^o) + F_{AB} \sin 75^o + (0.3 \times 100 \cos 60^o) - 100 \sin 60^o = 0$$

$$F_{AB} (0.3 \cos 75^o + \sin 75^o) = 100 \sin 60^o - (0.3 \times 100 \cos 60^o)$$

$$F_{AB} = \frac{100 \sin 60^o - (0.3 \times 100 \cos 60^o)}{(0.3 \cos 75^o + \sin 75^o)}$$

F_{AB} = 68.61 N (rod is under compression)

(ii) Consider the FBD of Joint *B*



FBD of Block *B*

$$\Sigma F_y = 0$$

$$N_B - W - F_{AB} \sin 45^o = 0$$

$$N_B = W + F_{AB} \sin 45^o$$

$$\Sigma F_x = 0$$

$$F_{AB} \cos 45^o - \mu N_B = 0$$

$$68.61 \cos 45^o - 0.3(W + 68.61 \sin 45^o) = 0$$

$$0.3\, W = 33.96$$

W = 113.2 N ***Ans.***

Problem 8

Two identical blocks A and B are connected by a rod and rest against vertical and horizontal planes, respectively, as shown in Fig. 8.8. If sliding impends when $\theta = 45^o$, determine the coefficient of friction μ, assuming it to be the same at both floor and wall.

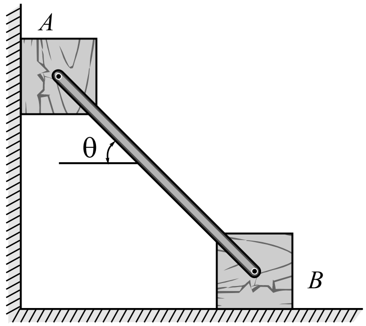
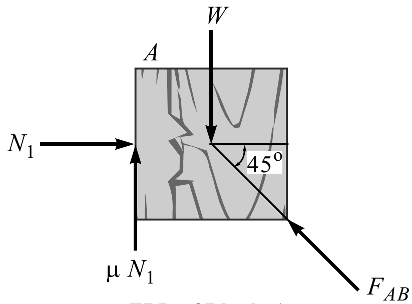


Fig. 8.8

Solution

(i) Consider the FBD of Block ***A***



FBD of Block ***A***

$$\Sigma F_x = 0$$

$$N_1 - F_{AB} \cos 45^o = 0$$

$$N_1 = F_{AB} \cos 45^o$$

$$\Sigma F_y = 0$$

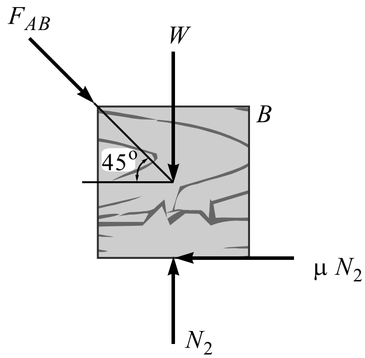
$$\mu N_1 + F_{AB} \sin 45^o - W = 0$$

$$W = \mu N_1 + F_{AB} \sin 45^o$$

$$W = \mu F_{AB} \cos 45^o + F_{AB} \sin 45^o$$

$$W = F_{AB} (\mu \cos 45^o + \sin 45^o) \quad \ldots \text{(I)}$$

(ii) Consider the FBD of Block ***B***



FBD of Block ***B***

$$\Sigma F_y = 0$$

$$N_2 - W - F_{AB} \sin 45^o = 0$$

$$N_2 = W + F_{AB} \sin 45^o$$

$$\Sigma F_x = 0$$

$$F_{AB} \cos 45^o - \mu N_2 = 0$$

$$F_{AB} \cos 45^o - \mu (W + F_{AB} \sin 45^o) = 0$$

$$F_{AB} \cos 45^o - \mu W - \mu F_{AB} \sin 45^o = 0$$

$$W = \frac{F_{AB} \cos 45^o - \mu F_{AB} \sin 45^o}{\mu}$$

$$W = \frac{F_{AB} (\cos 45^o - \mu \sin 45^o)}{\mu} \quad \ldots \text{(II)}$$

Equating Eqs. (I) and (II),

$$F_{AB} (\mu \cos 45^o + \sin 45^o) = \frac{F_{AB} (\cos 45^o - \mu \sin 45^o)}{\mu}$$

$$\mu^2 \cos 45^o + \mu \sin 45^o = \cos 45^o - \mu \sin 45^o$$

$$\mu^2 \cos 45^o + 2\mu \sin 45^o - \cos 45^o = 0$$

$$\mu^2 + 2\mu - 1 = 0$$

Solving the quadratic equation, we get $\mu = 0.414$ ***Ans.***

Problem 9

Determine the force P to cause motion to impend. Take masses of blocks A and B as 9 kg and 4 kg respectively and the coefficient of sliding friction as 0.25. The force P and the rope are parallel to the inclined plane as shown in Fig. 8.9. Assume pulley to be frictionless.

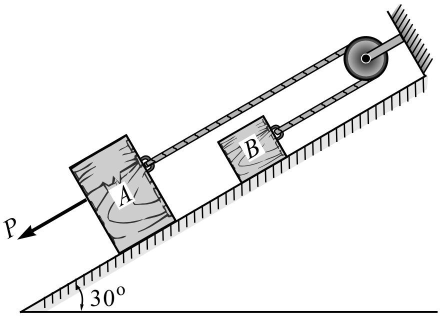


Fig. 8.9

Solution

(i) Consider the FBD of Block *B*

$$\Sigma F_y = 0$$

$$N_B - (4 \times 9.81 \cos 30^\circ) = 0$$

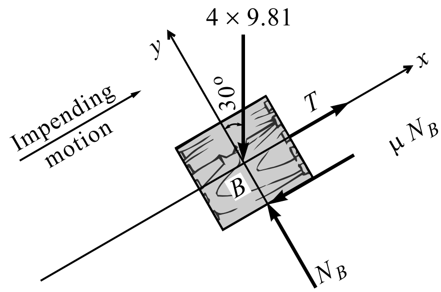
$$N_B = 33.98 \text{ N}$$

$$\Sigma F_x = 0$$

$$T - \mu N_B - (4 \times 9.81 \sin 30^\circ) = 0$$

$$T = (0.25 \times 33.98) + (4 \times 9.81 \sin 30^\circ)$$

$$T = 28.12 \text{ N}$$



FBD of Block *B*

(ii) Consider the FBD of Block *A*

$$\Sigma F_y = 0$$

$$N_A - (9 \times 9.81 \cos 30^\circ) = 0$$

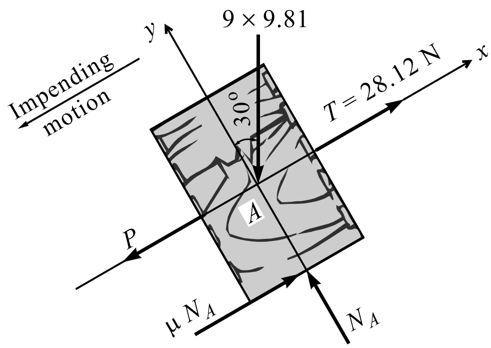
$$N_A = 76.46 \text{ N}$$

$$\Sigma F_x = 0$$

$$T + \mu N_A - P - (9 \times 9.81 \sin 30^\circ) = 0$$

$$P = 28.12 + (0.25 \times 76.46) - (9 \times 9.81 \sin 30^\circ)$$

$P = 3.09$ N ***Ans.***



FBD of Block *A*

Problem 10

Two blocks A and B of weight 500 N and 750 N, respectively are connected by a cord that passes over a frictionless pulley, as shown in Fig. 8.10. The coefficient of friction between the block A and the inclined plane is 0.4 and that between the block B and the inclined plane is 0.3. Determine the force P to be applied to block B to produce the impending motion of block B down the plane.

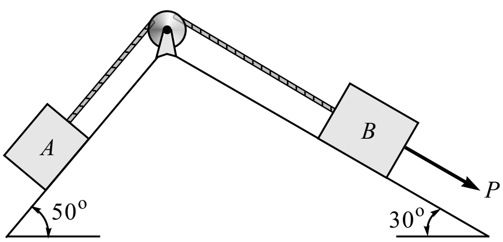


Fig. 8.10

(i) Consider the FBD of Block *A*

$\Sigma F_y = 0$

$N_A - 500 \cos 50^o = 0$

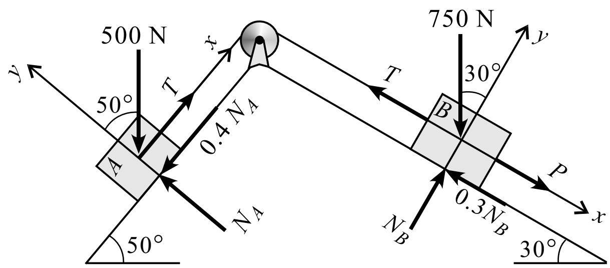
$N_A = 500 \cos 50^o$

$\Sigma F_x = 0$

$T - 0.4\, N_A - 500 \sin 50^o = 0$

$T = 0.4 \times 500 \cos 50^o + 500 \sin 50^o$

$T = 511.58$ N



FBD of Block *A* and Block *B*

(ii) Consider the FBD of Block *B*

$\Sigma F_y = 0$

$N_B - 750 \cos 30^o = 0$

$N_B = 750 \cos 30^o$

$\Sigma F_x = 0$

$P - 0.3\, N_B - T + 750 \sin 30^o = 0$

$P = 0.3 \times 750 \cos 30^o + 511.58 - 750 \sin 30^o$

$P = 331.44$ N $\left(\searrow 30^o\right)$ ***Ans.***

Problem 11

Find the value of θ if the blocks *A* and *B* shown in Fig. 8.11 have impending motion. Given block *A* = 20 kg, block *B* = 20 kg, $\mu_A = \mu_B = 0.25$.

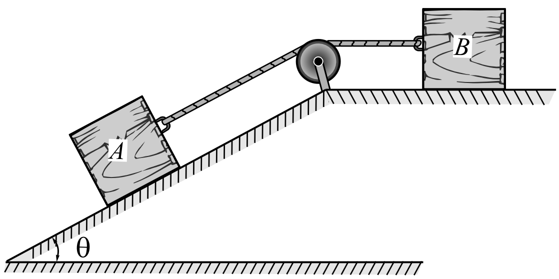


Fig. 8.11

Solution

(i) Consider the FBD of Block *B*

$\Sigma F_y = 0$

$N_B - (20 \times 9.81) = 0$

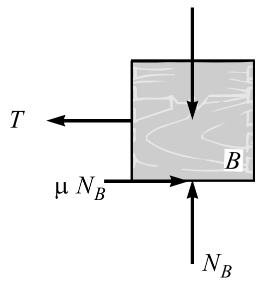
$N_B = 20 \times 9.81$

$\Sigma F_x = 0$

$\mu\, N_B - T = 0$

$(0.25 \times 20 \times 9.81) - T = 0$

$T = 0.25 \times 20 \times 9.81$



FBD of Block *B*

(ii) Consider the FBD of Block *A*

$\Sigma F_y = 0$

$N_A - (20 \times 9.81)\cos\theta = 0$

$N_A = (20 \times 9.81)\cos\theta$

$\Sigma F_x = 0$

$\mu N_A + T - (20 \times 9.81)\sin\theta = 0$

$(0.25 \times 20 \times 9.81)\cos\theta + (0.25 \times 20 \times 9.81)$

$\quad - (20 \times 9.81)\sin\theta = 0$

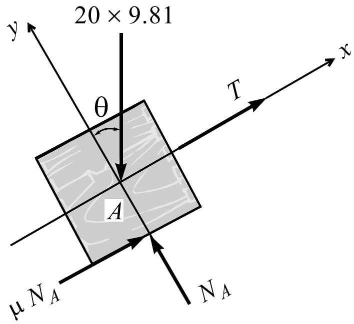
$0.25\cos\theta + 0.25 - \sin\theta = 0$ (Multiply by 4)

$\cos\theta + 1 = 4\sin\theta$

$2\cos^2(\theta/2) = 4\,[2\sin(\theta/2)\,.\cos(\theta/2)]$

$\frac{1}{4} = \tan\frac{\theta}{2} \Rightarrow \frac{\theta}{2} = \tan^{-1} 0.25$

$\therefore \theta = 28.07^o$ ***Ans.***



FBD of Block *A*

Problem 12

Find force P required to pull block B (shown in Fig. 8.12). Coefficient of friction between A and B is 0.3 and between B and floor is 0.25. Weights of A = 20 kg and B = 30 kg.

