Module-3 : Practice Problems

1)

x	y	y^2	xy^2	x^2
0	1.2	1.44	0	0
2	2.8	7.84	15.68	4
4	3.7	13.69	54.76	16
6	4.5	20.25	121.5	36
8	5.1	26.01	208.08	64
10	5.7	32.49	324.9	100
Σ		101.72	725.92	220

$n = 6$

(a) $y^2 = a + bx$

$$\therefore b = \frac{n\Sigma xy^2 - \Sigma x \Sigma y^2}{n\Sigma x^2 - (\Sigma x)^2} = \frac{6 \times 725.92 - 30 \times 101.72}{6 \times 220 - 900}$$

$$= 3.10$$

$$a = \frac{\Sigma y^2 - b\Sigma x}{n} = \frac{101.72 - 3.10 \times 30}{6}$$

$$= 1.45$$

$\therefore$ Regression model is, $y^2 = 1.45 + 3.1x$

(B) Regression line, $y^2 = 1.5 + 3x$

(C) Given, $x = 9$

$\therefore$ $y^2 = 1.5 + 3 \times 9$

$y^2 = 28.5$

$y = 5.33 \ / \ 5.3$

2)

x	y	$\log(y)$	$x \log y$	x^2
0.1	15.8	1.19	0.119	0.01
0.2	25.1	1.39	0.278	0.04
0.3	39.8	1.59	0.477	0.09
0.4	63.1	1.8	0.72	0.16
0.5	100	2	1	0.25
Σ 1.5		7.97	2.59	0.55

(a) fitted regression line -

$$b \Rightarrow \frac{n\Sigma x\log(y) - \Sigma x \Sigma \log(y)}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{5 \times 2.59 - 1.5 \times 7.97}{5 \times 0.55 - (1.5)^2}$$

$$= \frac{0.995}{0.5} = 1.99$$

$$a \Rightarrow \frac{\Sigma \log y - b\Sigma x}{n}$$

$$\Rightarrow \frac{7.97 - 1.99 \times 1.5}{5} = 0.99$$

(b) model is, $\log(y) = 0.99 + 1.99x$

(c) • Given, $x = 0.6$

$$\therefore \log(y) = 0.99 + 1.99 \times 0.6$$

$$\Rightarrow \log(y) = 0.99 + 1.194$$

$$\Rightarrow \log(y) = 2.184$$

$$\Rightarrow y = 10^{2.184}$$

$$\Rightarrow \boxed{y = 152.7}$$

3)

x	y	x^2	x^2y	x^4
0	16	0	0	0
1	15	1	1	1
2	12	4	8	16
3	7	9	27	81
4	0	16	64	256
Σ	50	30	100	354

(a) $y = a + bx^2$

$\therefore b = \dfrac{n\Sigma x^2y - \Sigma x^2 \Sigma y}{n\Sigma x^4 - (\Sigma x^2)^2}$

$= \dfrac{5 \times 100 - 30 \times 50}{5 \times 354 - 900}$

$= -\dfrac{1000}{870} = -1.15$

$a = \dfrac{\Sigma y - b\Sigma x^2}{n} = \dfrac{50 - 1.15 \times 30}{5}$

$= 3.1$

(b) fitted regression model is -

$y = 3.1 - 1.15x^2$

(c) Given, $x = -2$, $x^2 = 4$

$\therefore y = 3.1 - (1.15 \times 4)$

$= -1.5$

(d)

x	y	x^2	x^2y	x^4
1	3	1	3	1
2	9	4	36	16
3	19	9	171	81
4	33	16	528	256
5	51	25	1275	625
	115	55	2013	979

(a) $y = a + bx^2$

$\therefore b = \dfrac{n\Sigma x^2y - \Sigma x^2 \Sigma y}{n\Sigma x^4 - (\Sigma x^2)^2}$

$= \dfrac{5 \times 2013 - 55 \times 115}{5 \times 979 - (55)^2}$

$= 2$

$a = \dfrac{\Sigma y - b\Sigma x^2}{n}$

$= \dfrac{115 - 2 \times 55}{5}$

$= 1$

(b) we got, $b = 2$

$a = 1$

∴ regression line equation is,

$$y = 1 + 2x^2$$

(c) Given, $x = 6$, $x^2 = 36$

$$\therefore y = 1 + 2 \times 36$$

$$= 73$$

5)

x	y	y^2	xy^2	x^2
0	1.2	1.44	0	0
2	2.8	7.84	15.68	4
4	3.7	13.69	54.76	16
6	4.5	20.25	121.5	36
8	5.1	26.01	208.08	64
10	5.7	32.49	324.9	100
Σ 30		101.72	724.92	220

a) $$b = \frac{n\Sigma xy^2 - \Sigma x \Sigma y^2}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{6 \times 724.92 - 30 \times 101.72}{6 \times 220 - 900}$$

$$= 3.09 \approx 3.1$$

$$a = \frac{\Sigma y^2 - b\Sigma x}{n}$$

$$= \frac{101.72 - 3.1 \times 30}{6}$$

$$= 1.45$$

(b) Regression line is :-

$$y = 1.45 + 3.1x$$

(c) Given, $x = 9$

$$\therefore \quad y = 1.45 + 3.1 \times 9$$

$$y = 5.41$$

6)

