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Solutions to practice problems on Electromgnetism

① $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$r = |\bar{r}| = (x^2+y^2+z^2)^{\frac{1}{2}}$

$$\bar{\nabla} r = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(x^2+y^2+z^2)^{\frac{1}{2}}$$

$$= \hat{i}\frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}} \cdot 2x + \hat{j} \cdot \frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}} \cdot 2y$$

$$+ \hat{k}\frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}} \cdot 2z$$

$$= \frac{x\hat{i} + y\hat{j} + z\hat{k}}{(x^2+y^2+z^2)^{1/2}} = \frac{\bar{r}}{r} = \hat{r} \text{ where}$$

$\hat{r}$ is unit vector along $\bar{r}$

Interpretation : The gradient of magnitude of any vector gives unit vector along the source vector.

② $\bar{E} = \frac{c\bar{r}}{\epsilon_0 a^3}$ $\quad \bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$

According to Gauss' law in differential form,

$$\bar{\nabla} \cdot \bar{E} = \frac{1}{\epsilon_0} \cdot \rho \; ; \; \rho : \text{charge density}$$

$$\bar{\nabla} \cdot \bar{E} = \bar{\nabla} \cdot \left(\frac{c\bar{r}}{\epsilon_0 a^3}\right) = \frac{c}{\epsilon_0 a^3} \bar{\nabla} \cdot \bar{r}$$

$$\bar{\nabla}\cdot\bar{r} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$$

$$= 1 + 1 + 1 = 3$$

$$\therefore \bar{\nabla}\cdot\bar{E} = \frac{3c}{\epsilon_0 a^3}$$

$$\therefore \rho = \epsilon_0 \cdot \frac{3c}{\epsilon_0 a^3} = \frac{3c}{a^3} \text{ units.}$$

(3)

$$\bar{E} = c\left(xy\hat{i} + 2yz\hat{j} + 3xz\hat{k}\right)$$

For an electrostatic field, its curl vanishes
we check $\bar{\nabla}\times\bar{E}$ here.

$$\bar{\nabla}\times\bar{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy & 2yz & 3xz \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}(3xy) - \frac{\partial}{\partial z}(2yz)\right] + \hat{j}\left[\frac{\partial}{\partial z}(xy) - \frac{\partial}{\partial x}(3xz)\right]$$
$$+ \hat{k}\left[\frac{\partial}{\partial x}(2yz) - \frac{\partial}{\partial y}(xy)\right]$$

$$= \hat{i}\left[3x - 2y\right] + \hat{j}\left[0 - 3z\right] + \hat{k}\left[0 - x\right]$$

$$= (3x - 2y)\hat{i} - 3z\hat{j} - x\hat{k}$$

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since $\bar{\nabla}\times\bar{E}\neq 0$, given $\bar{E}$ is not an electrostatic field.

(4) $\bar{v} = c(-3x\hat{i} + 3y\hat{j} - 3z\hat{k})$

For ~~a~~ magnetic field, its divergence is always zero.

we check $\bar{\nabla}\cdot\bar{v}$ here

$$\bar{\nabla}\cdot\bar{v} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(-3x\hat{i} + 3y\hat{j} - 3z\hat{k}\right)$$

$$= -3 + 3 - 3 = -3 \neq 0$$

since $\bar{\nabla}\cdot\bar{v}\neq 0$, given $\bar{v}$ does not represent a magnetic field.

(5) $V(x,y,z) = -\frac{V_0}{a^4}(x^2yz + xy^2z + xyz^2)$ volt

Electric field is related to electric potential through the equation :

$$\bar{E} = -\bar{\nabla}V$$

we find $-\bar{\nabla}V$ here

$$-\bar{\nabla}V = -\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left[-\frac{V_0}{a^4}(x^2yz + xy^2z + xyz^2)\right]$$

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$$= \frac{V_0}{a^4}\left[(2xyz + y^2z + yz^2)\hat{i} + (x^2z + 2xyz + xz^2)\hat{j} + (x^2y + xy^2 + 2xyz)\hat{k}\right]$$

To find electric field at $(a, a, 0)$, put $x = a$, $y = a$, $z = 0$ in above expression

$$\therefore \bar{E} = \frac{V_0}{a^4}\left[(2a \times a \times 0 + a^2 \times 0 + a \times 0^2)\hat{i} + (a^2 \times 0 + 2 \times a \times a \times 0 + a \times 0^2)\hat{j} + (a^2 \times a + a \times a^2 + 2a \times a \times 0)\hat{k}\right]$$

$$= \frac{V_0}{a^4}\left[(0) + (0) + (a^3 + a^3)\hat{k}\right]$$

$$\therefore \bar{E} = \frac{2V_0}{a}\hat{k} \quad V/m$$

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