Discrete Probability Distributions

- It may be mentioned that a theoretical probability distributions gives us a law according to which different values of the random variable are distributed with specified probabilities according to some define law which can be expressed mathematically.

- It is possible to formulate such laws either on the basis of given conditions (a prior considerations) or on the basis of the results (a posterior inferences) of an experiment.

- The present study will also enable us to fit a mathematical model or a function of the form $\boldsymbol{y = p(x)}$ to the observed data.

Bernoulli Distribution Function

Definition: A random variable $\boldsymbol{X}$ is said to have a Bernoulli distribution with parameter $\boldsymbol{p}$ if its probability mass function is given by:

$$P(X = x) = \begin{cases} p^x\,(1-p)^{1-x} & ; \quad for \quad x = 0, 1 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Where p satisfies $0 \leq p \leq 1$ and $(1-p) = q$

A random experiment whose outcomes are of two types, success ***S*** and Failure ***F***, occurring with probabilities ***p*** and ***q*** respectively, is called a ***Bernoulli trial***. If for this experiment, a *r.v.* *X* is defined such that it takes value *1* when *S* occurs and *0* if *F* occurs, then *X* follows a *Bernoulli distribution*.

Moments

$$\mu'_r = E(X^r) = \sum_{x=0,1} X^r P(X = x_i) = 0^r \cdot q + 1^r \cdot p \quad = \; p$$

$$\mu'_1 = E(X^1) = p \qquad\qquad \mu'_2 = E(X^2) = p$$

$$\mu_2 = \; Var(X) = \; \mu'_2 - (\mu'_1)^2 = \; p - \; p^2 = pq$$

$$M_X(t) = E(e^{tX}) \quad = \; e^{t\times 0}\, P(X = 0) + \; e^{t\times 1}\, P(X = 1)$$

$$M_X(t) = \; q + pe^t$$

Binomial Distribution Function

Consider a set of $\boldsymbol{n}$- independent Bernoulli trials(n being finite) in which the probability '$\boldsymbol{p}$' of success in any trial is constant for each trial, then $q = 1 - p$ is the probability of failure in any trial.

$$S\ F\ S\ S\ F\ F\ S\ S\ \ldots\ldots\ldots\ldots F\ S\ F\ S\ S\ F\ F\ S\ F\ S\ F\ S\ F\ S$$

$$P(S\ F\ S\ S\ F\ F\ S\ S\ \ldots\ldots F\ S\ F\ S\ ) = P(S)P(F)P(S)\ldots\ldots\ldots\ldots.P(F)P(S)$$

$$= p\ .\ q\ .\ p\ .\ p\ldots\ldots\ldots q.\ p$$

$$= \underbrace{p\ .\ p\ .\ p\ .\ \ldots.p}_{\{x\ -factors\}}\ \ .\ \ \underbrace{q.q.q\ \ldots.q}_{\{(\mathrm{n}-x)\ -factors\}}$$

$$= p^x\ q^{n-x}$$

But $\boldsymbol{x} -$ successes in $\boldsymbol{n}$- trails can occur in $\binom{n}{x}$ ways and the probability for each of these ways is same…*i.e.*, $p^x\ q^{n-x}$.

Binomial Distribution Function

Definition: A r.v. ***X*** is said to follow binomial distribution if it assumes only non-negative values and its p.m.f. is given by:

$$P(X = x) = \begin{cases} \binom{n}{x} p^x q^{n-x} & ; \quad x = 0, 1,2 \ldots\ldots n \\ 0 & ; \quad \text{otherwise} \end{cases}$$

The two independent constants $\boldsymbol{n}$ and $\boldsymbol{p}$ in the distribution are known as the ***parameters*** of the distribution.

Physical conditions of B.D.:

1. Each trial results in two exhaustive and mutually disjoint outcomes, termed as ***success*** and ***failure***.
2. The number of trials ‘$\boldsymbol{n}$’ is finite.
3. The trials are independent to each other.
4. The probability of success ‘p’ is constant for each trial.

Example 1:

Ten coins are tossed simultaneously. Find the probability of getting at least seven heads.

