INTRODUCTION TO OPERATIONS RESEARCH

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INTRODUCTION

Operational Research is a **systematic** and **analytical approach** to **decision making** and **problem solving.**

O.R. is an Branch of **applied mathematics** that uses techniques and statistics to arrive at **Optimal solutions** to solve complex problems.

It is typically concerned with determining the **<u>maximum</u> profit, sale, output, crops yield and efficiency** And **<u>minimum losses</u>, risks, cost, and time** of some objective function.

Some of the PRIMARY TOOLS used by operation researchers are-

STATISTICS

GAME THEORY

PROBABILITY THEORY, etc.


Define OR ?
Operations Research is the application of analytical methods designed to help the decision makers choose between various courses of action available to accomplish specified objectives

History of Operations Research

There is no clear history that marks the Birth if O.R., it is generally accepted that the field originated in **England** during the **World War II**.

Some say that **Charles Babbage** (1791-1871) is the **Father of O.R** because his research into the cost of transportation and sorting of mail led to **England's University Penny Post** in 1840.

Modern Operations Research originated at the **Bowdsey Research Station in U.K**. in 1937 to analyse and improve the working of the **UK's Early Warning Rador System**.

During the **Second World War** about 1000 Men and Women were engaged to work for British Army.

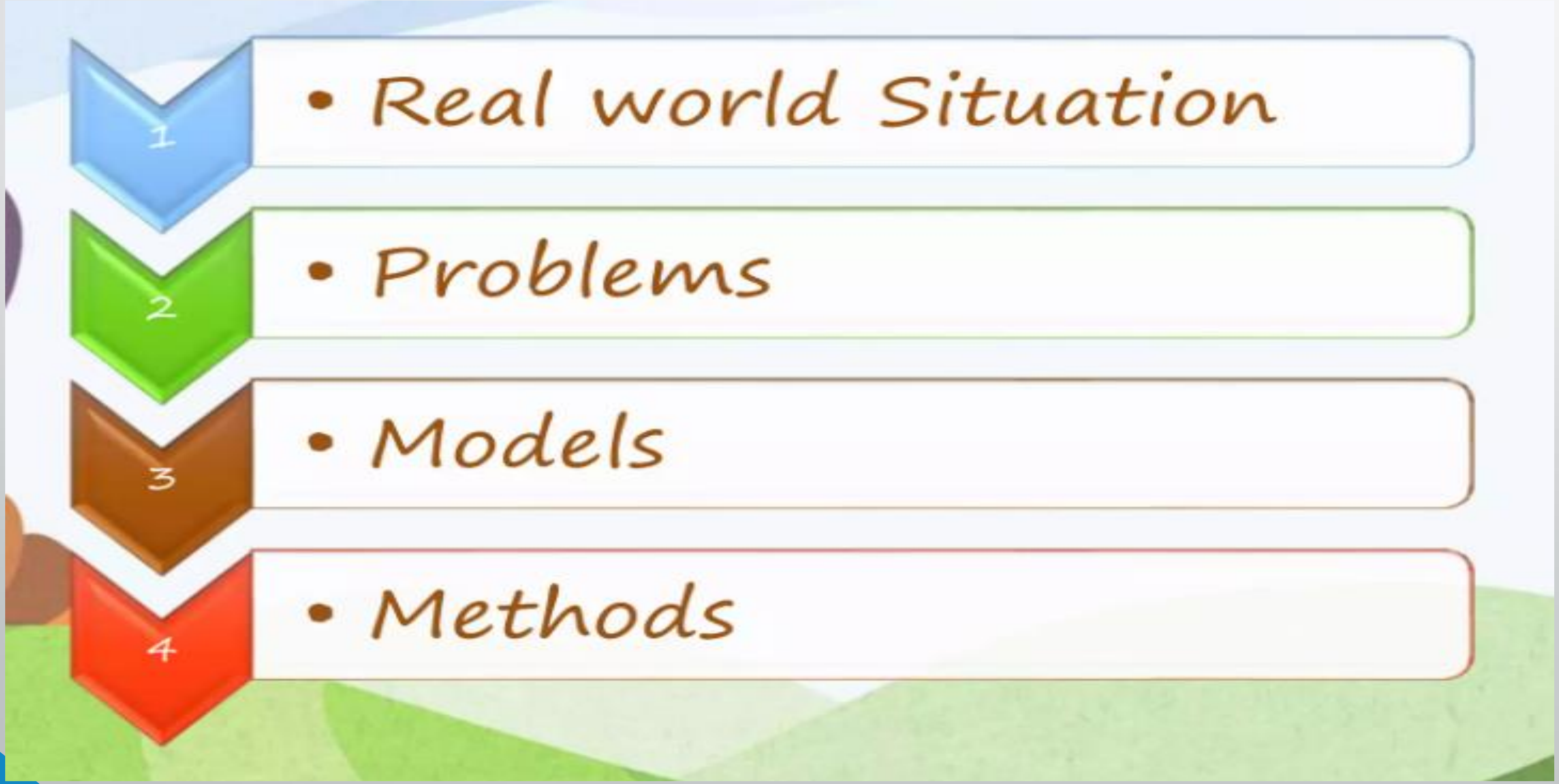
After World War II, **Military Operational Research in U.K. became Operational Analysis (OA) within the U.K**. Ministry of Defence with expanded techniques and graving awareness.

