3.2 Equation of AM Wave

Let the equation of modulating signal be

$$v_m = V_m \sin \omega_m t$$

Let the equation of carrier signal be

$$v_c = V_c \sin \omega_c t$$

Now, when this carrier is amplitude modulated,

- Frequency remains same
- Phase remains same
- Amplitude changes in accordance with modulating signal

The general equation of the AM wave is

$$v_{AM} = V \sin \omega_c t \quad \text{..... (i)}$$

where, V = Peak voltage and

$V \propto V_c + v_m \quad \therefore$ (Amplitude changes in accordance with modulation signal)

$v = V_c + V_m \sin \omega_m t$ (Assuming constant of proportionality to be one)

Substituting in (i)

$$v_{AM} = (V_c + V_m \sin \omega_m t) \sin \omega_c t$$

$$\therefore \; v_{AM} = V_c \sin \omega_c t + V_m(\sin \omega_c t \sin \omega_m t)$$

$$\therefore \; v_{AM} = V_c \sin \omega_c t + \frac{V_m}{2} [2 \sin \omega_c t \cdot \sin \omega_m t]$$

$$v_{AM} = V_c \sin \omega_c t + V_m [\cos (\omega_c - \omega_m)t - \cos (\omega_c + \omega_m)t]$$

$[\therefore \; 2 \sin A \sin B = \cos(A - B) \cos (A + B)]$

Taking V_c common we get,

$$v_{AM} = V_c\left\{ \sin \omega_c t + \frac{V_m}{2V_c} [\cos (\omega_c - \omega_m)t - \cos (\omega_c + \omega_m)t] \right\} \quad \text{..... (ii)}$$

Now, we define a new term

$$V_m/V_c = m_a \text{, modulation index}$$

∴ Equation (ii) is rewritten as

$$\boxed{\underset{}{v_{AM}} = \underset{\text{(A)}}{V_c \sin \omega_c t} + \underset{\text{(B)}}{\frac{m_a V_c}{2} \cos (\omega_c - \omega_m)t} - \underset{\text{(C)}}{\frac{m_a V_c}{2} \cos (\omega_c + \omega_m)t}}$$

The equation contains three terms.

(A) $V_c \sin \omega_c t$ i.e. carrier itself

- Frequency = f_c
- Amplitude = V_c

(B) $\frac{m_a V_c}{2} \cos (\omega_c - \omega_m)t$ i.e. Lower Side Band (LSB)

- Frequency = $f_c - f_m$
- Amplitude = $\frac{m_a V_c}{2}$

(C) $\frac{m_a V_c}{2} \cos (\omega_c + \omega_m)t$ i.e. Upper Side Band (USB)

- Frequency = $f_c + f_m$
- Amplitude = $\frac{m_a V_c}{2}$

3.3 Frequency Spectrum of AM Wave

Q. *Explain frequency spectrum of AM wave.*

Frequency spectrum of any signal is just the graph of amplitude versus frequency.

Its significance is that it shows what all frequencies are present in the signal with their amplitudes.

Now, we know that the equation of an AM wave is

$$v_{AM} = V_c \sin \omega_c t + \frac{m_a V_c}{2} \cos (\omega_c - \omega_m)t + \frac{m_a V_c}{2} \cos (\omega_c + \omega_m)t$$

Thus the frequency spectrum of AM wave is as shown in *figure 3.2*.