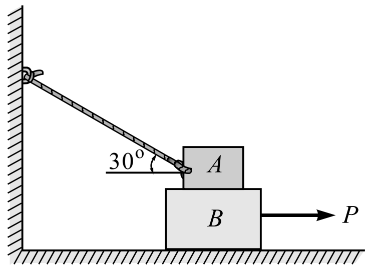


Fig. 8.12

Solution

(i) Consider the FBD of Block *A*

$\Sigma F_y = 0$

$N_A + T\sin 30^o - 20 \times 9.81 = 0$

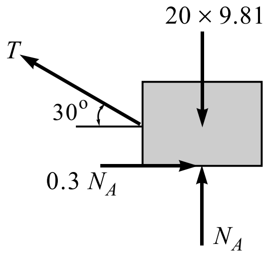
$N_A = 20 \times 9.81 - T\sin 30^o$

$\Sigma F_x = 0$

$T\cos 30^o - 0.3\,N_A = 0$

$T\cos 30^o - 0.3(20 \times 9.81 - T\sin 30^o) = 0$

$T = 57.93$ N and $N_A = 167.235$ N



FBD of Block *A*

(ii) Consider the FBD of Block *B*

$\Sigma F_y = 0$

$N_B - 30 \times 9.81 - 167.06 = 0$

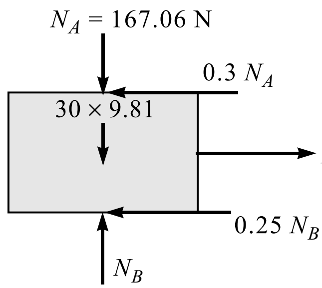
$N_B = 461.535$ N ***Ans.***

$\Sigma F_x = 0$

$P - 0.3\,N_A - 0.25\,N_B = 0$

$P = (0.3 \times 167.06) + (0.25 \times 461.36)$

$P = 165.55$ N ***Ans.***



FBD of Block *B*

Problem 13

Two blocks A = 100 N and B = 150 N are resting on ground as shown in Fig. 8.13. Coefficient of friction between ground and block B is 0.10 and that between block B and A is 0.30. Find the minimum value of weight P in the pan so that motion starts. Find whether B is stationary w.r.t. ground and A moves or B is stationary w.r.t. A.

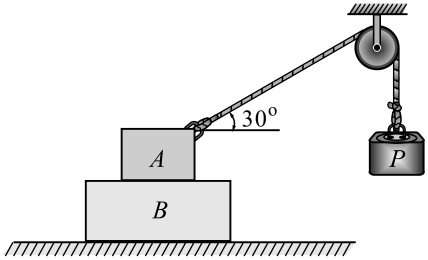
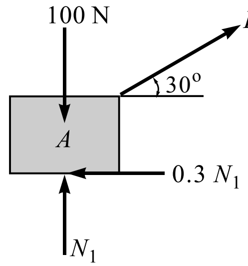


Fig. 8.13

Solution

Case I : B is stationary w.r.t. ground and A moves.

Consider given A is in limiting equilibrium which means block A moves over the surface of B.



FBD of Block *A*

Consider the FBD of Block *A*

$$\Sigma F_y = 0$$

$$N_1 + P \sin 30^o - 100 = 0$$

$$N_1 = 100 - P \sin 30^o$$

$$\Sigma F_x = 0$$

$$P \cos 30^o - 0.3\, N_1 = 0$$

$$P \cos 30^o - 0.3\,(100 - P \sin 30^o) = 0$$

$$P \cos 30^o - (0.3 \times 100) + 0.3\, P \sin 30^o = 0$$

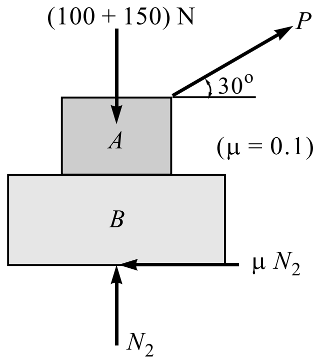
$$P\,(\cos 30^o + 0.3 \sin 30^o) = 0.3 \times 100$$

$$P = \frac{0.3 \times 100}{\cos 30^o + 0.3 \sin 30^o} \quad \therefore\ P = 29.53 \text{ N} \quad \textbf{\textit{Ans.}}$$

Case II : B is stationary w.r.t. A

Consider both blocks A and B moving together.

Consider the FBD of *A* and *B* together



FBD of Block *A* and *B* Together

$$\Sigma F_y = 0$$

$$N_2 - 250 + P \sin 30^o = 0$$

$$N_2 = 250 - P \sin 30^o$$

$$\Sigma F_x = 0$$

$$P \cos 30^o - \mu\, N_2 = 0$$

$$P \cos 30^o - 0.1\,(250 - P \sin 30^o) = 0$$

$$P \cos 30^o - (0.1 \times 250) + 0.1\, P \sin 30^o = 0$$

$$P\,(\cos 30^o + 0.1 \sin 30^o) = 0.1 \times 250$$

$$P = \frac{0.1 \times 250}{\cos 30^o + 0.1 \sin 30^o} \quad \therefore\ P = 27.29 \text{ N} \quad \textbf{\textit{Ans.}}$$

Referring to the answers of both the cases, we can declare 'Case II' is initiated first and therefore, minimum value of P_{min} = 27.29 N.

Problem 14

Block A of mass 30 kg rests on block B of mass 40 kg, as shown in Fig. 8.14. Block A is restrained form moving by a horizontal rope tied at point C. What force P applied parallel to the plane inclined at 30° with horizontal is necessary to start block B sliding down the plane. Take coefficient of friction for all surfaces as 0.35.

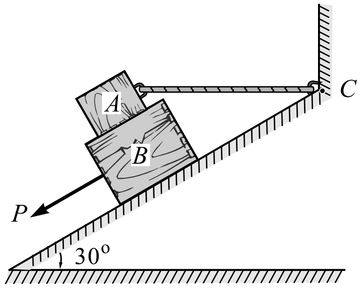


Fig. 8.14

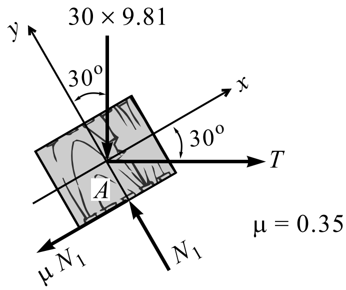
Solution

(i) Consider the FBD of Block *A*

$$\Sigma F_y = 0$$

$$N_1 - 30 \times 9.81 \cos 30^\circ - T \sin 30^\circ = 0$$

$$N_1 = 30 \times 9.81 \cos 30^\circ + T \sin 30^\circ \quad \ldots \text{(I)}$$



FBD of Block *A*

$$\Sigma F_x = 0$$

$$T \cos 30^\circ - \mu N_1 - 30 \times 9.81 \sin 30^\circ = 0$$

From Eq. (I)

$$T \cos 30^\circ - 0.35\,(30 \times 9.81 \cos 30^\circ + T \sin 30^\circ) - 30 \times 9.81 \sin 30^\circ = 0$$

$$T = 342.04 \text{ N}$$

Substituting T in Eq. (I)

$$N_1 = 30 \times 9.81 \cos 30^\circ + 342.04 \sin 30^\circ$$

$$N_1 = 425.89 \text{ N}$$

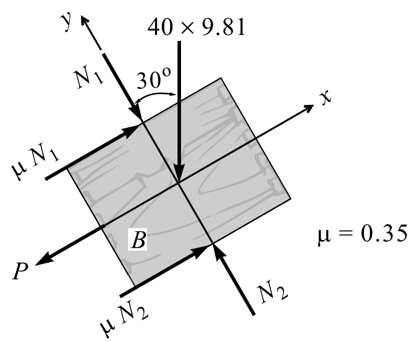
(ii) Consider the FBD of Block *B*

$$\Sigma F_y = 0$$

$$N_2 - N_1 - 40 \times 9.81 \cos 30^\circ = 0$$

$$N_2 = 425.89 + 40 \times 9.81 \cos 30^\circ$$

$$N_2 = 765.72 \text{ N}$$



FBD of Block *B*

$$\Sigma F_x = 0$$

$$\mu N_1 + \mu N_2 - 40 \times 9.81 \sin 30^\circ - P = 0$$

$$P = 0.35\,(425.89 + 765.72) - 40 \times 9.81 \sin 30^\circ$$

$P = 220.86$ N ***Ans.***

Problem 15

Three blocks are placed on the surface one above the other as shown in Fig. 8.15. The static coefficient of friction between the blocks and block *C* and surface is also shown. Determine the maximum value of *P* that can be applied before any slipping takes place.

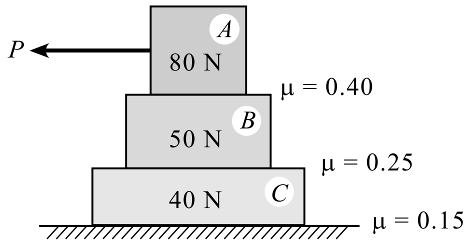


Fig. 8.15

Solution

For *P* there are three possibilities.

(i) Block *A* has impending motion and blocks *B* and *C* remain intact with each other and surface.

Consider the FBD of Block *A*

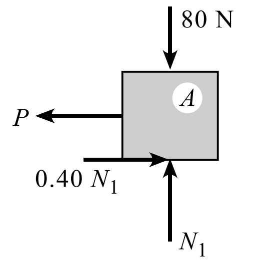
$\Sigma F_y = 0$

$N_1 - 80 = 0 \qquad \therefore\ N_1 = 80\text{ N}$

$\Sigma F_x = 0$

$0.4N_1 - P = 0$

$P = 0.4 \times 80 \qquad \therefore\ P = 32\text{ N} (\leftarrow)$



FBD of Block *A*

(ii) Blocks *A* and *B* together have impending motion and block *C* remains intact with surface.

FBD of Blocks *A* and *B* together

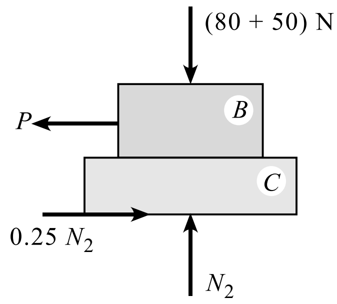
$\Sigma F_y = 0$

$N_2 - (80 + 50) = 0 \qquad \therefore\ N_2 = 130\text{ N}$

$\Sigma F_x = 0$

$0.25N_2 - P = 0$

$P = 0.25 \times 130 \qquad \therefore\ P = 32.5\text{ N} (\leftarrow)$



FBD of Block *A* and Block *B* Together

(iii) All the three blocks *A*, *B* and *C* together have impending motion.

FBD of Blocks *A*, *B* and *C* together

$\Sigma F_y = 0$

$N_3 - (50 + 80 + 40) = 0$

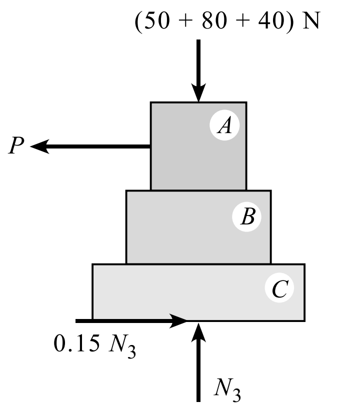
$N_3 = 170\text{ N}$

$\Sigma F_x = 0$

$0.15\ N_3 - P = 0$

$P = 0.15 \times 170$

$P = 25.5\text{ N}$



FBD of Block *A* and Block *B* and Block *C* Together

(iv) Comparing all three cases, we conclude that $P_{max} = 25.5$ N before any slipping takes place. ***Ans.***

Problem 16

Find the maximum height at which P should be applied so that the body would just slide without tipping. Also state magnitude of P. Refer to Fig. 8.16.

