PMDS504L: Regression Analysis and Predictive Models

Introduction to Simple Linear Regression

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Learning Goals

- Deterministic Relationship
- Non-Deterministic Relationship
- Simple Linear Regression Model

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Regression

- A statistical technique for modeling and analyzing numerical data.
- Exploits the relationship between two or more variables.
- Helps gain information about one variable through known values of others.

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Deterministic vs. Non-Deterministic Relationships

Deterministic Relationships:

• Occurs when one variable's value completely determines the other's value.

Example:

- A cab service charges a fixed base fare of Rs. 450, plus Rs. 15 per kilometer.
- Total fare y = 450 + 15x.
- For 10 kilometers, $y = 450 + 15 \times 10 = 600$.

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Linear Deterministic Relationship

- The simplest deterministic mathematical relationship is linear: $y = b_0 + b_1 x$.
- b_0 : Y-intercept (value of y when x = 0).
- b_1 : Slope (change in y for a one-unit change in x).
- Forms a straight line on a graph.

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Non-Deterministic Relationships

- More complex, where the relationship is not exact but shows a discernible pattern.
- Example: High school GPA X and college GPA Y.
- Predicts trends but not exact values.

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Non-Deterministic Relationships: Examples

- Child's age and vocabulary size.
- Engine size and fuel efficiency in automobiles.
- Generalize deterministic linear relationships into a probabilistic model.

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Non-Deterministic Relationships: Error Term ε

- Represents random variation or noise.
- Accounts for deviations from the deterministic path.
- $Y = \beta_0 + \beta_1 X + \varepsilon$.

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Key Variables

Independent Variable (X):

• Predictor or explanatory variable.

Dependent Variable (Y):

• Outcome or response variable.

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The Simple Linear Regression Model

• Relates *Y* and *X* through the model equation:

$$Y = \beta_0 + \beta_1 x + \varepsilon$$

Where:

- Y: Dependent variable (response).
- X: Independent variable (predictor).
- β_0 : Y-intercept.
- β_1 : Slope.
- ε : Random error term.

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Error Term (ε)

- Randomly distributed with mean 0 and variance σ^2 .
- Accounts for deviations from the true regression line.
- Points scatter around the true regression line.

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Regression Models

- Simple Linear Regression Model.
- Multiple Linear Regression Model.

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Summary

- Bivariate data.
- Deterministic linear relationships.
- Non-deterministic linear relationships.
- Simple linear regression model.
- Example.

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Abstract on Next Session

- Simple linear regression model.
- Estimation of regression coefficients.

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Thank You!

Thank you for your attention!

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