Unit-1 Questions

1. Reduce the following matrices to *Echelon* form and *Row-Reduced Echelon* form, and hence determine the rank:

   a) $A=\begin{bmatrix} 0 & 1 & 2 & 0 & 3 \\ 2 & 4 & 8 & 2 & 4 \\ 1 & 2 & 4 & 2 & 2 \\ 1 & 3 & 6 & 1 & 5 \end{bmatrix}$

   b) $A=\begin{bmatrix} 10 & 8 & 6 & 4 \\ 2 & 0 & -2 & -4 \\ -6 & -8 & -10 & -12 \\ -2 & -4 & -6 & -8 \end{bmatrix}$

2. Test for the consistency and if possible, solve the following system of equations:

$$x - y + 2z = 2,$$
$$2x + y + 4z = 7,$$
$$4x - y + z = 4$$

3. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & \alpha \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 3 \\ \beta \end{bmatrix}$. Then for what values of α and β, the system $AX = b$ has (i) no solution (ii) unique solution (iii) infinitely many solutions.

4. Let $A = \begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{bmatrix}$. For which triples $Y = (y_1, y_2, y_3)$ does the system $AX = Y$ have a solution?

5. For what values of $\lambda \in R$, the following system of equations has (i) no solution, (ii) a unique solution, and (iii) infinitely many solutions ?

$$(5 - \lambda)x + 4y + 2z = 4,$$
$$4x + (5 - \lambda)y + 2z = 4,$$
$$2x + 2y + (2 - \lambda)z = 2.$$

6. List all possible reduced row echelon form of each of a 2×2 and a 3×3 matrix.

7. Determine all 2×2 matrices A such that A has eigen values 2 and -1 with corresponding eigenvectors $[1\ 0]^T$ and $[2\ 1]^T$, respectively.

8. Let $A = \begin{bmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{bmatrix}$. For which X does there exist a scalar c such that $AX = cX$.

9. Determine the eigenvalues and eigenvectors of the matrix:

$$A = \begin{bmatrix} 2 & -2 & -2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

10. Let A be a 3×3 matrix with real entries such that $\det(A - I) = 0$. If the $trace(A) = 13$ and $\det(A) = 32$, then find the sum of squares of the eigenvalues of A.

11. If the characteristic polynomial of a 3×3 real matrix A is $\lambda^3 - 4\lambda^2 + a\lambda + 30, a \in \mathbb{R}$, and one eigenvalue of A is 2, then find the other eigenvalues.

12. In the give matrix $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$, one of the eigenvalues is 1. Find the eigenvectors corresponding to the eigenvalue 1.

13. Using Gauss – Elimination method, solve the system of equations.

$$\begin{aligned} y + z - 2w &= -3 \\ x + 2y - z \quad &= 2 \\ 2x + 4y + z - 3w &= -2 \\ x - 4y - 7z - w &= -19. \end{aligned}$$

14. Using Gauss – Elimination method, solve the system of equations.

$$\begin{cases} x \qquad\quad - 3z = -5 \\ 3x + y - 2z = -4 \\ 2x + 2y + z = -2 \end{cases}$$

15. Applying the Gauss-Elimination method, solve the following system:

$$2x_1 - x_2 + 2x_3 + 2x_4 = 14$$
$$x_1 + 2x_2 - x_3 + x_4 = 6$$
$$-x_1 + x_2 + 2x_3 - x_4 = 3$$
$$x_1 + x_2 - x_3 + 2x_4 = 8$$

16. Solve the system of equations by Gauss elimination method:

$$10x + y + 2z = 13,$$
$$3x + 10y + z = 14,$$
$$2x + 3y + 10z = 15.$$

17. Applying Gauss-Jordan Method, solve the following system of equations

$$x_2 + 2x_3 = 3$$
$$x_1 + 2x_2 + 4x_3 + 2x_4 = 2$$
$$x_1 + 3x_2 + 6x_3 + x_4 = 5$$
$$2x_1 + 4x_2 + 8x_3 + 2x_4 = 4$$