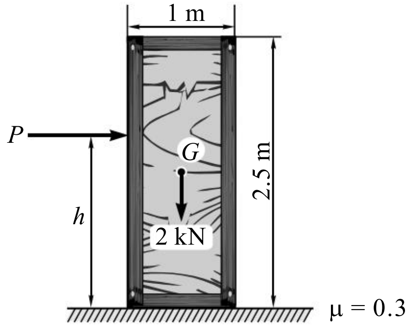
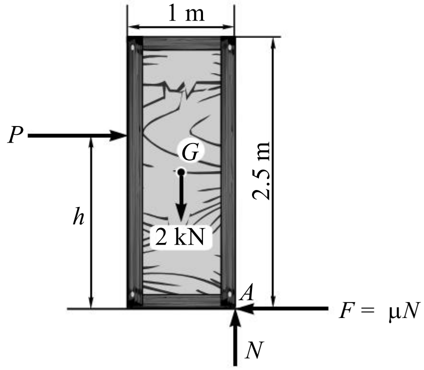


Fig. 8.16

Solution

Consider the FBD of the block

$$\Sigma F_y = 0$$
$$N - 2 = 0$$
$$N = 2 \text{ kN}$$
$$\Sigma F_x = 0$$
$$P - \mu N = 0$$
$$P = 0.3 \times 2$$
$$P = 0.6 \text{ kN} \qquad \textbf{\textit{Ans.}}$$
$$\Sigma M_A = 0$$
$$2 \times 0.5 - P \times h = 0 \quad \Rightarrow \quad h = \frac{2 \times 0.5}{0.6}$$
$$h = 1.67 \text{ m} \qquad \textbf{\textit{Ans.}}$$



FBD of Block

Problem 17

A homogeneous block A of weight W rests upon an inclined plane as shown in Fig. 8.17. $\mu = 0.3$. Determine the greatest height at which a force P parallel to the inclined plane may be applied so that the block will slide up the plane without tipping over.

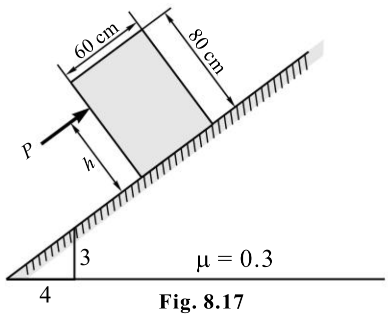
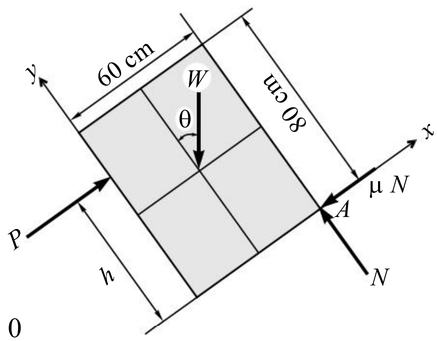


Fig. 8.17

Solution

Consider the FBD of Block *A*

$$\Sigma F_y = 0$$
$$N - W \cos \theta = 0$$
$$N = W \cos 36.87^{\circ}$$
$$\Sigma F_x = 0$$
$$P - \mu N - W \sin \theta = 0$$
$$P = 0.3 \times W \cos 36.87^{\circ} + W \sin 36.87^{\circ}$$
$$P = 0.84 \, W \text{ N}$$
$$\Sigma M_A = 0$$
$$W \cos \theta \times 30 + W \sin \theta \times 40 - P \times h = 0$$
$$W \cos 36.87^{\circ} \times 30 + W \sin 36.87^{\circ} \times 40 - 0.84 \, W \times h = 0$$
$$h = 57.14 \text{ cm} \qquad \textbf{\textit{Ans.}}$$



FBD of Block *A*

8.8 Solved Problems Based on Wedge, Ladder and Screw Jack

Wedge

A tapper shaped block (with very less angle of inclination) which are used for lifting or shifting or holding the heavy block by very less effort is called a *wedge*. The lifting or shifting of the distance is very small. While installing heavy machinery horizontal levelling is required with zero error. It is possible to adjust the small height by inserting wedge as a packing. Sometimes, combination of wedge is also used to push or shift heavy bodies by little distance. Simple lifting machine such as screw jack is based on principle of wedge which is used to raise or lower the heavy load by small effort.

Problem 18

A block of mass 150 kg is raised by a 10^o wedge weighing 50 kg under it and by applying a horizontal force at it as shown in Fig. 8.18. Taking coefficient of friction between all surfaces of contact as 0.3, find minimum force that should be applied to raise the block.

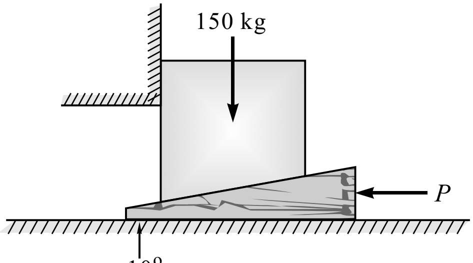
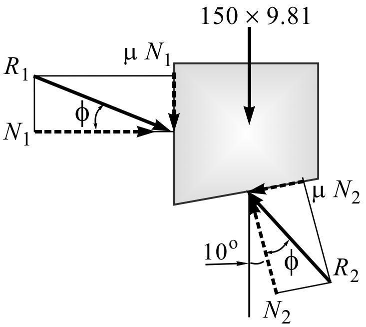


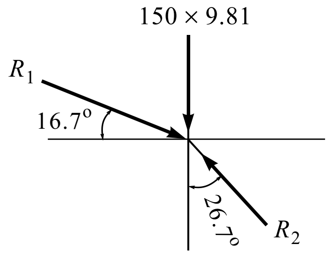
Fig. 8.18

Solution

(i) Consider the FBD of 150 kg block



FBD of Block



$$\tan \phi = \mu = 0.3$$

$$\therefore \phi = 16.7^o$$

By Lami's theorem, we have

$$\frac{R_2}{\sin (90 - 16.7)^o} = \frac{150 \times 9.81}{\sin (90 + 16.7 + 26.7)^o}$$

$$\therefore R_2 = 1939.84 \text{ N}$$

(ii) Consider the FBD of the wedge

$$\Sigma F_y = 0$$

$$N_2 - (50 \times 9.81) - 1929.84 \cos 26.7^o = 0$$

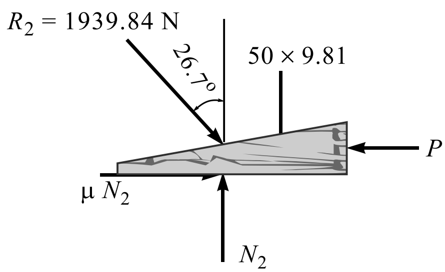
$$N_2 = 2223.5 \text{ N}$$

$$\Sigma F_x = 0$$

$$\mu N_2 + 1939.84 \sin 26.7^o - P = 0$$

$$P = (0.3 \times 2223.5) + 1939.84 \sin 26.7^o$$

$P = 1538.66$ N ***Ans.***



FBD of Wedge

Problem 19

The block, as shown in Fig. 8.19, supports a load W = 5000 N and is to be raised by forcing the wedge B under it. The angle of friction for all surface for contact is ϕ = 15°. Determine the force P which is necessary to start the wedge under the block. The block and wedge have negligible weight.

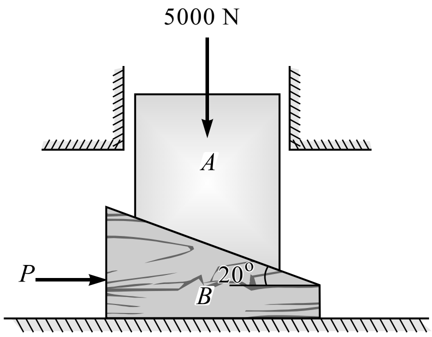
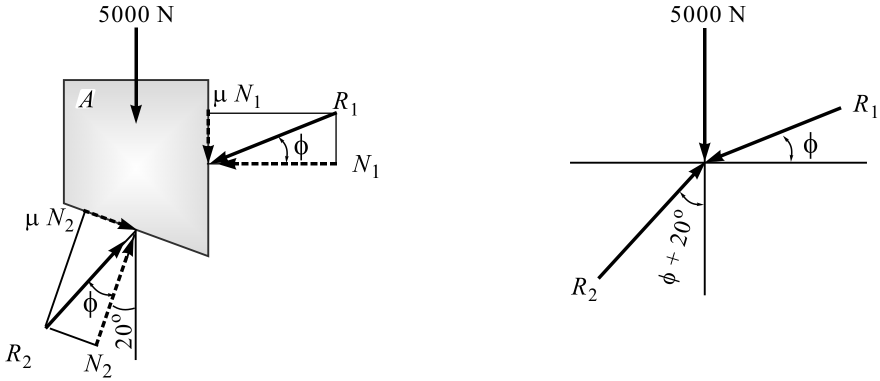


Fig. 8.19

Solution

(i) Consider the FBD of Block *A*



FBD of Block *A*

By Lami's theorem, we have

$$\frac{5000}{\sin(\phi + 20 + 90 + \phi)^{\circ}} = \frac{R_2}{\sin(90 - \phi)^{\circ}}$$

$$\therefore\ R_2 = \frac{5000 \sin 75^{\circ}}{\sin 140^{\circ}}$$

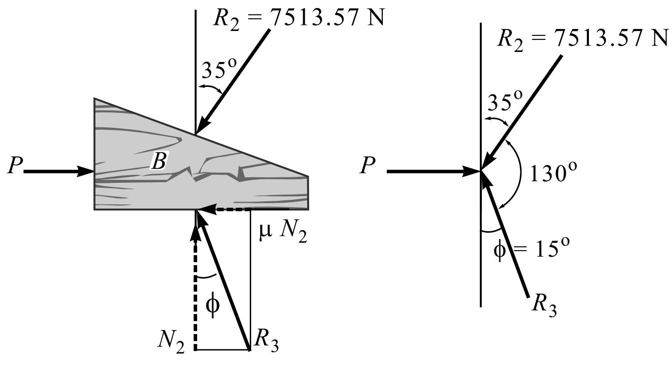
$$\therefore\ R_2 = 7513.57 \text{ N}$$

(ii) Consider the FBD of Wedge *B*

By Lami's theorem, we have

$$\frac{P}{\sin 130^{\circ}} = \frac{7513.57}{\sin 105^{\circ}}$$

$\therefore\ P$ = 5958.77 N ***Ans.***



FBD of Wedge

Problem 20

Two 6° wedges are used to push a block horizontally, as shown in Fig. 8.20. Calculate the minimum force required to push the block of weight 10 kN. Take μ = 0.25 for all contact surfaces.

