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1)

a) Bernoulli Distribution:

→ MLE:

the bernoulli distribution as follows =

$$P(X=x) = \begin{cases} p^x (1-p)^{1-x}, & x=0,1 \\ 0, & \text{elsewhere} \end{cases}$$

likelihood —

$$L = \prod_{i=1}^{n} p^{x_i} (1-p)^{(1-x_i)}$$

$$= p^{x_1}(1-p)^{(1-x_1)} \cdot p^{x_2}(1-p)^{(1-x_2)} \cdots p^{x_n}(1-p)^{(1-x_n)}$$

$$= p^{x_1+x_2+x_n} (1-p)^{(1-x_1)+(1-x_2)+\cdots(1-x_n)}$$

$$= p^{\Sigma x}(1-p)^{\Sigma(1-x)}$$

log likelihood —

$$\log L = \log p^{\Sigma x} + \log(1-p)^{\Sigma(1-x)}$$

$$\text{or, } \log L = \Sigma x \log p + \Sigma(1-x)\log(1-p)$$

$$\text{or, } \log L = \Sigma x \log p + (n-\Sigma x)\log(1-p)$$

$$\frac{d \log L}{d p} = \frac{1}{p}\Sigma x - (n-\Sigma x)\frac{1}{1-p} = 0$$

$$\text{or, } \frac{d\log L}{dp} = \frac{\Sigma x}{p} - \frac{n-\Sigma x}{1-p} = 0$$

$$\text{or, } \frac{\Sigma x}{p} = \frac{n-\Sigma x}{1-p}$$

or, $\sum x - \sum xp = np - \sum xp$

or, $np = \sum x$

or, $\boxed{p = \frac{\sum x}{n}}$

→ Method of moments:

The population moment of bernoulli distribution is →

x	1	0
fx	p	$(1-p)$

$$E(x) = p \times 1 + (1-p)$$
$$= p + (1-p) \times 0$$
$$= p$$

Sample moment $\frac{1}{n} \sum x_i = \bar{x}$

Equating sample & population moment,

$$P_{MME} = \bar{x}$$

b) Binomial Distribution:

MLE the distribution as follows,

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

→ Liklihood →

$$L = \prod_{i=1}^{n} \binom{n}{x} p^x (1-p)^{n-x}$$
$$= \sum \binom{n}{x} p^{\sum x} (1-p)^{(n^n - \sum x)}$$

→ log likelihood –

$$\log L = \sum \log \binom{n}{x} + \log p^{\sum x} + \log (1-p)^{(n^n - \sum x)}$$

$$\log L = \sum \log \binom{n}{x} + \sum x \log p + (n^n - \sum x) \log (1-p)$$

or, $\log L = \sum \log \binom{n}{x} +$

$$\frac{d\log(L)}{dp} = 0 + \frac{1}{p}\Sigma x - \frac{(n^2 - \Sigma x)}{1-p} = 0$$

or, $$\frac{1}{p}\Sigma x - \frac{(n^2 - \Sigma x)}{(1-p)} = 0$$

or, $$\frac{\Sigma x}{p} = \frac{n^2 - \Sigma x}{1-p}$$

or, $$\Sigma x - \Sigma x p = n^2 p - \Sigma x p$$

or, $$\Sigma x = n^2 p$$

or, $$\boxed{p = \frac{\Sigma x}{n^2}}$$

→ method of moments :

1st moment of Binomial Dist. –

$$E(x) = \sum_{x=1}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

Now Binomial can be written as sum of independent bernoulli r.v. –

$x = x_1 + x_2 + \cdots + x_n$, where $x_i \sim$ Bernoulli(p)

&, x_i take value 1 with prob. p & 0 with prob. $(1-p)$

$$\therefore E(x) = E\left[\sum_{i=0}^{n} x_i\right] = \sum_{i=0}^{n} E[x_i]$$

for bernoulli r.v. we have –

$$E[x_i] = 1 \cdot p + 0(1-p) = p$$

$$\therefore E(x) = \sum_{i=1}^{n} p = np = \bar{x}$$

$$\therefore \boxed{p = \frac{\bar{x}}{n}}$$

c) Poisson Distribution

the poisson distribution as follows -

$$f(x) = \frac{e^{-\lambda}\lambda^{x_i}}{x_i!} \quad ; \quad x = 0, 1, 2, 3 \cdots n.$$