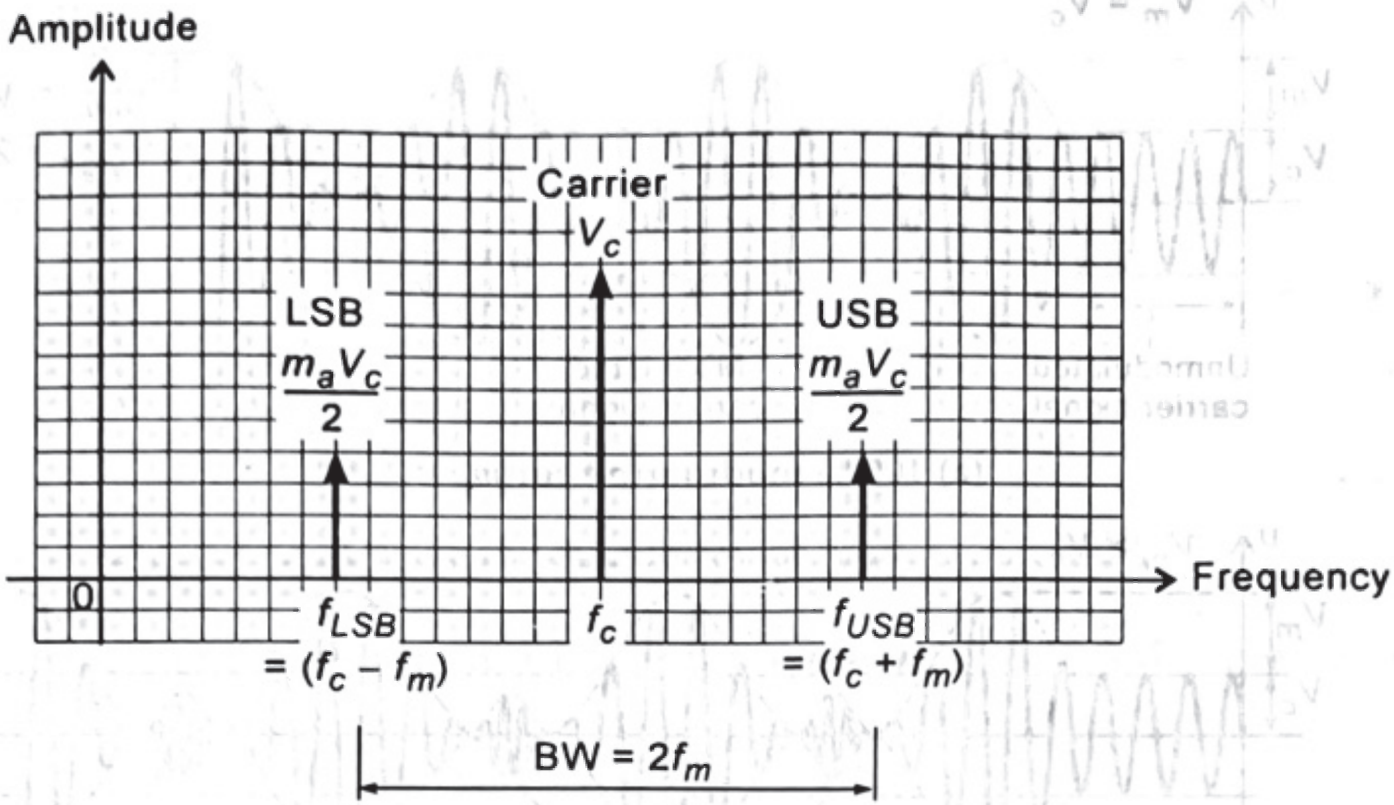


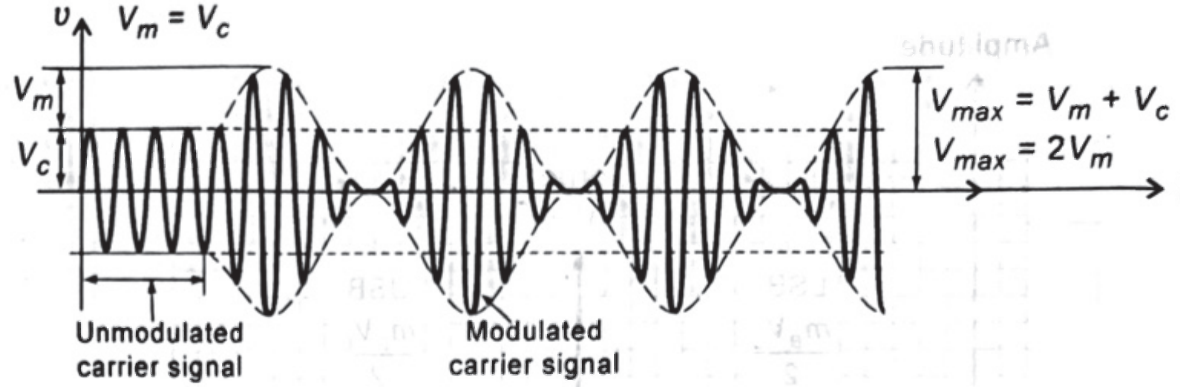
Fig. 3.2 : Single sided frequency spectrum of AM wave.

