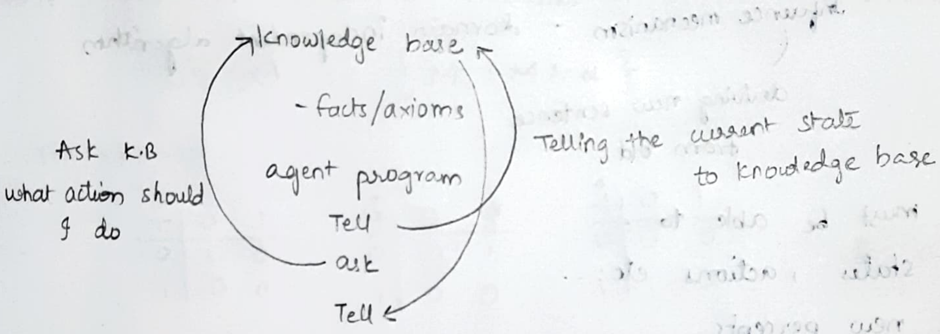
MODULE : 04 KNOWLEDGE REPRESENTATION AND REASONING

Logical Agents :

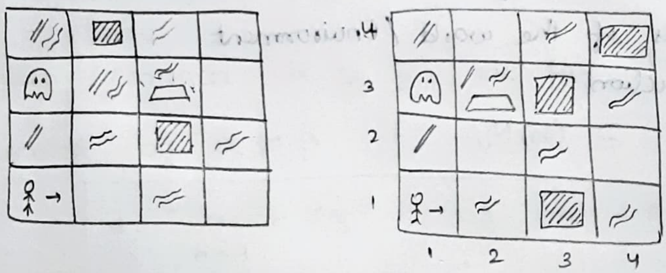
Agents with some representation of complex knowledge about the world / its environment & uses inference to derive new information from the knowledge combined with new inputs

Knowledge base :

set of sentences in a formal language representing facts about the world.



WUMPUS World :



1. Stench
2. Breeze
3. Glitter
4. Bump
5. Scream

at 1,1 → $\langle -, -, -, -, - \rangle$, $\langle N, N, N, N, N \rangle$

2,1 → $\langle -$, Breeze, $-, -, - \rangle$, $\langle N$, Breeze, $N, N, N \rangle$

Logic

- Propositional Logic, Syntax / Semantics = 'meaning

- sentences : " John is eating an Apple "

  " There is a real no. whose square is negative "

  " 1+1 = 2 "

  " 1+1 = 0 "

⇒ A model of α, M is a structure where α is true

NOTE : $M \models \alpha$ read : M satisfies α or M is a model of α

$$\mathcal{M}(\alpha) := \{ M : M \models \alpha \}$$

For 1+1=0

$\dot{+}$	0	1
0	1	0
1	0	0

$\dot{+}$	0	1
0	0	1
1	0	0

$\dot{+}$	0	1
0	0	0
1	1	0

.. etc are models

$$M = \langle \{0,1\} , \dot{+} \rangle$$

⇒ Given sentences α & β, we say that " α entails β "

denoted by $\alpha \models \beta$ if $\mathcal{M}(\alpha) \subseteq \mathcal{M}(\beta)$.

α : " percept seq. at (2, 1) is $\langle$ N, Breeze , N , N, N $\rangle$ "

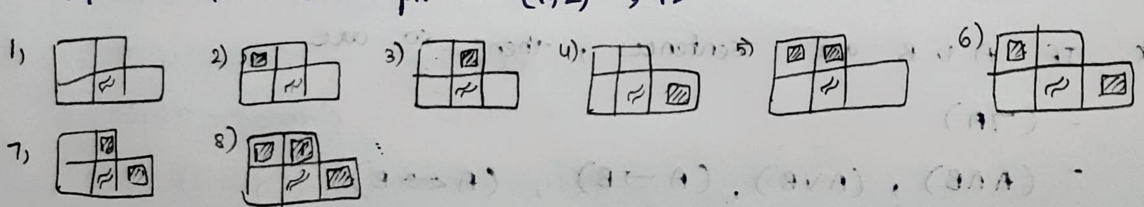
β : " There is a pit at (3,1) "
"There is a pit at (2,2) " or both

Knowledge-Base (K B)

→ $KB \models \beta$ iff $\mathcal{M}(KB) \subseteq \mathcal{M}(\beta)$

* K.B ⇒ " There is no breeze / stench / bump / glitter / scream at (1,1) "
and " There is only a breeze at (2,1) ".

β_1 : " There is no pit at (1,2) " , β_2 : " There is no pit at (2,2) ".



K.B : { (3) , (4), (7) }

β_1 : { (1) , (3) , (4) , (7) }

β_2 : { (1) , (2) , (4), (6) }

$\therefore \mathcal{M}(KB) \subseteq \mathcal{M}(\beta_1)$

but $\mathcal{M}(K.B) \nsubseteq \mathcal{M}(\beta_2)$

→ Revised K.B : Previous K.B + "There is no pit at (1,2)".

* An inference algorithm i deriveds α from K.B denoted by

→ $K \vdash_i \alpha$

Syntactical inference $\vdash$

completeness ↑ ↓ soundness

Semantic entailment $\models$

Grounding : "Is K.B true in real world?"

Godel learning :

PROPOSITIONAL LOGIC [Either True or False, not both]

propositional logic — Syntax / Semantic

* Propositional logic - syntax

1. Atomic sentences : Propositional variable denoted by : P, Q, $R_{1,2}$, South etc...

2. Compound / Complex sentences :

→ Negation : $\neg$ Ex: $(\neg P)$

→ Conjunction / and : $\wedge$ Ex: $(P \wedge Q)$

→ Disjunction / or : $\vee$ Ex: $(P \vee Q)$

→ material Implication / conditional : $\rightarrow$ Ex: $(P \rightarrow Q)$ "if P then Q"

If there is rain then road are wet.