Operations Research in India

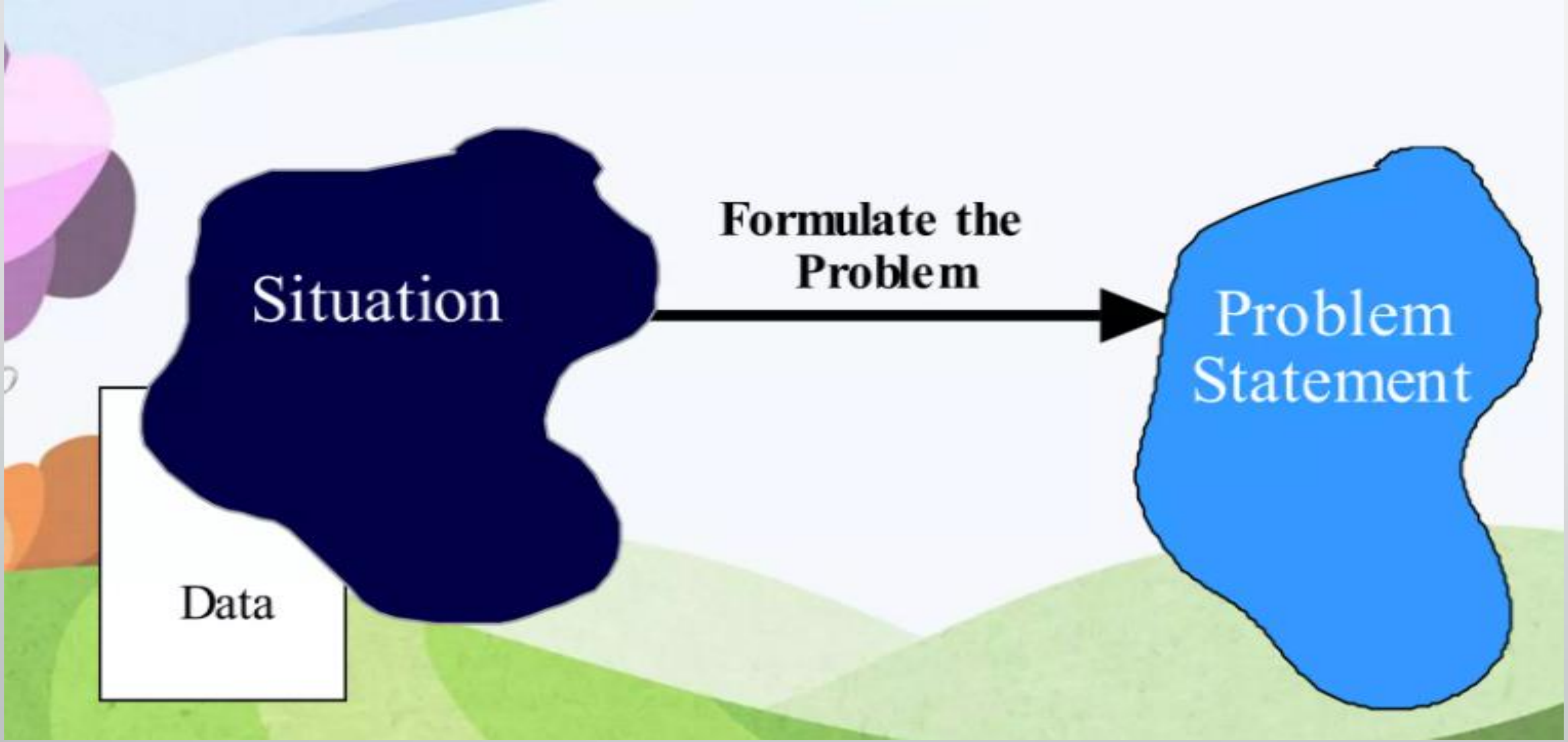
The **Operational Research Society of India** was founded in **1957** to provide a forum for the Operational Research Scientists as well as an avenue to widen their horizon by exchange of knowledge and application of techniques from outside the country. The Society is affiliated to the **International Federation of Operational Research Societies** (IFORS).