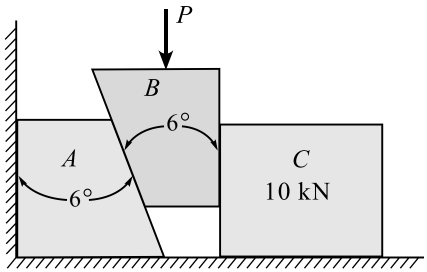
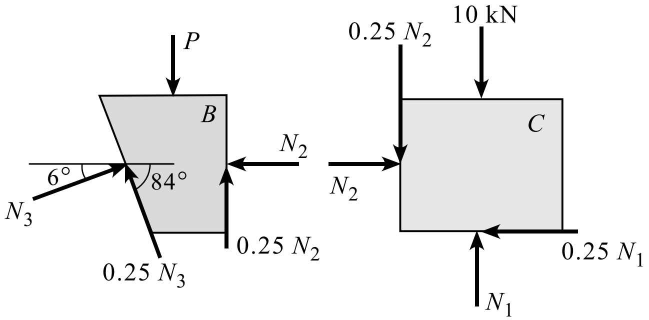


Fig. 8.20

Solution



FBD of Wedge *B* and Block *C*

(i) Consider the FBD of Block *C*

$\Sigma F_y = 0$

$N_1 - 10 - 0.25N_2 = 0$

$\therefore N_1 = 10 + 0.25N_2$

$\Sigma F_x = 0,$

$N_2 - 0.25N_1 = 0$

$N_2 - 0.25(10 + 0.25N_2) = 0$

$\therefore N_2 = 2666.7$

(ii) Consider the FBD of Wedge *B*

$\Sigma F_x = 0$

$N_3 \cos 6^\circ - N_2 - 0.25N_3 \cos 84^\circ = 0$

$0.9684N_3 = N_2$

$N_3 = 2.754$

$\Sigma F_y = 0$

$N_3 \sin 6^\circ + 0.25N_3 \sin 84^\circ + 0.25\ N_2 - P = 0$

$P = 1.639$ kN ***Ans.***

Problem 21

Determine the force P required to move the block A of weight 5000 N up the inclined plane. Coefficient of friction between all contact surfaces is 0.25. Neglect the weight of the wedge and the wedge angle is 15 degrees.

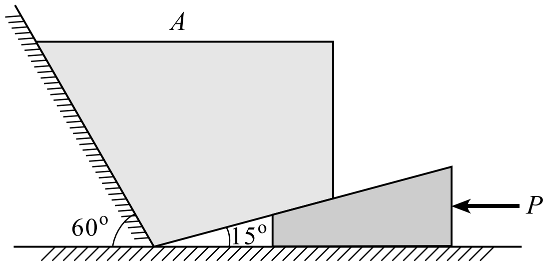


Fig. 8.21

Solution

(i) Consider the FBD of Block A

$$\Sigma F_x = 0$$

$$N_1 \cos 30^\circ + 0.25N_1 \cos 60^\circ - 0.25N_2 \cos 15^\circ - N_2 \cos 75^\circ = 0$$

$$0.860N_1 + 0.125N_1 - 0.241N_2 - 0.2588N_2 = 0$$

$$0.991N_1 = 0.499N_2$$

$$\therefore N_1 = 0.5165N_2$$

$$\Sigma F_y = 0$$

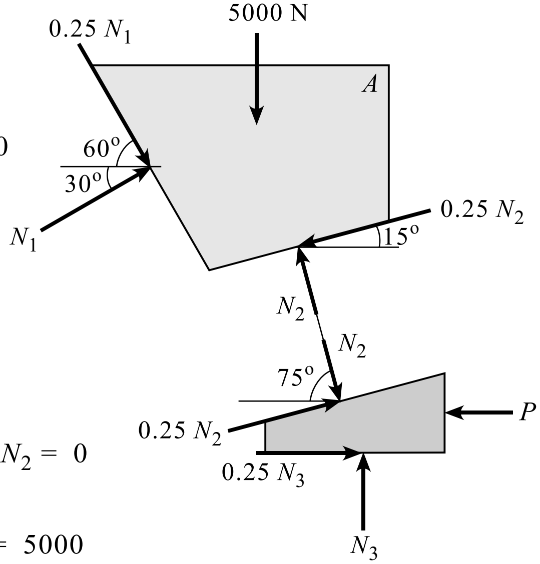
$$N_2 \sin 75^\circ + N_1 \sin 30^\circ - 5000 - 0.25N_1 \sin 60^\circ - 0.25N_2 \sin 15^\circ = 0$$

$$0.966N_2 + 0.5N_1 - 5000 - 0.2165N_1 - 0.0647N_2 = 0$$

Substituting the value of N_1

$$0.966N_2 + 0.2583N_2 - 0.1118N_2 - 0.0647N_2 = 5000$$

$$\therefore N_2 = 4772.3585 \text{ N}$$



FBD of Block A and Wedge

(ii) Consider the FBD of Wedge B

$$\Sigma F_y = 0$$

$$N_3 + 0.25N_2 \sin 15^\circ - N_2 \sin 75^\circ = 0$$

$$N_3 + 308.79 - 4609.744 = 0$$

$$N_3 = 4300.95 \text{ N}$$

$$\Sigma F_y = 0$$

$$0.25N_3 + 0.25N_2 \cos 15^\circ + N_2 \cos 75^\circ - P = 0$$

$$1075.23 + 1152.436 + 123.17 = P$$

$P = 3462.84$ N ***Ans.***

Ladder

Many a times, we come across the uses of ladder for reaching the higher height. Ladders are used by painters and carpenters who want to peg a nail in the wall for mounting a photo frame. We observe that care is taken to place the ladder at an appropriate angle with respect to ground and wall. We try to adjust the friction offered by the ground and wall in contact with ladder. Also, sometimes we prefer to hold the ladder by a person for safety purposes. The forces acting on the ladder are normal reactions; frictional forces between the ground, the wall and the ladder, weight of the ladder and the weight of the man climbing the ladder. Considering the free body diagram of ladder, we get general force system. The system may be simplified by considering that equilibrium condition can be worked out by following equations :

$$\Sigma F_x = 0 ; \quad \Sigma F_y = 0 \text{ and } \Sigma M = 0$$

Problem 22

A uniform ladder weighing 100 N and 5 meters long has lower end *B* resting on the ground and upper end *A* resting against a vertical wall as shown in Fig. 8.22. The inclination of the ladder with horizontal is 60°. If the coefficient of friction at all surfaces of contact is 0.25, determine how much distance (up along the ladder) a man weighing 600 N can ascent without causing it to slip.

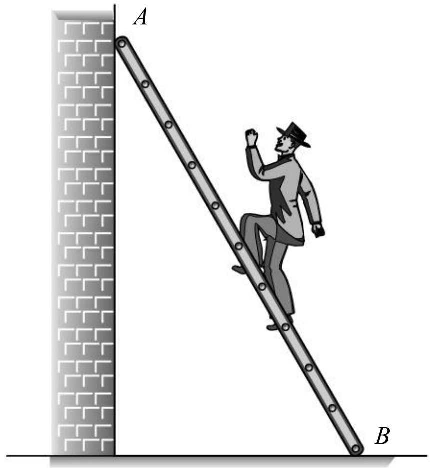


Fig. 8.22

Solution

Consider the FBD of the ladder

$$\Sigma F_x = 0$$

$$N_A - \mu N_B = 0$$

$$N_B = 4 N_A$$

$$\Sigma F_y = 0$$

$$\mu N_A + N_B - 100 - 600 = 0$$

$$0.25 N_A + 4 N_A = 700$$

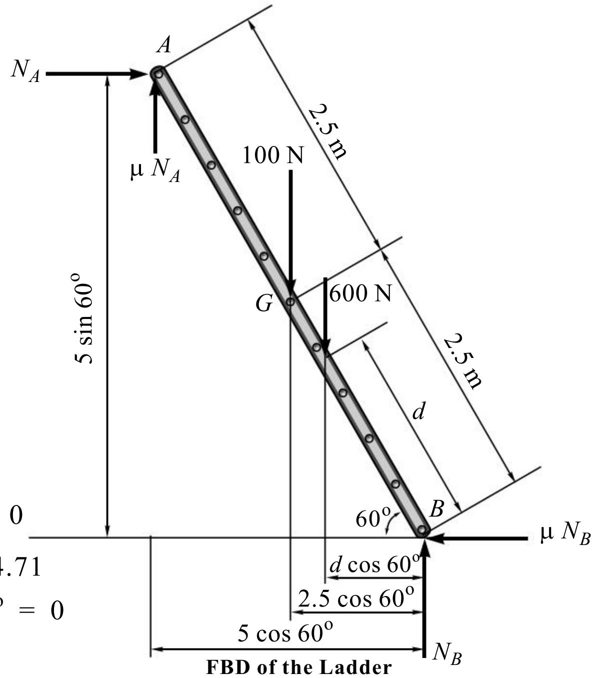
$$N_A = 164.71$$

$$\Sigma M_B = 0$$

$$100 \times 2.5 \cos 60^\circ + 600 \times d \cos 60^\circ - N_A \times 5 \sin 60^\circ - \mu N_A \times 5 \cos 60^\circ = 0$$

$$100 \times 2.5 \cos 60^\circ + 600 \times d \cos 60^\circ - 164.71 \times 5 \sin 60^\circ - 0.25 \times 164.71 \times 5 \cos 60^\circ = 0$$

$d = 2.304$ m ***Ans.***



FBD of the Ladder

Problem 23

A 100 N uniform rod AB is held in the position as shown in Fig. 8.23. If coefficient of friction is 0.15 at A and B. Calculate range of values of P for which equilibrium is maintained.

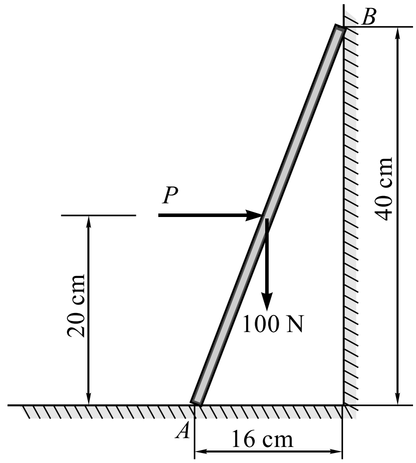


Fig. 8.23

Solution

Case I : For $P_{minimum}$

Consider FBD of rod AB when P is minimum and in limiting equilibrium condition the tendency of rod will be to slip in downward direction.

$$\Sigma F_x = 0$$

$$P_{min} + \mu N_A - N_B = 0$$

$$P_{min} = N_B - 0.15 N_A \qquad \text{... (I)}$$

$$\Sigma F_y = 0$$

$$N_A + \mu N_B - 100 = 0$$

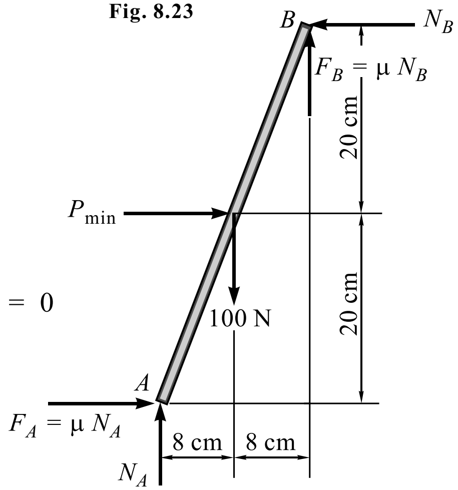
$$N_A + 0.15 N_B = 100 \qquad \text{... (II)}$$

$$\Sigma M_A = 0$$

$$\mu N_B \times 16 + N_B \times 40 - 100 \times 8 - P_{min} \times 20 = 0$$

$$0.15 N_B \times 16 + N_B \times 40 - 800 - (N_A - 0.15 N_A) \times 20 = 0$$

$$3 N_A + 22.4 N_B = 800 \qquad \text{... (III)}$$



FBD of Rod AB for P_{min}

Solving Eqs. (II) and (III),

$$N_A = 96.58 \text{ N}$$

$$N_B = 22.78 \text{ N}$$

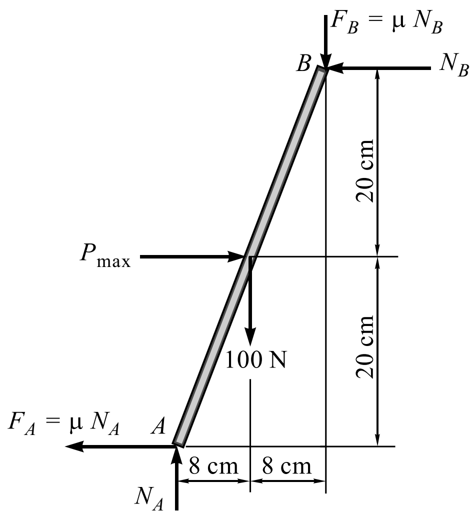
From Eq. (I),

$$P_{min} = 22.78 - 0.15 \times 96.58$$

$$P_{min} = 8.29 \text{ N} \quad \textit{Ans.}$$

Case II : For $P_{maximum}$

Consider FBD of rod AB when P is maximum and in limiting equilibrium condition the tendency of rod will be to slip in upward direction.