→ Bicondition : $\leftrightarrow$ Ex: $(P \leftrightarrow Q)$ "P iff Q"

Def : A sentence is defined by the following rules, inductively or recursively.

* If P is a propositional variable then P is also a sentence.

* TRUE and FALSE are sentences

* If A & B are sentences, then so are

- $(\neg A)$

- $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$, $(A \leftrightarrow B)$.

are also sentences.

Q: Which of the following are sentences ?

1. $\neg P \wedge Q$ — ✗
2. $\neg(P \wedge Q)$ — ✗
3. $(\neg P) \wedge Q$ — ✗
4. $\wedge \neg P$ — ✗
5. $(Q \rightarrow (\neg P))$ ✓

* Propositional logic - Semantics.

A	$(\neg A)$
F	T
T	F

A	B	$(A \wedge B)$	$(A \vee B)$	$(A \rightarrow B)$	$(A \leftrightarrow B)$
F	F	F	F	T	T
F	T	F	T	T	F
T	F	F	T	F	F
T	T	T	T	T	T

Q: $((\neg((P_1 \vee Q) \rightarrow R)) \wedge Never)$ is a sentence ?

Ans: P_1 is a propositional variable - so P_1 is a sentence.
Similarly, Q, R, Never are P.V's - so they are sentences.
$(P_1 \vee Q)$ - sentence
$((P_1 \vee Q) \rightarrow R)$ - sentence
$(\neg((P_1 \vee Q) \rightarrow R))$ — sentence
$((\neg((P_1 \vee Q) \rightarrow R)) \wedge Never)$ — sentence.

Q: $(((\neg P) \vee South) \leftrightarrow (North \rightarrow East))$
P, South, North, East are P.Vs' so they are sentences.
$(\neg P) \rightarrow$ sentence
$((\neg P) \vee south)$ — Sentence
$(North \rightarrow East)$ — sentence
$(((\neg P) \vee south) \leftrightarrow (North \rightarrow East)) \rightarrow$ Sentence.

A truth Assignment / Valuation :

$\mathcal{P}$: set of all propositional Variables.

$\mathcal{S}$: set of all sentences

$$\mathcal{P} \subseteq \mathcal{S}$$

It is a function $v : \mathcal{P} \rightarrow \{T, F\}$

An extended truth Valuation / assignment :

with respect to v is a function

$\bar{v} : \mathcal{S} \rightarrow \{T, F\}$ defined inductively as follows :

- $\bar{v}(P) = v(P)$ if $P \in \mathcal{P}$
- $\bar{v}(\text{TRUE}) = T$, $\bar{v}(\text{FALSE}) = F$
- Let A & $B \in \mathcal{S}$

* $\bar{v}((\neg A)) = \begin{cases} T, & \text{if } \bar{v}(A) = F \\ F, & \text{if } \bar{v}(A) = T \end{cases}$

* $\bar{v}((A \wedge B)) = \begin{cases} T, & \text{if } \bar{v}(A) = \bar{v}(B) = T \\ F, & \text{if } \bar{v}(A) = \bar{v}(B) = F \text{ (or) otherwise} \end{cases}$

* $\bar{v}((A \vee B)) = \begin{cases} F, & \text{if } \bar{v}(A) = F \text{ and } \bar{v}(B) = F \\ T, & \text{otherwise} \end{cases}$

* $\bar{v}((A \rightarrow B)) = \begin{cases} F, & \text{if } \bar{v}(A) = T \text{ and } \bar{v}(B) = F \\ T, & \text{otherwise} \end{cases}$

* $\bar{v}((A \leftrightarrow B)) = \begin{cases} T, & \text{if } \bar{v}(A) = \bar{v}(B) \\ F, & \text{otherwise} \end{cases}$

Q: Suppose : $V : \mathcal{P} \to \{T, F\}$ given by

$$V(x) = \begin{cases} T, & \text{if } x = P \text{ or } x = North \\ F, & \text{if } x = East \\ T, & \text{if } x = John, \\ F, & \text{otherwise} \end{cases}$$

Find : $\bar{V}(((North \to East) \wedge (Q \leftrightarrow John)))$.

Ans:- North, East, Q & John are sentences.

- $\bar{V}(North) = T$ $\quad$ ($\because \bar{V}(North) = V(North)$ $\because North \in \mathcal{P}$)
- $\bar{V}(East) = F$
- $\bar{V}(Q) = F$
- $\bar{V}(John) = T$
- $\bar{V}(\underset{T}{North} \to \underset{F}{East}) = F$
- $\bar{V}(\underset{F}{Q} \leftrightarrow \underset{T}{John}) = F$
- $\bar{V}((North \to East) \wedge (Q \leftrightarrow John)) = F$

Q: $V : \mathcal{P} \to \{T, F\}$ given by

$$V(x) = \begin{cases} F, & x = P \\ T, & x = P_1 \\ F, & o\omega \end{cases}$$

Find i) $\bar{V}(((P \vee Queen) \to (\neg P)))$.