The Headquarters of the Society is located in **Kolkata** at 39, Mahanirvan Road, Kolkata 700029, India. At present the **Society has 12 Operating Chapters located in Agra, Ahmedabad, Ajmer, Bangalore, Chennai, Delhi, Durgapur, Jamshedpur, Kolkata, Madurai, Mumbai and Tirupati.**

Introduction cont….



PROBLEM SOLVING



Constructing a Model

1 Problem must be translated from verbal, qualitative terms to logical, quantitative terms

2 A logical model is a series of rules, usually embodied in a computer program

3 A mathematical model is a collection of functional relationships by which allowable actions are delimited and evaluated

Solving a Mathematical Model

1 Many tools are available as discussed in this course
2 Some lead to "optimal" solutions
3 Others only evaluate candidates to trial and error to find "best" course of action

Operations Research Models

Deterministic Models

- Linear Programming
- Network Optimization
- Integer Programming
- Nonlinear Programming

Stochastic Models (or) Probabilistic Model

- Discrete-Time Markov Chains
- Continuous-Time Markov Chains
- Queuing Theory
- Decision Analysis

SCOPE OF OR


Finance Budgeting and investments
Purchasing Procurement and Exploration
Production Management
Marketing Management
Personal Management

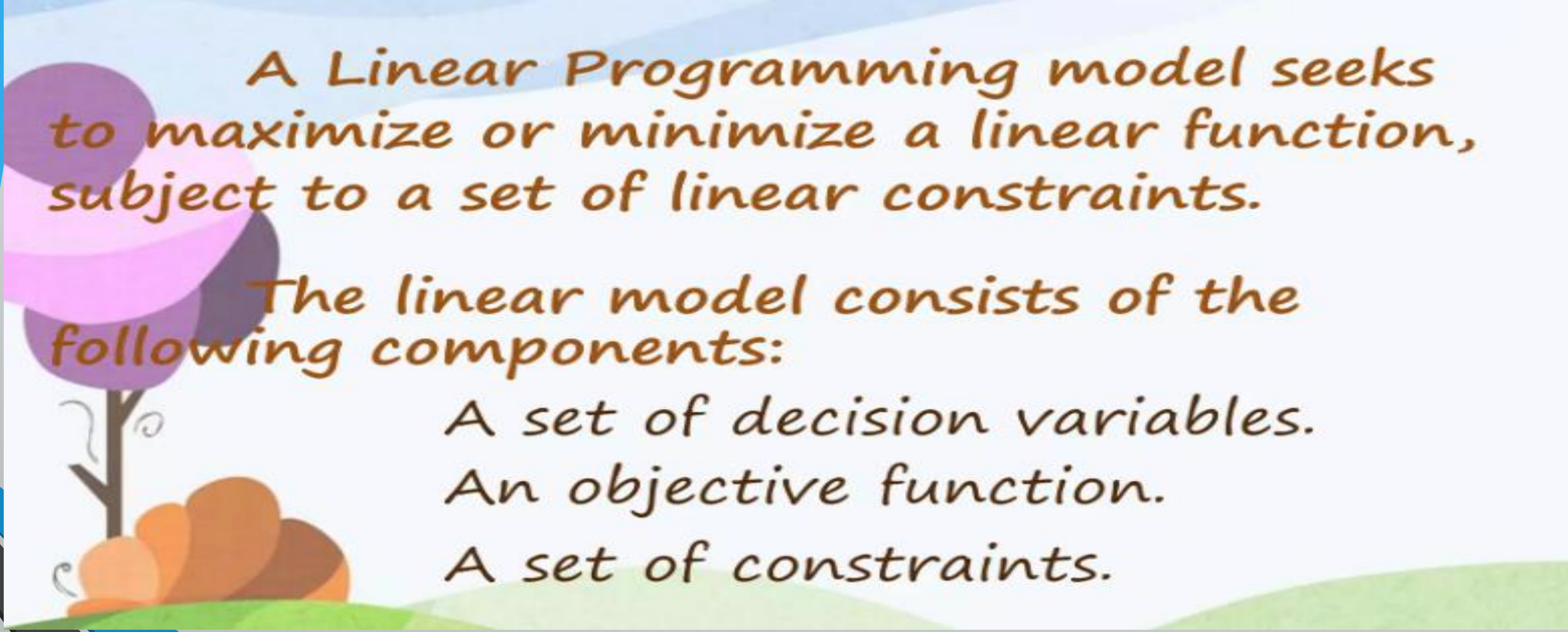
Applications of OR in Different Fields

Limitations of OR

- Problems are often simplified / assumptions are incorporated
- Model may not represent the actual behavior of the system
- Lack of decision makers awareness
- Many real world problems can not have OR solution

INTRODUCTION OF LINEAR PROGRAMMING PROBLEM

INTRODUCTION TO LINEAR PROGRAMMING PROBLEM



$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le b_m,$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \quad \ldots, x_n \ge 0.$$

