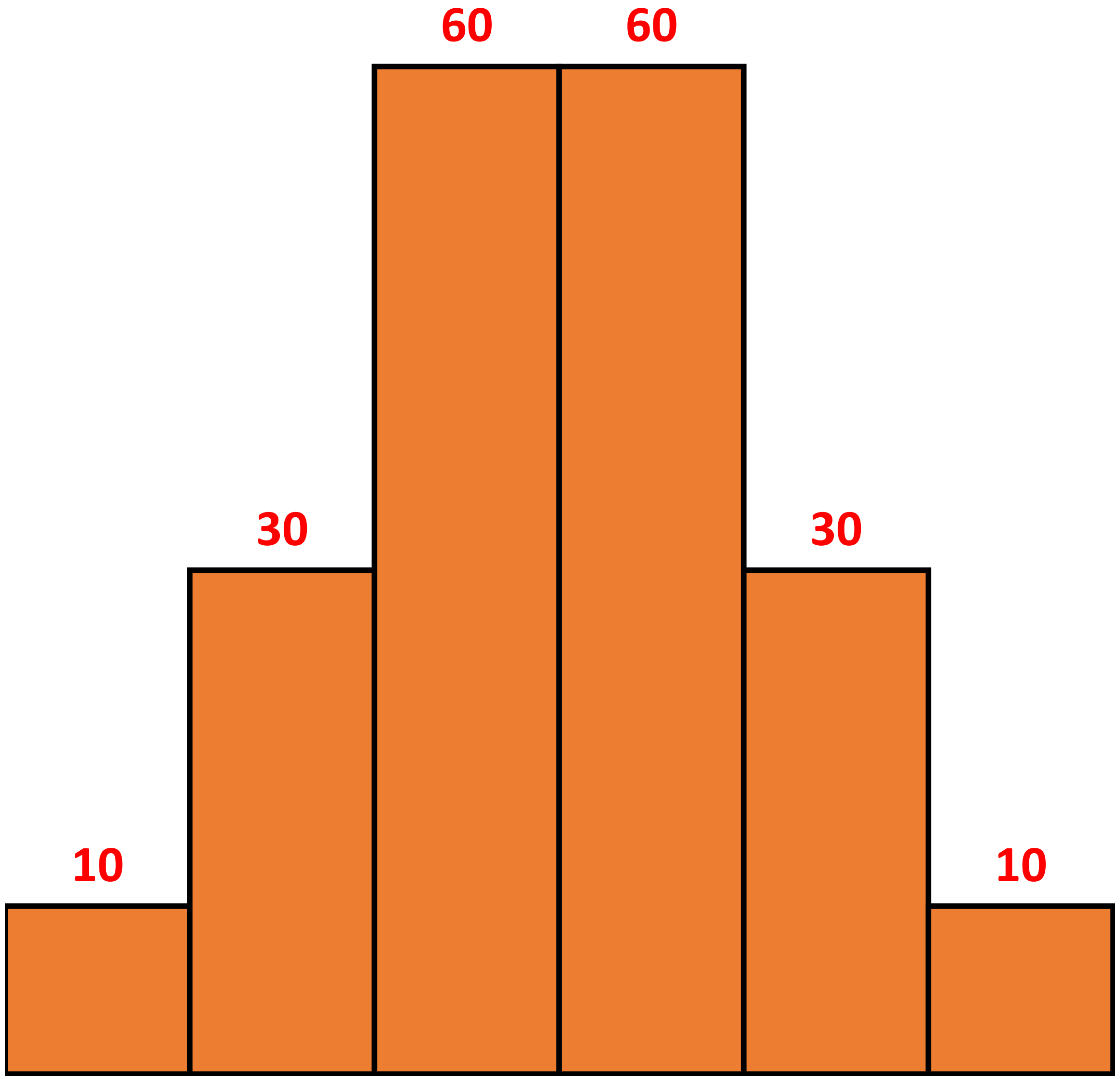
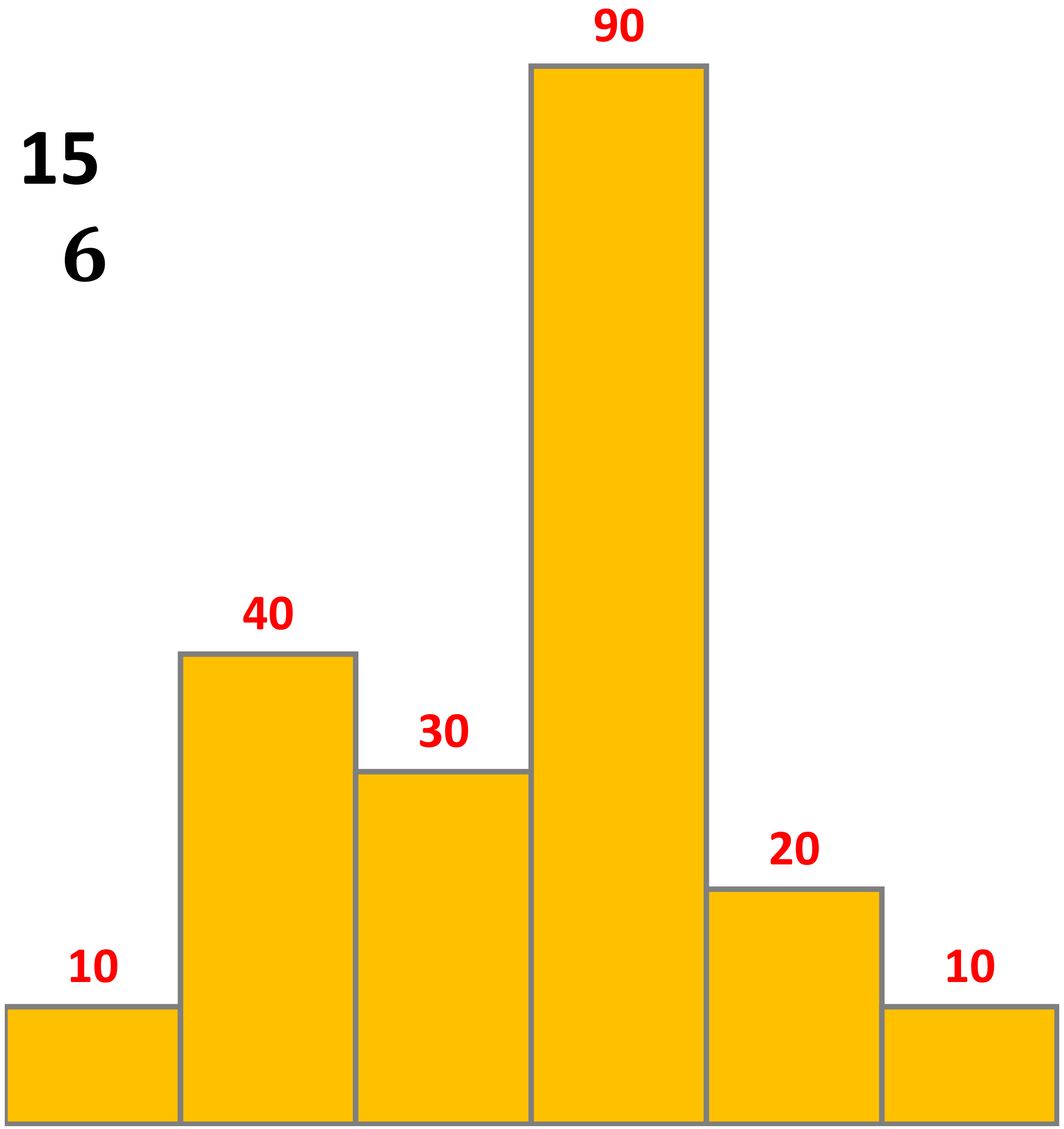
C.I.	f
0-5	10
5-10	30
10-15	60
15-20	60
20-25	30
25-30	10

