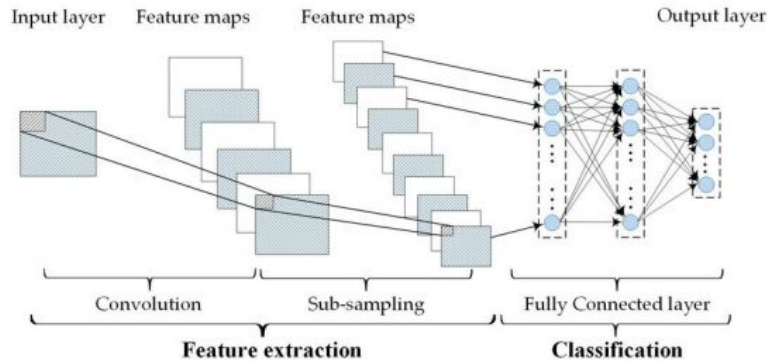




# Genetic Algorithm In Echo State Network

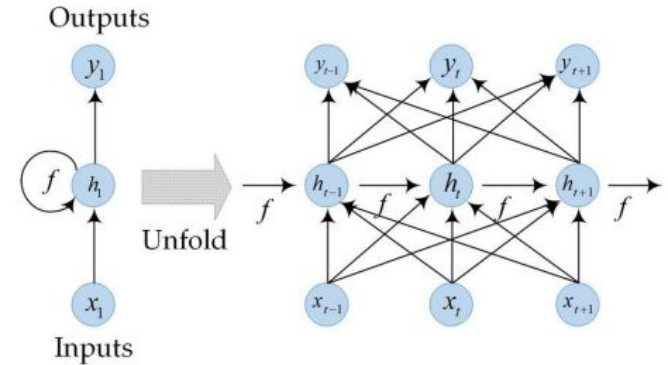
Soumyadeep Jana  
Roll - 21D070075  
Guide - Prof. Udayan Ganguly  
Mentor - Anmol Biswas