Process to formulate a Linear Programming problem

Let us look at the steps of defining a Linear Programming problem generically:

1. Identify the decision variables
2. Write the objective function
3. Mention the constraints
4. Explicitly state the non-negativity restriction

For a problem to be a linear programming problem, the decision variables, objective function and constraints all have to be linear functions.

If the all the three conditions are satisfied, it is called a **Linear Programming Problem**.

FORMULATION OF LPP

Structure of all LPP has three important components.

(1) Decision variables (activities) : These are activities for which we want to determine a solution. These are usually denoted by $x_1, x_2, \ldots, x_n$.

(2) The objective function (goal) : This is a function which is expressed in terms of decision variables and we want to optimize (maximize or minimize) this function.

(3) The constraints : These are limiting conditions on the use of resources. The solution of LPP must satisfy all these constraints.

Max/min $\quad z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

subject to:

$$
\begin{cases}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \ (\leq, =, \geq)\ b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \ (\leq, =, \geq)\ b_2 \\
\quad\quad\quad \vdots \\
a_{m1}x1 + a_{m2}x_2 + \ldots + a_{mn}x_n \ (\leq, =, \geq)\ b_m
\end{cases}
$$

x_j = decision variables
b_i = constraint levels
c_j = objective function coefficients
a_{ij} = constraint coefficients

x_1, x_2, x_3---------- x_n to MAX or MIN the objective function.

$$\text{Max } Z = c_1x_1 + c_2x_2 + \ldots\ldots + c_nx_n \qquad \text{(a)} \longrightarrow \text{Objective Function}$$