FBD of Rod AB for P_{max}

$$\Sigma F_x = 0$$

$$P_{max} - \mu N_A - N_B = 0$$

$$P_{max} = 0.15 N_A + N_B \qquad \text{... (IV)}$$

$$\Sigma F_y = 0$$

$$N_A - \mu N_B - 100 = 0$$

$$N_A - 0.15 N_B = 100 \qquad \text{... (V)}$$

$$\Sigma M_A = 0$$

$$N_B \times 40 - \mu N_B \times 16 - P_{\max} \times 20 - 100 \times 8 = 0$$

$$N_B \times 40 - 0.15 N_B \times 16 - (0.15 N_A + N_B) \times 20 - 800 = 0$$

$$17.6 N_B - 3 N_A = 800 \qquad \text{... (VI)}$$

Solving Eqs. (V) and (VI),

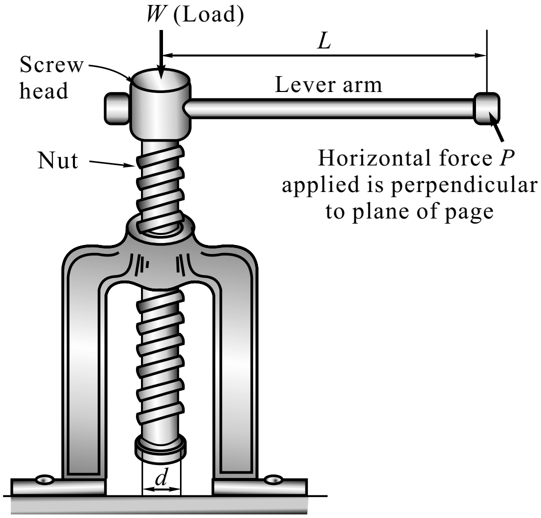
$$N_A = 109.62 \text{ N} \text{ and } N_B = 64.14 \text{ N}$$

From Eq. (IV),

$$P_{\max} = 80.58 \text{ N} \quad \textbf{\textit{Ans.}}$$

Screw Jack

It is a simple lifting machine, which is used for lifting heavy loads and its principle is same as that of inclined plane.



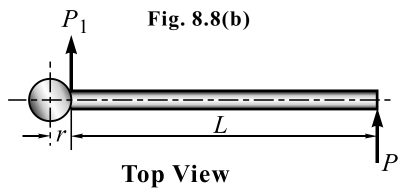
Front View
Fig. 8.8(b)

The machine consist of a screw and nut. The screw head carries the load *W*. The nut is an integral part of the screw jack. The screw is rotated by means of a lever at the end of which effort *P* is applied [Fig. 8.8(b)].

Lead : The axial advancement of screw when it completes one revolution is called *lead*.

Pitch : The distance between consecutive threads is called *pitch*. If the screw is single threaded, then lead of the screw is equal to the pitch. If the screw is double threaded then lead of the screw is twice the pitch, and so on.

Let *L* be the length of lever and *r* be the mean radius of the screw [see Fig. 8.8(c)].



Top View
Fig. 8.8(c)

If an effort *P* is applied at the end of lever, it is equivalent to an effort P_1 applied at the surface of the screw and is given by

$$P_1 \times r = P \times L$$

$$P = \frac{r}{L} P_1$$

Now, consider one complete revolution of the lever. The load *W* is lifted up by a distance *p* (pitch) equal to the lead of the screw (considering single threaded screw).

This can be compared to the case of an inclined plane on which a load *W* is moved up by a horizontal force P_1.

The inclination of this inclined plane with the horizontal is given by θ [Fig. 8.8(d)].

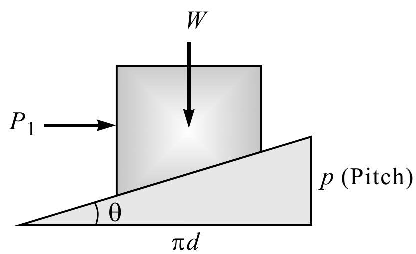
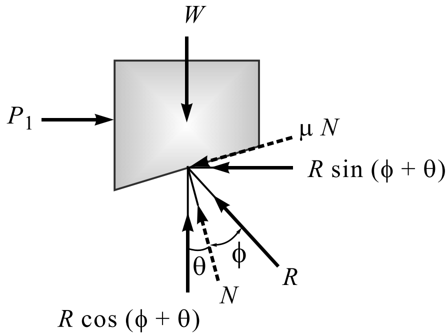


Fig. 8.8(d)

Consider FBD of Block with Load W



FBD of Block

$$\Sigma F_x = 0$$

$$P_1 = R \sin (\phi + \theta) \qquad \text{... (I)}$$

$$\Sigma F_y = 0$$

$$W = R \cos (\phi + \theta) \qquad \text{... (II)}$$

Dividing equation (I) by (II) gives

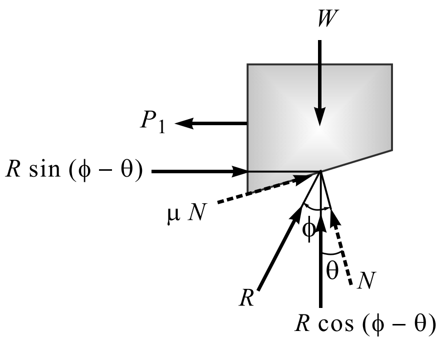
$$\frac{P_1}{W} = \tan (\phi + \theta)$$

or $P_1 = W \tan (\phi + \theta)$

From equation (I)

$$P_{(\text{Lift})} = \frac{r}{L} \times W \tan (\phi + \theta)$$

Consider FBD of Block with Load W



FBD of Block

$$\Sigma F_x = 0$$

$$P_1 = R \sin (\phi - \theta) \qquad \text{... (III)}$$

$$\Sigma F_y = 0$$

$$W = R \cos (\phi - \theta) \qquad \text{... (IV)}$$

Dividing equation (III) by (IV) gives

$$P_1 = W \tan (\phi - \theta)$$

But

$$P = \frac{r}{L} P_1$$

$$P_{(\text{Lower})} = \frac{r}{L} \times W \tan (\phi - \theta)$$

where P is the effort required to lower the load.

Note : If the load W lifted by the screw jack moves in the reverse direction on removal of effort P, it is called as *reversible* otherwise *self locking*. Exceptionally, if the screw jack is reversible, some effort is necessary to apply to the jack to prevent the load from moving down.

Check for reversibility of screw jack

If $\phi > \theta$, screw jack is self locking

$\phi < \theta$, screw jack is reversible

$\phi = \theta$, screw jack is at the point of reversing or self locking

For maximum size of pitch if screw jack is self-locking, then $\phi = \theta$.

Differential Screw Jack

Compared to simple screw jack, differential screw jack has advantages and more efficiency.

The arrangement is with two screws, screw inside the main screw. Main screw passes through the nut which is fixed to the body. Another screw runs inside the main screw, which means main screw is having external as well as internal threading. External threading matches with fixed nut of body and smaller screws external thread matches with internal threading on main screw.

A special lever mechanism is used with attached movable nut. When movable nut is rotated with the help of lever, main screw rotates. Rotation of inner smaller screw is prevented by special mechanism.

Consider D and d, the mean diameter of the main screw and smaller screw respectively. Let P and p be the pitch of main screw and smaller screw respectively.

For one complete rotation of lever, the axial advancement of main screw will be P, whereas simultaneously smaller screw will move axially in reverse direction by p. Since P is greater than p, therefore the net axial advancement of lifting of load will be $P - p$.

If effort is applied at the end of lever with radial length L from the axis of screw the velocity ratio is given by the relation

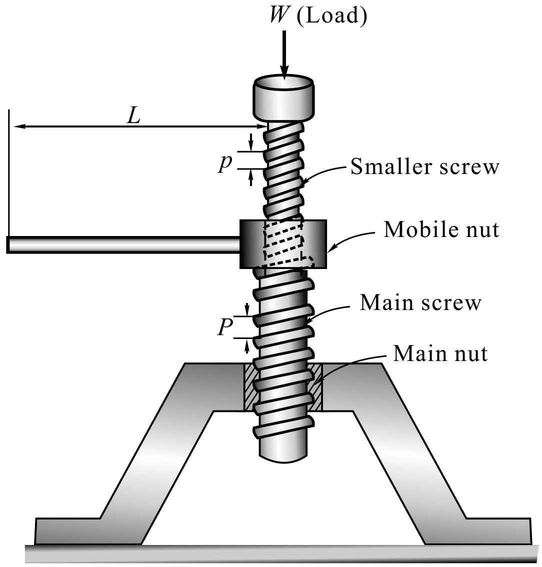


Fig. 8.8(iii)

$$VR = \frac{2\pi L}{P - p}$$

$$\text{Velocity Ratio } VR = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}$$

$$\text{Mechanical Advantage } MA = \frac{\text{Load}}{\text{Effort}}$$

$$\text{Efficiency} = \frac{MA}{VR}$$

Problem 24

The screw press, shown in Fig. 8.24, is used in book binding. The screw bas a mean radius of 10 mm and its pitch is 5 mm. The static coefficient of friction between the threads is 0.18. If a clamping force of 1000 N is applied to the book, determine **(1)** The torque that was applied to the handle of the press. **(2)** The torque required to loosen the press.

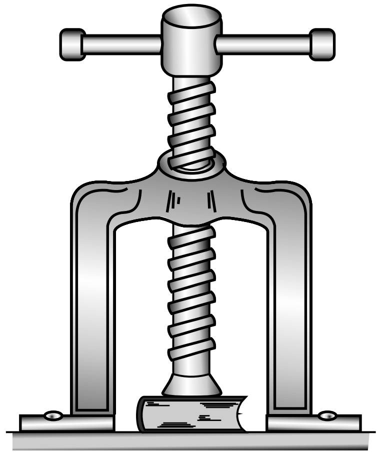


Fig. 8.24

Solution

The lead angle of the screw is

$$\theta = \tan^{-1}\frac{p}{2\pi r} = \tan^{-1}\frac{5}{2\pi \times 10} = 4.550^\circ$$

The friction angle is

$$\phi_S = \tan^{-1}\mu_S = \tan^{-1} 0.18 = 10.204^\circ$$

(1) Torque required to apply the force $W = 1000$ N

$$C_0 = W\,r\tan(\phi_S + \theta)$$
$$= 1000\,(0.01)\tan(10.204^\circ + 4.550^\circ)$$
$$= 2.63 \text{ N-m}$$ ***Ans.***

(2) Torque needed to loosen the press

$$C_0 = W\,r\tan(\phi_S - \theta)$$
$$= 1000\,(0.01)\tan(10.204^\circ - 4.550^\circ)$$
$$= 0.990 \text{ N-m}$$ ***Ans.***

Problem 25

A screw jack has square threads with a mean radius 38 mm and pitch of 15 mm. Consider $\mu = 0.06$. If the lever is of length 0.4 m find the force that has to be applied at the end of the lever to lift up a load of 8 kN. Is the screw jack self locking ? If not, find the force required at the end of the lever to prevent the load from descending.

Solution

Given : $W = 8000$ N, $\mu = 0.06$, $l = 0.4$ m,

pitch $p = 15$ mm, $r = 38$ mm.

$\therefore$ Helix angle, $\theta = \tan^{-1}\dfrac{p}{2\pi r} = \tan^{-1}\dfrac{15}{2\pi \times 38} = 3.59^\circ$

Angle of friction,

$$\phi = \tan^{-1}\mu = \tan^{-1} 0.06 = 3.43^\circ$$

Since $\theta > \phi$, the screw jack is 'not self locking'.

$$P_{\text{lift}} = \frac{W\,r}{l} \tan(\phi + \theta)$$

$$= \frac{8000 \times 0.038}{0.4} \tan(3.43^{\circ} + 3.59^{\circ}) = 93.58 \text{ N} \quad \textbf{\textit{Ans.}}$$

To prevent the load from descending,

$$P = \frac{W\,r}{l} \tan(\theta - \phi)$$

$$= \frac{8000 \times 0.038}{0.4} \tan(3.59^{\circ} - 3.43^{\circ}) = 2.12 \text{ N} \quad \textbf{\textit{Ans.}}$$

Problem 26

A screw jack with single start square threads has outside and inside diameter of the thread as 68 mm and 52 mm respectively. The coefficient of friction is 0.1 for all pairs of surfaces in contact. If the length of the lever is 0.5 m, find the force required to lift a load of 2 kN.

