1. Find the QR decomposition of the following matrices:

i. $\begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix}$

ii. $\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$

iii. $\begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix}$

iv. $\begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 5 \\ 0 & 3 & 6 \end{bmatrix}$

2. Find the singular-value decomposition of the following matrices:

i. $\begin{bmatrix} 4 & 0 \\ 3 & -5 \end{bmatrix}$

ii. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

iii. $\begin{bmatrix} 1 & 1 & 1 \\ -1 & -3 & -3 \\ 2 & 4 & 4 \end{bmatrix}$

iv. $\begin{bmatrix} 2 & 3 \\ 4 & 10 \end{bmatrix}$

3. Diagonalize the following matrices if possible:

i. $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

ii. $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

iii. $\begin{bmatrix} 1 & 1 & 1 \\ -1 & -3 & -3 \\ 2 & 4 & 4 \end{bmatrix}$

iv. $\begin{bmatrix} 2 & 3 \\ 4 & 10 \end{bmatrix}$

4. Given the following quadratic equations. Find the matrices of the quadratic form, state whether they are positive/negative/semi definite. Find ranks and signatures:

1. $x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_1x_3$
2. $7x_1^2 + 4x_2^2 + 7x_3^2 - 2x_1x_2 - 4x_1x^3 - 2x_{23}^{x_1}$