C.I.	f
0-5	10
5-10	40
10-15	30
15-20	90
20-25	20
25-30	10

$$\bar{X} = 15$$
$$\sigma = 6$$



$$\bar{x} = 15$$
$$\sigma = 6$$



Skewness

When a series is not symmetrical it is said to be asymmetrical or skewed.

A distribution is said to be ‘skewed’ when the ***mean*** and ***median*** fall at different points in the distribution, and the balance (or centre of gravity) is shifted to one side or the other to left or right.

Measure of skewness is the lack of symmetry of a distribution.

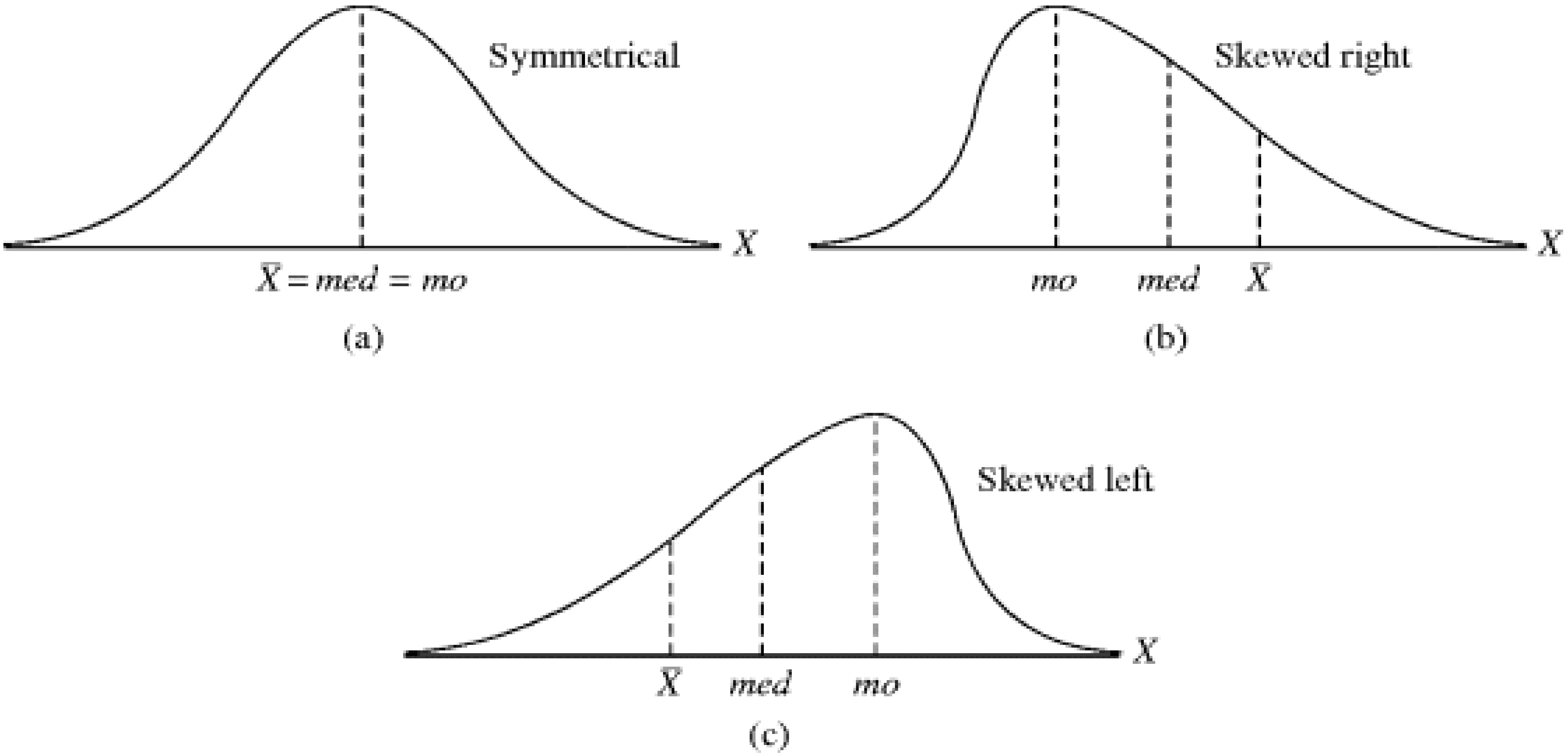
Dispersion is concerned with the amount of variation rather than with its direction.

