PMDS504L: Introduction to Time Series

Components of Time Series

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Table of Contents

1. Introduction to Time Series
   - Trend
   - Seasonal Variations
   - Cyclic Fluctuations
   - Irregular Variations
2. Measurement of Trend
   - Free Hand Method
   - Semi Averages Method
   - Moving Average Method
   - Least Squares Method
3. Seasonal Variations
   - Method of Simple Averages
   - Ratio to Moving Average Method

Introduction

A **time series** is a sequence of data points recorded over time. The key components that influence the patterns in time series data are:

- **Secular Trend (Trend)**
- **Seasonal Variations**
- **Cyclic Fluctuations**
- **Irregular Variations**

Secular Trend (Trend)

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- The steady rise in **population growth** over decades.

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Graphical Representation:

- Trends can be **upward** (economic growth) or **downward** (decline in birth rates).
- Typically modeled using *linear regression* or *exponential smoothing*.

Seasonal Variations

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- Higher **retail sales** during festive seasons like Christmas or Diwali.

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Characteristics:

- **Fixed time intervals:** Weekly, monthly, or annual cycles.
- **Weather and cultural factors:** Holidays, climate, and traditions play a major role.

Cyclic Fluctuations

Definition: Cyclic variations occur in time series when data exhibits fluctuations over a longer period (more than a year), but without a fixed seasonal pattern.

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- The **real estate market** goes through cycles of booms and recessions.
- **Stock market trends** fluctuate due to economic policies, global events, or investor sentiment.

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Key Differences from Seasonal Variation:

- **Not fixed in time**-cycles may last several years.
- **Economic and business cycles** often follow this pattern.

Irregular Variations

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Example:

- A sudden spike in sales due to a **viral social media campaign**.
- A stock market crash caused by **political instability**.
- The impact of a **natural disaster** on economic activity.

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- A stock market crash caused by **political instability**.
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Characteristics:

- **No fixed pattern** ? occurs due to unforeseen events.
- Can cause **short-term shocks** in data.

Components

- Time series analysis helps in understanding **patterns and trends** in data.
- Identifying **secular, seasonal, cyclic, and irregular variations** allows for better forecasting and decision-making.
- Different statistical techniques, such as **regression models, moving averages, and decomposition methods**, are used to analyze these components.

Measurement of Secular Trend

Methods:

- Free Hand Method (Graphical Method)
- Method of Semi-Averages
- Method of Moving Averages
- Method of Least Squares

Free Hand Method

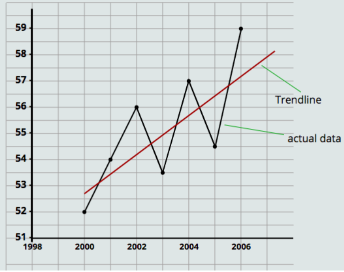
Steps:

1. Plot the data points on a graph with time on the x-axis and the variable on the y-axis.
2. Identify the general trend (upward, downward, or flat).
3. Draw a smooth trend line that captures the overall direction of the data.

Example

Determine a trend line from the following:

Year	2000	2001	2002	2003	2004	2005	2006
Sales	52	54	56	53.5	57	54.5	59



Note: A trendline is a line that represents the overall direction of a dataset. It is not intended to perfectly match every data point, but rather to capture the general trend.

Trend Line Representation

Types of Trends:

- **Upward trend:** Line with a positive slope.
- **Downward trend:** Line with a negative slope.
- **Flat trend:** Line with near-zero slope.

Semi Averages Method

Steps:

1. **Divide the Data:** Split your time series data into two equal halves. If you have an odd number of data points, exclude the middle one.
2. **Calculate Averages:** Compute the average (arithmetic mean) of the values in each half. These are the "semi-averages."
3. **Plot the Points:** Assign the first semi-average to a point midway through the first half of the data, and the second semi-average to a point midway through the second half. Plot these on a graph.
4. **Draw the Trend Line:** Connect the two plotted points with a straight line to estimate the trend.