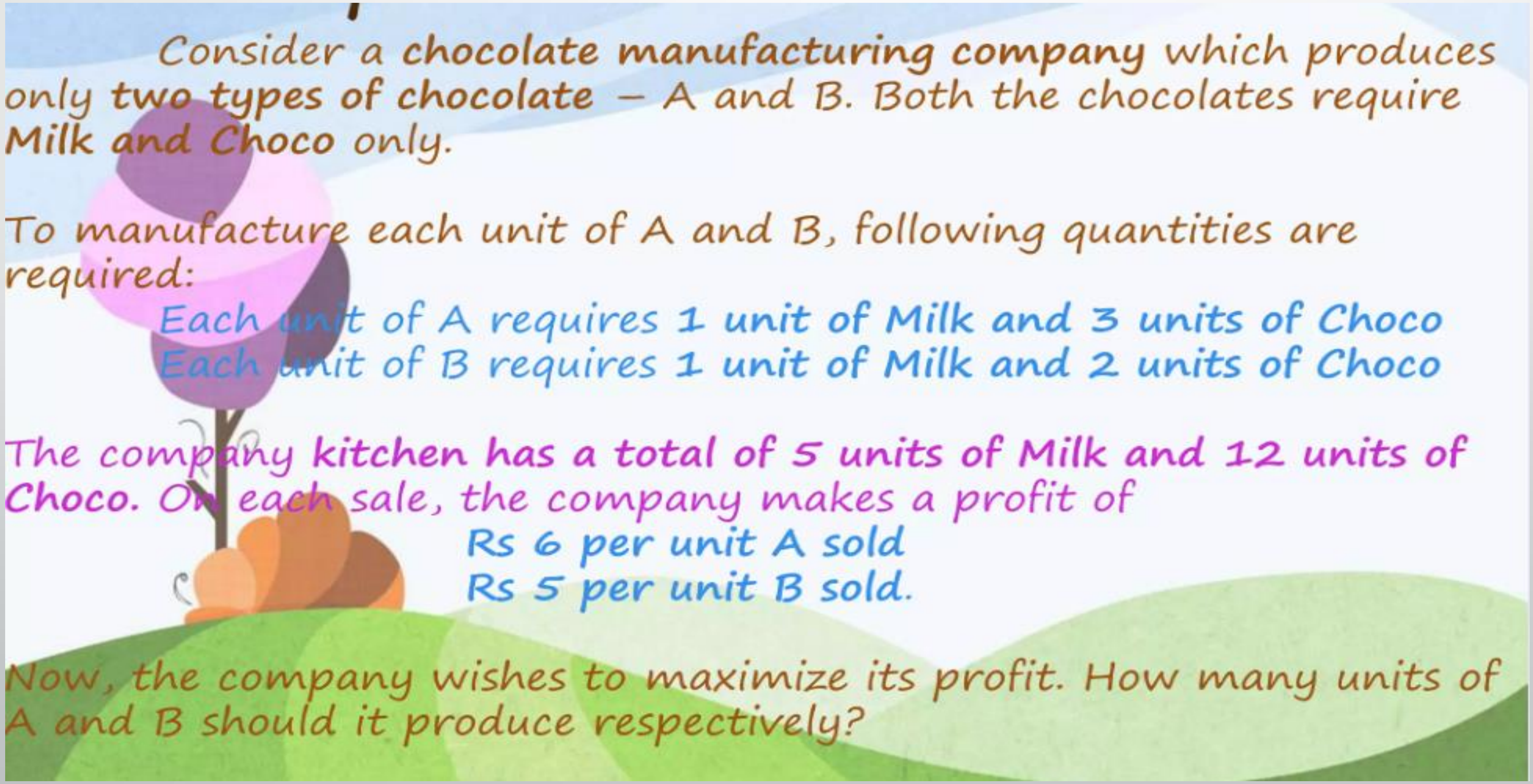
Also satisfy m – constraints or Subject to Constraint

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots\ldots + a_{2n}x_n = b_2 \\ |\ | \\ |\ | \\ a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots + a_{mn}x_n = b_n \\ x_1 >= 0 \text{ , } x_2 >= 0, \ldots\ldots + x_n >= 0 \end{array}\right\} \begin{array}{l} \text{(b)} \longrightarrow \text{Constant} \\ \\ \text{(c)} \longrightarrow \text{Non Negative Restriction} \end{array}$$

c_j (j = 1,2----n) is the objective function in equations (a) are called cost coefficient (max profit or min cost)

b_i (i= 1,2,----m) defining the constraint requirements or available in equation(b) or available in equation (b) is called stipulations and the constants a_{ij} (i=1,2,----m; j= 1,2,---n)are called structural co-efficient in equation (c) are known as non-negative restriction

Problem 1


Consider a **chocolate manufacturing company** which produces only **two types of chocolate** – A and B. Both the chocolates require **Milk and Choco** only.
To manufacture each unit of A and B, following quantities are required:
Each unit of A requires 1 unit of Milk and 3 units of Choco
Each unit of B requires 1 unit of Milk and 2 units of Choco
The company **kitchen has a total of 5 units of Milk and 12 units of Choco.** On each sale, the company makes a profit of
Rs 6 per unit A sold
Rs 5 per unit B sold.
Now, the company wishes to maximize its profit. How many units of A and B should it produce respectively?

Solution

	Milk	Choco	Profit per unit
A	1	3	Rs 6
B	1	2	Rs 5
Total	5	12	

Let the total number of units produced of A be = X

Let the total number of units produced of B be = Y

Now, the total profit is represented by Z

The total profit the company makes is given by the total number of units of A and B producec multiplied by its per unit profit Rs 6 and Rs 5 respectively.

Profit: Max $Z = 6X+5Y$

which means we have to maximize Z.