Solution:

$$X = \text{No. of heads}, \; n = 10,$$

$$p = \text{ Probability of getting head} = \frac{1}{2}$$

$$q = \text{ Probability of not getting head} = \frac{1}{2} = 1 - p$$

The probability of getting $\boldsymbol{x}$ heads in a random throw of 10 coins is

$$P(X = x) = \binom{10}{x} \left(\frac{1}{2}\right)^{x} \left(\frac{1}{2}\right)^{10-x} = \binom{10}{x} \left(\frac{1}{2}\right)^{10} ; \quad X = 1, 2, \ldots, 10$$

$\therefore$ Probability of getting at least seven heads is given by,

$$P(X \geq 7) = p(7) + p(8) + p(9) + p(10)$$

$$= \left(\frac{1}{2}\right)^{10} \left\{\binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10}\right\}$$

$$P(X \geq 7) = \frac{176}{1024} = 0.17$$

Example

It is believed that approximately 75% of American youth now have insurance due to the health care law. A random sample of 15 American youth with private health insurance is taken in a survey.

(a) What is the probability that at least 10 ?

(b) What is the probability that at most 10 ?

(c) What is the probability that exactly 10 ?

of these youth have private health insurance?

Example 2

An irregular six-faced die is thrown and the expectation that in 10 throws it will give five even numbers is twice the expectation that it will give four even numbers. How many items in 10,000 sets of 10 throws each, would you expect it to give no even number.

Solution:

Let $\boldsymbol{p}$ be the probability of getting an even number in a throw of a die. Then, the probability of getting $\boldsymbol{x}$ even numbers in ten throws of a die is,

$$P(X = x) = \binom{10}{x}\, p^x q^{10-x}\ ; \quad X = 0, 1, 2, \dots, 10$$

Given , $$P(X = 5) = 2\, P(X = 4)$$

$$i.e., \quad \binom{10}{5}\, p^5 q^{10-5} = 2\, \binom{10}{4}\, p^4 q^{10-4}$$

$$\frac{p}{5} = \frac{2q}{6} = \frac{q}{3} \implies 8p = 5 \implies p = \frac{5}{8} \text{ and } q = \frac{3}{8}$$

$$\therefore \quad P(X = x) = \binom{10}{x}\, \left(\frac{5}{8}\right)^x \left(\frac{3}{8}\right)^{10-x}$$

Hence, the required number of times that in 10,000 sets of 10 throws each, we get no even number

$$= 10000 \times P(X = 0) = 10000 \times \left(\frac{3}{8}\right)^{10} = 1 \text{ (approx).}$$

Moments of B.D.

Non- Central moments: $\mu_1' = E(X) = \sum_{x=0}^{n} x \cdot \binom{n}{x} p^x q^{n-x} = np$

$$\mu_2' = E(X^2) = \sum_{x=0}^{n} x^2 \cdot \binom{n}{x} p^x q^{n-x} = n(n-1)p^2 + np$$

$$\mu_3' = E(X^3) = n(n-1)(n-2)p^3 + 3n(n-1)p^2 + np$$

$$\mu_4' = E(X^4) = n(n-1)(n-2)(n-3)p^4 + 6n(n-1)(n-2)p^3$$

$$+7\mathrm{n}(\mathrm{n}-1)p^2 + np$$

Central moments:

$$\mu_2 = V(x) = \mu_2' - (\mu_1')^2 = npq$$

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3 = npq(q-p)$$

$$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4 = npq\{1 + 3(n - 2pq\}$$

$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(1-2p)^2}{npq}$ and $\gamma_1 = \sqrt{\beta_1} = \frac{(1-2p)^2}{\sqrt{npq}}$

$\beta_2 = \frac{\mu_4}{\mu_2^2} = 3 + \frac{1-6pq}{npq}$ and $\gamma_2 = \beta_2 - 3 = \frac{1-6pq}{npq}$

Moment Generating Function of B.D.

Let $X \sim B(n, p)$,

$$M_X(t) = E(e^{tX}) = \sum_{x=0}^{n} e^{tx} \binom{n}{x} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \binom{n}{x} (pe^t)^x q^{n-x}$$

$$M_X(t) = (q + pe^t)^n$$

Recurrence relation for B.D.

$$P(X = x) = \binom{n}{x} p^x\, q^{n-x} \qquad\qquad P(X = x + 1) = \binom{n}{x+1} p^{x+1}\, q^{n-x-1}$$

$$\frac{\mathrm{p}(x+1)}{\mathrm{p}(x)} = \frac{\binom{n}{x+1} p^{x+1}\, q^{n-x-1}}{\binom{n}{x} p^x\, q^{n-x}} \Rightarrow \frac{n-x}{x+1} \cdot \frac{p}{q}$$

$$\mathrm{p}(x + 1) = \left\{\frac{n-x}{x+1} \cdot \frac{p}{q}\right\} \mathrm{p}(x)$$

$$\mathrm{p}(0) = \mathrm{q}^{\mathrm{n}} \quad ; \quad \bar{x} = np \implies \hat{p} = \frac{\bar{x}}{n}$$

