Ex : $\begin{bmatrix} 0 & 1 & 3 & 4 & 5 \\ 0 & 0 & 1 & 2 & 9 \\ 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Non-Zero Row :- If there is atleast one non-zero element in a row, then it is called a non-zero row.

Ex : $\begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & 0 & -6 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Problems :

1. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{bmatrix}$ by reducing it to the Echelon form.

Sol: Given matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{bmatrix}_{3\times3}$

Applying Row transformations $R_2 \to R_2 - 3R_1$
$R_3 \to R_3 - 7R_1$

$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -5 \\ 0 & -4 & -9 \end{bmatrix}$ $R_3 \to R_3 - 2R_2$ $4-6$ $12-21$

$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -5 \\ 0 & 0 & 1 \end{bmatrix}$ $-9+10$

The last equivalent matrix is in Echelon form

$\therefore$ Rank of A = number of non-zero rows = 3.

$\therefore$ $\boxed{\rho(A) = 3}$

2. Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$ into Echelon form and hence find its rank.

Sol: Given matrix $A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}_{3\times 4} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$

Applying row transformations $R_2 \rightarrow R_2 + 2R_1$

$R_3 \rightarrow R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 8 & 5 & 0 \\ 0 & -2 & 1 & -8 \end{bmatrix}$$

$R_3 \rightarrow 4R_3 + R_2$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 8 & 5 & 0 \\ 0 & 0 & 9 & -32 \end{bmatrix}$$

This is in Echelon form. The number of non-zero rows is 3

$\boxed{\therefore \; \rho(A) = 3}$

3. Assignment:- Find the rank of matrix $A = \begin{bmatrix} -1 & 2 & 1 & 8 \\ 2 & 1 & -1 & 0 \\ 3 & 2 & 1 & 7 \end{bmatrix}$ by reducing it to the Echelon form. Ans: $\rho(A) = 3$

7. Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ into echelon form and hence find its rank.

Sol: Given matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

Applying row transformations $R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 6R_1$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix} \quad R_2 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix} \quad R_4 \to R_4 - R_2$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix} \quad R_4 \to R_4 - R_3$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This in Echelon form and the number of non-zero rows is '3'.

$$\boxed{\therefore \ \rho(A) = 3}$$

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5. Reduce the matrix to Echelon form and find its rank $\begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$

Sol:- Given matrix $A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$ $R_2 \to R_2 + R_1$, $R_3 \to R_3 + 2R_1$, $R_4 \to R_4 - R_1$

$$\sim \begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & -11 & 8 & -5 \\ 0 & 4 & -3 & 2 \end{bmatrix} \quad R_3 \to 2R_3 - 11R_2, \quad R_4 \to R_4 + 2R_2$$

$$\sim \begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & 0 & -6 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad R_4 \to 6R_4 + R_3$$

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$$\sim \begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & 0 & -6 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This is in Echelon form. Number of non-zero rows is '4'

$\therefore$ $\boxed{\rho(A) = 4}$

Assignment: Find the rank of the matrix of

$A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ by reducing it into Echelon form.

6. Reduce the matrix $A = \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$ into Echelon form and hence find its rank

Sol:- let $A = \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$ $R_2 \to R_2 + 2R_1$, $R_3 \to R_3 + R_1$, $R_4 \to R_4 + 3R_1$

$$\sim \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 5 & 5 & 0 & 5 \\ 0 & 10 & 10 & 0 & 10 \\ 0 & 15 & 15 & 0 & 15 \end{bmatrix} \quad R_3 \to R_3 - 2R_2,\ R_4 \to R_4 - 3R_2$$

$$\sim \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 5 & 5 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Number of non-zero rows is '2'

$$\boxed{\therefore \rho(A) = 2}$$

7. Find the rank of the matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ by reducing to echelon form.

Sol:- Given Matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

Applying row transformations $R_1 \leftrightarrow R_2$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix} \quad R_3 \to R_3 - 3R_1,\ R_4 \to R_4 - R_1$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix} \quad \begin{matrix} R_3 \to R_3 - R_2 \\ R_4 \to R_4 - R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is in Echelon form. Number of non-zero rows is '2' $\boxed{\therefore \rho(A) = 2.}$

System of linear Simultaneous Equations :

Definition :- An equation of the form

$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b$, where $x_1, x_2 \ldots x_n$ are unknowns and $a_1, a_2 \ldots a_n, b$ are constants is called a linear equation in n unknowns.

Non-Homogeneous linear equations :

Consider m linear non-homogeneous equations in n unknowns as given below

$$a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n = b_1$$
$$a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n = b_2$$
$$\cdots \cdots \cdots$$
$$a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n = b_m$$

The above system of equations in matrix notation can be written as $AX = B$.

where A is the coefficient matrix formed by coefficients of unknowns.