GRAPHICAL METHOD

Steps to Solve Linear Programming Problems Using Graphical Method:

When a LPP has only two variables in the objective function and constraints, it can be easily solved using the graphical method. The given information of a LPP can be plotted on the graph and the optimal solution can be obtained from the graph.

The steps to solve an Linear Programming Problem using Graphical method is given below:

Step 1:

Identify the decision variables, the objective function and the restrictions for the given Linear Programming Problem (LPP).

Step 2:

Write the Mathematical Formulation of the problem.

Step 3:

Plot the points on the graph representing all the constraints of the problem. Find the feasible region or solution space. The intersection of all the regions represented by the constraints of the problem is called the feasible region and is restricted to the first quadrant only.

Step 4:

The Feasible region obtained in the step 3 may be bounded or un bounded. Determine the Co-ordinates (x, y) values of all the corner points of the feasible region.

Step 5:

Find the value of the objective function at each corner points (solution) determined in step 3.

Step 6:

Select a point from all the corner points that optimises (Maximises or Minimises) the values of the objective function. It gives the Optimum Feasible Solution.

Some Exceptional Cases of Linear Programming Problem:

There may be an LPP for which no solution exists or for which the only solution obtained is an unbounded one. The exceptional cases arise in the application of graphical method are

- Alternative Optima
- Unbounded Solution
- Infeasible Solution or Non existing Solution

Alternative Optima:

When the objective function is parallel to the binding constraint, the objective function will assume the same optimal value at more than one solution point, because of this reason, they are called as Alternative Optima.

Unbounded Solution:

When the values of the decision variables may be increased in definitely without violating any of the constraints, the feasible region is unbounded. In such cases, the value of the objective function may increase (for maximisation) or decrease (for minimisation) in definitely. Thus, both the solution space and the objective function value are unbounded.

Infeasible Solution:

When the constraints are not satisfied simultaneously, the LPP has no feasible solution. This solution can never occur, if all the constraints are of less than or equal to type.

Example Problem for Solving Graphical Method:

Example problem for solving graphical method are as follows:

1. Determine the optimal solution of the LPP.

$$\text{Minimize } Z = 3x + 8y$$

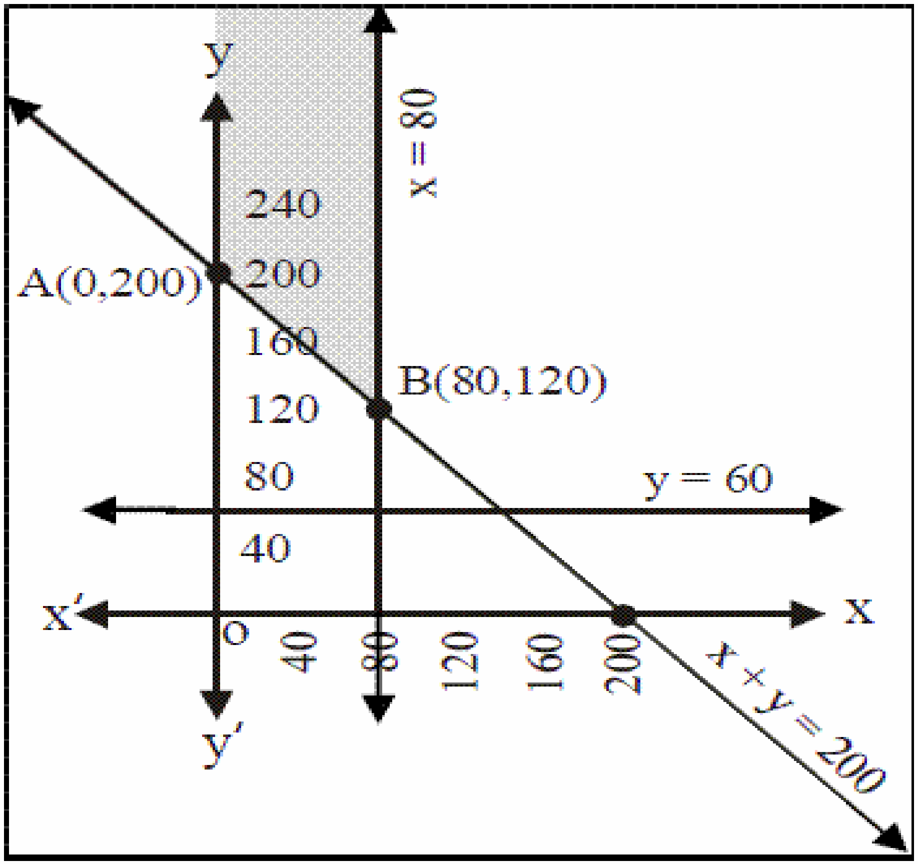
$$\text{Subject to } x + y > 200,$$

$$x < 80,\ y > 60,$$

$$x > 0,\ y > 0$$

Solution:

Since $x > 0$ and $y > 0$ the solution set is restricted to the First quadrant


y
x = 80
240
A(0,200)
200
160
B(80,120)
120
80
y = 60
40
x′
x
O
40
80
120
160
200
x + y = 200
y′

i) x+ y > 200 Draw the graph of x + y = 200

X + y = 200 => y = 200 –x

X	200	0
Y	0	200

Determine the region represented by x + y > 200

ii) X < 80 Draw the graph of x = 80

Determine the region for x < 80

iii) Y > 60 Draw the graph of y = 60

Determine the region for y > 60

Shade the intersection of the three regions. The shaded region is the feasible region.

B (80, 120) is the point of intersection of x + y = 200 and x = 80.

The corner points are A (0,200) and B (80, 120).

Corners	A (0,200)	B (80,120)
Z = 3x + 8y	1600	1200

Graphical Linear Programming Problem 1

Find feasible solution for graphical linear programming problem to maximize profit function.

$Z = 20x + 50y$ ...(1)

Subject to the constraints,

$3x+y \leq 90$

$x+y \leq 80$

$x \geq 0, \ y \geq 0$

Solution:

$3x + y = 90$ ...(2)

$x + y = 80$...(3)

Determine two points on the straight line $3x + y = 90$.

Put $y = 0$, $3x = 90$

$x = 30$

$(30, 0)$ is a point on the line (2),

put $x = 0$ in (2), $y = 90$

$(0, 90)$ is the other point on the line (2),

Which point represent the line $3x + y = 90$.

Determine two points on the straight line $x + y = 80$.

Put $y = 0$, $x = 80$

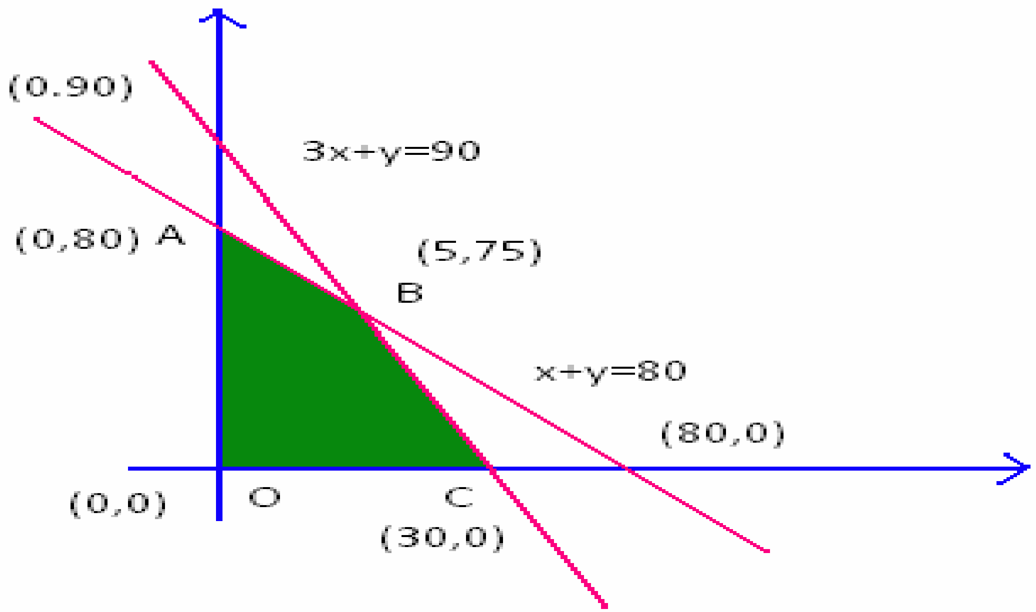
$x = 80$

$(80, 0)$ is a point on the line (3),

put $x = 0$ in (3), $y = 100$

$(0, 80)$ is the other point on the line (3),

Which points represent the line $x + y = 80$.



(0.90)
3x+y=90
(0,80) A
(5,75)
B
x+y=80
(80,0)
(0,0)
O
C
(30,0)

Corners of the feasible region are,

O (0, 0), A (0, 80), B(5, 75), C(30, 0)

At (0, 0) $Z = 0$

At (0, 80) $Z = 20 (0) + 50(80)$

$= 4000$

At (5, 75), $Z = 20 (5) +50 (75)$

$= 100 + 3750 = 3850$

At (30, 0) $Z = 20 (30) + 50 (0)$

$= 600$

Z maximum, at (0,80)

Optimal solution $x = 0$ and $y = 80$.

Optimal value= 4000.