18. Determine the inverse of the following matrices using Gauss-Jordan method:

(a) $A = \begin{pmatrix} 11 & 12 & 8 \\ 10 & 5 & 4 \\ 13 & 6 & 14 \end{pmatrix}$

(b) $B = \begin{bmatrix} 2 & 1 & -1 \\ 5 & 2 & -3 \\ 0 & 2 & 1 \end{bmatrix}$

(c) $C = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$

(d) $D = \begin{bmatrix} 3 & 3 & 0 & -3 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

(e) $E = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ -3 & 4 & -4 \end{bmatrix}$

19. Let L , D and U be a lower triangular matrix, diagonal matrix, and the upper triangular matrix, respectively. Decompose the following matrix

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 15 & 20 \\ 2 & 18 & 26 \end{bmatrix}$$

into a product of L , D and U.

20. Let a, b be real numbers such that $a \neq 0$ and $a \neq b$. Determine whether the following matrix is invertible over $\mathbb{R}$ and if so, find its inverse:

$$\begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}$$

21. Use an LU factorization of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

to solve the system of equations $Ax = b$ for

$$b = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, b = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

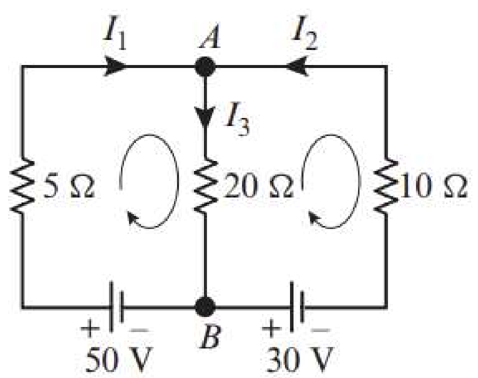
Find A^{-1} after finding the LDU factorization of A.

22. For what values of c and k, the following systems

   a. $x + y + z = 3, x + 2y + cz = 4, 2x + 3y + 2cz = k.$
   b. $x + y + z = 3, x + y + 2cz = 7, x + 2y + 3cz = k.$

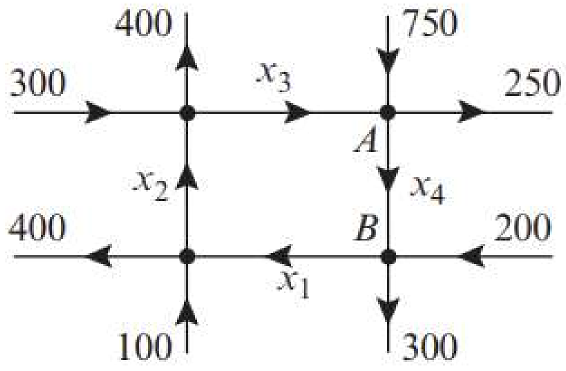
   have i) no solution, ii) a unique solution and iii) infinite number of solutions.

23. Determine the currents I_1, I_2, and I_3 in the circuit shown in Figure

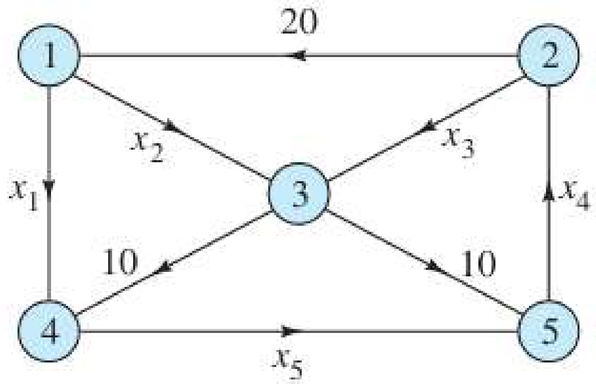


24. The accompanying figure shows a network of one-way streets with traffic flowing in the directions indicated. The flow rates along the streets are measured as the average number of vehicles per hour.
   a. Set up a linear system whose solution provides the unknown flow rates.
   b. Solve the system for the unknown flow rates.