Skewness tell us about the ***direction of the variation*** or the departure from symmetry.

Types of Skewness: (i) Symmetrical Distribution

(ii) Positively Skewed Distribution (**Mean > Mode**)

(iii) Negatively Skewed Distribution (**Mean < Mode**)


Symmetrical
X
X̄ = med = mo
(a)
Skewed right
X
mo
med
X̄
(b)
Skewed left
X
X̄
med
mo
(c)

Measure of Skewness

Absolute measures of Skewness(Sk)

$$Sk = \bar{X} - \text{Mode}$$

Relative measures of Skewness:

(i) Karl pearson's coefficient of skewness.

(ii) Bowley's coefficient of skewness.

(iii) Measure of skewness based on moments.

(i) Karl pearson's coefficient of skewness.

Coefficient of Skewness : $Sk = \frac{Mean - Mode}{Standard\ deviation} = \frac{\bar{X} - Mo}{\sigma}$

But using , $Mode = 3\ Median – 2\ Mean$

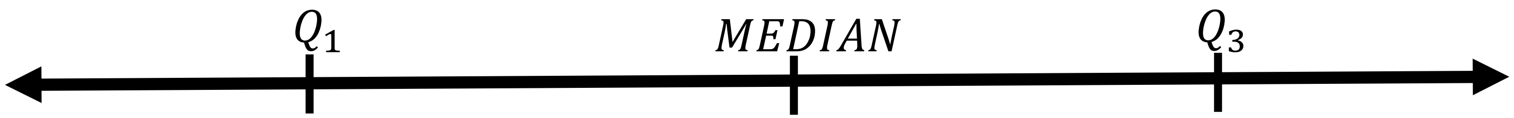
$$Sk = \frac{3(\bar{X} - Median)}{\sigma}$$

this measure can vary between $\pm\mathbf{3}$;

however, in practice, it is rare that the value of ***Sk*** exceed the limits of $\pm\mathbf{1}$**.**

(ii) Bowley's coefficient of skewness

It is based on Quartiles. In a symmetrical distribution first and third quartiles are equidistant from the median :



In a symmetrical distribution the third quartile is the same distance above the median as the first quartile is below it, i.e.,

$$Q_3 - Median = Median - Q_1$$

$$or \qquad Q_3 + Q_1 - 2\,Median = 0$$

$$\therefore \qquad Sk = \frac{Q_3 + Q_1 - 2\,Median}{Q_3 - Q_1}$$

This measure is called the ***quartile measure of skewness*** and varies between ± 1.

(iii) Moments

Moment is a measure of a force with respect to its tendency to provide rotation.

The strength of the tendency depends on the amount of force and the distance from the origin of the point at which the force is exerted.

Definition: Let the symbol $\boldsymbol{x_i} = (\boldsymbol{X_i} - \bar{\boldsymbol{X}})$ be used to represent the deviation of any item in a distribution from the arithmetic average of that distribution.

The arithmetic mean of the various powers of these deviations in any distribution are called the ***moments*** of the distribution.

Central Moments (Moments about the Arithmetic Mean):

First Moment $\mu_1 = \frac{\sum(X_i - \bar{X})}{N}$: (sum of the deviations from A.M. is always zero. $\boldsymbol{\mu_1 = 0}$)

Second Moment $\mu_2 = \frac{\sum(X_i - \bar{X})^2}{N}$: σ^2 = Variance

Third Moment $\mu_3 = \frac{\sum(X_i - \bar{X})^3}{N}$

Fourth Moment $\mu_4 = \frac{\sum(X_i - \bar{X})^4}{N}$

For a frequency distribution:

First Moment $\mu_1 = \frac{\sum f_i(X_i - \bar{X})}{N}$

Second Moment $\mu_2 = \frac{\sum f_i(X_i - \bar{X})^2}{N}$: σ^2 = Variance

Third Moment $\mu_3 = \frac{\sum f_i(X_i - \bar{X})^3}{N}$ or $\frac{\sum f_i \, x_i^3}{N}$

Fourth Moment $\mu_4 = \frac{\sum f_i(X_i - \bar{X})^4}{N}$ or $\frac{\sum f_i \, x_i^4}{N}$

$$\beta_1 \text{ (beta one) } = \frac{\mu_3^2}{\mu_2^3} \quad \text{(Coeff. of Skewness)}$$

$$\beta_2 \text{ (beta two) } = \frac{\mu_4}{\mu_2^2} \quad \text{(Coeff. of Kurtosis)}$$

$$\gamma_1 \text{(Gamma one)} = \sqrt{\beta_1} \quad \text{(Coeff. of Skewness)}$$

$$\gamma_2 \text{ (Gamma two)} = \beta_2 - 3 \quad \text{(Coeff. of Kurtosis)}$$

Non-central Moments
(Moments about the Assumed Mean)

Where the actual mean is in fractions it is difficult to calculate moments by applying the above formulae.

In such cases we can first compute moments about an **arbitrary origin (*A*)**.

$$\mu_1' = \frac{\sum(X_i - A)}{N} \text{ (Mean)} \quad ; \quad \mu_1' = \frac{\sum f_i(X_i - A)}{N}$$

$$\mu_2' = \frac{\sum(X_i - A)^2}{N} \quad ; \quad \mu_2' = \frac{\sum f_i(X_i - A)^2}{N}$$

$$\mu_3' = \frac{\sum(X_i - A)^3}{N} \quad ; \quad \mu_3' = \frac{\sum f_i(X_i - A)^3}{N}$$

$$\mu_4' = \frac{\sum(X_i - A)^4}{N} \quad ; \quad \mu_4' = \frac{\sum f_i(X_i - A)^3}{N}$$

Conversion of moments about an Arbitrary origin into Moments about mean

$$\mu_1 = \mu_1' - \mu_1'$$

$$\mu_2 = \mu_2' - (\mu_1')^2$$

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3$$

$$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$

Marks	***Frequency***
0-10	5
10-20	20
20-30	15
30-40	45
40-50	10
50-60	5

Find the Measure of skewness based on moments.

Marks	*Mid point (m)*	f	$d = (m - 35)/10$	fd	fd^2	fd^3
0-10	5	5	-3	-15	45	-135
10-20	15	20	-2	-40	80	-160
20-30	25	15	-1	-15	15	-15
30-40	35	45	0	0	0	0
40-50	45	10	+1	+10	10	+10
50-60	55	5	+2	+10	20	+40
		N =100		$\sum fd$ = -50	$\sum fd^2$ = 170	$\sum fd^3$ = -260

$$\mu_1' = \frac{\sum f_i d_i}{N} \times i = -5$$

$$\mu_2' = \frac{\sum f_i d_i^2}{N} \times i^2 = 170$$

$$\mu_3' = \frac{\sum f_i d_i^3}{N} \times i^3 = 2600$$

$$\mu_2 = \mu_1' - (\mu_1')^2 = 145$$

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3 = -300$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = 0.02952$$

$$\gamma_1 = \sqrt{\beta_1} = -0.172$$

Since γ_1 - is negative, the distribution is negatively skewed.

Kurtosis

Kurtosis enables us to have an idea about the '*flatness*' or '*peakedness*' of the frequency curve.