The remaining probabilities can be obtained as:

$$\mathrm{p}(1) = [\mathrm{p}(x+1)]_{x=0} = \left\{\frac{n-x}{x+1} \cdot \frac{p}{q}\right\}_{x=0} \mathrm{p}(0)$$

$$\mathrm{p}(2) = [\mathrm{p}(x+1)]_{x=1} = \left\{\frac{n-x}{x+1} \cdot \frac{p}{q}\right\}_{x=1} \mathrm{p}(1)$$

$$\mathrm{p}(3) = [\mathrm{p}(x+1)]_{x=2} = \left\{\frac{n-x}{x+1} \cdot \frac{p}{q}\right\}_{x=2} \mathrm{p}(2)$$

Ex:3 - Seven coins are tossed and number of heads noted. The experiment is repeated 128 times and the following distribution is obtained:

No. of heads	0	1	2	3	4	5	6	7	**Total**
Frequencies	7	6	19	35	30	23	7	1	**128**

Fit a binomial distribution assuming that the *coin is unbiased*.

Solution:

(i)When the coin is unbiased:

In fitting Binomial distribution, first of all the mean and variance of the data are equated to $\boldsymbol{np}$ and $\boldsymbol{npq}$ respectively.

Then the expected frequencies are calculated from these values of $\boldsymbol{n}$ and $\boldsymbol{p}$.

Here $n = 7$ and $N = 128$.

When the coin is unbiased, $p = q = \frac{1}{2}, \ \left(\frac{\boldsymbol{p}}{\boldsymbol{q}} = \mathbf{1}\right)$

Now, $p(0) = q^n = \left(\frac{1}{2}\right)^7 = \frac{1}{128}$

$$f(0) = N\,q^n = 128\left(\frac{1}{2}\right)^7 = 1$$

Using the recurrence formula, the various probabilities, *viz.*, $p(1), p(2), \ldots$ can be easily calculated as shown below.

x	f	$\frac{n-x}{x+1}$	$\frac{n-x}{x+1} \cdot \frac{p}{q}$	Expected frequency $f(x) = N\,P(x)$
0	7	7	7	$f(0) = N\,P(0) = 1$
1	6	3	3	$f(1) = 1 \times 7 = 7$
2	19	$\frac{5}{3}$	$\frac{5}{3}$	$f(2) = 7 \times 3 = 21$
3	35	1	1	$f(3) = 21 \times \frac{5}{3} = 35$
4	30	$\frac{3}{5}$	$\frac{3}{5}$	$f(4) = 35 \times 1 = 35$
5	23	$\frac{1}{3}$	$\frac{1}{3}$	$f(5) = 35 \times \frac{3}{5} = 21$
6	7	$\frac{1}{7}$	$\frac{1}{7}$	$f(6) = 21 \times \frac{1}{3} = 7$
7	1			$f(7) = 7 \times \frac{1}{7} = 1$

Solution:

(ii) When the nature of the coin is not known,

$$\text{Mean} = np = \bar{X} = \frac{\sum_{i=1}^{n} f_i x_i}{N} = \frac{433}{128} = 3.3828 \quad and \quad n = 7$$

$$\therefore \quad p = \frac{\bar{X}}{n} = \frac{3.3828}{7} = 0.48326 \quad \text{and} \quad q = 1 - p = 0.51674$$

$$\boldsymbol{\frac{p}{q} = 0.9352}$$

Now, $p(0) = q^n = (0.51674)^7 = 0.009839$

$$f(0) = N\,q^7 = 128(0.51674)^7 = 1.2593 \cong 1$$

Using the recurrence formula, the various probabilities, *viz.*, $p(1), p(2), \ldots$ can be easily calculated as shown below.