P, Queen, P_1 are sentences

$\bar{V}(P) = \bar{V}(P) = F$

$\bar{V}(Queen) = F$

$\bar{V}(P_1) = T$

$\bar{V}(\underset{F}{P} \vee \underset{F}{Queen}) = F$

$\bar{V}(\neg P_1) = F$

so $\bar{V}(((\underset{F}{P \vee Queen}) \to (\underset{F}{\neg P}))) =$ True

ii) $\bar{V}((TRUE \leftrightarrow Queen))$

TRUE & Queen are sentences

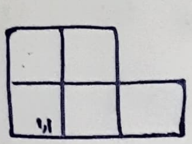
$\bar{V}(TRUE) = T$

$\bar{V}(Queen) = F$

so, $\bar{V}((TRUE \leftrightarrow Queen)) =$ False

$P_{m,n}$ = "There is a pit at (m,n)"

$B_{m,n}$ = "There is a breeze at (m,n)"



K.B :

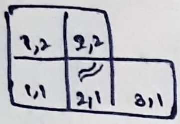
α_1 : "There is no pit at (1,2)" → $(\neg P_{1,2})$

$P_{1,1}$: "There is a pit at (1,1)" → $P_{1,1}$

$B_{1,1}$: "There is a breeze at (1,1)" → $B_{1,1}$

For 1st cell $((\neg P_{1,1}) \wedge (\neg B_{1,1}))$

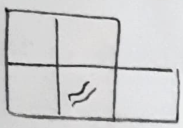
$2^5 = 32$ Models

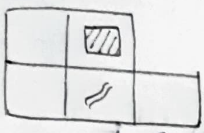


K.B: $B_{2,1}$, $((\neg P_{1,1}) \wedge (\neg B_{1,1}))$

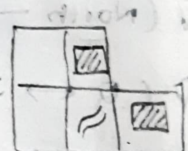
α_1 : "There is no pit at (1,2)" : $(\neg P_{1,2})$

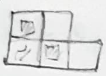
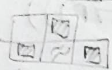
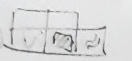
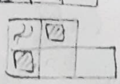
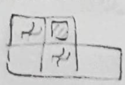
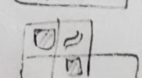
Models :

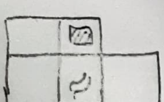
1) 

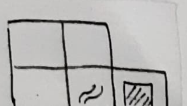
2) 

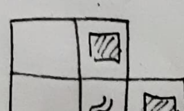
3) 

4) 

- $(\overset{F}{P_{1,1}} \rightarrow (B_{2,1} \wedge B_{1,2}))$  T
- $(\overset{F}{B_{1,1}} \rightarrow (P_{2,1} \vee P_{1,2}))$  → T
- $(\overset{F}{P_{2,1}} \rightarrow (B_{1,1} \wedge (B_{2,2} \wedge B_{3,1})))$  → T
- $(\overset{T}{B_{2,1}} \rightarrow (\overset{F}{P_{1,1}} \vee (\overset{T}{P_{2,2}} \vee \overset{T}{P_{3,1}})))$  → T
- $(\overset{T}{P_{3,1}} \leftrightarrow \overset{T}{B_{2,1}})$  → T
- $(\overset{F}{B_{3,1}} \leftrightarrow P_{2,1})$  → T
- $(\overset{F}{P_{1,2}} \rightarrow (B_{1,1} \wedge B_{2,2}))$  → T
- $(\overset{F}{B_{1,2}} \rightarrow (P_{2,2} \vee P_{1,1}))$  → T
- $(\overset{T}{P_{2,2}} \rightarrow (\overset{F}{B_{1,2}} \wedge B_{2,1}))$  → f
- $(\overset{F}{B_{2,2}} \rightarrow (P_{1,2} \vee P_{2,1}))$  → T

1) 

2) 

3) 

Basic Implications "implies" $A \Rightarrow B$: $(A \rightarrow B)$ is a tautology!

1. $(A \wedge B) \Rightarrow A$
2. $(A \wedge B) \Rightarrow B$

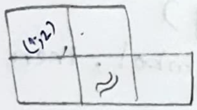
} simplification.

3. $A, B \rightarrow (A \wedge B)$
4. $A \Rightarrow (A \vee B)$
5. $B \Rightarrow (A \vee B)$

} conjunction

6. $(\neg A), (A \vee B) \Rightarrow B$. ~~(Modus ponens)~~ [Disjunctive syllogism]
7. $A, (A \rightarrow B) \Rightarrow B$ (Modus ponens)
8. $(\neg B), (A \rightarrow B) \Rightarrow (\neg A)$ (Modus tollens)
9. $(A \rightarrow B), (B \rightarrow C) \Rightarrow (A \rightarrow C)$ [Hypothetical syllogism]

Syntactic proof.



* $KB \vDash \alpha$ $(\alpha : (\neg P_{1,2})$

step	Statement	Rule
1.	$(B_{2,1} \wedge (\neg P_{2,1}))$	(P)
2.	$B_{2,1}$	1, (1)
3.	$(\neg P_{2,1})$	1, (2)
4.	$((\neg B_{1,1}) \wedge (\neg P_{1,1}))$	(P)
5.	$(\neg B_{1,1})$	4, (1)
6.	$(\neg P_{1,1})$	4, (2)
7.	$(B_{1,2} \rightarrow (P_{1,1} \vee P_{2,2}))$	(P)
8.	$(P_{1,2} \rightarrow (B_{1,1} \wedge B_{1,2}))$	
9.	$((\neg B_{1,1}) \vee (\neg B_{2,2}))$	5, (4)
10.	$(\neg (B_{1,1} \wedge B_{2,2}))$	9, (DM) Demorgan law
11.	$(\neg P_{1,2})$	10, 8 (MT)

$\{(\neg(B_{1,1} \wedge B_{2,2})), (P_{1,2} \rightarrow (B_{1,1} \wedge B_{2,2})) \Rightarrow (\neg P_{1,2})\}$

$\downarrow$

$(\neg P_{1,2})$

[Propositional Theorem proving]

FIRST ORDER LOGIC :

O : set of object symbols — constant symbols (O_c), Variable symbols (O_v)

R : set of relation symbols

F : set of function symbols

Language of Natural Numbers ($\mathcal{L}_N$) :

$O_c = \{0, 1\}$

$O_v = \{x_1, x_2, x_3 \ldots\}$

$R = \{\underbrace{\leq}_{2}\}$ Arity

$F = \{\underbrace{+}_{2}, \underbrace{\cdot}_{2}\}$

terms : (defined recursively)

- If c is a constant symbol, then c is a term
- If x is a variable symbol, then x is a term
- If f is an $\underbrace{n\text{-ary}}_{\text{arity is } n}$ function symbol, and $t_1, t_2, \ldots, t_n$ are terms then so is $f(t_1, t_2, t_3, \ldots, t_n)$

Q: Which of the following are terms in $\mathcal{L}_N$?