Solution

Given : $W = 2000$ N, $\mu = 0.1$, $l = 0.5$ m

For the screw jack, $d_0 = 68$ mm and $d_i = 52$ mm

For square threads,

$$\text{Mean diameter, } d = \frac{d_0 + d_i}{2} = \frac{68 + 52}{2} = 60 \text{ mm}$$

$\therefore$ Mean radius, $r = 30$ mm

Pitch $= d_0 - d_i = 68 - 52 = 16$ mm

$$\therefore \text{ Helix angle, } \theta = \tan^{-1} \frac{p}{2\pi\,r} = \tan^{-1} \frac{16}{2\pi \times 30} = 4.859^{\circ}$$

Angle of friction,

$$\phi = \tan^{-1} \mu = \tan^{-1} 0.1 = 5.713^{\circ}$$

Since $\phi > \theta$, the screw jack is 'self locking'.

$$P_{\text{lift}} = \frac{W\,r}{l} \tan(\phi + \theta)$$

$$= \frac{2000 \times 0.03}{0.5} \tan(5.71^{\circ} + 4.85^{\circ}) = 22.37 \text{ N} \quad \textbf{\textit{Ans.}}$$

Problem 27

A screw jack has threads with mean diameter 58 cm. Considering the coefficient of friction between the screw and the nut to be 0.12, find the maximum pitch of the thread so that the screw jack is self locking.

Solution

Given : $r = 29$ cm, $\mu = 0.12$

For a self locking screw jack $\phi > \theta$

$$\therefore \tan^{-1} \mu > \tan^{-1} \frac{p}{2\pi\,r}$$

For the range of values of μ commonly encountered,
$y = \tan^{-1} x$ is an increasing function.

$$\therefore \quad \mu > \frac{p}{2\pi r}$$

$$\therefore \quad 0.12 > \frac{p}{2\pi \times 30}$$

$$\therefore \quad 0.12 > p \text{ (Pitch)}$$

Hence for being self-locking, the maximum permissible pitch is 21.87 cm.

Exercises

[I] Problems

1. A wooden block rests on a horizontal plane, as shown in Fig. 8.E1. Determine the force P required to just impend motion. Assume the weight of block as 100 N and the coefficient of friction $\mu = 0.4$.

$\left[\textit{\textbf{Ans.}}\ \ P = 37.4 \text{ N}\right]$

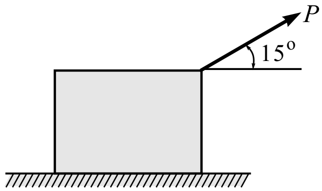


Fig. 8.E1

2. A 100 N force acts, as shown in Fig. 8.E2, on a 30.6 kg block on a inclined plane. The coefficient of friction (static and kinetic) between the block and the plane are $\mu_s = 0.25$ and $\mu_k = 0.20$, respectively. Determine whether the block is in equilibrium and find the value of the friction force. Take $g = 9.81$ m/sec^2.

$\left[\textit{\textbf{Ans.}}\ \ F = 48 \text{ N, block will slide down.}\right]$

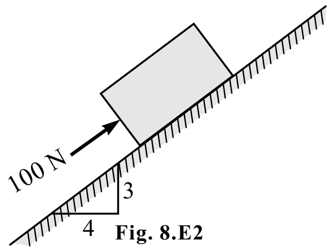


Fig. 8.E2

3. Block of weight 1000 N is kept on an inclined plane as shown in Fig. 8.E3. A force P is applied parallel to plane to keep the body in equilibrium. Determine range of values of P for which the block will be in equilibrium.

$\left[\textit{\textbf{Ans.}}\ \ 370.1 \le P \le 629.9 \text{ N}\right]$

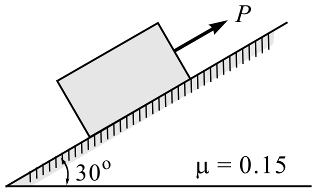


Fig. 8.E3

4. Block A of weight 2000 N is kept on an inclined plane at 35° as shown in Fig. 8.E4. It is connected to weight B by an inextensible string passing over smooth pulley. Determine the weight of B so that B just moves down. $\mu = 0.2$.

$\left[\textit{\textbf{Ans.}}\ \ 1463.1 \text{ N}\right]$

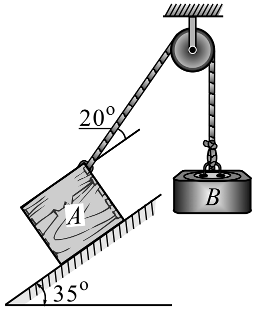


Fig. 8.E4

5. In Fig. 8.E5, the two blocks (W_1 = 30 N and W_2 = 50 N) are placed on rough horizontal plane. Coefficient of friction between the block A and plane is 0.3 and that between bock B and plane is 0.2. Find the minimum value of the force P to just move the system. Also find the tension in the string.

[**Ans.** P = 19.67 N and T = 9 N.]

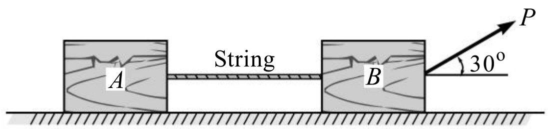


Fig. 8.E5

6. In Fig. 8.E6, weights of two blocks A and B are 100 N and 150 N respectively. Find the smallest value of the horizontal force F to just move the lower block B if

(a) the block is restrained by a string and

(b) when string is removed.

[**Ans.** (a) 82.5 N and (b) 62.5 N.]

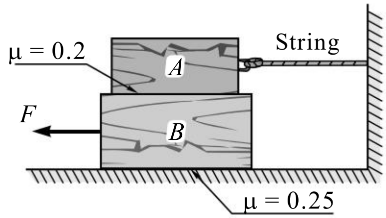


Fig. 8.E6

7. Determine the necessary force P acting parallel to the plane to cause motion to impend, as shown in Fig. 8.E7. Assume coefficient of friction as 0.25 and the pulley to be smooth.

[**Ans.** P = 98.85 N]

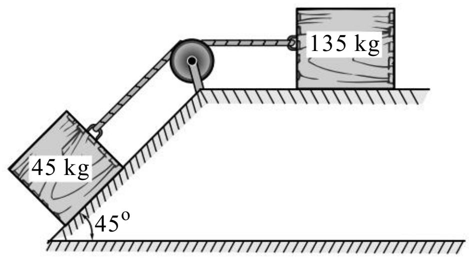


Fig. 8.E7

8. A weight 500 N just starts moving down a rough inclined plane supported by a force of 200 N acting parallel to the plane and it is at the point of moving up the plane when pulled by a force of 300 N parallel to the plane. Find the inclination of the plane and the coefficient of friction between the inclined plane and the weight.

[**Ans.** 30° and 0.1155.]

9. A cord connects two bodies A and B of weights 450 N and 900 N. The two bodies are laced on an inclined plane and the cord is parallel to inclined plane. The coefficient of friction for body - A is 0.16 and that for B is 0.42. Determine the inclination of the plane to the horizontal and tension in the cord when motion is about to take place down the plane.

[**Ans.** $\theta = 18.434^\circ$ and T = 73.98 N.]

10. Two rectangular blocks of weights W_1 and W_2 are connected by a flexible cord and rest upon a horizontal and inclined plane respectively with the cord passing as shown in Fig. 8.E10. Taking a particular case, where $W_1 = W_2$ and coefficient of friction μ is same for all contact surfaces, find the angle of inclination of the inclined plane at which motion of the system will impend.

[**Ans.** $\alpha = 2 \tan^{-1} \mu$]

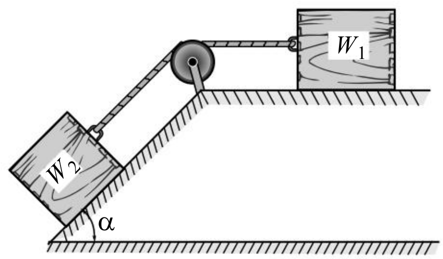


Fig. 8.E10

11. Two inclined planes AC and BC inclined at 60^o and 30^o to the horizontal meet at a ridge C, as shown in Fig. 8.E11. A mass of 100 kg rests on the inclined plane BC and is tied to a rope, which passes over a smooth pulley at the ridge, the other end of the rope, being connected to a block of W kg mass resting on the plane AC. Determine the least and greatest value of W for the equilibrium of the whole system.

[**Ans.** $W_{min} = 243.88$ N and $W_{max} = 973.05$ N.]

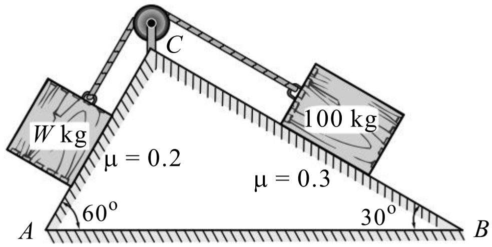


Fig. 8.E11

12. Find the tensions in the cords of the inclined plane system shown in Fig. 8.E12.

[**Ans.** $T_1 = 165.36$ kN and $T_2 = 80.51$ kN.]

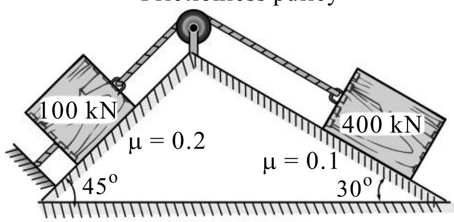


Fig. 8.E12

13. A 2.04 kg block A and a 3.06 kg block B are supported by an inclined plane, which is held in position shown in Fig. 8.E13. Knowing that the coefficient of friction is 0.15 between the two blocks and zero between block B and the incline, determine the value of θ for which motion is impending.

[**Ans.** $\theta = 31^o$]

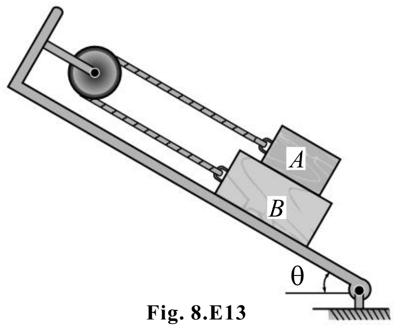


Fig. 8.E13

14. Find the least value of P that will just start the system of blocks shown in Fig. 8.E14. moving to the right. The coefficient of friction under each block is 0.30.

[**Ans.** $P_{min} = 247.12$ N at $\alpha = 16.7^o$]

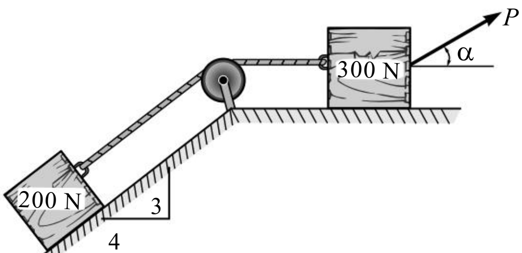


Fig. 8.E14

15. Find the weight W_B if the weight $W_A = 20$ kN is to be kept in equilibrium with pin-connected rod AB in horizontal position. Find also maximum value of W_B for the same purpose. Find, therefore, the range of values of axial force in the rod AB. Refer to Fig. 8.E15.

[**Ans.** $W_{B(min)} = 4.511$ kN,
$W_{B(max)} = 26.365$ kN and
6.766 kN (C) to 17.577 kN (C).]