→ Liklihood -

$$L = \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$$

→ log liklihood -

$$\log(L) = \sum_{i=1}^{n} \left[\log\left(e^{-\lambda}\lambda^{x_i}\right) - \log(x_i!)\right]$$

or, $\log(L) = \sum_{i=1}^{n} \left[\log e^{-\lambda} + \log \lambda^{x_i} - \log x_i!\right]$

or, $\log(L) = \sum_{i=1}^{n} \left[-\lambda \log e + x_i \log \lambda - \log_e x_i!\right]$

$$= \sum_{i=1}^{n} -\lambda + \log\lambda \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} (x_i!)$$

$$= -n\lambda + \log\lambda \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} (x_i!)$$

$$\frac{d \log(L)}{d\lambda} = -n + \frac{1}{\lambda}\sum_{i=1}^{n} x_i = 0$$

or, $\sum x_i = n\lambda$

or, $\boxed{\lambda = \frac{\sum x_i}{n}}$

MME:

1st moment is, $E(x) = \sum_{x=0}^{\infty} x \cdot \frac{\lambda^x e^{-\lambda}}{x!}$

$$= \sum_{x=0}^{\infty} \frac{\cancel{x} \cdot \lambda \times \lambda^{x-1} \times e^{-\lambda}}{\cancel{x}(x-1)!}$$

$$= \lambda e^{-\lambda} \sum \frac{\lambda^k}{k!} = \lambda e^{-\lambda} \cdot e^{\lambda}$$

$$\Rightarrow E(x) = e^{-\lambda} \sum_{x=1}^{\infty} \lambda \cdot \frac{\lambda^{x-1}}{(x-1)!}$$

$$= \lambda \cdot e^{-\lambda+\lambda}$$

$$= \lambda$$

Now sample mean is $\bar{x} = \frac{1}{n} \sum x_i$

setting sample population moment.

$$\boxed{\bar{x} = \lambda \text{ MME}}$$

d) Geometric Distribution:

The Geometric Distn. as follows -

$$f(x) = (1-p)^{(x-1)} p$$

Liklihood -

$$L = \prod_{i=1}^{n} (1-p)^{(x_i-1)} p$$

$$= (1-p)^{(x_1-1)} \cdot p \cdot (1-p)^{(x_2-1)} p \ldots (1-p)^{(x_n-1)} p$$

$$= (1-p)^{(\sum x_i - n)} \cdot p^n$$

$\therefore$ log liklihood -

$$\log L = (\sum x - n) \log(1-p) + n \log p$$

$\therefore \frac{d \log L}{dp} = -\frac{\Sigma x - n}{1-p} + \frac{n}{p} = 0$

or, $\frac{n}{p} = \frac{\Sigma x - n}{1-p}$

or, $n - np = p\Sigma x - np$

=>, $n = p \Sigma x$

=> $\boxed{p = \frac{n}{\Sigma x}}$

MME:

1st moment of GD ⟶

$E(x) = \sum_{x=1}^{\infty} x(1-p)^{x-1} p$

$= p \sum_{x=1}^{\infty} x(1-p)^{x-1}$

use the identity, $\sum_{x=1}^{\infty} x q^{x-1} = \frac{1}{(1-q)^2}$ for $0<q<1$

let $q = 1-p$ then, $E(x) = p \sum_{x=1}^{\infty} x(1-p)^{x-1}$

$= p \times \frac{1}{p^2} = \frac{1}{p}$

Now sample mean, $\bar{x} = \frac{\Sigma x}{n}$

Now, $\bar{x} = \frac{1}{p}$ $\therefore p = \frac{1}{\bar{x}}$

$\therefore \boxed{p_{MME} = \frac{1}{\bar{x}}}$

7) Exponential Distribution:

MLE exponential distribution as follows-

$$f(x) = \lambda e^{-\lambda x} ; x \geq 0$$

Likelihood-

$$L = \prod_{i=1}^{n} \lambda e^{-\lambda x_i}$$

$$= \lambda e^{-\lambda x_1} \cdot \lambda e^{-\lambda x_2} \cdot \dots \lambda e^{-\lambda x_n}$$

$$= \lambda^n e^{(-\lambda x_1 - \lambda x_2 - \dots \lambda x_n)}$$

$$\Rightarrow \lambda^n e^{-\lambda(x_1 + x_2 + \dots x_n)}$$

$$= \lambda^n e^{-\lambda \Sigma x_i}$$

log likelihood-

$$\log L = n \log \lambda - \lambda \Sigma x_i \log e$$

$$= n \log \lambda - \lambda \Sigma x_i$$

$$\frac{d \log L}{d \lambda} = \frac{n}{\lambda} - \Sigma x_i = 0$$

$$\Rightarrow \frac{n}{\lambda} = \Sigma x_i$$

$$\Rightarrow \boxed{\lambda = \frac{n}{\Sigma x_i}}$$

MME 1st moment $= E(x) = \int_0^\infty x \lambda e^{-\lambda x} dx$

$$d v = e^{\lambda x} x \lambda \, d x$$

let $u = x$

$du = dx \rightarrow \int dv = \int e^{-\lambda x} x \lambda \, dx = -e^{-\lambda x}$

$$\therefore E(x) = \left[-x \cdot e^{-\lambda x}\right]^\infty + \int_0^\infty e^{-\lambda x} dx$$

$$= 0 - \left[\frac{e^{-\lambda x}}{\lambda}\right]_0^\alpha = -\frac{1}{\lambda}[0-1] = \frac{1}{\lambda}$$

$\therefore E(x) = \frac{1}{\lambda}$

set $\bar{x} = \frac{1}{\lambda}$ $\therefore \boxed{\lambda = \frac{1}{\bar{x}}}$

e) Normal Distribution:

Normal Distribution as follows:-

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x_i-\mu)^2/2\sigma^2}$$

→ Liklelihood:

$$L = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x_i-\mu)^2/2\sigma^2}$$

$$L = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x_i-\mu)^2/2\sigma^2} \times \cdots \times \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x_n-\mu)^2/2\sigma^2}$$

$$L = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-(x_1-\mu)^2/2\sigma^2-(x_2-\mu)^2/2\sigma^2-\cdots-(x_n-\mu)^2/2\sigma^2}$$

$$L = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{\Sigma(x_i-\mu)^2}{2\sigma^2}}$$

→ log Liklihood:

$$\log L = \log\left(\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{\Sigma(x_i-\mu)^2}{2\sigma^2}}\right)$$

$$= n\log 1 - n\log(\sqrt{2\pi\sigma^2}) + -\frac{\Sigma(x-\mu)^2}{2\sigma^2}\log e$$

$$= 0 - \frac{n}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}\Sigma(x-\mu)^2$$

to estimate σ^2.

$$\frac{d\log L}{d\sigma^2} = -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}\Sigma(x_i-\mu)^2 = 0$$

$$\text{or, } \frac{1}{\cancel{2\sigma^4}_{\sigma^2}}\Sigma(x_i-\mu)^2 = \frac{n}{\cancel{2\sigma^2}}$$

$$\text{or, } \Sigma(x_i-\mu)^2 = n\sigma^2$$

$$\Rightarrow \boxed{\sigma^2 = \frac{\Sigma(x-\mu)^2}{n}}$$

To estimate μ:

$$\frac{d \log}{d\mu} = -\frac{2}{2\sigma^2} \sum (x-\mu) \times (-1) = 0$$

$$\Rightarrow \sum (x-\mu) = 0$$

$$\Rightarrow \sum x - n\mu = 0$$

$$\Rightarrow \sum x = n\mu$$

$$\Rightarrow \boxed{\mu = \frac{\sum x}{n}}$$

MME:

$\bar{x} = \frac{1}{n} \sum x_i$ is the 1st moment

2nd moment $\rightarrow \frac{1}{n} \sum x_i^2$

~~At~~ we know, $E(x) = \mu$

$$\Rightarrow \mu = \frac{\sum x_i}{n} = \bar{x}$$

var, $\sigma^2 = E[x^2] - (E(x))^2$

$$= E[x^2] - \mu^2$$

$$\therefore E[x^2] = \frac{1}{n} \sum x_i^2$$

$$\therefore \boxed{\mu^2 = \bar{x}^2} \text{ — (i)}$$

$$\Rightarrow \boxed{\hat{\sigma}^2_{MME} = \frac{1}{n} \sum x_i^2 - \bar{x}^2} \text{ — (ii)}$$

g) Gamma Distribution:

the Gamma Distribution as follows -

$$f_x(x \mid \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x_i^{\alpha-1} e^{-\beta x_i} \; ; \; x_i > 0$$

