# Learning Goals

1. Bernoulli Trial
2. Binomial Distribution
3. Summary

# Bernoulli Trial

- A trial or an experiment, whose outcome can be classified as either a **“success”** or a **“failure”** is called a Bernoulli trial.

# Bernoulli Trial

- Tossing a coin

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# Bernoulli Trial

- Tossing a coin
- Whether it will rain today or not?
- Is the newborn child a girl or boy?
- Whether a disk drive crashed?
- Whether someone likes a Netflix movie?
- Success of a medical treatment
- Student result in an exam
- Transmittance of a disease
- Sale person selling an object
- Testing the effectiveness of a drug
- Opinion poll

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- Non-Bernoulli Trial?
- Randomly choosing a person and asking their age.

# Bernoulli Experiment

- The number of trials should be **finite**.
- Each trial should be **independent**.
- Each trial should have only two possible outcomes - **success and failure**.
- The **probability** of each outcome should be the **same in every trial**.

# Bernoulli Random Variable

- A Bernoulli random variable is the simplest kind of random variable.
- It can take on two values, 1 and 0.
- It takes on a 1 if an experiment with probability $p$ resulted in success and a 0 otherwise.
- If $X$ is a Bernoulli random variable, denoted $X \sim Ber(p)$.

# PMF of Bernoulli Random Variable

- The probability mass function associated with Bernoulli random variable is given as

$$P(X = x) = \begin{cases} p, & \text{if } x = 1 \\ 1 - p, & \text{if } x = 0 \\ 0, & \text{otherwise} \end{cases}$$

where $0 \leq p \leq 1$.

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- $X$: number of successes in $n$ independent Bernoulli trials.
- What is probability of getting $k$ successes out of $n$ trials? or, what is $P(X = k)$ where $k \leq n$?
- What are the possible outcomes?

# Binomial Distribution

| $k$ | **Outcome** | **Probability** | **No. of combinations** |
|---|---|---|---|
| 0 | $\underbrace{FF\ldots F}_{n \text{ times}}$ | $(1-p)^n$ | $\binom{n}{0}$ |

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| $\vdots$ | | | |
| i | $\underbrace{SS\dots S}_{i \text{ times}}\ \underbrace{FF\dots F}_{n-i \text{ times}}$ | $p^i(1-p)^{n-i}$ | $\binom{n}{i}$ |

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| n | $\underbrace{SS\ldots S}_{n \text{ times}}$ | $p^n$ | $\binom{n}{n}$ |

# Binomial Distribution

- The probability mass function associated with Binomial random variable is given as

$$P(X = k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k}, & \text{if } k = 0, 1, \ldots, n, \quad k < n \\ 0, & \text{otherwise} \end{cases}$$

where $0 \leq p \leq 1$.

# Binomial Distribution

## Definition

A binomial random variable is random variable that represents the number of successes in $n$ successive independent trials of a Bernoulli experiment. If $X$ is a Binomial random variable, we denote this $X \sim Bin(n, p)$, where $p$ is the probability of success in a given trial. Then the PMF of $X$ when $X = k$ is

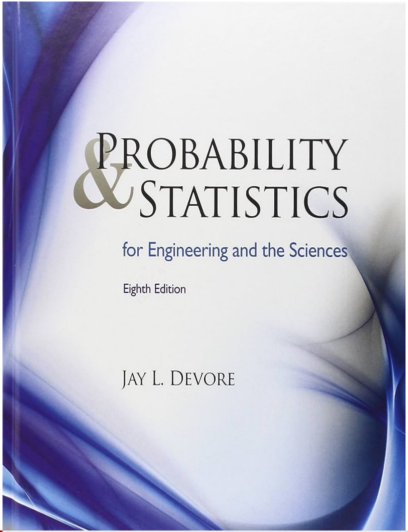
$$P(X = k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k}, & \text{if } k \in \mathbb{N}, \quad 0 \leq k \leq n \\ 0, & \text{otherwise} \end{cases}$$

where $0 \leq p \leq 1$.

# Summary

- Discussed Bernoulli random variable.
- Derived Binomial distribution from $n$ independent Bernoulli trials.

# Reference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*Thank You...*