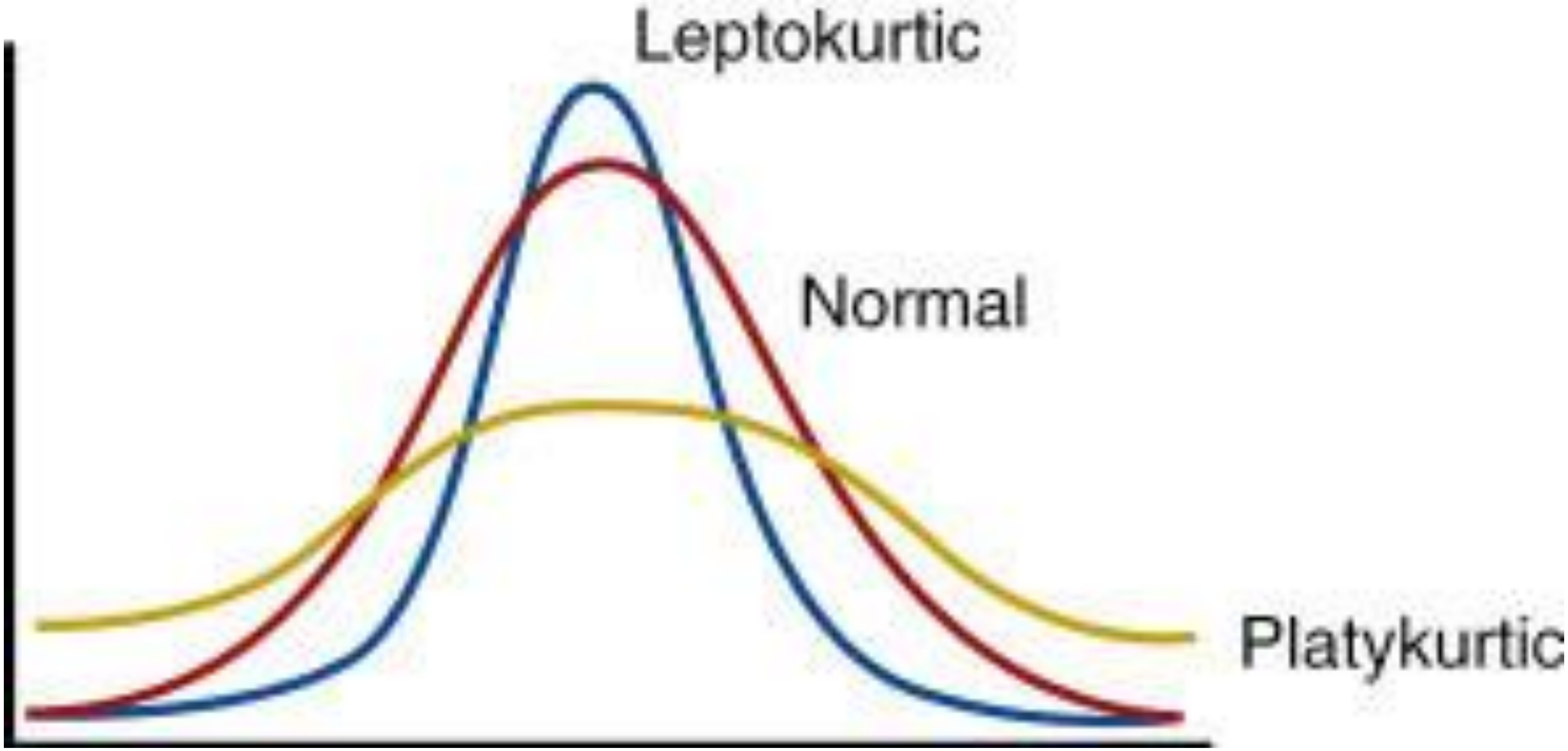
"Convexity of the frequency curve"

It is measured by the Coefficient of $\beta_2 = \frac{\mu_4}{\mu_2^2}$ or $\gamma_2 = \beta_2 - 3$

(i) $\beta_2 = 3$, *i.e.*, $\gamma_2 = 0$: mesokurtic curve

(ii) $\beta_2 < 3$, *i.e.*, $\gamma_2 < 0$: platykurtic curve

(iii) $\beta_2 > 3$, *i.e.*, $\gamma_2 > 0$: leptokurtic curve


Leptokurtic
Normal
Platykurtic