1. 1 ✓
2. 0 ✓
3. 2 ✗
4. $(0+1)$ ✓
5. $((0+1)\cdot(1\cdot(1\cdot 1)))$ ✓
6. $((0+1)\cdot(1\cdot(1\cdot 1))) \leq \$$ ✗ depends on functions not on relation dependencies.

5 Examples of terms.

1. $((1+0)+(1+1))$
2. $((1\cdot 1)\cdot(0\cdot 1))$
3. $+(1, 0)$
4. $\cdot(1, 1)$
5. $((1\cdot 1)+(0\cdot 1))$
6. $((x_1+x_2)+x_3)$

5 Examples of not terms

1. $(1+0) \leq 2$
2. $(1\cdot 1 + (1\cdot 0))$
3. $0\cdot x_1$
4. x_1+x_2
5. x_1+5

~~knowles~~ Language of king of England : $L_{K.E}$

O_c = { Richard, John, Crown, Richard's left leg, John's left leg }

O_v = { $x_1, x_2, x_3, \ldots\ldots$ }

R = { OnHead } ② ~~Arit~~ Arity ②

F = { Left leg }

Examples:

1. OnHead (Richard, Leftleg (John))
2. OnHead (Richard, Crown)
3. OnHead (X_1, John)
4. (Richard ~~(~~left leg (John's left leg)))
5. OnHead (X_1, X_2)

} Not terms.

Examples for terms.

1. Crown
2. (Crown ~~(~~left leg)
3. (left leg (left leg (x_1)))

ATOMIC SENTENCES :-

* If t_1 and t_2 are terms then ($t_1 = t_2$) is an atomic sentence
* If R is an n-ary relational symbol, $t_1; t_2, \ldots t_n$ are terms then ($R(t_1, t_2, \ldots t_n)$) is an atomic sentence.

Examples of Atomic Sentences

1. ((leftleg (John)) = (leftleg (Richard)))
2. (Onhead (Crown, John)) .
3. (Onhead (leftleg (x_1, x_2)) ~~)~~, x_2))
4. (Crown = X_1)
5. (Onhead (X_1, X_2))
6. (~~(~~ ((leftleg (John)) = (leftleg (Richard))))

Examples of Not Atomic Sentences

1. Onhead (leftleg (John), Crown);
2. Crown = X_1
3. leftleg (leftleg (Richard))
4. (Onhead (leftleg (John))
5. (Onhead (leftleg (Ril), leftleg (x_1))

SENTENCES (Recursively)

* If A is an Atomic sentence, then A is a sentence.
* If A & B are sentences then so are
  i) $(\neg A)$
  ii) $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$, $(A \leftrightarrow B)$.
* If X is a variable & A is a sentence then the following are sentences.

i) $(\forall x (A))$ $\forall \rightarrow$ for each / for any.

ii) $(\exists x (A))$ $\exists \rightarrow$ There Exists.

Ex:

1. $(\neg(\text{leftleg}(\text{John})) = (\text{leftleg}(\text{Richard})))$
2. $(\neg(\text{crown} = x_1))$
3. $((x_1 = \text{John}) \wedge (x_2 = \text{Richard}))$
4. $(\neg((\text{R'll} \vee \text{John'll}) \wedge (\text{Richard} \vee \text{John})))$
5. $((\text{crown}, \text{John}) \longrightarrow (x_1, x_2))$
6. $((\text{OnHead}(x_1, x_2)) \longrightarrow (x_1 = x_2))$

→ Let us modify the $L_{K.E}$.

O_c = {Richard, John, Crown, Richard's left leg, John's left leg}

O_v = {$x_1, x_2, x_3 \ldots x_n$}

R = {OnHead, Brother, King, person}

F = {left leg}.

Ex:

1. (Brother (John, Richard))
2. (King (leftleg (R'sll)))
3. $(\exists x (\text{Brother}(x, \text{Richard})))$
4. $(\exists x (\exists y (\text{Brother}(x, y))))$
5. $(\forall x ((\text{King}(x)) \rightarrow (\text{Person}(x))))$
6. $(\forall x ((\text{King}(x)) \wedge (\text{Person}(x))))$

H.W Language of People : Lpeople

O_c = { every single person }

O_v = $\{x_1, x_2, \ldots, x_n\}$

f = { }

R = { person, brother, sibling, parent, father, Mother, male, female, Caresfor }

person ①, brother ②, sibling ②, parent ②, father ②, Mother ②, male ①, female ①, Caresfor ②, sister ②

1. If X is a brother of Y then Y has to be brother of X

then define
- father interms of parent
- Mother interms of parent
- Brother interms of sibling
- Everyone cares for someone.
- sister in terms of sibling

→ ((brother (x, Y)) → (brother (Y, x)))

* father interms of parent

→ (∀x (∀Y ((father (x, Y)) ↔ ((parent (x, Y)) ∧ (Male (x))))))

* Mother interms of parent

→ (∀x (∀Y ((mother (x, Y)) ↔ ((parent (x, Y)) ∧ (female (x))))))

* Brother interms of sibling

→ (∀x (∀Y ((brother (x, Y)) ↔ ((sibling (x, Y)) ∧ (Male (x))))))

* Everyone cares for someone.