The company will try to produce as many units of A and B to maximize the profit. But the resources Milk and Choco are available in limited amount.

As per the above table, each unit of A and B requires 1 unit of Milk. The total amount of Milk available is 5 units. To represent this mathematically,

$$X+Y \leq 5$$

Also, each unit of A and B requires 3 units & 2 units of Choco respectively. The total amount of Choco available is 12 units. To represent this mathematically,

$$3X+2Y \leq 12$$

Also, the values for units of A can only be integers.

So we have two more constraints, $X \geq 0$ & $Y \geq 0$

Problem 2

Example: A farmer has recently acquired an 110 hectares piece of land. He has decided to grow Wheat and barley on that land. Due to the quality of the sun and the region's excellent climate, the entire production of Wheat and Barley can be sold. He wants to know how to plant each variety in the 110 hectares, given the costs, net profits and labor requirements according to the data shown below:

Variety	Cost (Price/Hec)	Net Profit (Price/Hec)	Man-days/Hec
Wheat	100	50	10
Barley	200	120	30

The farmer has a budget of US$10,000 and an availability of 1,200 man-days during the planning horizon. Find the optimal solution and the optimal value.

Formulation of Linear Problem

Step 1: Identify the decision variables

The total area for growing Wheat = X (in hectares)

The total area for growing Barley = Y (in hectares)

X and Y are my decision variables.

Step 2: Write the objective function

Since the production from the entire land can be sold in the market. The farmer would want to maximize the profit for his total produce. We are given net profit for both Wheat and Barley. The farmer earns a net profit of US$50 for each hectare of Wheat and US$120 for each Barley.

Our objective function (given by Z) is, **Max Z = 50X + 120Y**

Step 3: Writing the constraints

1. It is given that the farmer has a total budget of US$10,000. The cost of producing Wheat and Barley per hectare is also given to us. We have an upper cap on the total cost spent by the farmer. So our equation becomes:

$$100X + 200Y \leq 10{,}000$$

2. The next constraint is, the upper cap on the availability on the total number of man-days for planning horizon. The total number of man-days available are 1200. As per the table, we are given the man-days per hectare for Wheat and Barley.

$$10X + 30Y \leq 1200$$

3. The third constraint is the total area present for plantation. The total available area is 110 hectares. So the equation becomes,

$$X + Y \leq 110$$

Step 4: The non-negativity restriction

The values of X and Y will be greater than or equal to 0. This goes without saying.

$$X \geq 0, Y \geq 0$$

Problem 3

- Product mix problem - Beaver Creek Pottery Company
- How many bowls and mugs should be produced to maximize profits given labor and materials constraints?
- Product resource requirements and unit profit:

	Resource Requirements		
Product	Labor (Hr./Unit)	Clay (Lb./Unit)	Profit ($/Unit)
Bowl	1	4	40
Mug	2	3	50