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} \, dt = (\alpha-1)! \; ; \; \alpha \in \mathbb{N}$$

→ Liklihood:

$$L = \prod_{i=1}^{n} \left(\frac{\beta^\alpha}{\Gamma(\alpha)} x_i^{\alpha-1} e^{-\beta x_i} \right)$$

$$= \left[\frac{\beta^\alpha}{\Gamma(\alpha)} \right]^n \times \left(\prod_{i=1}^{n} x_i \right)^{\alpha-1} \times e^{-\beta \sum_{i=1}^{n} x_i}$$

→ log liklihood:

$$\log L = \log \left(\frac{\beta^{\alpha n}}{\Gamma(\alpha)^n} \times \left(\prod_{i=1}^{n} x_i \right)^{\alpha-1} e^{-\beta \sum x_i} \right)$$

$$= \log \left(\frac{\beta^{\alpha n}}{\Gamma(\alpha)^n} \right) + \log \left(\prod_{i=1}^{n} x_i \right) - \beta n \bar{x}$$

$$= \alpha n \log \beta - n \log(\Gamma(\alpha)) + (\alpha-1) \sum \log(x_i) - \beta n \bar{x}$$

$$\therefore \frac{\partial \log(L)}{\partial \beta} = \frac{\alpha n}{\beta} - n\bar{x} = 0$$

or, $\frac{\alpha n}{\beta} = n\bar{x}$

or, $\boxed{\beta = \frac{\alpha}{\bar{x}}}$

→ MME: $E(x) = \frac{\alpha}{\beta}$, $var(x) = \frac{\alpha}{\beta^2}$

Sample mean: $\bar{x}$, Sample var. $= S^2$

$$\therefore S^2 = \frac{\beta \bar{x}}{\beta^2} = \frac{\bar{x}}{\beta} \Rightarrow \beta = \frac{\bar{x}}{S^2}$$

$$\therefore \boxed{\alpha = \beta \bar{x} = \frac{\bar{x}^2}{S^2}}$$

h) <u>Uniform Distribution:</u>

the uniform Distribution as follows,

$$f(x \mid a, b) = \frac{1}{b-a}$$

→ <u>liklihood:</u>

$$L = \prod_{i=1}^{n} \cancel{fo} \frac{1}{b-a}$$

$$= \frac{1}{(b-a)^n}$$

→ <u>log liklihood</u> –

$$\log L = \log\left[\frac{1}{(b-a)^n}\right]$$

$$= \log(1) - n\log(b-a)$$

$$= -n\log(b-a)$$

→ partial derivative with respect to a.

$$\frac{\partial \log L}{\partial a} = \cancel{n\log} \frac{n}{(b-a)} = 0$$

→ partial derivative with respect to b.

$$\frac{\partial \log L}{\partial b} = -\frac{n}{(b-a)}$$

noticed that derivative wrt. a is monotonically increasing, MLE of a is $\min(x_1, x_2, x_3 \cdots x_n)$

for b it is monotonically decreasing,

∴ MLE of b is $\max(x_1, x_2, x_3 \cdots x_n)$

MME:

the 1st moment is, $\int_a^b x\, f(x)\, dx = \int_a^b \frac{x}{b-a}\, dx$

$$= \frac{1}{2} \frac{b^2-a^2}{b-a}$$

$$= \frac{b+a}{2}$$

the 2nd moment is,

$$\int_a^b x^2 f(x)\, dx = \int_a^b \frac{x^2}{b-a}\, dx = \frac{1}{3} \frac{b^3-a^3}{b-a}$$

$$= \frac{b^2-ba+a^2}{3}$$

equate sample moment with population moment

$$M1 = \frac{x_1+x_2+x_3+\cdots+x_n}{n} = \bar{x} = \frac{b+a}{2} \quad \text{or, } b = 2\bar{x}-a$$

$$M2 = \frac{x_1^2+x_2^2+x_3^2+\cdots+x_n^2}{n} = \frac{b^2-ba+a^2}{3}$$

$$\text{or, } M2 = \frac{1}{3}\left[4\bar{x}^2-2a\bar{x}+a^2\right]$$

$$\Rightarrow 3M2 = a^2-2a\bar{x}+4\bar{x}^2$$

$$\Rightarrow a^2-2a\bar{x}+(4\bar{x}^2-3M_2) = 0$$

$$\therefore a = \frac{2\bar{x} \pm \sqrt{(-2\bar{x})^2-4(4(\bar{x})^2-3M2)}}{2}$$

