



**SCHOOL OF ADVANCED SCIENCES**

**Fall Semester 2024-2025**

**Digital Assignment 1**

**Programme Name & Branch : M.Sc. & Data Science**  
**Course Name & code : Probability and Distribution Models & PMDS502L**  
**Faculty Name : Dr. Jisha Francis**  
**Due Date & Max. Marks : 20 September, 2024 & 10 Marks**

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1. Let the random variable  $\mathbf{X} = (X_1, X_2)$  have the joint probability mass function (p.m.f.):

$$f_X(x_1, x_2) = \begin{cases} c(x_1 + 2x_2), & \text{if } x_1 = 1, 2 \text{ and } x_2 = 1, 2 \\ 0, & \text{otherwise} \end{cases}$$

where  $c$  is a real constant.

- (a) Find the constant  $c$ .
  - (b) Find the marginal p.m.f.s of  $X_1$  and  $X_2$ .
  - (c) Find the conditional variance of  $X_2$  given  $X_1 = x_1$ , where  $x_1 = 1, 2$ .
  - (d) Find  $P(X_1 < \frac{X_2}{3})$ ,  $P(X_1 = X_2)$ ,  $P(X_1 \geq \frac{X_2}{2})$ , and  $P(X_1 + X_2 \leq 3)$ .
  - (e) Find the correlation coefficient  $\rho(X_1, X_2)$ .
  - (f) Are  $X_1$  and  $X_2$  independent?
2. Let  $\mathbf{X} = (X_1, X_2, X_3)$  be a random vector with joint probability density function (p.d.f.):

$$f_X(x_1, x_2, x_3) = \frac{1}{(2\pi)^{3/2}} \exp\left(-\frac{1}{2}(x_1^2 + x_2^2 + x_3^2)\right) \left(1 + x_1 x_2 x_3 \exp\left(-\frac{1}{2}(x_1^2 + x_2^2 + x_3^2)\right)\right),$$

where  $-\infty < x_i < \infty$  for  $i = 1, 2, 3$ .

- (a) Are  $X_1, X_2, X_3$  independent?
  - (b) Are  $X_1, X_2, X_3$  pairwise independent?
3. Let  $X$  and  $Y$  be jointly distributed random variables with  $E(X) = E(Y) = 0$ ,  $E(X^2) = E(Y^2) = 2$ , and  $\text{Corr}(X, Y) = \frac{1}{3}$ . Find the correlation  $\text{Corr}(\frac{X}{3} + \frac{2Y}{3}, \frac{2X}{3} + \frac{Y}{3})$ .