Question 1

Fit a trend line by the method of semi-averages to the data given below:

Years	2000	2001	2002	2003	2004	2005	2006	2007
Sales (in lakhs)	412	438	444	454	470	482	490	500

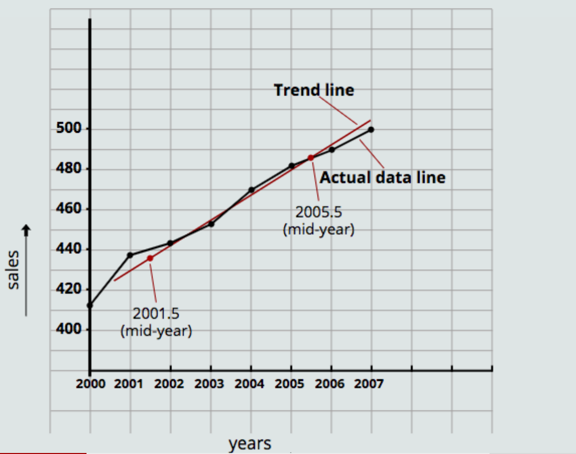
Solution

Here we have even number of years so, these can be divided into two equal parts.

Year	Sales	Semi-Total	Semi-Avg	Mid-Year
2000	412			
2001	438	1748	1748/4 = 437	2001.5
2002	444			
2003	454			
2004	470			
2005	482	1742	1742/4 = 485.5	2005.5
2006	490			
2007	500			

Graph

Year	2000	2001	2002	2003	2004	2005	2006	2007
Sales	412	438	444	454	470	482	490	500



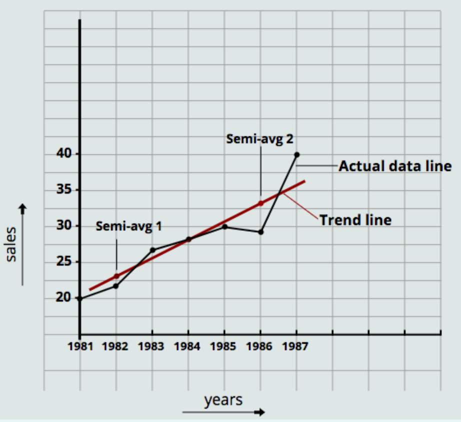
Question 2

Find a trend line by the method of semi-averages.

Year	Sales	Semi-Total	Semi-Avg	Mid-Year
1981	20			
1982	22	69	69/3 = 23	1982
1983	27			
1984	26			
1985	30			
1986	29	99	99/3 = 33	1986
1987	40			

Graph

Year	1981	1982	1983	1984	1985	1986	1987
Sales	20	22	27	26	30	29	40



Moving Average Method

Definition: This method smooths out short-term fluctuations in the data to reveal the underlying long-term trend (secular trend).

Steps:

1. **Given Moving Average Size:** You will be provided with a specific number of data points to be included in the moving average calculation. A larger moving average size results in a smoother trend but may be less responsive to recent changes.
2. **Calculate Moving Averages:**
   - **Odd Moving Average Size:** Slide the window across your dataset and compute the average for each window position.
   - **Even Size (Centered Moving Average):** Compute the centered moving average by averaging the two neighboring points when using an even-numbered window size.
3. **Plotting the Moving Averages:** Plot the original data points along with the computed moving averages to visualize the smoothed trend.

Question 3

Find the 5 yearly moving average for the following.

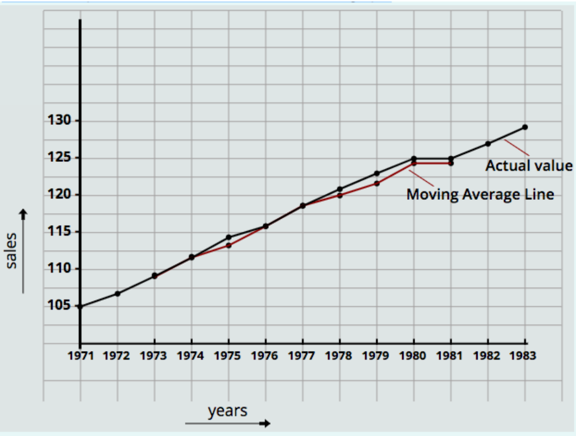
Year	Sales
1971	105
1972	107
1973	109
1974	112
1975	114
1976	116
1977	118
1978	121
1979	123
1980	124
1981	125
1982	127
1983	129

Solution

Year	Sales	5 yearly Sum	5 yearly MA	Trend value (Round off)
1971	105			
1972	107			
1973	109	547	109.4	109
1974	112	558	111.6	112
1975	114	569	113.8	113
1976	116	581	116.2	116
1977	118	592	118.4	118
1978	121	602	120.4	120
1979	123	611	122.2	122
1980	124	620	124.0	124
1981	125	622	124.4	124
1982	127			
1983	129			

Graph

Now, we will plot actual value and trend value on the graph.



Even Size (Centered Moving Average)

Centered Moving Average (CMA) is a smoothing technique used for time series data.