Let $s^2 = M_2-\bar{x}^2$ (sample variance)

then,

$$\boxed{\begin{aligned} a_{MME} &= \bar{x}-\sqrt{3s^2} \\ b_{MME} &= \bar{x}+\sqrt{3s^2} \end{aligned}}$$

2) Let $x_1, x_2, \cdots x_n$ are r.s from the pdf, $f(x,\theta) = \frac{1}{2\theta} e^{-|x|/\theta}$; $-\infty < x < \infty$

given, $H_0 : \theta = 1$

$H_1 : \theta = 2$

<u>Liklihood:</u>

$$\frac{L(\theta=1)}{L(\theta=2)} = \frac{\prod_{i=1}^{n} \frac{1}{2} e^{-|x|}}{\prod_{i=1}^{n} \frac{1}{4} e^{-|x|/2}}$$

$$= \frac{\left(\frac{1}{2}\right)^n e^{-\Sigma|x|}}{\left(\frac{1}{4}\right)^n e^{-1/2 \Sigma |x_i|}}$$

$$= \left(\frac{1}{2^n}\right) 4^n e^{-\Sigma|x_i| + \frac{1}{2}|\Sigma x_i|}$$

$$= 2^n e^{-1/2 \Sigma |x_i|}$$

rejects H_0 when,

$$2^n e^{1/2 \Sigma |x_i|} < R$$

$$\Rightarrow e^{-1/2 \Sigma |x_i|} < \frac{R}{2^n}$$

take log →

$$-\frac{1}{2}\Sigma|x_i| \log e < \log(R) - n \log 2$$

$$\Rightarrow -\frac{1}{2}\Sigma|x_i| < \log(R) - n\log 2 \quad \boxed{\Sigma|x_i| > C}$$

(C is a consts)

The best critical region for testing $H_1 : \theta = 2$ is of the ~~for~~ form -

reject H_0 if $\sum_{i=1}^{n} |x_i| > C$

for some critical value c determined by the significance level α.

3) x_i Melting Point	f_i Frequency	$x_i f_i$	$(x-\bar{x})^2$
320	5	1600	0.04
326	1	326	33.64
325	2	650	23.04
318	3	954	4.84
322	3	966	3.24
329	3	987	77.44
317	3	951	10.24
316	2	632	17.64
331	1	331	116.6
308	2	616	148.8
321	1	321	0.64
319	2	638	1.44
335	1	335	219.04
313	2	626	51.84
327	2	654	46.24
314	3	942	38.44
323	2	646	7.84
324	4	1296	14.44
305	1	305	231
328	2	956	60.84
330	1	330	96.04
310	2	620	104.04
312	1	312	67.24
311	1	311	84.64
$\sum_{i=1}^{24}$	50	16005	2232.5

$\therefore$ mean $\bar{x} = \dfrac{\sum_{i=1}^{24} x_i f_i}{\sum_{i=1}^{24} f_i} = \dfrac{16005}{50} = \cancel{338.42}\ 320.2$

$\sigma = \sqrt{\frac{1}{49}(2232.5)} \Rightarrow \sqrt{43.21} = 6.75$

devide the dataset in 5 intervals —

where, min = 305 max = 335

no of bins → $k = 1 + \log_2(n)$

or, $k = 1 + 3.32 \log_{10}(50)$

or, $k = 1 + 3.32 \times 1.699$

$= 6.64 \approx 7$ bins

length of each interval $\dfrac{\text{max} - \text{min}}{\text{no of bins}}$

$\Rightarrow \dfrac{30}{7} \Rightarrow 4.29 \approx 5$

Interval	observed frequency	Expected frequency
305 – 309	3	$1.84 \approx 2$
310 – 314	9	$5.86 \approx 6$
315 – 319	10	$10.64 \approx 11$
320 – 324	15	$14.3 \approx 14$
325 – 329	10	$6.96 \approx 7$
330 – 334	2	$2.55 \approx 3$
335 – 339	1	$0.5495 \approx 1$