x	f	$\frac{n-x}{x+1}$	$\frac{n-x}{x+1} \cdot \frac{p}{q}$	Expected frequency $f(x) = N\,P(x)$
0	7	7	6.5464	$f(0) = N\,P(0) = 1.2593 \cong 1$
1	6	3	2.8056	$f(1) = 1.2593 \times 6.5464 = 8.2438 \cong 8$
2	19	$\frac{5}{3}$	1.5586	$f(2) = 2.8056 \times 8.2438 = 23.129 \cong 23$
3	35	1	0.9352	$f(3) = 1.5586 \times 23.129 = 36.05 \cong 36$
4	30	$\frac{3}{5}$	0.5611	$f(4) = 0.9352 \times 36.05 = 33.715 \cong 34$
5	23	$\frac{1}{3}$	0.3117	$f(5) = 0.5611 \times 33.7155464 = 18.918 \cong 19$
6	7	$\frac{1}{7}$	0.1336	$f(6) = 0.3117 \times 18.918 = 5.897 \cong 6$
7	1			$f(7) = 0.1336 \times 5.897 = 0.788 \cong 1$

Poisson Distribution Function

(i) $\boldsymbol{n}$, the number of trials is indefinitely large, i.e., $\mathbf{n} \to \infty$.

(ii) $\boldsymbol{p}$, the constant probability of success for each trial is indefinitely small, i.e., $\boldsymbol{p} \to \mathbf{0}$.

(iii) $\boldsymbol{np = \lambda}$ is finite, where $\boldsymbol{\lambda} -$ is a positive real number.

Thus, $p = \frac{\lambda}{n}$ $\quad q = 1 - \frac{\lambda}{n}$

The probability of $\boldsymbol{x}$ successes in a series of $\boldsymbol{n}$ independent trials is

$$b(x; n, p) = \binom{n}{x} p^x \, q^{n-x} \quad ; \quad x = 0, 1,2 \ldots\ldots n$$

Poisson Distribution as a limiting form the B.D.

$$b(x; n, p) \quad = \binom{n}{x} p^x q^{n-x}$$

$$= \binom{n}{x} p^x (1-p)^{n-x} \quad (\because\ q = 1-p)$$

$$= \binom{n}{x} p^x (1-p)^n (1-p)^{-x}$$

$$= \binom{n}{k} \left(\frac{p}{1-p}\right)^x (1-p)^n$$

let $np = \lambda \Rightarrow p = \lambda/n$

$$= \frac{n(n-1)(n-2)\ldots.(n-x+1)}{x\,!} \frac{\left(\frac{\lambda}{n}\right)^x}{\left(1-\frac{\lambda}{n}\right)^x} \left(1-\frac{\lambda}{n}\right)^n$$

$$= \frac{n \cdot n\left(1-\frac{1}{n}\right) \cdot n\left(1-\frac{2}{n}\right) \ldots.\, n\left(1-\frac{x-1}{n}\right)}{x\,!\left(1-\frac{\lambda}{n}\right)^x} \frac{\lambda^x}{n^x} \left(1-\frac{\lambda}{n}\right)^n$$

Limiting form of the B.D. under the above conditions:

As $\quad n \to \infty \ \Rightarrow \frac{1}{n} \to 0$ and $\left(1-\frac{\lambda}{n}\right)^n \to e^{-\lambda}$

$$\lim_{\substack{n \to \infty \\ p \to 0}} b(x; n, p) = \frac{e^{-\lambda}\lambda^{x}}{x\,!} \qquad ; \quad x = 0, 1,2 \; \ldots\ldots \infty$$

Definition: A r.v. X is said to follow ***Poisson distribution*** if it assumes only non-negative values and its p.m.f. is given by:

$$P(X=x)=\begin{cases}\dfrac{e^{-\lambda}\lambda^{x}}{x\,!} & ; \quad for \quad x=0,1,2\,\ldots\ldots\infty; \quad \lambda>0 \\ 0 & ; \quad otherwise\end{cases}$$

λ - is the parameter of the distribution.

1. Poisson distribution occurs when there are events which do not occur as outcomes of a definite number of trials of an experiment but which occur at random points of time and space where in our interest lies only in the number of occurrences of the event, not in the non-occurrences.
2. **Some instances**:
   (i) Number of deaths from a disease.
   (ii) Number of defective materials in a packing manufactured by a good concern.
   (iii) Number of air accidents in a unit of time.
   (iv) Number of printing mistakes at each page of the book.
   (v) The emission of radioactive particles.

Moments of Poisson D.F.