→ (∀x (∃Y ((person (x)) ↔ (caresfor (x, Y))))).

* sister interms of sibling

(∀x (∀Y ((sister (x, Y)) ↔ ((sibling (x, Y)) ∧ (female (x))))))

Ex. of ~~terms~~ atomic sentences
1. (person (X))
2. (brother (x_1, x_2))
3. (parent (x_1, x_2))
4. (Sibling (john, mary))
5. (female (mary))
6. (mother (x_1, mary))
7. (caresfor (john, mary))

Ex. of terms
1. John
2. ananya
3. x_1
4. x_2
5. potter

Ex: of Sentences.
1. ((sibling (john, mary)) = (sibling (x, Y)))
2. ((caresfor (x, Y)))
3. ((x_1 = John) → (x_2 = Richard))
4. (Parent(x_1, x_2) ∧ Male(x_1)) → (father (x_1, x_2)))

Write sentences in FOL over $\mathcal{L}_{people}$ for:

1. Richard cares for John.

   Caresfor (Richard, John).

2. Richard cares for all his children.

   $(\forall x(\text{Parent}(\text{Richard}, x)) \longrightarrow (\text{Caresfor}(\text{Richard}, x)))$

3. Everyone cares for someone.

   $(\forall x(\exists y(\text{caresfor}(x,y))))$

4. There is a person who is universally cared for.

   $(\exists x(\forall y(\text{Caresfor}(y,x))))$

5. Every parent cares for their child.

   $(\forall x(\forall y(\text{Parent}(x,y) \longrightarrow \text{Caresfor}(x,y))))$

Theory of Inference in FOL:

* $\exists x\ A(x)$

  for some $c \in \mathcal{O}_c$ $A(c)$

  for Existential Instantiation (EI)

  (to eliminate $\exists$)

for some $c \in \mathcal{O}_c$, $A(c)$

$\exists x\ A(x)$

Existential Generalization (EG)

(to introduce $\exists$)

* $\forall x\ A(x)$

  for any arbitary $c \in \mathcal{O}_c$, $A(c)$

  Universal Instantiation (UI)

  (To eliminate $\forall$)

* for each $c \in \mathcal{O}_c$, $A(c)$

  $\forall x\ A(x)$

  Universal Generalization (UG)

  (To introduce $\forall$)

Prove in FOL: All men are Mortal, Socrates is a man. ∴ Socrates is mortal.

Attempt: O_c : { c : c is a human being }
O_v : { $x_1, x_2 \ldots$ }
R : { Mortal, man }
   1 1
f : { }

K.B : $\forall x(Man(x) \rightarrow Mortal(x))$
Man(Socrates)
α : Mortal(socrates)

step		Sentence	Rule
1.		Man(Socrates)	(P)
2.		$\forall x\,(Man(x) \rightarrow Mortal(x))$	(P)
3.	for some Socrates $\in O_c$	$man(socrates) \rightarrow Mortal(socrates)$	2, (UI)
4.		Mortal(Socrates)	(T)[1,3,MP] Tautological Inference

Every person who took the AI course has pass the exam.

→ There is a person who took this course ~~has pass the exam~~. that did not read the book. Therefore, the person who did not read the book passed the exam.

Attempt: O_c = { c : c is a human being }
R = { TookAICourse, PassedExam, ReadBook }, f = { ϕ
   1 1 1

K.B : $\forall x(TookAICourse(x) \rightarrow PassedExam(x))$, $\exists x(TookAICourse(x) \land \neg ReadBook(x))$
α : $\exists x(\neg ReadBook(x) \land PassedExam(x))$

step		Sentence	Rule
1.		$\forall x(TAC(x) \rightarrow PE(x))$	(P)
2.		$\exists x(TAC(x) \land RB(x))$	(P)
3.	for some $c_0 \in O_c$	$TAC(c_0) \land \neg RB(c_0)$	2, E.I
4.	for some $c_0 \in O_c$	$TAC(c_0) \rightarrow PE(c_0)$	1, UI
5.		$TAC(c_0)$	(T) [3, Simplification]
6.		$\neg RB(c_0)$	(T) [3, Simplification]
7.		$PE(c_0)$	(T) [4, 5, MP]
8.		$\neg RB(c_0) \land PE(c_0)$	[6, 7, Conjunction]
9.		$\exists x(\neg RB(x) \land PE(x))$	[8, EG]

step		Sentence	Rule
1.		$\forall x(TAC(x) \rightarrow PE(x))$	(P)
2.		$\exists x(TAC(x) \land \neg RB(x))$	(P)
3.	for some $c_0 \in O_c$	$TAC(c_0) \rightarrow PE(c_0)$	[1, UI]
4.	for some $c_0 \in O_c$	$TAC(c_0) \land \neg RB(c_0)$	[2, EI]
5.		$TAC(c_0)$	[4, Simplification]
6.		$\neg RB(c_0)$	[4, Simplification]
7.		$PE(c_0)$	[3, 5, MP]
8.		$\neg RB(c_0) \land PE(c_0)$	[6, 7, Conjunction]
9.		$\exists x(\neg RB(x) \land PE(x))$	[8, EG]

Wrong
Not possible
Always apply EI before UI

* Always apply EI before UI.

Unification : term by term matching

UNIFY (knows (john, x), knows (john, mary))

= { x | mary }

UNIFY (knows (john, x), knows (Y, mary))

= { x | mary , Y | john }

UNIFY (knows (john, x), knows (Y, Z))

① = { Y | john , x | z } ✓ → Most General Unification (unifier)

② = { Y | john , x | mary , z | mary }

③ = { Y | john , x | john , z | john }

④ = { Y | john ; x | richard , z | richard }

UNIFY (knows (john, X), knows (mary, Y))

⇒ FAIL !!

