Functions – Formula Sheet:

Average Rate of Change:

$$\frac{f(b)-f(a)}{b-a}$$

The Difference Quotient:

$$\frac{f(x+h)-f(x)}{h}$$

Vertical Line Test:

If a vertical line intersects a graph at more than one point, then the relation does not represent a function.

Horizontal Line Test:

If a horizontal line intersects a graph at only one point, then the function is ***one-to-one***. In addition, the ***inverse function*** is also a function.

Even Functions:

$$f(-x)=f(x)$$

Odd Functions:

$$f(-x)=-f(x)$$

Composite Functions:

$$(f\ o\ g)(x)=f(g(x))$$

$$(g\ o\ f)(x)=g(f(x))$$

Inverse Functions:

$$If\ f\big(g(x)\big)=x\ and\ g\big(f(x)\big)=x,\ \ then\ ....$$

$$g(x)=f^{-1}(x)\ and\ f(x)=g^{-1}(x)$$

The Distance Formula:

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

The Midpoint Formula:

$$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Transformations – Vertical Shifts:

$y=f(x)+c$ $\quad shift\ up$
$y=f(x)-c$ $\quad shift\ down$

Transformations – Horizontal Shifts:

$y=f(x+c)$ $\quad Shift\ left$
$y=f(x-c)$ $\quad Shift\ right$

Vertical Stretch:

$$y=c\cdot f(x) \qquad c>1$$

Vertical Shrink:

$$y=c\cdot f(x) \qquad 0<c<1$$

Horizontal Shrink:

$$y=f(cx) \qquad c>1$$

Horizontal Stretch:

$$y=f(cx) \qquad 0<c<1$$

Reflection about the x-axis:

$$y=-f(x)$$

Reflection about the origin:

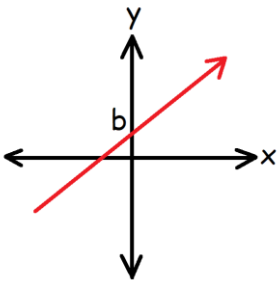
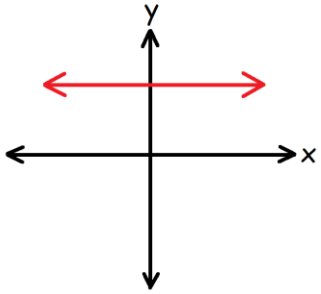
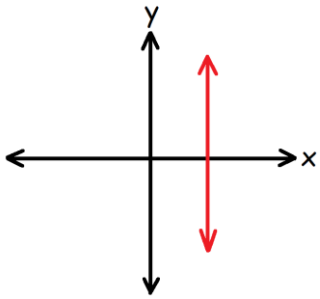
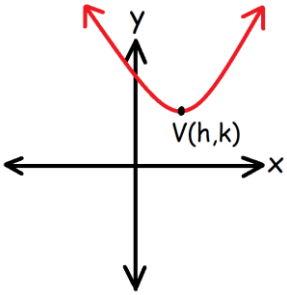
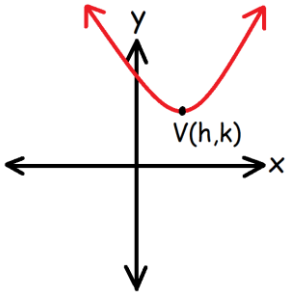
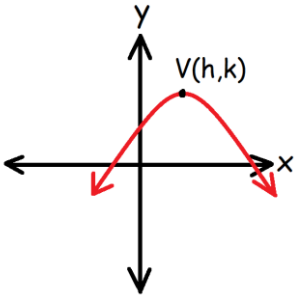
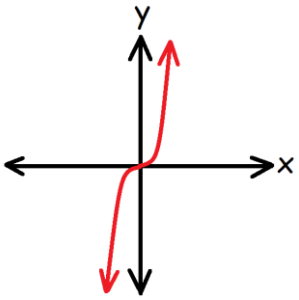
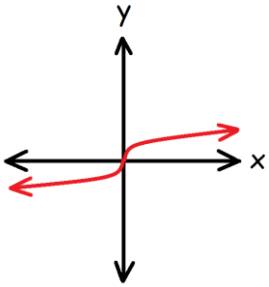
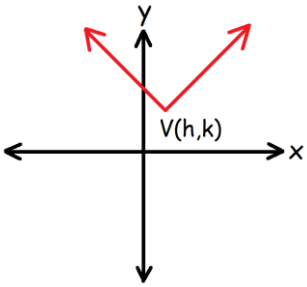
$$y=-f(-x)$$