When using an **even** window size:

- Compute the **Simple Moving Average (SMA)** over the window.
- Since there is no single middle value, take the **average of two consecutive SMAs**.
- This ensures proper alignment of the moving average with the data points.

Steps for Computing CMA

1. **Define the Window Size (W)**: Choose an even-numbered moving average window (e.g., 2, 4, 6, etc.).
2. **Compute the SMA**: For each window, compute the average of the values.
3. **Compute the CMA**: Take the average of two consecutive SMAs.

Example: 4-Point Centered Moving Average

Consider the following dataset:

Time	Value
1	10
2	20
3	30
4	40
5	50
6	60

Step 1: Compute the SMA

For a 4-point window:

$$SMA_1 = \frac{10 + 20 + 30 + 40}{4} = 25$$

$$SMA_2 = \frac{20 + 30 + 40 + 50}{4} = 35$$

$$SMA_3 = \frac{30 + 40 + 50 + 60}{4} = 45$$

Step 2: Compute the CMA

The CMA is obtained by averaging two consecutive SMAs:

$$CMA_1 = \frac{SMA_1 + SMA_2}{2} = \frac{25 + 35}{2} = 30.$$

This 30 assigned to time 2.5.

$$CMA_2 = \frac{SMA_2 + SMA_3}{2} = \frac{35 + 45}{2} = 40$$

This 40 assigned to time 3.5.

- For even-sized windows, the SMA is first computed.
- To align the values properly, the **average of two consecutive SMAs** is taken.
- This adjustment ensures that the moving average is correctly centered.

Least Squares Method

Problem: Fit a straight-line trend by the method of least squares (taking 1978 as the year of origin) for the given data.

Year	1979	1980	1981	1982	1983	1984
Production (in Lakhs)	5	7	9	10	12	17

Least Squares Method: Solution

Step 1: Trend Equation

- The trend equation is given by:

$$y = a + bx$$

- Normal equations:

$$\sum y = Na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

- Here, N is the number of years of data given.

Least Squares Method: Solution

Step 2: Create a Table

- Compute values for x, y, xy, and x.
- If the origin year is given, calculate x as the row year minus the origin year.
- If not given, assume the origin based on the following rule:
  - If N is odd, take the middle year as the origin.
  - If N is even, take the average of the mid two years.
- Example: For years 1980, 1981, 1982, 1983 (N=4), take 1981.5 as the origin.

Least Squares Method: Solution

Step 3: Compute Summations

x is deviation from the origin year.
(Row year - 1978)

Year	y	x	xy	x^2
1979	5	1	5	1
1980	7	2	14	4
1981	9	3	27	9
1982	10	4	40	16
1983	12	5	60	25
1984	17	6	102	36

Least Squares Method: Solution

- Compute $\sum x = 21$, $\sum y = 60$, $\sum xy = 248$, $\sum x^2 = 91$.
- Substitute these values into normal equations:

$$60 = 6a + 21b$$
$$248 = 21a + 91b$$

- Solve for $a = 2.40$ and $b = 2.17$.

Step 4: Final Trend Equation

$$y = 2.40 + 2.17x$$

Using the trend equation, we can find the trend values.

Least Squares Method: Solution

Trend Values

Year	y	x	trend value
1979	5	1	2.40 + 2.17 (1) = 4.57
1980	7	2	2.40 + 2.17 (2) = 6.74
1981	9	3	2.40 + 2.17 (3) = 8.91
1982	10	4	2.40 + 2.17 (4) = 11.08
1983	12	5	2.40 + 2.17 (5) = 13.25
1984	17	6	2.40 + 2.17 (6) = 15.42

Seasonal Variations

Definition: Seasonal variations are short-term fluctuations in recorded values due to different circumstances. These variations are measured through their indices called seasonal indices.

Explanation: In seasonal variation analysis, we examine how data changes with seasons. For example, in summer or winter, a business may see changes in profit. By understanding these variations, a business person can plan for the future. All of this comes under seasonal variations.

Methods for Finding the Seasonal Index:

- Method of Simple Averages
- Ratio to Moving Average Method
- Ratio to Trend Method
- Link Relative Method

Method of Simple Averages

Calculate the seasonal index for the following data using the simple average method, assuming the trend is absent.

Year	1st Quater	2nd Quater	3rd Quater	4th Quater
1987	3.7	4.1	3.3	3.5
1988	3.7	3.9	3.6	3.6
1989	4.0	4.1	3.3	3.1
1990	3.3	4.4	4.0	4.0
Total	14.7	16.5	14.2	14.2
Average	14.7/4 = 3.675	16.5/4 = 4.125	14.2/4 = 3.55	14.2/4 = 3.55

Note: If the question does not explicitly state *"use simple average method"* but mentions *"trend is absent,"* then we should use the simple average method.