(x) Diameter (m)	Number of people (y)	x^2	x^2y	x^4
0.5	1	0.25	0.25	0.0625
0.7	2	0.49	0.98	0.2401
0.85	3	0.72	2.16	0.5184
1	4	1	4	1
1.1	5	1.21	6.05	1.4641
	15	3.67	13.44	3.2851

(a) Number of people = ~~slope +~~ intercept + slope $\times x^2$

$$\therefore \text{slope} = \frac{5 \times 13.44 - 3.67 \times 15}{5 \cdot 3.851 - (3.67)^2}$$

$$= 2.1$$

$$\text{Intercept} = \frac{15 - 2.1 \times 3.67}{5} = 1.458 \approx 1.46$$

$\therefore$ Regression line equation :-

Number of people $= 1.46 + 2.1 \times (\text{Diameter})^2$

(b) Given diameter $= 1.3$

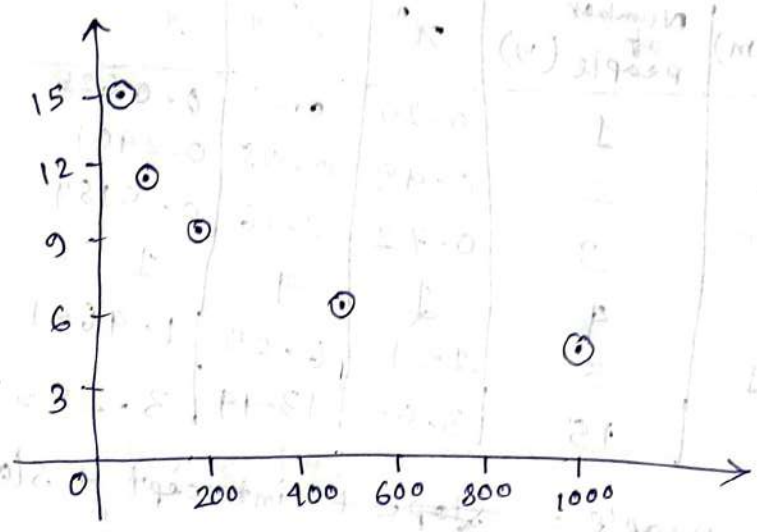
$\therefore$ Number of people $= 1.46 + 2.1 \times (1.3)^2$

$= 5.009 \approx 5$

7)

x	y	$\log(x)$	$\log(x)y$	$(\log x)^2$
10	15	1	15	1
44	11.8	1.64	19.35	2.68
132	9.4	2.12	19.92	4.5
436	6.8	2.63	17.88	6.91
981	5	2.99	14.95	8.94
	48	10.38	86.8	24.03

(a)



y decreases as x increases, the relationship is negative and non-linear.

(b) $y = a + b\log(x)$

$$b = \frac{5 \times 86.8 - 10.38 \times 48}{5 \times 24.03 - (10.38)^2}$$

$$= -5.17$$

$$a = \frac{48 + 5.17 \times 10.38}{5}$$

$$= 20.33$$

(c) $\therefore y = 20.33 - 5.17 \times \log(x)$

(d) Given, $x = 1000$

$$\therefore y = 20.33 - 5.17 \times 3$$

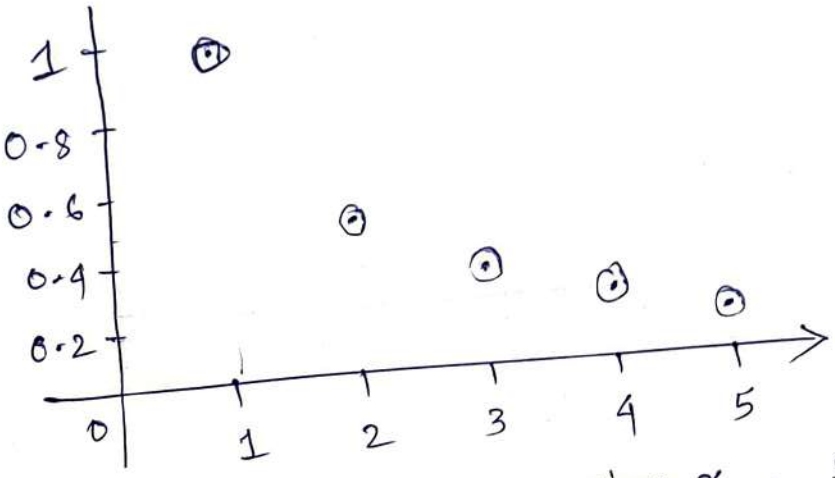
$$= 20.33 - 15.5$$

$$= 4.82$$

8)

x	y	$1/y$	x^2	x/y
1	1	1	1	1
2	0.5	2	4	4
3	0.33	3.03	9	9.09
4	0.25	4	16	16
5	0.2	5	25	25
15		15.03	55	55.09

(a)



y is decreasing on increasing x, so the relationship is negative & non-linear.

(b) $$b = \frac{5 \times 55.09 - 15 \times 15.03}{5 \times 55 - (15)^2}$$

$$= 2.5$$

$$\therefore a = \frac{15.03 - 2.5 \times 15}{5} = -4.5$$

(c) Regression line equation is -

$$\frac{1}{y} = -4.5 + 2.5x$$

(d) Given, $x = 0.25$

$$\frac{1}{y} = -4.5 + 2.5 \times 0.25$$

$$\Rightarrow \frac{1}{y} = -3.875$$

$$\Rightarrow y = -0.25$$