The following information can be inferred from the above graph :

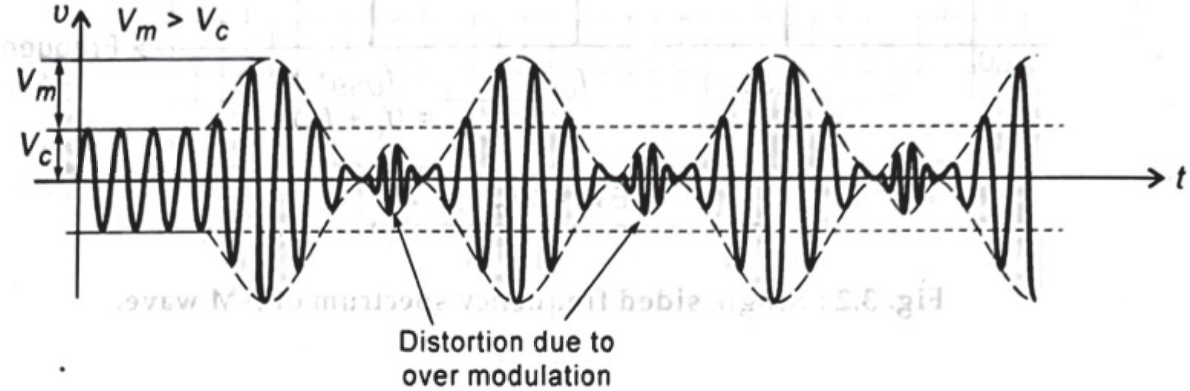
(i) The amplitudes of LSB and USB are equal.

(ii) The LSB and USB are equidistant from the location of carrier signal. Thus, f_c is also called *central frequency*.

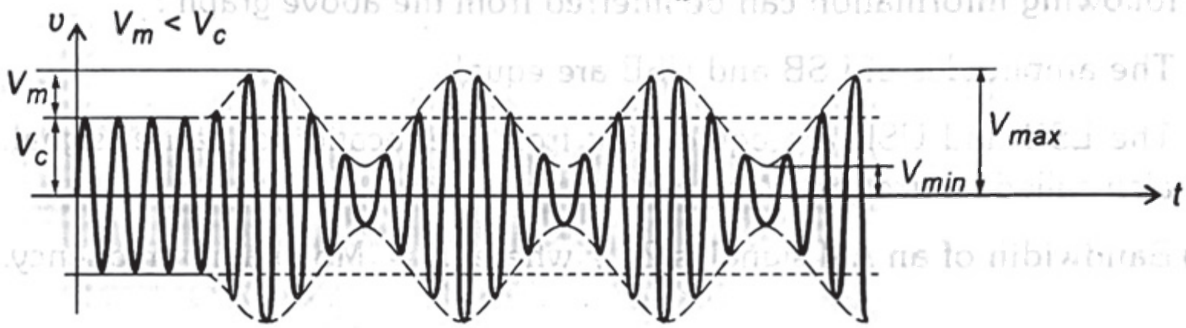
(iii) Bandwidth of an AM signal is $2f_m$, where f_m = Modulating frequency.



(a) 100 % modulation for m_a = 1.



(b) Over modulation for m_a > 1.



(c) Under modulation for m_a < 1.

Fig. 3.3 : AM waveform for various values of m_a.