Non- Central moments: $\mu_1' = E(X) = \lambda$

$$\mu_2' = E(X^2) = \lambda^2 + \lambda$$

$$\mu_3' = E(X^3) = \lambda^3 + 3\lambda^2 + \lambda$$

$$\mu_4' = E(X^4) = \lambda^4 + 6\lambda^3 + 7\lambda^2 + \lambda$$

Central moments:

$$\mu_2 = V(x) = \mu_2' - (\mu_1')^2 = \lambda$$

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3 = \lambda$$

$$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4 = 3\lambda^2 + \lambda$$

$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{1}{\lambda}$ and $\gamma_1 = \sqrt{\beta_1} = \frac{1}{\sqrt{\lambda}}$

$\beta_2 = \frac{\mu_4}{\mu_2^2} = 3 + \frac{1}{\lambda}$ and $\gamma_2 = \beta_2 - 3 = \frac{1}{\lambda}$

M.G.F. of Poisson Distribution Function

$$M_X(t) = E(e^{tX}) = \sum_{x=0}^{\infty} e^{tx}\, P(X = x)$$

$$= \sum_{x=0}^{\infty} e^{tx}\, \frac{e^{-\lambda}\lambda^x}{x\,!}$$

$$= e^{\lambda(e^t - 1)}$$

Example

For the case of the thin copper wire, suppose that the number of flaws follows a Poisson distribution with a mean of 2.3 flaws per millimeter. Determine the probability of exactly 2 flaws in 1 millimeter of wire.

Also find the probability of 10 flaws in 5 millimeters of wire.

Solution

Let X denote the number of flaws in 1 millimeter of wire.

Then, $E(X) = \lambda = 2.3$ and probability of exactly 2 flaws in 1 millimeter of wire is

$$P(X = 2) = \frac{e^{-2.3}\ 2.3^2}{2\,!} = 0.265$$

Determine the probability of 10 flaws in 5 millimeters of wire.

Let Y denote the number of flaws in 5 millimeters of wire. Then, Y has a Poisson distribution with

$$E(X) = \lambda = 5mm \times 2.3 flaws/mm = 11.5\ flaws$$

$$\therefore \qquad P(Y = 10) = \frac{e^{-11.5}\ 11.5^{10}}{10\,!} = 0.113$$

Example

If the average number of claims handled daily by an insurance company is 5, what proportion of days have less than 3 claims? What is the probability that there will be 4 claims in exactly 3 of the next 5 days? Assume that the number of claims on different days is independent.

Solution:

Because the company probably insures a large number of clients, each having a small probability of making a claim on any given day, it is reasonable to suppose that the number of claims handled daily, call it X, is a Poisson random variable.

Since $E(X) = 5$,

the probability that there will be fewer than 3 claims on any given day is

$$P\{X < 3\} = P\{X = 0\} + P\{X = 1\} + P\{X = 2\}$$

$$= e^{-5} + e^{-5}\frac{5^1}{1!} + e^{-5}\frac{2}{2!}$$

$$P\{X < 3\} \approx 0.1247$$

Since any given day will have fewer than 3 claims with probability .125, it follows, from

the law of large numbers, that over the long run 12.5 percent of days will have fewer than 3 claims.

It follows from the assumed independence of the number of claims over successive days that the number of days in a 5-day span that has exactly 4 claims is a binomial random variable $\boldsymbol{Y}$, with parameters $\boldsymbol{n} = \mathbf{5}$ and

$\boldsymbol{p} = \boldsymbol{P}\{\boldsymbol{X} = \mathbf{4}\}$.

$$P\{X = 4\} = e^{-5}\frac{5^4}{4\,!} \approx 0.1755$$

it follows that the probability that 3 of the next 5 days will have 4 claims is,

$$Y \sim B(5, 0.1755)$$

$$\mathrm{P(Y = 3)} = \binom{5}{3}(0.1755)^3(0.8245)^2 = 0.0367$$

Example

It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in a packet of 20, find the number of packets containing at least, exactly and at most 2 defective items in a consignment of 1000 packets using (i) Binomial distribution and (ii) Poisson approximation.

Example

Find the probability of 5 or more telephone calls arriving in a 9 min period in switch-board, if the telephone calls that are received at the rate of 2 calls in every 3 minute period.