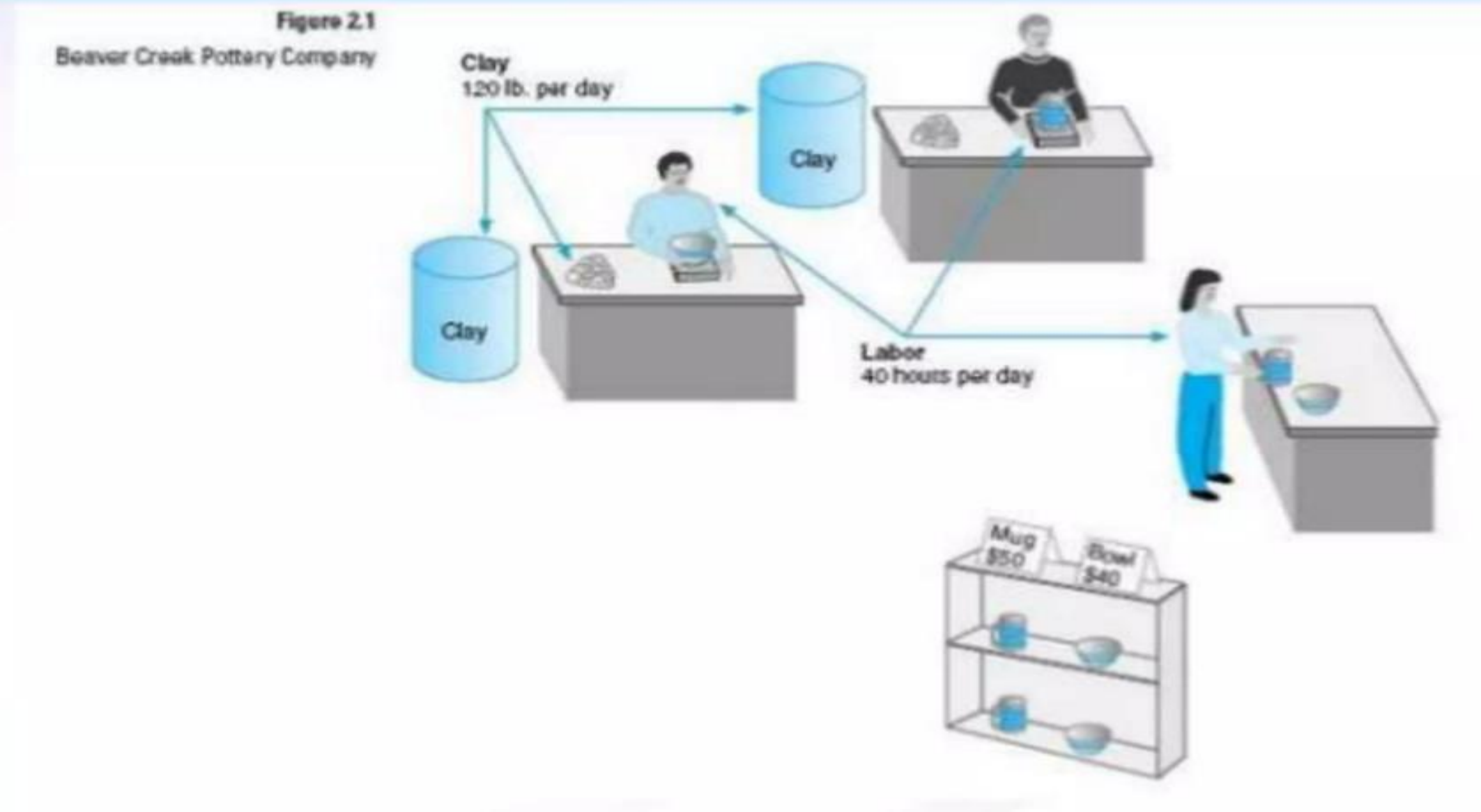


Figure 2.1
Beaver Creek Pottery Company
Clay
120 lb. per day
Clay
Clay
Labor
40 hours per day
Mug
$50
Bowl
$40

Resource Availability:	40 hrs of labor per day 120 lbs of clay
Decision Variables:	x_1 = number of bowls to produce per day x_2 = number of mugs to produce per day
Objective Function:	Maximize $Z = \$40x_1 + \$50x_2$ Where Z = profit per day
Resource Constraints:	$1x_1 + 2x_2 \leq 40$ hours of labor $4x_1 + 3x_2 \leq 120$ pounds of clay
Non-Negativity Constraints:	$x_1 \geq 0$; $x_2 \geq 0$

$$
\begin{aligned}
\text{Maximize} \quad & Z = \$40x_1 + \$50x_2 \\
\\
\text{subject to:} \quad & 1x_1 + 2x_2 \leq 40 \\
& 4x_2 + 3x_2 \leq 120 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

I. **Feasible solution:** If all the constraints of the given LP model are satisfied by the solution of the model, then that solution is known as feasible solutions.

Several such solutions are possible for a given LP model.

II. **Optimal solution:** If there is no other superior solution to the solution obtained for a given LP model, then the solution obtained is treated as the optimal solution.

III. **Alternate optimal solution :** For some LP model, there are may be more than one combination of values of the decision variables yielding the best objective function value. Such combinations of the values of the decision variables are know as alternate optimal solution.

IV. **Unbounded solution :** For some LP model, the objective function value can be increased/decreased infinitely without any limitation. Such solution is know as unbounded solution.

V. **Infeasible solution:** If there is no combination of the values of the decision variables satisfying all the constraints of the LP model, the that model is said to have infeasible solution. This means that there is no solution for the given model which can be implemented.

VI. **Degenerate solution:** In LPP, intersection of two constraints will define a corner point of the feasible region. But if more than two constraints pass through any one of the corner points of the feasible region, excess constraints will not serve any purpose, and therefore they become as redundant constraints. Under such situation, degeneracy will occur.

Thank you