The modulation index decides the physical appearance of an AM wave (*figure 3.3*)

- $m_a = 1$, when $V_m = V_c$ implies 100% modulation
- $m_a > 1$, when $V_m > V_c$ implies over modulation
- $m_a < 1$, when $V_m < V_c$ implies under modulation

Practically feasible value of m_a is $0 < m_a < 1$

Calculation of Modulation Index

As we already know, the modulation index m_a is given by

$$m_a = \frac{V_m}{V_c} \quad ..... \text{(i)}$$

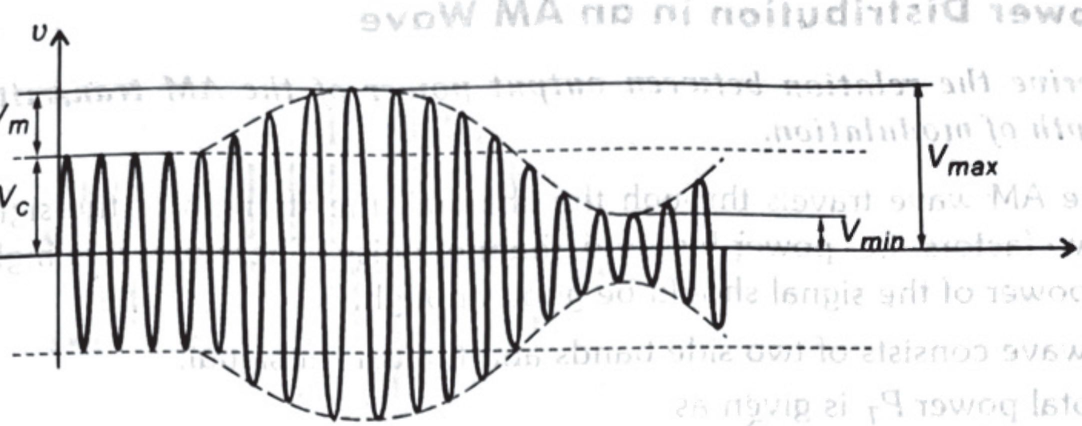


Fig. 3.4 : AM wave for calculation of modulation index *m*.

For the *figure 3.4* the maximum and minimum peak values of AM wave are obtained as :

$$V_{max} = V_c + V_m \quad \text{..... (ii)}$$

$$V_{min} = V_c - V_m \quad \text{..... (iii)}$$

Note : *Modulation index of AM, m_a is also denoted by m.*

Referring to *figure 3.4.*

Subtracting equation (iii) from (ii)

$$V_{max} - V_{min} = 2V_m$$

$$\therefore V_m = \frac{V_{max} - V_{min}}{2}$$

Adding equation (ii) and (iii)

$$V_{max} + V_{min} = 2\,V_c$$

$$\therefore V_c = \frac{V_{max} + V_{min}}{2}$$

Substituting in equation (i), we get

$$m_a = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

or the percentage modulation is given as

$$\%\, m_a = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} \times 100$$

Note : *This technique is used for calculating m_a by viewing an AM wave on a CRO.*

3.5 Power Distribution in an AM Wave

Q. ***Derive the relation between output power of the AM transmitter and the depth of modulation.***

As the AM wave travels through the channel, the strength of the signal degrades due to two factors, i.e. power loss and channel noise. Therefore, it is highly desirable that the power of the signal should be good enough.

The AM wave consists of two side bands and the carrier signal.

$\therefore$ The total power P_T is given as

$$P_T = \text{Carrier power} + \text{Power in LSB} + \text{Power in USB}$$

$$\therefore P_T = P_c + P_{LSB} + P_{USB}$$

Note : *If R is the resistance of antenna and the signal applied is V, then power dissipated in antenna is given as*

$$\text{Power} = \frac{V_{rms}^2}{R} \; ; \text{ where } R = \text{Resistance of antenna}$$