# Convolutional vs Recurrent Neural Network



## Convolutional Neural Net (CNN)

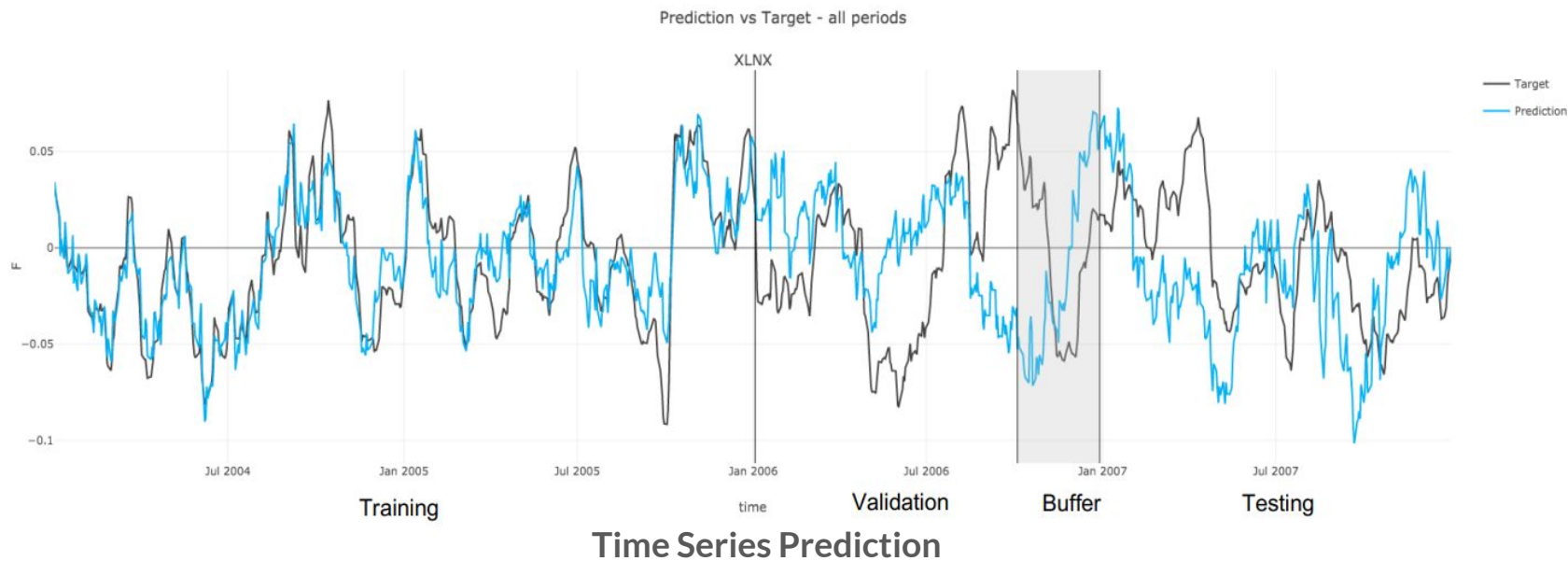
Spatial Structure matters



## Recurrent Neural Net (RNN)

Sequential Context matters

# Future Trend...



# Limitations of RNN

- Vanishing Gradient = Gradient becomes too small → Hard to learn long-term patterns.
- Exploding Gradient = Gradient becomes too big → Training becomes chaotic. These are like trying to balance between not losing your message and not making it overwhelming!
- Computationally intensive and hard to tune

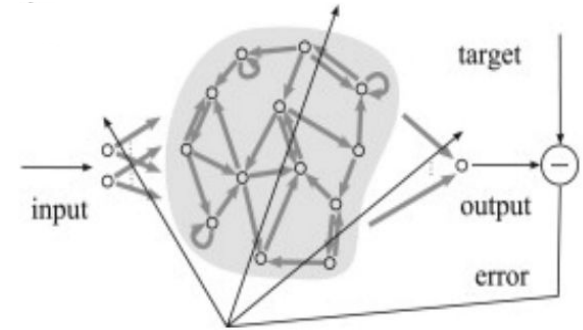


Fig - Traditional RNN. Bold arrows indicate trained weights

# Reservoir Computing

- Fast! Accurate! Still challenging to tune
- Only output weights are adapted, no need of long back propagation

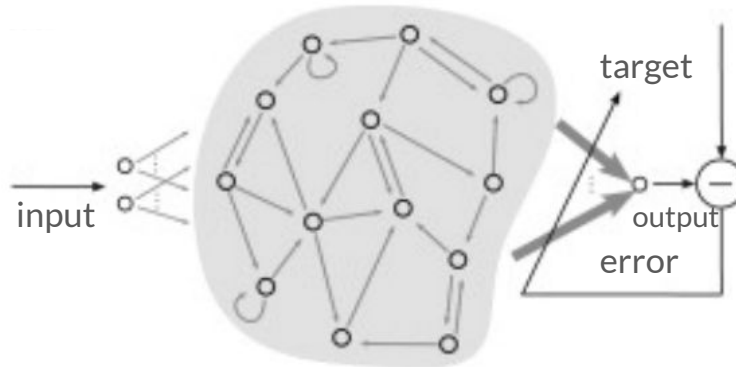


Fig - Traditional Reservoir computing .Bold arrows indicate trained weights



# Types of Reservoir Computing

- Liquid State Machine
- **Echo State Network**
- Context Reverberation
- Nonlinear Transient Computation
- Deep Reservoir Computing

# Echo State Network

ESNs are a recurrent neural network

- Captures dynamic behavior of a time series or sequential data.
- Internal state (memory) to process sequences of inputs and remember context