H0 : Normal dist. is good fit for the dataset

H1 : Normal dist. is not a good fit for the dataset

calculate expected frequencies :

(i) 305 – 309

a = 305 b = 309

$Z_a = \dfrac{305 - 320.2}{6.75} = -2.24$

$Z_b = \dfrac{309 - 320.1}{6.75} = -1.64$

$\phi(Z_b) - \phi(Z_a)$

$\rightarrow 0.050 - 0.0125$

$= 0.0379$

$\therefore Ef \Rightarrow 0.0379 \times 50$

$\Rightarrow 1.84$

ii) $\underline{310 - 314}$

$a = 310 \quad b = 314$

$Z_a = \dfrac{310 - 320.2}{6.75} = -1.496$

$Z_b = \dfrac{314 - 320.1}{6.75} = -0.909$

$\phi(-1.496) - \phi(-0.909)$

$\Rightarrow 0.117$

$Ef \Rightarrow 50 \times 0.117$

$\Rightarrow 5.86$

iii) $\underline{315 - 319}$

$a = 315 \quad b = 319$

$Z_a = \dfrac{315 - 320.2}{6.75} = -0.76$

$Z_b = \dfrac{315 - 320.2}{6.75} = -0.16$

$\phi(-0.76) - \phi(-0.16)$

$\Rightarrow 0.212$

$\therefore Ef = 0.212 \times 50$

≈ 10.641

iv) $\underline{320 - 324}$

$a = 320 \quad b = 324$

$Z_a = \dfrac{320 - 320.2}{6.75} = -0.015$

$Z_b = \dfrac{324 - 320.2}{6.75} = 0.6$

$\phi(-0.015) - \phi(0.6)$

$\Rightarrow 0.28$

$\therefore Ef : 50 \times 0.28 = 14$

v) $\underline{325 - 329}$

$a = 325 \quad b = 329$

$Z_a = \dfrac{325 - 320.2}{6.75} = 0.73$

$Z_b = \dfrac{329 - 320.2}{6.75} = 1.32$

$\phi(0.73) - \phi(1.32)$

$\Rightarrow 0.139$

$Ef = 0.139 \times 50 = 6.96$

vi) $\underline{330 - 334}$:

$a = 330 \quad b = 334$

$Z_a = \dfrac{330 - 320.2}{6.75} = 1.47$

$Z_b = \dfrac{334 - 320.2}{6.75} = 2.06$

$\phi(1.47) - \phi(2.06)$

$\Rightarrow 0.0510$

$Ef \Rightarrow 0.051 \times 50$

$\Rightarrow 2.554$

vii) $\underline{335 - 339}$

$a = 335, \quad b = 339$

$Z_a = \frac{335 - 320.2}{6.75} = 2.21$

$Z_b = \frac{339 - 320.2}{6.75} = 2.8$

$\phi(2.21) - \phi(2.8)$

$\Rightarrow 0.0109$

$Ef \rightarrow 0.0109 \times 50$

$\rightarrow 0.549$

Now, expected frequency sum $41 < 50$

merged bins where expected frequency < 5

Adjusted Interval	Observed frequency	Expeted frequency
305 - 314	12	8
315 - 319	10	11
320 - 329	15	14
325 - 339	13	11

$\therefore$ chi square test statistic -

$$\Sigma \frac{(Ob_i - E_i)^2}{E_i}$$

$$\Rightarrow \frac{(12-8)^2}{8} + \frac{(10-11)^2}{11} + \frac{(15-14)^2}{14} + \frac{(13-11)^2}{11}$$

$$\Rightarrow 2.53 = \chi^2$$

Degree of freedom, $4(\text{intervals}) - 1 - 2$

$\Rightarrow 4 - 1 - 2 = 1$

Critical value -

$$\chi^2_{0.05, 1} = 3.841$$

$\therefore \chi^2 < \chi^2_{0.05, 1}$

$\therefore$ Accept null hypothesis, so the dataset follows normal Distribution.

P-value :

at $\alpha = 0.05$, dof = 1 right tailed χ^2 :-

2.53 lies between (0.016, 2.706)

$\rightarrow$ $0.016 < 2.53 < 2.706$

$0.1 < \text{p-value} < 0.9$

as $\alpha = 0.05 < \text{P value} \Rightarrow$ fail to reject Ho

$\therefore$ So, data follows a normal Distribution.