$$V_{rms} = \text{Root mean square voltage.}$$

Carrier Power is given by

$$P_c = \frac{V_{c\,rms}^2}{R}$$

$$P_c = \frac{\left[V_c/\sqrt{2}\right]^2}{R}$$

$$\boxed{P_c = \frac{V_c^2}{2R}}$$

Power in Sidebands

$$P_{USB} = P_{LSB} = \frac{\left[\frac{mV_c}{2}\Big/\sqrt{2}\right]^2}{R}$$

$$\text{as peak amplitude} = \frac{mV_c}{2}$$

$$\boxed{\therefore P_{USB} = P_{LSB} = \frac{m^2V_c^2}{8R}}$$

$\therefore$ Total power in both sidebands is

$$\boxed{P_{SB} = \frac{m^2V_c^2}{4R}}$$

Total Power

The total power is given by

$$P_T = P_c + P_{USB} + P_{LSB}$$

$$\therefore\ P_T = P_c + \frac{m^2}{4} P_c + \frac{m^2}{4} P_c$$

$$\boxed{\therefore\ P_T = \left[1 + \frac{m^2}{2}\right] P_c}$$

Transmission Efficiency

Transmission efficiency of an AM wave is the ratio of transmitted power which contains the information (i.e. total sideband power) to the total power transmitted.

$$\eta = \frac{P_{LSB} + P_{USB}}{P_T} = \frac{\left[\frac{m^2}{4} P_c + \frac{m^2}{4} P_c\right]}{\left[1 + \frac{m^2}{2}\right] P_c} = \frac{m^2}{2 + m^2}$$

AM Power in Terms of Current

$$\boxed{I_T = I_C\left[1 + \frac{m^2}{2}\right]^{\frac{1}{2}}}$$

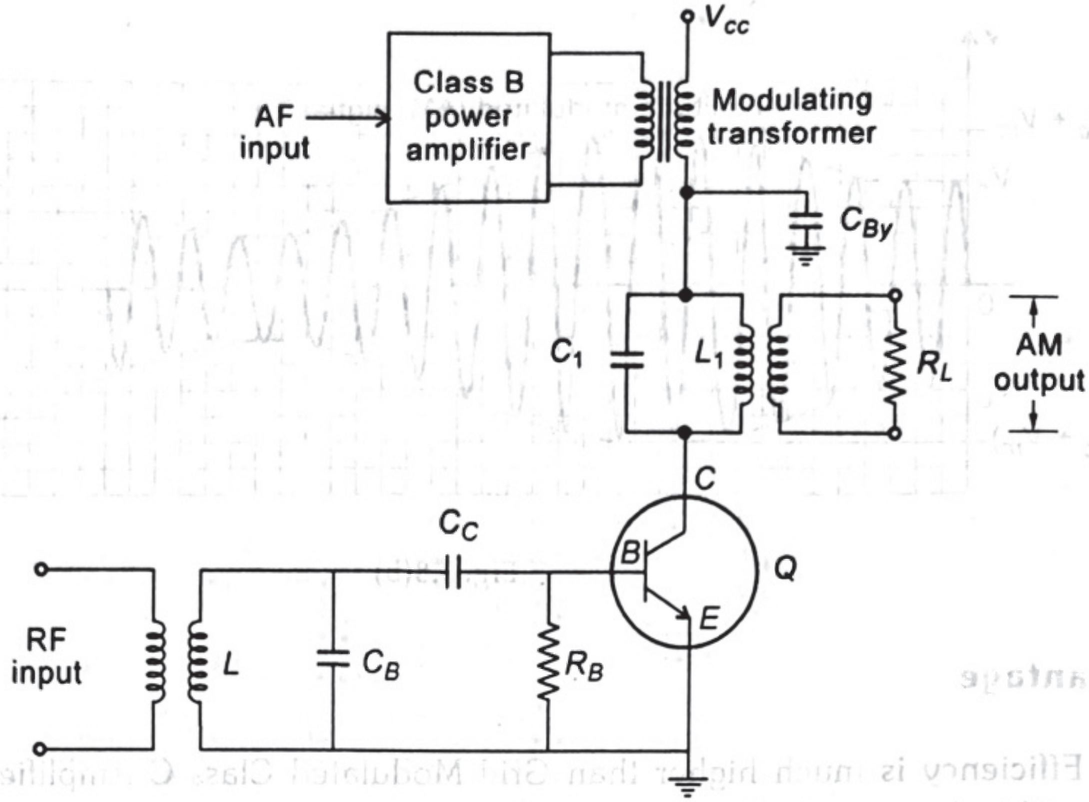


Fig. 3.9(a)

Operation

- RF carrier signal is applied at the base of transistor Q.