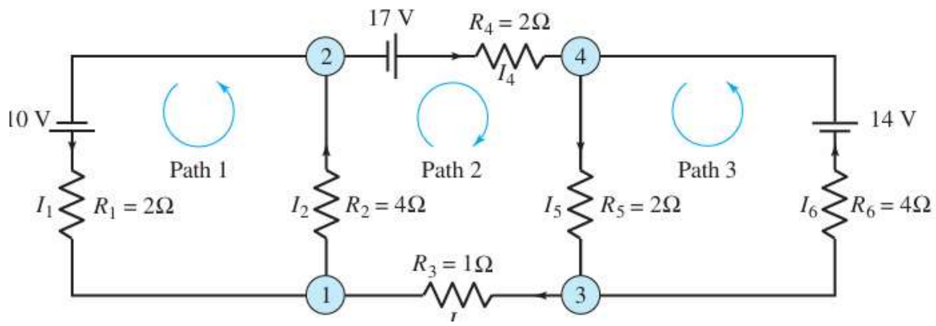
c. If the flow along the road from A to B must be reduced for construction, what is the minimum flow that is required to keep traffic flowing on all roads?



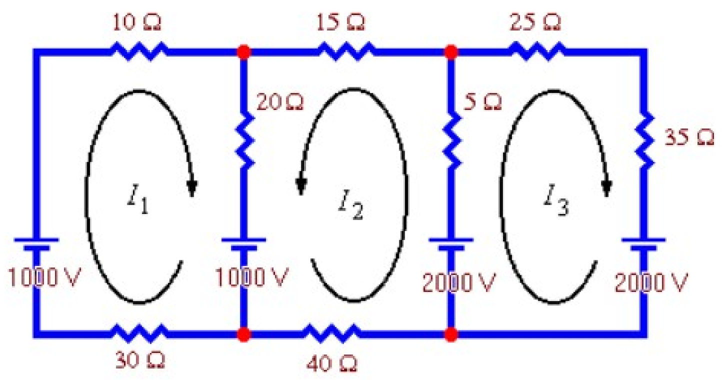
25. Write a balanced equation for the given chemical reaction and solve by matrix methods
    - $C_3H_8 + O_2 \rightarrow CO_2 + H_2O$ (propane combustion)
    - $C_6H_{12}O_6 \rightarrow CO_2 + C_2H_5OH$ (fermentation of sugar)
    - $H_3PO_4 + Ca(OH)_2 \rightarrow Ca_3(PO_4)_2 + H_2O.$
26. Set up a system of linear equations to represent the network shown in Figure, below. Then solve the system.



27. Determine the currents I_1, I_2, I_3, I_4, I_5, and I_6 for the electrical network shown below.



28. Determine the current i_1 , i_2 , i_3 in the following electrical circuit using any matrix method.



Answer. $i_1 = -4.56$, $i_2 = 13.7$, $i_3 = -1.05$.

29. Find the equation of the second order parabola that passes through the points $(-1, 9)$, $(1, 5)$, and $(2, 12)$.

30. Associate the numbers with their corresponding letters from the following table

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
P	Q	R	S	T	U	V	W	X	Y	Z	*Blank*	?	!	
$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	
15	16	17	18	19	20	21	22	23	24	25	26	27	28	

a) Apply the matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ to encode the message: "**WELCOME**".

b) Apply the matrix $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ to encode the message: "ATTACK NOW!"

c) Encode "THANK UFO" using the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$.

d) Encode the following Message "HAPPY FAMILY" by using key matrix $\begin{bmatrix} -3 & -3 & -4 \\ 0 & 1 & 1 \\ 4 & 3 & 4 \end{bmatrix}$

e) Decrypt the following Message "GOOD MORNING TO ALL". By using key matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$.