Recurrence relation for Poisson Distribuiton

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x\,!} \quad ; \quad x = 0, 1, 2, \dots \dots \dots, \infty$$

$$P(X = x + 1) = \frac{e^{-\lambda}\lambda^{x+1}}{(x+1)\,!} \quad ; \quad x = 0, 1, 2, \dots \dots \dots, \infty$$

$$\therefore \quad \frac{\mathrm{p}(x+1)}{\mathrm{p}(x)} = \frac{\lambda}{(x+1)}$$

$$\mathrm{p}(x + 1) = \left\{\frac{\lambda}{x+1}\right\} \mathrm{p}(x)$$

$$\mathrm{p}(0) = e^{-\lambda} \quad \text{or} \quad \bar{x} = \lambda$$

The remaining probabilities can be obtained as:

$$\mathrm{p}(1) = [\mathrm{p}(x+1)]_{x=0} = \left\{\frac{\lambda}{x+1}\right\}_{x=0} \mathrm{p}(0)$$

$$\mathrm{p}(2) = [\mathrm{p}(x+1)]_{x=1} = \left\{\frac{\lambda}{x+1}\right\}_{x=1} \mathrm{p}(1)$$

$$\mathrm{p}(3) = [\mathrm{p}(x+1)]_{x=2} = \left\{\frac{\lambda}{x+1}\right\}_{x=2} \mathrm{p}(2)$$

and so on.

Expected frequencies:

$$f(0) = N \times \; P(X = 0)$$

$$f(1) = N \times \; P(X = 1)$$

$$f(2) = N \times \; P(X = 2)$$

and so on..

Example

After correcting 50 pages of the proof of a book, the proof reader finds that there are on the average, 2 errors per 5 pages. How many pages would one expect to find with 0, 1, 2, 3 and 4 errors in 1000 pages of the first print of the book ?

Let, $\boldsymbol{X}$ – denote the number of errors per page.

Average number of errors per page is given by:

$$\lambda = \frac{2}{5} = 0.4$$

$$\therefore \quad X \sim P(0.4) \;\Rightarrow\; P(X = x) = \frac{\boldsymbol{e^{-\lambda}\lambda^{x}}}{\boldsymbol{x}\,!} \quad ; \quad \boldsymbol{x = 0, 1, 2, \ldots \ldots \ldots}$$

Expected number of pages with $\boldsymbol{X} -$ errors per page in a book of 1000 pages are :

$$1000 \times P(X = x) = 1000 \times \frac{e^{-0.4}(0.4)^{x}}{x\,!} \quad ; x = 0, 1, 2, \ldots \ldots$$

Using the recurrence relation of P.D.,

No. of errors per page (X)	**Probability $P(X = x)$**	**Expected number of pages $1000 \times p(x)$**
0	$p(0) = e^{-0.4} = 0.6703$	$670.3 \simeq 670$
1	$p(1) = \left\{\frac{0.4}{0+1}\right\} p(0) = 0.26812$	$268.12 \simeq 268$
2	$p(2) = \left\{\frac{0.4}{1+1}\right\} p(1) = 0.05362$	$53.624 \simeq 54$
3	$p(3) = \left\{\frac{0.4}{2+1}\right\} p(2) = 0.007129$	$7.1298 \simeq 7$
4	$p(4) = \left\{\frac{0.4}{3+1}\right\} p(3) = 0.00071298$	$0.71298 \simeq 1$

Example Problems

1. Past Experience reveals that Mr. Rana can hit the target 3 times out of 5 shots. Find the probability of hitting the target,
   (i) 4 out of 6 shots. [Ans:0.31104]
   (ii) at least 2 times out of six shots. [Ans:0.959]
   (iii) not more than once out of six shots. [Ans:0. 0409]

2. A box contains 20 capacitors, 4 of which are defective. Five capacitors are selected randomly. What is the chance that,
   (i) 3 out of 5 selected capacitors are defective? [Ans:0.0512]
   (ii) There is no defective capacitors among these 5? [Ans:0.3276]
   (iii) There is at least one defective? [Ans:0.6723]

3. The fist print of a new book consisting of 750 pages has on an average 1 error in five pages. Find the number of pages which have 0, 1, 2, 3 errors per page in the whole book.

Ans:

X	0	1	2	3
No. of Pages	614	123	12	1

4. A razor blade manufacturing company has a chance of 1 defective blade in 500 blades. The blades are marketed in packets of 5 blades. One hundred packets are supplied to retailer. Find the number of packets which are likely to have *no*, 1 or 2 defective blades.

Ans:

X	0	1	2
No. of Packets	99	1	0

5. Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant. Find the probability that in the next 18 samples, (i) exactly 2 contain the pollutant, (ii) atleast four samples contain the pollutant. [Ans: 0.284, 0.098]