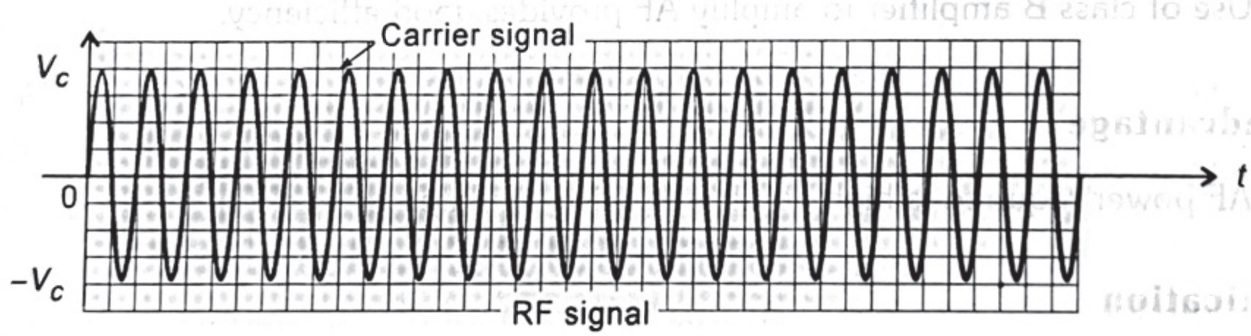


Fig. 3.9(b)

- The transistor will conduct only for positive half cycle of carrier signal.
- The AF input is given to the class B amplifier which increases power level of AF signal.
- This modulating signal is the applied in series with V_{CC}.

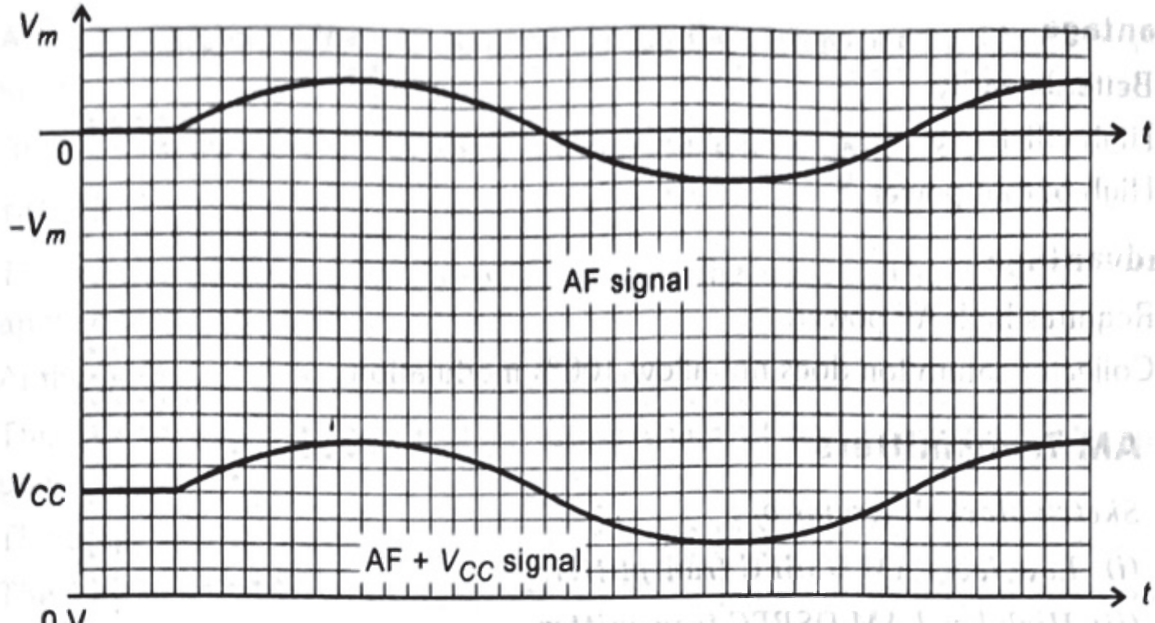


Fig. 3.9(c)