UNIFY (knows (john, x), knows (x, mary))

⇒ FAIL !!

Forward Chaining in FOL :-

* The law states that It is a crime for an American to sell weapons to hostile nations.

* The Country NoNo, and enemy of America, has some missiles, and all of its missiles were sold to it by ~~Correct~~ colonel West, an American.

Goal / Conclusion : Colonel West is a criminal.

criminal (colonel West).

O_c : { America , Nono , missile , Colonel West }

O_v = { x_1, x_2 .... }

R = { criminal (1), enemy (2), hostile-nation (1), has_weapon (1), sell (3), American (1), weapon (1) }

(1) $\forall x \forall Y \forall z$ (American(x) ∧ Weapen (Y) ∧ Hostile (z) ∧ Sells(x, Y, z))

→ Criminal (x).

3 – 5 – 4 – M.P

(2) Enemy (Nono, America) ∧ Weapon (missile) ∧ has_missile (Nono)

→ Sells (Colonel west , missile , Nono) .

$\exists$ x (Enemy (Nono, America) ∧ ~~Weapon~~ Missile (X) ∧ has_missile (Nono)

→ sells (Colonel west , ~~Missile~~ X , Nono))

L_{law} over FOL

O_c = { c : c is a country } ∪ { h : h is human } ∪ { w : w is a weapon }

O_v = ϕ

R = { Criminal (1), American (1), Sells (3), Missile (1), Weapon (1), Hostile (1), Enemy (2), Owns (2) }

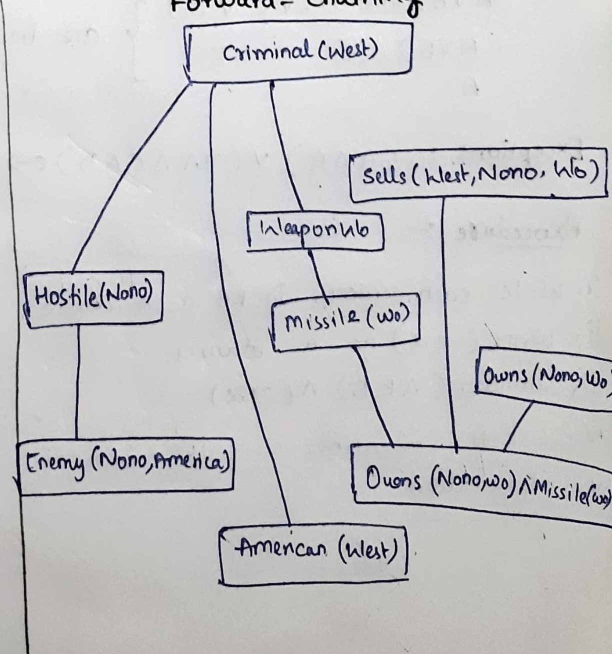
F = ϕ

K.B :

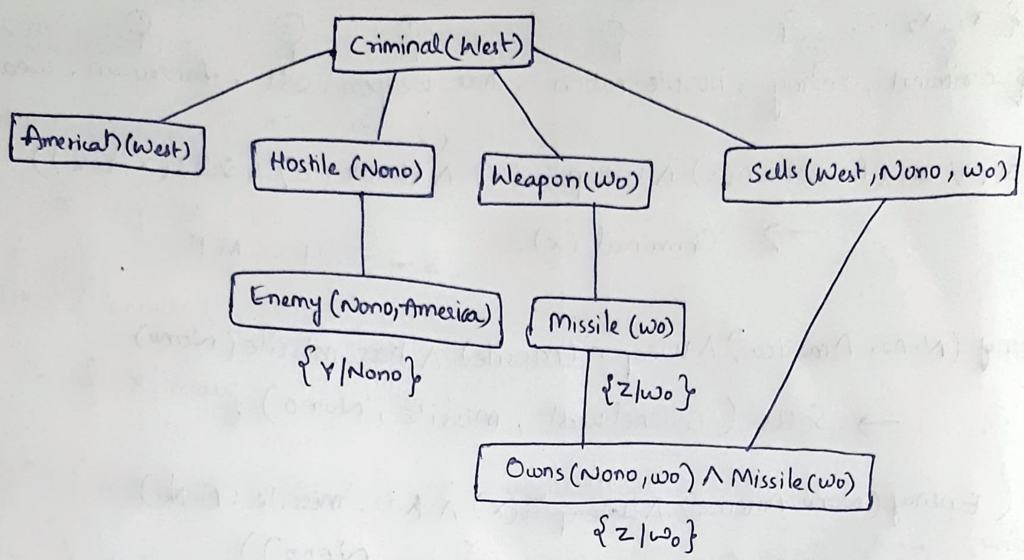
1. $\forall x \forall Y \forall z$ ((American (x) ∧ Hostile(Y) ∧ Weapon (z) ∧ Sells (x , Y, z)) → Criminal (x))
2. Enemy (Nono, America)
3. $\exists$ x (Owns (Nono , x) ∧ Missile (x))
4. $\forall$x (missile (x) ∧ Owns (Nono, x) → Sells (West, Nono, x))
5. $\forall$x (missile (x) → Weapon (x))
6. $\forall$x (Enemy (x, America) → Hostile (x))
7. American (West)

Conclusion : α : Criminal (West) .