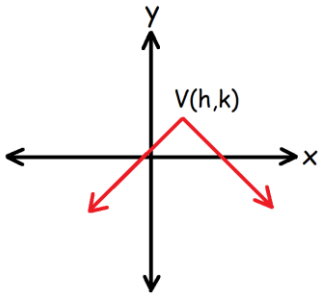
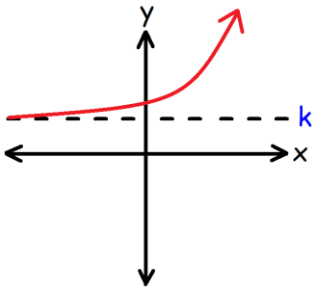
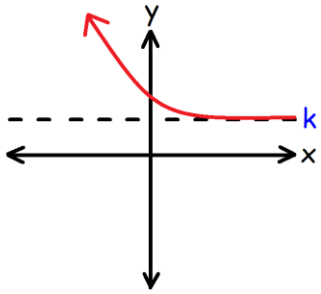
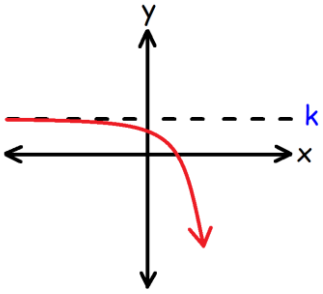
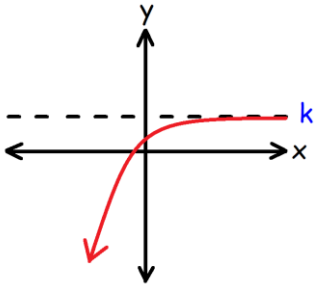
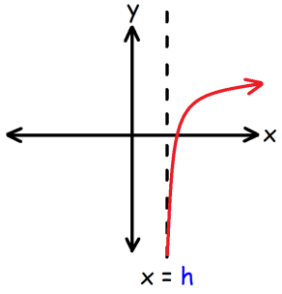
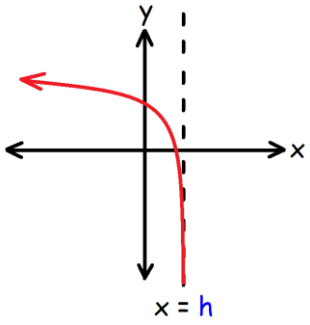
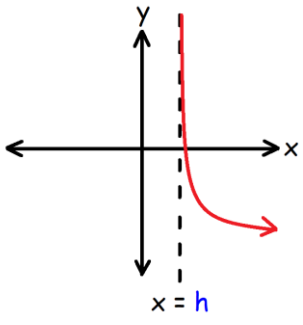
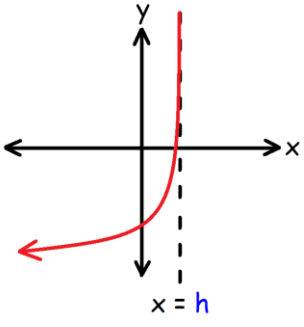
Reflection about the y-axis:

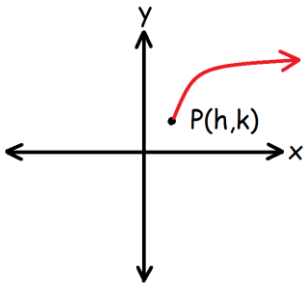
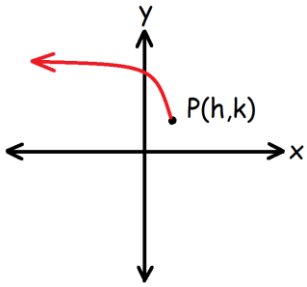
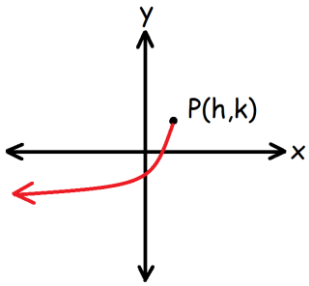
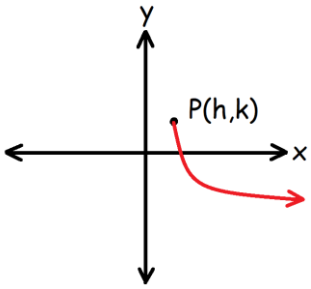
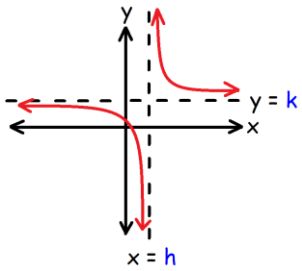
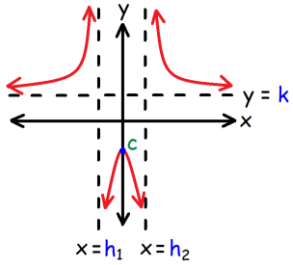
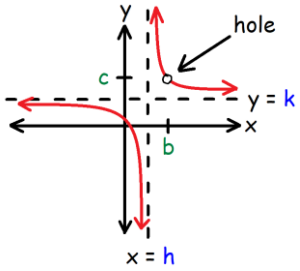
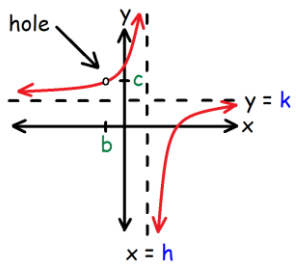
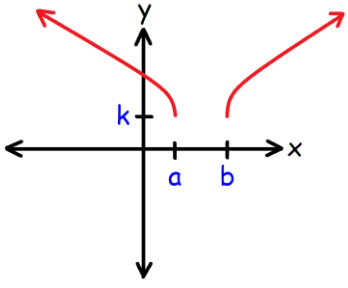
$$y=f(-x)$$