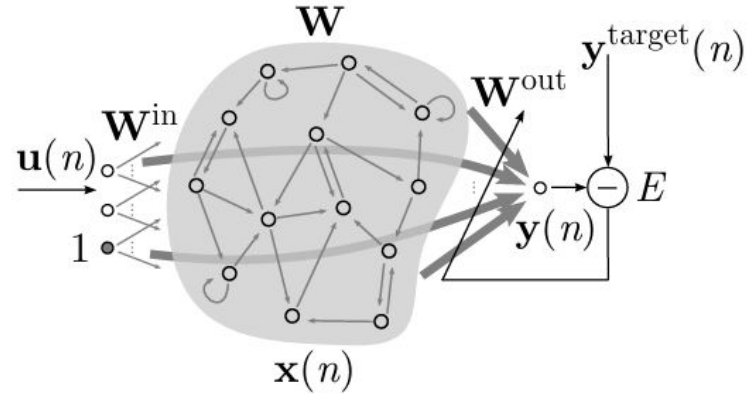
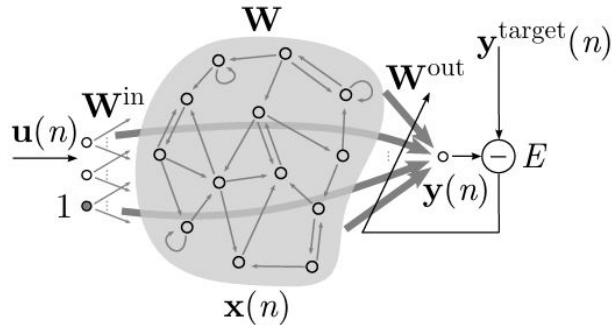


Fig :- An Echo State Network

# Overview



**For each time step  $n$**

$u(n)$ : input

$x(n)$ : reservoir activations

$y(n)$ : network output

$y^{target}(n)$ : desired output

**Randomly Initialized & fixed**

$W^{in}$ : input weights. Linearly map input into reservoir

$W$ : recurrent weights. How are the reservoir neurons connected to each other?

**Learned parameters**

$W^{out}$ : output weights

- Linearly map  $(u(n), x(n))$  to  $y(n)$
- Minimize  $\text{Error}(y^{target}(n), y(n))$





## Dimensions and Update Equation

$W^{in} \in \mathbb{R}^{N_x \times (1+N_u)}$ : input weights

$W \in \mathbb{R}^{N_x \times N_x}$  : recurrent weights

$\alpha \in (0,1]$ : leak rate

$x(n) \in \mathbb{R}^{N_x}$  : neuron activations

$\tilde{x}(n) \in \mathbb{R}^{N_x}$  : neuron updates

$$x(n) = (1 - \alpha)x(n - 1) + \alpha\tilde{x}(n)$$

$$\tilde{x}(n) = \tanh(W^{in}[1; u(n)] + Wx(n - 1))$$



## Reservoir Parameter:- Input Weights

$$\mathbf{W}^{in} \in \mathbb{R}^{N_x \times (1+N_u)}$$

$$W_{ij}^{in} \sim \text{Unif}(-a, a)$$

**Tune/scale input weights by varying  $a$**

- Input scaling determines how nonlinear reservoir responses are
  - $a < 1$  (generally)
- Scaling input weights helps remove the influence of outliers
- Taken  $a = 0.5$



## Reservoir Parameter:- Spectral Radius

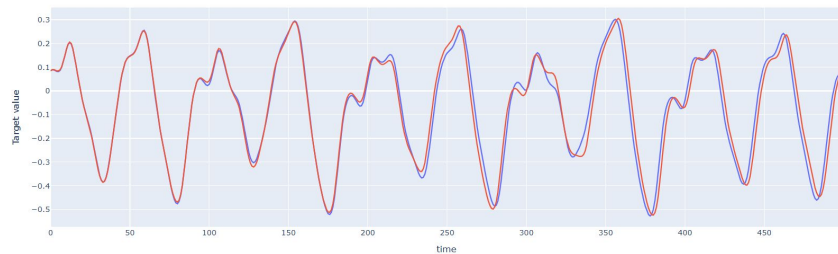
$$W \in \mathbb{R}^{N_x \times N_x}$$

$$\rho(W) = \text{spectral radius of } W$$

- Spectral radius = eigenvalue of  $W$  with greatest magnitude
- The value of  $\rho$  is related to the variable memory length and the degree of contractivity of reservoir dynamics, with larger values of  $\rho < 1$  resulting in longer memory length
- Taken  $\rho = 0.95$