- Therefore the supply voltage varies according to the amplitude of modulating signal.
- Such varying voltage is applied to the collector which results in variation in amplitude of current pulses at the collector of transistor.

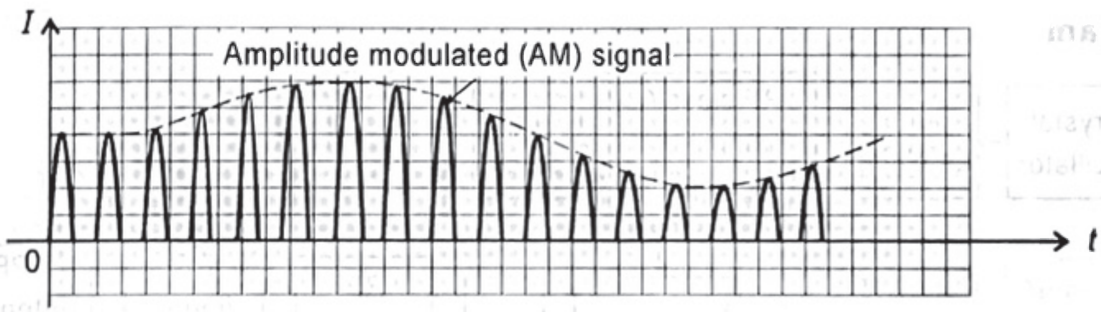


Fig. 3.9(d)

- These pulses are applied to the tank circuit and we get the AM signal across load resistor R_L.

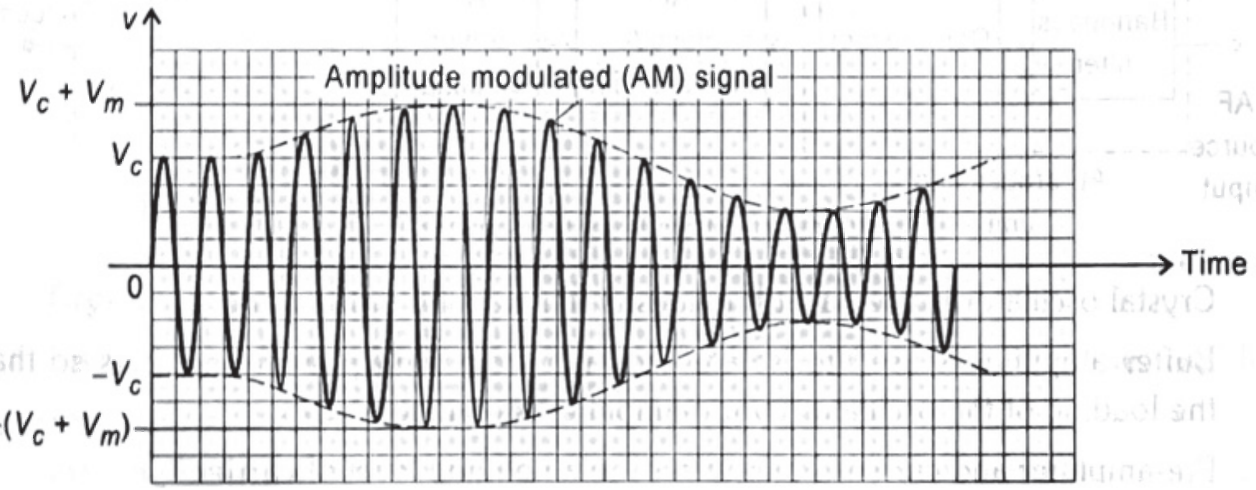


Fig. 3.9(e)