Forward- Chaining



Backward Chaining



Proof by Resolution

* De Morgan's Law in First order Logic :-

$\neg(\forall x\ A(x)) \Longleftrightarrow \exists x(\neg A(x))$

$\neg(\exists x\ A(x)) \Longleftrightarrow \forall x(\neg A(x))$

$\neg(A \vee B) = \neg A \wedge \neg B$

$\neg(A \wedge B) = \neg A \vee \neg B$

* Conjuctive Normal form (CNF)

Conjunction of clauses → disjunction of literals (atomic/negation of atomic sentences)

$$\underbrace{(l_1 \vee l_2 \vee l_3 \vee \ldots l_n)} \wedge \underbrace{(r_1 \vee r_2 \vee r_3 \ldots \vee r_n)} \wedge \ldots \wedge \underbrace{(s_1 \vee s_2 \vee \ldots \vee s_n)}$$

combination of literals/clauses

Ex: $(A \vee B) \wedge (\neg A \vee C \vee D)$
$(A \vee B) \wedge T$
A,
} are in CNF

Exceptions : $(A \wedge B) \vee (\neg A \wedge C \wedge D) \Longleftrightarrow (A \vee \neg A) \wedge (A \vee C) \wedge (A \vee D) \wedge (B \vee \neg A) \wedge (B \vee C) \wedge (B \vee D)$

Procedure :-

i, Write each sentence in KB as a clause
ii, write $(\neg \alpha)$ as a clauses
iii, Consider $(\wedge KB) \wedge (\neg \alpha)$
iv, ~~skole-mization~~ – skolemization
v, Resolution

K.B : (i) $\forall x\,(TAC(x) \rightarrow PE(x))$
(ii) $\exists x\,(TAC(x) \wedge \neg RB(x))$

α : $\exists x\,(\neg RB(x) \wedge PE(x))$

$\rightarrow (\neg\alpha) \Leftrightarrow \forall x\,(RB(x) \vee (\neg PE(x)))$

$TAC(P_0) \wedge \neg RB(P_0)$

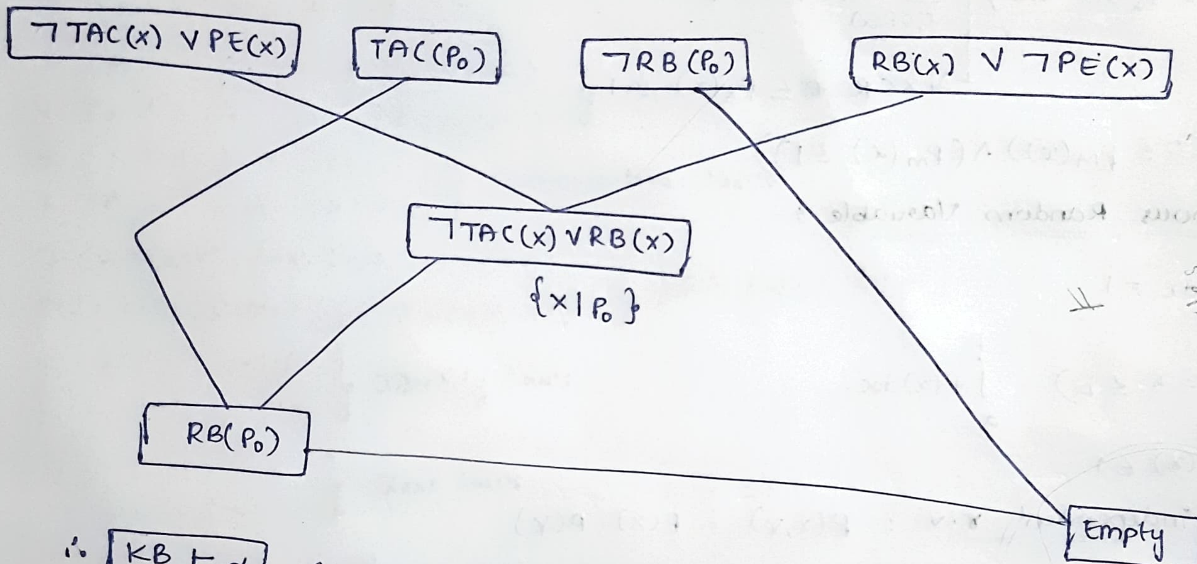
$\neg TAC(x) \vee PE(x)$

$\rightarrow TAC(P_0)$
$\longrightarrow \neg RB(P_0)$ } (ii)

$\longrightarrow RB(x) \vee \neg PE(x)$ — α (De-Morgan's)

$K.B \vdash \alpha \Leftrightarrow KB \models \alpha$

$\Leftrightarrow ((\wedge KB) \wedge (\neg\alpha))$
$\downarrow$
unsatisfiable.

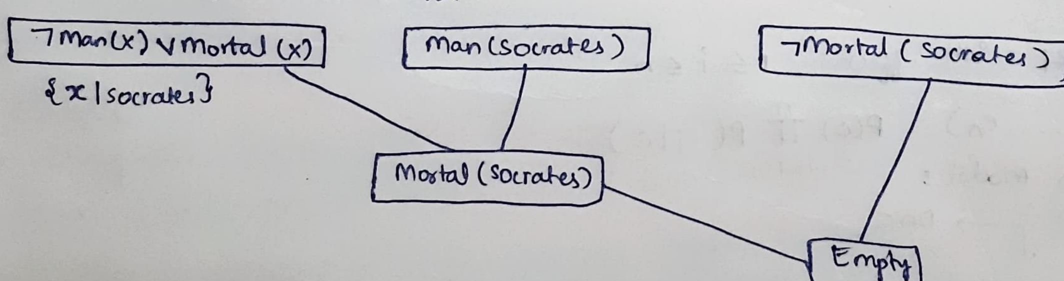


$\therefore$ $\boxed{KB \vdash \alpha}$ (knowledge base derives α)

$\vdash$	$\rightarrow$ derives
$\models$	$\rightarrow$ Entiles

Q: KB : $\forall x\,(man(x) \rightarrow mortal(x))$
man (Socrates)
α : mortal (socrates)