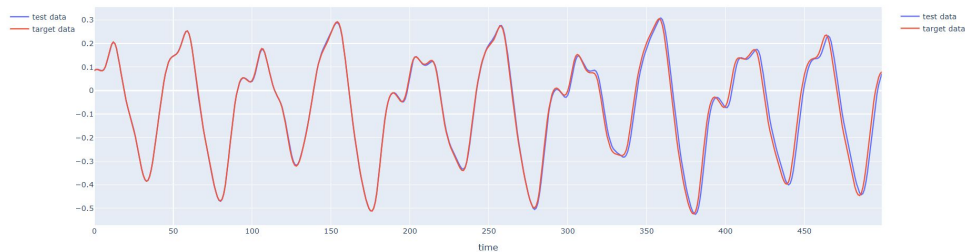
# Effect of Spectral Radius

test output vs target output



**Spectral radius = 0.75**  
**MSE = 0.001528**

test output vs target output



**Spectral radius = 0.95**  
**MSE = 0.000137**



## Reservoir Parameter:- Leaky Rate

$\alpha \in (0, 1]$ : leak rate

$x(n) = (1 - \alpha)x(n - 1) + \alpha\tilde{x}(n)$ : neuron activations

$\tilde{x}(n) = \tanh(W^{in}[1; u(n)] + Wx(n - 1))$ : neuron updates

- Captures temporal dynamics of system
- "speed of reservoir update dynamics discretized in time"
- Taken as 0.5



## Output Weight:-

$$\mathbf{y}(t) = \mathbf{W}_{out} \mathbf{x}(t)$$

$$\mathbf{y}(t) \in \mathbb{R}^{N_y} \quad \mathbf{W}_{out} \in \mathbb{R}^{N_y \times N_x}$$

$$\mathbf{W}_{out} = \mathbf{Y}^T \mathbf{X}^T (\mathbf{X} \mathbf{X}^T + \lambda \mathbf{I})^{-1}$$

where  $\mathbf{y} = [y(1), y(2), \dots, y(N_y)]^T \in \mathbb{R}^{N_y}$  is the target vector  
 $\mathbf{X} = [x(1), x(2), \dots, x(N_x)] \in \mathbb{R}^{N_y \times N_x}$



## Ensemble ESN

- Combines outputs of multiple ESNs for more accurate predictions
- Less sensitive to noisy or corrupted data compared to a single ESN
- Each ESN in the ensemble can have different reservoir parameters or connectivity, capturing a wider range of dynamics

But in what pattern should reservoirs be connected ?

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## Let's Take some examples...

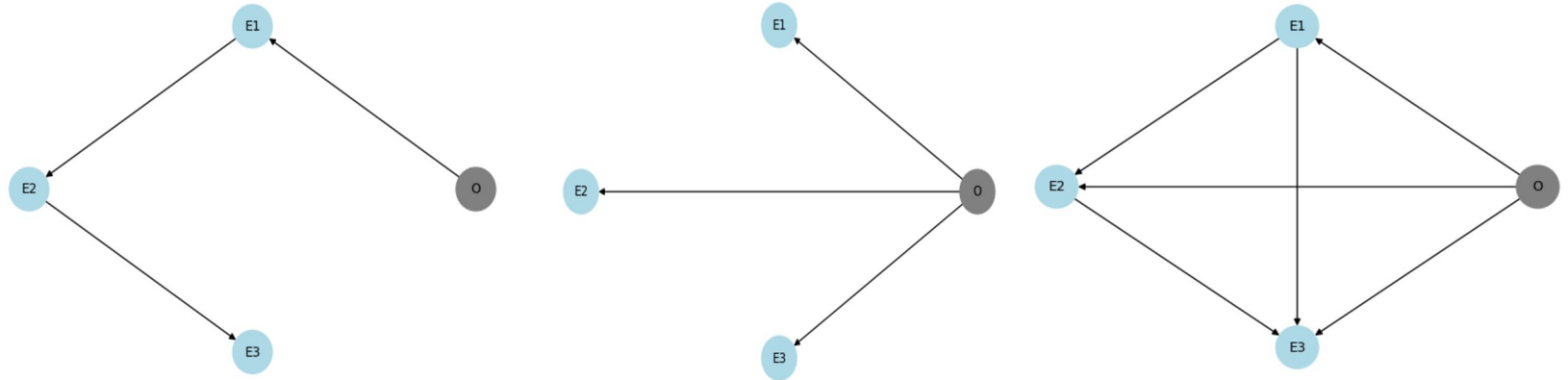


Fig - Some possible ensemble topology. Here O(input) and Ei(Reservoir)