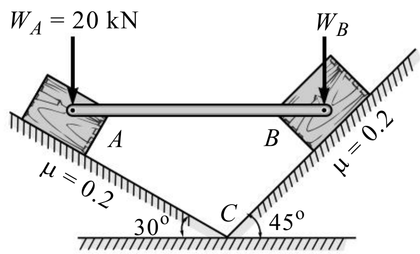


Fig. 8.E15

16. Two blocks *A* and *B* weighing 800 N and 1000 N, respectively rest on two inclined planes, each inclined at 30° to the horizontal. They are connected by a rope passing through a smooth pulley at the valley.

Ropes carrying loads W_1 and 5000 N (W_2) and passing over pulleys at the tops of the planes are also connected to the two blocks, as shown in Fig. 8.E16. Coefficient of friction μ may be taken as 0.1 and 0.2 for blocks *A* and *B*, respectively. Determine the least and greatest value of W_1 for the equilibrium of the whole system.

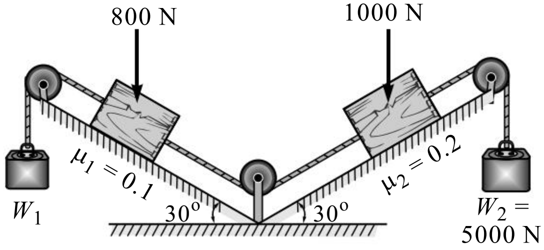


Fig. 8.E16

[***Ans.*** $W_{1(min)}$ = 4657.52 N and $W_{1(max)}$ = 5142.48 N.]

17. Two slender rods of negligible weight are pin-connected at *A* and attached to the 200 N block *B* and the 600 N block *C*, as shown. The coefficient of static friction is 0.6 between all surfaces of contact. Determine the range of values of *P* for which equilibrium is maintained. Refer to Fig. 8.E17.

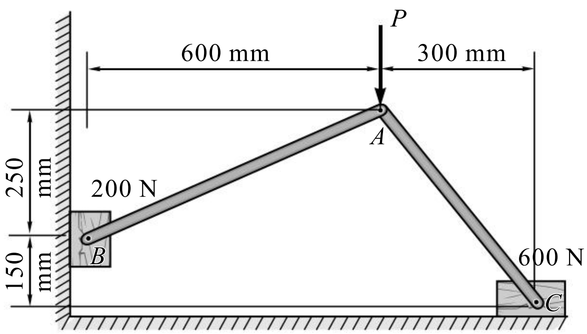


Fig. 8.E17

[***Ans.*** 1913.9 N to 3150 N]

18. Determine the range of values of *P* for which equilibrium of the block, shown in Fig. 8.E18, is maintained ($\mu_S = 0.25$, $\mu_K = 0.2$).

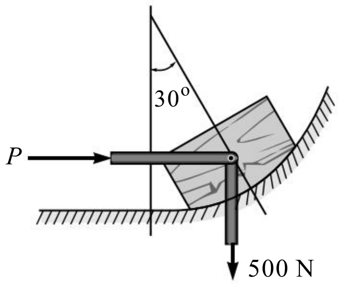


Fig. 8.E18

[***Ans.*** $143.03 \leq P \leq 483.46$ N]

19. Block *A* has a mass of 20 kg and block *B* has a mass of 10 kg in Fig. 8.E19. Knowing that μ_s = 0.15 between all surfaces of contact, determine the value of θ for which motion will impend. Take $g = 10$ m/s^2.

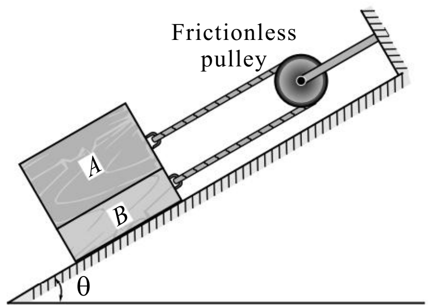


Fig. 8.E19

[***Ans.*** θ = 46.4°]

20. Find the least force P to start motion of any part of the system of three blocks resting upon one another, as shown in Fig. 8.E20. The weights of the blocks are W_A = 300 N, W_B = 100 N and W_C = 200 N. The coefficient of friction between A and B is 0.3, between B and C is 0.2 and between C and the ground is 0.1.

[***Ans.*** 57.143 N]

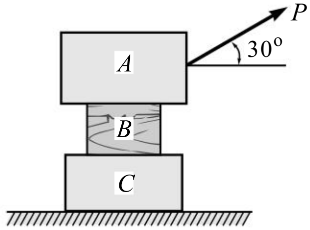


Fig. 8.E20

21. The three flat blocks are positioned on the 30° incline, as shown in Fig. 8.E21, and a force P parallel to the incline is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The coefficient of static friction for each of the three pairs of surfaces is shown. Determine the maximum value which P may have before any slipping takes place. Take g = 10 m/s^2.

[***Ans.*** P = 95.58 N]

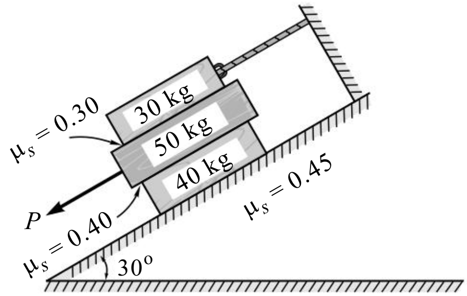


Fig. 8.E21

22. Two rectangular blocks of weight, W_1 = 150 N and W_2 = 100 N, are connected by a string and rest on an inclined plane and on a horizontal surface as shown in Fig. 8.E22. The coefficient of friction for all contiguous surfaces is μ = 0.2. Find the magnitude and direction of the least force P at which the motion of the blocks will impend.

[***Ans.*** P = 161.7 N and θ = 11.31°.]

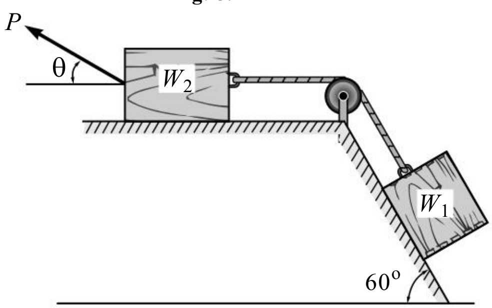


Fig. 8.E22

23. A 1500 N cupboard is to be shifted to the right by a horizontal force P, as shown in Fig. 8.E23. Find the force P required to just cause the motion and the maximum height up to which it can be applied. Take μ = 0.25.

[***Ans.*** P = 375 N and h = 1.75 m.]

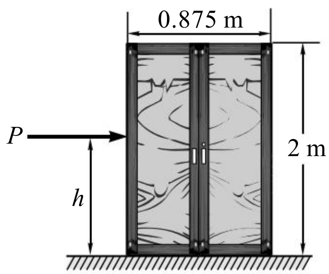


Fig. 8.E23

24. Two block A and B each weighing 1500 N are connected by a uniform horizontal bar which weighs 1000 N, as shown in Fig. 8.E24. If the angle of limiting friction under each block is 15°, find the force P directed parallel to the 60° inclined plane that will cause motion impending to the right.

[***Ans.*** P = 1856.4 N]

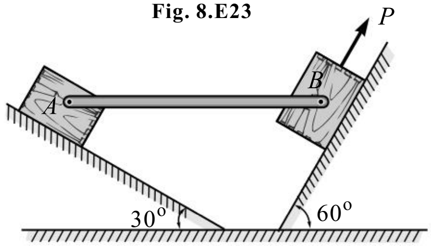


Fig. 8.E24

25. Refer to Fig. 8.E25, where a 8.15 kg block is attached to link *AB* and rests on a moving belt. Knowing that $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the magnitude of the horizontal force *P* which should be applied to the belt to maintain its motion (a) to the right, (b) and to the left.

[**Ans.** (a) $P = 18.08$ N and (b) $P = 14.34$ N.]

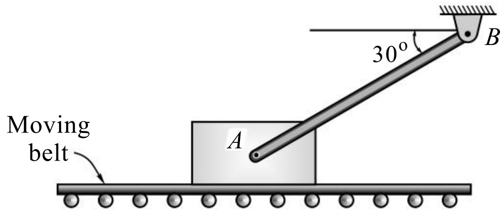


Fig. 8.E25

26. Block *A* has a mass of 20 kg and block *B* has a mass of 10 kg, as shown in Fig. 8.E26. Coefficient of static friction between the blocks is 0.15 and between the block *B* with the slope is zero. Find the existing frictional force between the blocks. What is the force in the string ?

[**Ans.** $T = 75$ N and $F = 25$ N.]

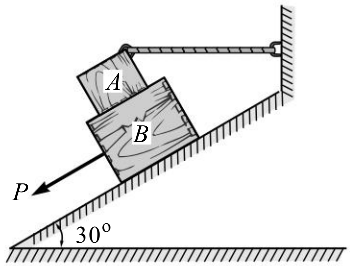


Fig. 8.E26

27. Refer to Fig. 8.E27, where a 45 kg disk rests on the surface for which the coefficient of static friction is $\mu = 0.2$, determine the largest couple moment *M* that can be applied to the bar without causing motion.

[**Ans.** $M = 77.3$ Nm]

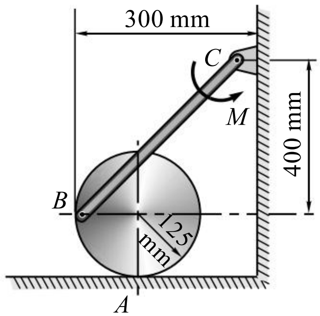


Fig. 8.E27

28. A uniform rod *AB* of length 10 m and weight 280 N is hinged at *B* and end *A* rests on a block weighing 400 N, as shown in Fig. 8.E28. If $\mu = 0.4$ for all contact surfaces, find horizontal force *P* required to start moving 400 N block.

[**Ans.** $P = 320$ N]

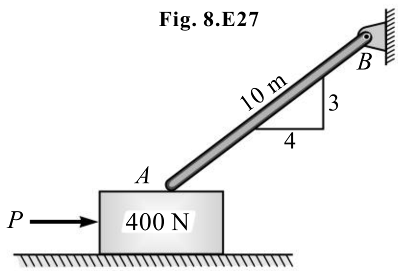


Fig. 8.E28

29. The beam *AB* has a negligible mass and thickness and is subjected to a triangular distributed loading. It is supported at one end by a pin and at the other end by a post having a mass of 50 kg and negligible thickness, as shown in Fig. 8.E29. Determine the minimum force *P* needed to move the post. The coefficients of static friction at *B* and *C* are $\mu_B = 0.4$ and $\mu_C = 0.2$, respectively.

[**Ans.** $P = 355$ N]

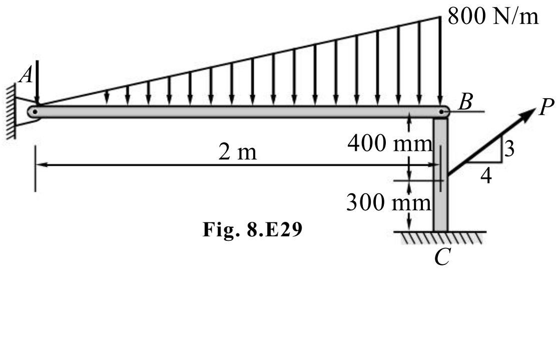


Fig. 8.E29

30. A 60 kg cupboard is to be shifted to the right, μ_s between cupboard and floor is 0.35, as shown in Fig. 8.E30. Determine

(a) the force P required to move the cupboard and

(b) The largest allowable value of h if the cupboard is not to tip over.

[**Ans.** P = 206 N and h = 714 mm.]

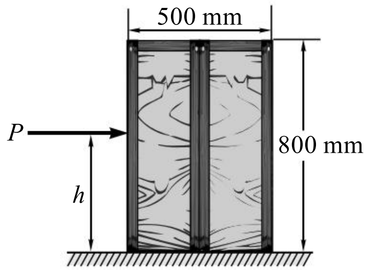


Fig. 8.E30

31. A cupboard of 750 N weight is placed over an inclined plane with μ = 0.20, as shown in Fig. 8.E31. Find the range of values of h where force P may be applied parallel to inclined plane to hold it in equilibrium.