Step 1: Compute Total Sum and Average

Find the total sum of all quarter values and then calculate their average.

Step 2: Compute General Average

The General Average is the mean of the quarterly averages:

$$\text{General Average} = \frac{\text{Average of Q1} + \text{Average of Q2} + \text{Average of Q3} + \text{Average of Q4}}{4} \tag{1}$$

For example,

$$\text{General Average} = \frac{3.675 + 4.125 + 3.55 + 3.55}{4} = \frac{14.9}{4} = 3.725 \tag{2}$$

Step 3: Compute Seasonal Index for Each Quarter

The Seasonal Index for a particular quarter is given by:

$$\text{Seasonal Index} = \left(\frac{\text{Quarter Average}}{\text{General Average}}\right) \times 100 \tag{3}$$

For example:

$$\text{Seasonal Index for Q1} = \left(\frac{3.675}{3.725}\right) \times 100 = 98.66$$

$$\text{Seasonal Index for Q2} = \left(\frac{4.125}{3.725}\right) \times 100 = 110.73$$

$$\text{Seasonal Index for Q3} = \left(\frac{3.55}{3.725}\right) \times 100 = 95.30$$

$$\text{Seasonal Index for Q4} = \left(\frac{3.55}{3.725}\right) \times 100 = 95.30$$

Ratio to Moving Average Method

In the ratio to moving average method, we use a moving average to smooth out the data and calculate the seasonal indices by comparing the actual values to the moving average.

Year	1st Quater	2nd Quater	3rd Quater	4th Quater
1985	68	62	61	63
1986	65	58	61	61
1987	68	63	63	67

Question:

From the following data, calculate the seasonal indices by the ratio to moving average method.

Example: Ratio to Moving Average Method

Steps:

1. Create a table with columns: 4-quarter moving total, 4-quarter centralized total, 4-quarter moving average trend (T), and the ratio of the moving average.
2. Find the total of the quarterly values and calculate the average for each quarter.
3. Calculate the General Average: General average $= \frac{106.2+96+97.8+99.3}{4} = 99.825$
4. Calculate the Seasonal Index: Seasonal Index $= \frac{\text{Quarter Average}}{\text{General Average}} \times 100$
   Example:
   - 1st Quarter: $\frac{106.2}{99.825} \times 100 = 106.39$
   - 2nd Quarter: $\frac{96}{99.825} \times 100 = 96.16$
   - 3rd Quarter: $\frac{97.8}{99.825} \times 100 = 97.97$
   - 4th Quarter: $\frac{99.3}{99.825} \times 100 = 99.47$

Example: Ratio to Moving Average Method

Year	Quater	value (O)	4 Q Moving total	4 Q Centralized total	4 Q M-A Trend (T)	Ratio to MA (O/T * 100)
1985	I	68		-	-	-
			-			
	II	62		-	-	-
			254			
	III	61		505	505/8 = 63.125	(61/63.125)*100 = 96.6
			251			
	IV	63		498	498/8 = 62.25	101.2
			247			
1986	I	65		494	494/8 = 61.75	105.3
			247			
	II	58		492	492/8 = 61.50	94.3
			245			
	III	61		493	493/8 = 61.625	99.0
			248			
	IV	61		501	501/8 = 62.625	97.4
			253			
1987	I	68		508	508/8 = 63.50	107.1
			255			
	II	63		516	516/8 = 64.50	97.7
			261			
	III	63		-	-	-
			-			
	IV	67		-	-	-

Ratio to Moving Average Method

Definition: The Ratio to Moving Average Method is used to isolate seasonal effects by removing the trend component from the data. This method helps in calculating seasonal indices by comparing actual values with smoothed values obtained through moving averages.

Steps:

1. **Construct a Table:** Create a table with the following columns:
    - 4-quarter moving total
    - 4-quarter centralized moving total
    - 4-quarter moving average (T)
    - Ratio of actual values to moving averages
2. **Compute the Average for Each Quarter:** Sum up the values for each quarter and determine their respective averages.
3. **Calculate the General Average:**

$106.2 + 96 + 97.8 + 99.2$

References

This presentation is adapted from:

- Montgomery, D. C., Peck, E. A., & Vining, G. G. (2012). Introduction to Linear Regression Analysis, Fifth Edition. Wiley.
- MTH 416 : Regression Analysis — Shalabh, IIT Kanpur

Thank You!

Thank you for your attention!