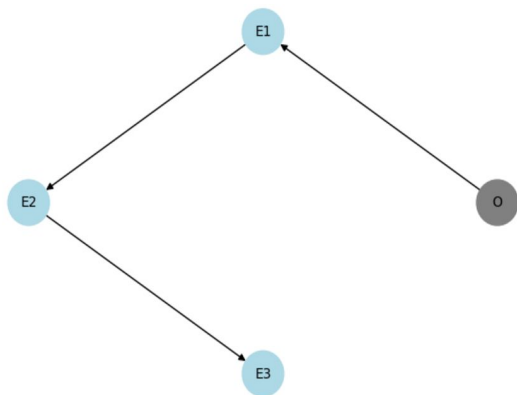
## Ensemble Case

$$\mathbf{x}^{(l)}(t) = (1 - a^{(l)})\mathbf{x}^{(l)}(t - 1) + a^{(l)} \tanh(\mathbf{W}_{in}^{(l)}\mathbf{i}^{(l)}(t) + \boldsymbol{\theta}^{(l)} + \hat{\mathbf{W}}^{(l)}\mathbf{x}^{(l)}(t - 1)),$$

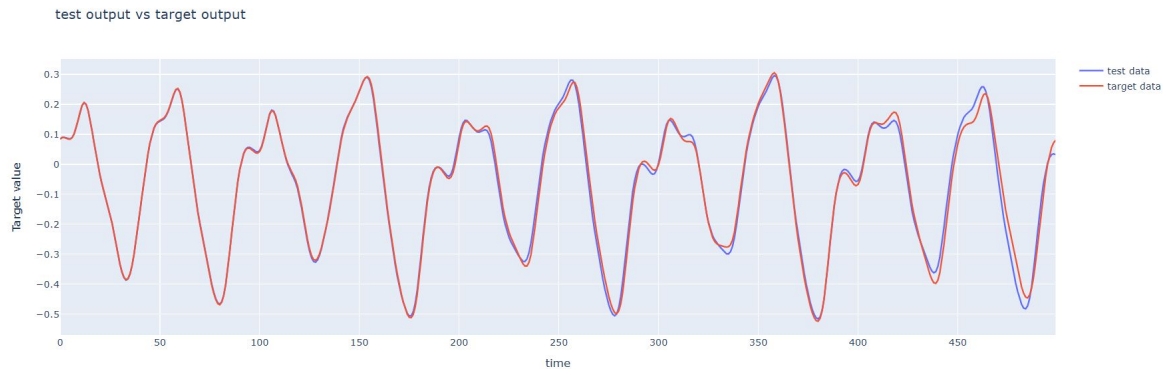
$$\mathbf{y}(t) = \mathbf{W}_{out} [\mathbf{x}^{(1)}(t) \mathbf{x}^{(2)}(t) \dots \mathbf{x}^{(N_L)}(t)]^T$$

$$\mathbf{i}^{(l)}(t) = \begin{cases} \mathbf{u}(t) & \text{if } l = 1 \\ [\mathbf{u}(t) \mathbf{x}^{(l-1)}(t)]^T & \text{if } l > 1 \end{cases}$$

