Module-3 (conti…)

1. Find the transition matrix from B to B' where the bases given below

(i) $B=\{(1,0),(0,1)\}$, $B'=\{(2,4),(1,3)\}$

(ii) $B=\{(1,0),(0,1)\}$, $B'=\{(1,1),(5,6)\}$

(iii) $B=\{(1,0,0),(0,1,0),(0,0,1)\}$, $B'=\{(1,3,-1),(2,7,-4),(2,9,-7)\}$

2. Find the transition matrix from B to B' where the bases given below

(i) $B=\{(2,5),(1,2)\}$, $B'=\{(2,1),(-1,2)\}$

(ii) $B=\{(-2,1),(3,2)\}$, $B'=\{(1,2),(-1,0)\}$

(iii) $B=\{(1,2,4),(-1,2,0),(2,4,0)\}$, $B'=\{(0,2,1),(-2,1,0),(1,1,1)\}$

3.Find the matrix A' for T relative to the bases B' and show that A' is similar to A where A is the standard matrix of T,

(i) $T:R^2 \to R^2$ defined by $T(x,y)=(2x-y,y-x), B'=\{(1,-2),(0,3)\}$

(ii) $T:R^2 \to R^2$ defined by $T(x,y)=(x+y,4y), B'=\{(-4,1),(1,-1)\}$

(iii) $T:R^3 \to R^3$ defined by $T(x,y,z)=(x,x+2y,x+y+3z)$,

$B'=\{(1,-1,0),(0,0,1),(0,1,-1)\}$.

4.let $B=\{(1,1,0),(1,0,1),(0,1,1)\}$ and $B'=\{(1,0,0),(0,1,0),(0,0,1)\}$ be the bases of R^3, and

let $A=\begin{bmatrix} \frac{3}{2} & -1 & -\frac{1}{2} \\ -\frac{1}{2} & 2 & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{5}{2} \end{bmatrix}$ be the matrix for $T:R^3 \to R^3$ relative to basis B.

Find (i) transition matrix from B' to B. (ii) $[v]_B$ *and* $[T(v)]_B$, where $[v]_{B'}=\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

(iii) matrix for T relative to B' and (iv) $[T(v)]_{B'}$.

5.The eight vertices of a rectangular box having sides of lengths 1, 2, and 3 are as follows.

V_1=(0,0,0); V_2=(1,0,0); V_3=(1,2,0); V_4=(0,0,3); V_5 =(0, 0, 3); V_6 =(1, 0, 3); V_7=(1, 2, 3), V_8=(0, 2, 3).

Find the coordinates of the box when it is rotated counterclockwise about the -axis through each angle. (i) $\theta=60^o$ (ii) $\theta=90^o$ (iii) $\theta=120^o$.

6.let the linear transformation T be represented by $T(X) = AX$ and A is given below. Find a basis for (a) the kernel of T and (b) the range of T. Also verify rank-nullity theorem.

(i)A=$\begin{bmatrix} 1 & 2 & -1 & 4 \\ 3 & 1 & 2 & -1 \\ -4 & -3 & -1 & -3 \\ -1 & -2 & 1 & 1 \end{bmatrix}$ (ii)$\begin{bmatrix} 2 & 2 & -3 & 1 & 13 \\ 1 & 1 & 1 & 1 & -1 \\ 3 & 3 & -5 & 0 & 14 \\ 6 & 6 & -2 & 4 & 16 \end{bmatrix}$ (iii)$\begin{bmatrix} 3 & -2 & 6 & -1 & 15 \\ 4 & 3 & 8 & 10 & -14 \\ 2 & -3 & 4 & -4 & 20 \end{bmatrix}$

(iv)$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Module-4

1 Let $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ and $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ be two vectors in R^2. Show that $\langle u, v \rangle = 2u_1v_1 + 3u_2v_2$ is inner product and also find its Gram Matrix.

2 Find the angle between 1 and x^2 in $C[0,1]$.

3 Consider the inner product space $C[0,1]$. Compute the following inner products:
(i) $\langle x, x \rangle$ (ii) $\langle x, 1 + x^2 \rangle$.

4 Show that the set $S = \left\{ \left(\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3} \right), \left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right), \left(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3} \right) \right\}$ is an orthonormal set in R^3 with standard inner product.

5 Apply Gram-Schmidt process, obtain an orthonormal basis of $R^3(R)$ from the basis $\{(1,0,1), (1,0,-1), (0,3,4)\}$.

6 In P_2, let $p(x) = a_0 + a_1x + a_2x^2$ and $q(x) = b_0 + b_1x + b_2x^2$, show that $\langle p(x), q(x) \rangle = a_0b_0 + a_1b_1 + a_2b_2$ is an inner product.

7 Consider the inner product on $C[0,1]$, if $f(x) = x$ and $g(x) = 3x - 2$ then find (i) $\|f\|$ (ii) $d(f, g)$ (iii) $\langle f, g \rangle$.

8 Apply Gram-Schmidt process, obtain an orthonormal basis of $R^3(R)$ from the basis $\{(2,1,3), (1,2,3), (1,1,1)\}$.

9 Find a unit vector orthogonal to $(4,2,3)$ in $R^3(R)$.

10 In the inner product space $R^3(R)$ with respect to the standard inner product, show that the set $X = \{(1,1,0),(1,-1,1),(-1,1,2)\}$ is an orthogonal set.

11 Construct an orthogonal basis for P_2 with respect to the inner product $\langle f, g\rangle = \int_{-1}^{1} f(x)\,g(x)\,dx$ by applying the Gram-Schmidt process to the basis $B = \{1, x, x^2\}$.

12 If $S = \{(1,1,0),(1,-1,1),(-1,1,2)\}$ is an orthogonal set of the inner product space of $R^3(R)$ then find orthonormal set.

13 Find the normalize $\alpha = \left(\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}\right)$ in the inner product space of $R^3(R) = V_3(R)$.

14 Find an orthonormal basis for the solution space of the homogeneous system of linear equations $x_1 + x_2 + 7x_4 = 0$, $2x_1 + x_2 + 2x_3 + 6x_4 = 0$.