Reflection about the line y=x:

$$(x,y)\rightarrow(y,x)$$

Linear Functions: $y = mx + b$  $Domain: (-\infty, \infty)$ $Range: (-\infty, \infty)$	Horizontal Line: (m = 0) $y = k$  $Domain: (-\infty, \infty)$ $Range: \{k\}$	Vertical Line: (m = undefined) $x = h$  $Domain: \{h\}$ $Range: (-\infty, \infty)$
Quadratic Functions: (a = +) $y = ax^2 + bx + c$  $Domain: (-\infty, \infty)$ $Range: [k, \infty)$ $h = -b/2a \quad k = f(-b/2a)$	Quadratic Functions: (a = +) $y = a(x-h)^2 + k$  $Domain: (-\infty, \infty)$ $Range: [k, \infty)$ $Vertex\ Form$	Quadratic Functions: (a = -) $y = ax^2 + bx + c$  $Domain: (-\infty, \infty)$ $Range: (-\infty, k]$ $Standard\ Form$
Cubic Functions: $y = ax^3 + bx^2 + cx + d$  $Domain: (-\infty, \infty)$ $Range: (-\infty, \infty)$	Cube Root Function: $y = \sqrt[3]{x}$  $Domain: (-\infty, \infty)$ $Range: (-\infty, \infty)$	Absolute Value Functions: (a = +) $y = a\|x-h\| + k$  $Domain: (-\infty, \infty)$ $Range: [k, \infty)$

Absolute Value Functions: (a = -) $y = a\|x - h\| + k$  $Domain$: $(-\infty, \infty)$ $Range$: $(-\infty, k]$	Exponential Functions: $y = a^x + k$  $Domain$: $(-\infty, \infty)$ $Range$: (k, ∞)	Exponential Functions: $y = a^{-x} + k$  $Domain$: $(-\infty, \infty)$ $Range$: (k, ∞)
Exponential Functions: $y = -a^x + k$  $Domain$: $(-\infty, \infty)$ $Range$: $(-\infty, k)$	Exponential Functions: $y = -a^{-x} + k$  $Domain$: $(-\infty, \infty)$ $Range$: $(-\infty, k)$	Logarithmic Functions: $y = \log_a(x - h)$  $Domain$: (h, ∞) $Range$: $(-\infty, \infty)$
Logarithmic Functions: $y = \log_a(h - x)$  $Domain$: $(-\infty, h)$ $Range$: $(-\infty, \infty)$	Logarithmic Functions: $y = -\log_a(x - h)$  $Domain$: (h, ∞) $Range$: $(-\infty, \infty)$	Logarithmic Functions: $y = -\log_a(h - x)$  $Domain$: $(-\infty, h)$ $Range$: $(-\infty, \infty)$

Radical Functions: $y = \sqrt{x-h} + k$  $Domain:\ [h, \infty)$ $Range:\ [k, \infty)$	Radical Functions: $y = \sqrt{h-x} + k$  $Domain:\ (-\infty, h]$ $Range:\ [k, \infty)$	Radical Functions: $y = -\sqrt{h-x} + k$  $Domain:\ (-\infty, h]$ $Range: (-\infty, k]$
Radical Functions: $y = -\sqrt{x-h} + k$  $Domain:\ [h, \infty)$ $Range:\ (-\infty, k]$	Rational Functions: (a > 0) $y = \frac{a}{x-h} + k$  $Domain:\ (-\infty, h)\ U\ (h, \infty)$ $Range:\ (-\infty, k)\ U\ (k, \infty)$	Rational Functions: (a > 0, k > c) $y = \frac{a}{(x-h_1)(x-h_2)} + k$  $D: (-\infty, h_1)\ U\ (h_1, h_2)\ U\ (h_2, \infty)$ $R: (-\infty, c)\ U\ (k, \infty)$
Rational Functions: $y = \frac{a\cancel{(x-b)}}{(x-h)\cancel{(x-b)}} + k$  $D: (-\infty, h)\ U\ (h, b)\ U\ (b, \infty)$ $R: (-\infty, k)\ U\ (k, c)\ U\ (c, \infty)$ Note: b > h and c > k.	Rational Functions: $y = \frac{a\cancel{(x-b)}}{(x-h)\cancel{(x-b)}} + k$  $D: (-\infty, b)\ U\ (b, h)\ U\ (h, \infty)$ $R: (-\infty, k)\ U\ (k, c)\ U\ (c, \infty)$ Note: h > b and c > k.	Complex Radical Functions: $y = \sqrt{(x-a)(x-b)} + k$  $Domain:\ (-\infty, a]\ U\ [b, \infty)$ $Range:\ [k, \infty)$