$\rightarrow$ a) $\forall x\,(man(x) \rightarrow mortal(x)) \cong (\neg Man(x)) \vee mortal(x)$
b) man (Socrates)
c) $\neg$ mortal (Socrates) $\cong \neg\alpha$



$\therefore$ $KB \vdash \alpha$

(knowledge base derives α)

Reosoning With Uncertainity

→ Boolean Random Variable , A

$P(A = \text{true}) = 0.2$

$P(a) = 0.2$

↑ Boolean propositional variable

Kolmogorov axioms :

$P(\neg a) = 1 - P(a)$

$P(a \vee b) = P(a) + P(b) - P(a \wedge b)$

$P(a|b) = \dfrac{P(a \wedge b)}{P(b)}$ where $b \neq 0$

8-oct-2024

$P(A = \text{true}) = P(a)$, $P(A = \text{false}) = P(\neg a)$

$P(X=a) = P_x(a) \longrightarrow \left[\begin{array}{l} \sum_{x_i \in R(x)} P_M(x_i) = 1 \\ \forall x \in R \;\; 0 \leq P_x(x) \leq 1 \end{array} \right.$

a) $\forall x \left((0 \leq P_M(x)) \wedge (P_m(x) \leq 1) \right)$

Continous Random Variable :-

i) $\int_{-\infty}^{\infty} f(x)dx = 1$

ii) $P(a \leq x \leq b) = \int_a^b f(x)dx.$

iii) $0 \leq f(x) \leq 1$

X, Y - independent r.v : $P(x,y) = P(x) \cdot P(y)$

* x and y are conditionally independent w.r.t Z if :

$P(x,y|z) = P(x|z) \cdot P(y|z)$

$P(x|y,z) = P(x|z)$

$P(y|x,z) = P(y|z)$

* If there are 'n' random variables, we have to make a table of entries 2^n.

C = cause , e_i : ith effect $1 \leq i \leq n$

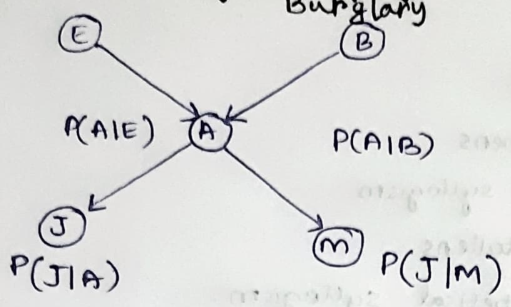
$P(c, e_1, e_2, \ldots e_n) = P(c) \prod_{i=1}^{n} P(e_i|c)$

Naive Bayesian Model :

i) Directed Graph } → DAG
ii) Acyclic
iii) Vertices boolean r.v's edge from A to B : A causes B
iv) Parents are drawn above children, and edges are downwards..
v) At each node 'child', state P(child | parents).

CAT II - Syllabus → Module 4 : 3 Q's , Module 5 → 2 Q's

\# r.v = { Earthquake happening (E), ~~Burglary~~ Burglary happening (B), Alarm rings (A), John calls (J), Mary calls (M) }



\# Basic Equivalent formulas :

1. $(A \wedge A) \Leftrightarrow A$
2. $(A \vee A) \Leftrightarrow A$

} idempotent laws

3. $(A \wedge (B \wedge C)) \Leftrightarrow ((A \wedge B) \wedge C)$
4. $(A \vee (B \vee C)) \Leftrightarrow ((A \vee B) \vee C)$

} Associative laws

5. $(A \wedge B) \Leftrightarrow (B \wedge A)$
6. $(A \vee B) \Leftrightarrow (B \vee A)$

} commutative laws

7. $(A \wedge (B \vee C)) \Leftrightarrow ((A \wedge B) \vee (A \wedge C))$
8. $(A \vee (B \wedge C)) \Leftrightarrow ((A \vee B) \wedge (A \vee C))$

} Distributive laws

9. $(A \wedge T) \Leftrightarrow A$
10. $(A \vee F) \Leftrightarrow A$

} Identity laws.

11. $(A \wedge F) \Leftrightarrow F$
12. $(A \vee T) \Leftrightarrow T$

} Zero laws

13. $(A \wedge (\neg A)) \Leftrightarrow F$
14. $(A \vee (\neg A)) \Leftrightarrow T$

} Negation laws

15. $(A \wedge (A \vee B)) \Leftrightarrow A$
16. $(A \vee (A \wedge B)) \Leftrightarrow A$

} Absorption laws

17. $(\neg (A \wedge B)) \Leftrightarrow ((\neg A) \vee (\neg B))$
18. $(\neg (A \vee B)) \Leftrightarrow ((\neg A) \wedge (\neg B))$

} De Morgan's laws.

19. $(A \rightarrow B) \Leftrightarrow ((\neg A) \vee B)$
20. $(A \leftrightarrow B) \Leftrightarrow (((\neg A) \vee B) \wedge ((\neg B) \vee A))$

Examples :

1. Using the previous basic equivalence laws we can show that for any well-formed-formula (wff) A, $A \Leftrightarrow (\neg(\neg A))$.

This is known as the double negation law.

21. $(A \wedge B) \Rightarrow A$
22. $(A \wedge B) \Rightarrow B$

$\Big\}$ Simplification

23. $A \Rightarrow (A \vee B)$
24. $B \Rightarrow (A \vee B)$

$\Big\}$ Addition

25 $A, B \Rightarrow (A \wedge B)$ — Conjunction

26. $A, (A \rightarrow B) \Rightarrow B$ — Moden ponens

27. $(\neg A), (A \vee B) \Rightarrow B$ — disjunctive syllogism

28. $(\neg B), (A \rightarrow B) \Rightarrow (\neg A)$ — modus tollens

29. $(A \rightarrow B), (B \rightarrow C) \Rightarrow (A \rightarrow C)$ — hypothetical syllogism.