[**Ans.** 0.213 m and 2.014 m.]

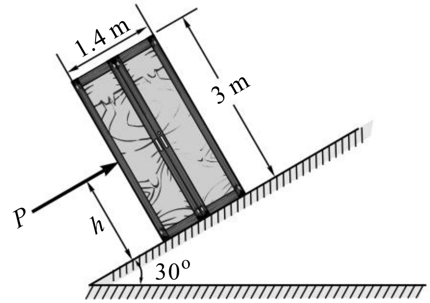


Fig. 8.E31

32. Referring to Fig. 8.E32, the coefficients of friction are as follows : 0.25 at the floor, 0.3 at the wall and 0.2 between the blocks. Find the minimum values of a horizontal force P, applied to the lower block that will hold the system in equilibrium.

[**Ans.** P = 81.02 N]

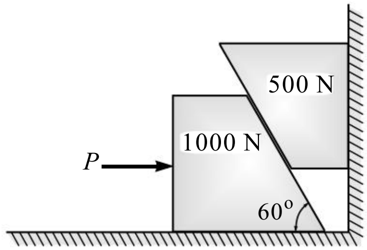


Fig. 8.E32

33. Block A weighs 25 kN and block B 18 kN in Fig. 8.E33. μ for all surfaces is 0.11. For what range of values of P will the system be in equilibrium.

[**Ans.** P_{min} = 45.8 kN and P_{max} = 124 kN.]

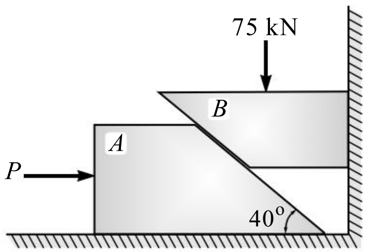


Fig. 8.E33

34. Refer to Fig. 8.E34 and draw the FBD for different bodies and find the minimum value of force F to move the block A up the plane.

[**Ans.** 134.6 N]

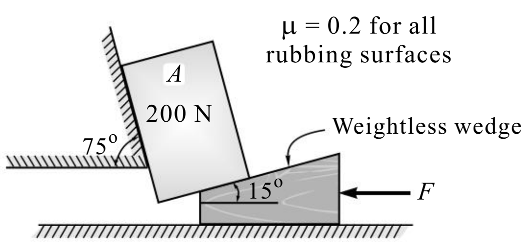


Fig. 8.E34

35. Determine the force P required to start the motion of wedge shown in Fig. 8.E35.

Take $\mu = 0.26$ for all surfaces.

[*Ans.* $P = 1464.33$ N]

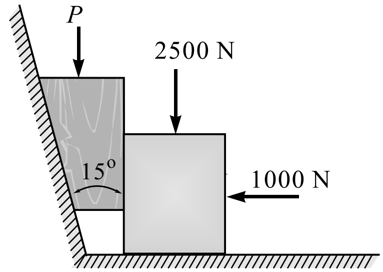


Fig. 8.E35

36. Calculate the force P required to initiate the motion of the 24 kg block up the 10° incline, as shown in Fig. 8.E36. The coefficient of static friction for each pair of surfaces is 0.3. Assume $g = 10$ m/s^2.

[*Ans.* $P = 224.36$ N]

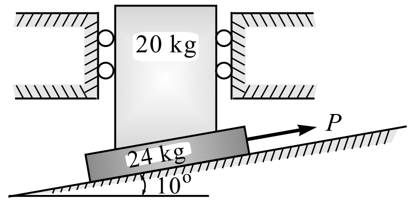


Fig. 8.E36

37. What horizontal force P on the wedge B and C is necessary to raise 200 kN resting on A, as shown in Fig. 8.E37? Assume that coefficient of friction μ between the wedges and the ground is 0.25 and between wedges and A is 0.2. Also assume symmetry.

[*Ans.* $P = 55.665$ N]

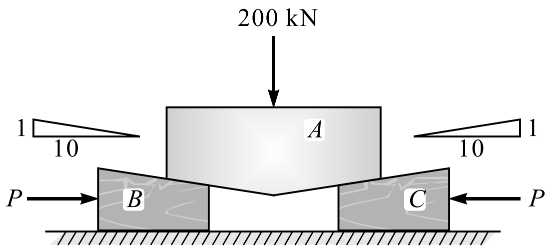


Fig. 8.E37

38. A horizontal force of 5 kN is acting on the wedge, as shown in Fig. 8.E38. The coefficient of friction at all rubbing surfaces is 0.25. Find the load W which can be held in position. The weight of block B may be neglected.

[*Ans.* $W = 22.89$ kN]

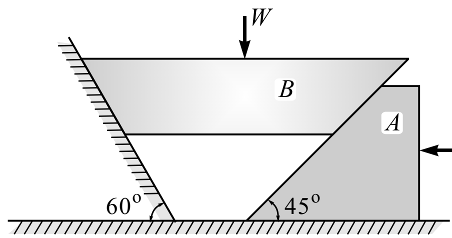


Fig. 8.E38

39. A 15° wedge of negligible weight is driven to tighten a body B which is supporting a vertical load of 1000 N, as shown in Fig. 8.E39. If the coefficient of friction for all contacting surfaces be 0.25, find the minimum force P required to drive the wedge.

[*Ans.* $P = 232$ N]

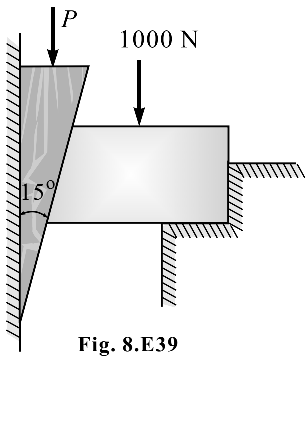


Fig. 8.E39

40. A ladder of length 4 m weighing 200 N is placed against a vertical wall, as shown in Fig. 8.E40. The coefficient of friction between the wall and the ladder is 0.2 and that between the ladder and the floor is 0.3. The ladder in addition to its own weight has to support a man weighing 600 N at a distance of 3 m from A. Calculate the minimum horizontal force to be applied at A to prevent slipping.

[**Ans.** $P = 61.76$ N]

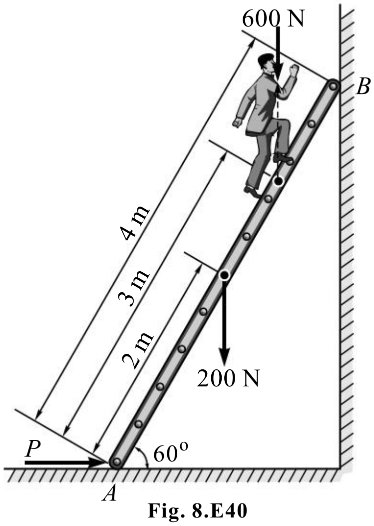


Fig. 8.E40

41. The ladder shown in Fig. 8.E41 is 6 m long and is supported by a horizontal floor and vertical wall. The coefficient of friction between the floor and the ladder is 0.4 and between wall and ladder is 0.25. The weight of ladder is 200 N and may be considered a concentrated at G. The ladder also supports a vertical load of 900 N at C, which is at a distance of 1 m from B. Determine the least value of α at which the ladder may be placed without slipping. Determine the reaction at that

[**Ans.** $N_A = 1000$ N, $F_A = 400$ N,
$N_B = 400$ N, $F_B = 100$ N and $\alpha = 61.927°$.]

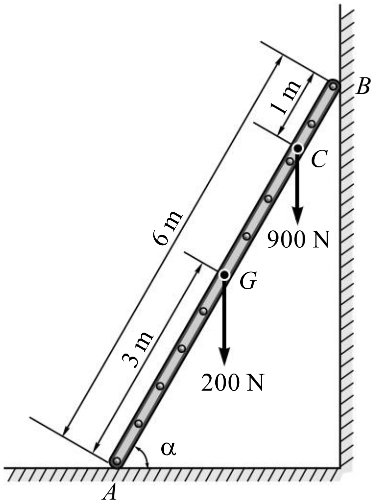


Fig. 8.E41

42. A horizontal bar 1 m long and of negligible weight rests on rough inclined plane, as shown in Fig. 8.E42. If the angle of friction is 15°, determine the minimum value of x at which the load $Q = 200$ N may be applied before slipping occurs.

[**Ans.** $x = 0.35$ m]

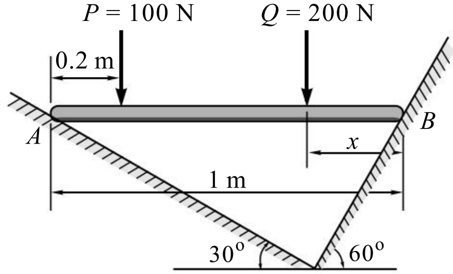


Fig. 8.E42

43. The pitch of the screw of a jack is 10 mm and mean diameter of the thread is 60 mm and the length of the lever is 500 mm. Find the effort required **(i)** to lift a load of 10 kN **(ii)** to lower the same load. Take $\mu = 0.08$.

[***Ans.*** **(i)** 80.17 N **(ii)** 16.1 N]

44. A screw jack carries a load of 4000 N. The mean diameter of the screw rod is 50 mm and the pitch of the square threads is 20 mm. If the coefficient of friction is 0.22, find the torque required, **(i)** to raise the load **(ii)** to lower the load.

[***Ans.*** **(i)** 35.73 Nm **(ii)** 9.015 Nm]

45. A square threaded screw press is used in book binding for compressing the books. The pitch of the screw is 5 mm and the mean diameter is 50 mm. The screw is double threaded. If a force of 100 N is applied horizontally, at the end of lever, 150 mm long, find the compressive force with which the books are pressed. Take $\mu_s = 0.08$.

[***Ans.*** 4155 N]

46. A screw jack has square threads with 75 mm mean diameter and 15 mm pitch. The load on the jack revolves with the screw. The coefficient of friction at the screw threads is 0.05.

Find the tangential force applied to the jack at 360 mm radius so as to lift a load of 6000 N.

State whether the jack is self locking. If it is, find the torque required to lower the load. If not, find the torque which must be applied to keep the load from descending.

[***Ans.*** 71.265 N, machine is not self locking, 30 64 Nmm]

[II] Review Questions

1. What is meant by frictional force ?

2. State the laws of friction.

3. Describe the following terms :

(a) Limiting friction (b) Static friction (c) Dynamic friction
(d) Coefficient of friction (e) Angle of friction (f) Angle of repose
(g) Cone of friction

4. Why is the coefficient of static friction greater than the coefficient of kinetic friction ?

5. Discuss the merits and demerits of friction.

6. What is the use of wedge ?

[III] Fill in the Blanks

1. A wedge is generally used to lift a heavy load by a ________ distance.

2. The inverted cone with semicentral angle equal to the limiting frictional angle ϕ is called the ________ .

3. The minimum angle of inclination of a plane with horizontal at which the body is about to slide on its own is called the ________.

[IV] Multiple-choice Questions

Select the appropriate answer from the given options.

1. The cause of friction between two surfaces is _________.
(a) material **(b)** roughness **(c)** material and roughness **(d)** None of these

2. Limiting frictional force is directly proportional to _________.
(a) weight **(b)** mass **(c)** area **(d)** normal reaction

3. Frictional force is independent of the _______.
(a) only area **(b)** only speed **(c)** area and speed **(d)** None of these

4. Coefficient of static friction is always ________ than the coefficient of kinetic friction.
(a) greater **(b)** less **(c)** equal **(d)** zero

5. Angle of repose is ________ to angle of friction.
(a) less **(b)** equal **(c)** greater **(d)** zero

6. Angle made by the resultant of the limiting frictional force and the normal reaction with normal reaction is called the ________.
(a) angle of friction **(b)** angle of repose
(c) angle of inclination **(d)** angle of limiting friction

7. If the inclination of the plane with horizontal is less than angle of friction then the block kept on the incline will ________ .
(a) move downward **(b)** move upward **(c)** be in equilibrium **(d)** be in motion