# Series Connection

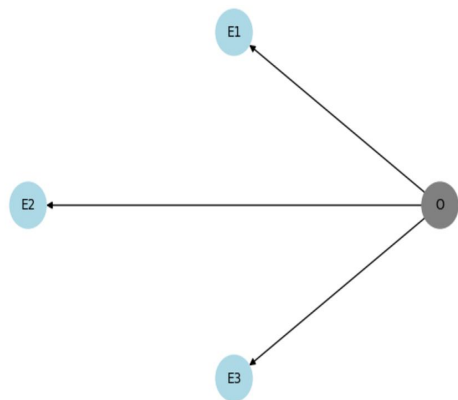


Series Connection topology: Deep ESN

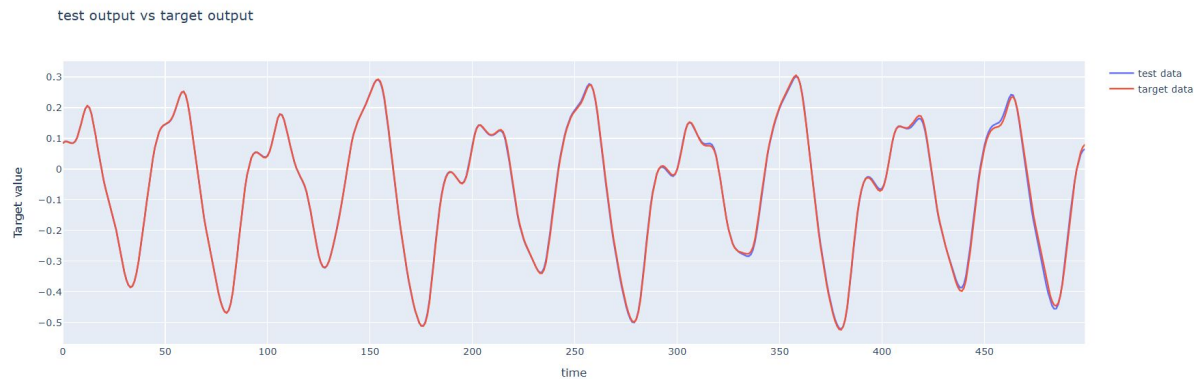


MSE: 0.000462

# Parallel Connection

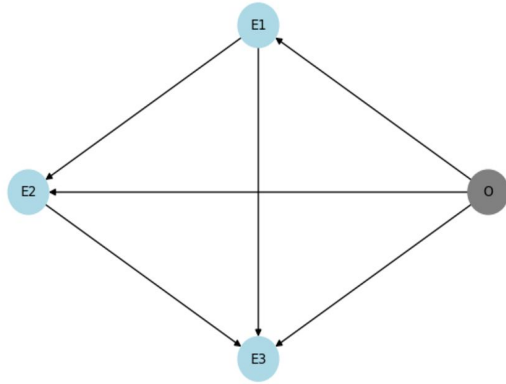


Parallel Connection Topology: GroupedESN

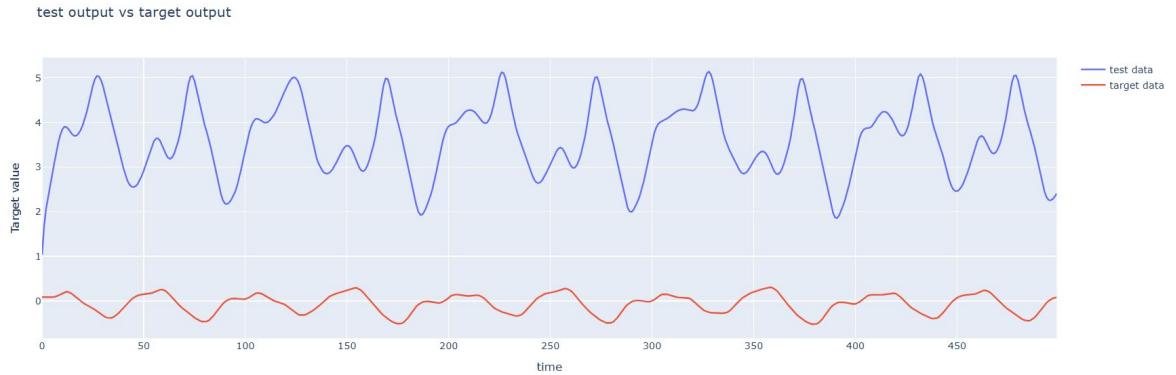


MSE: 3.8e-05

# Cross Connection



Cross Connection Topology



MSE: 14.205438

# Genetic Algorithm

- Represent reservoir combination as binary sequence

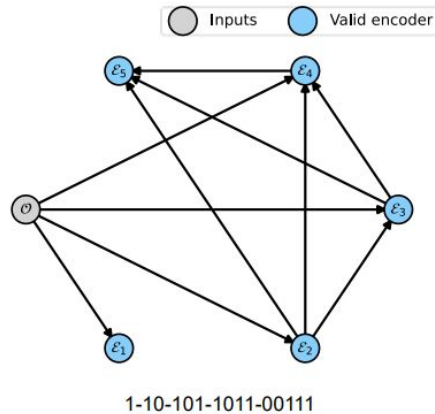


Fig: An example network topology of an MRESN (upper) and the corresponding binary sequence (bottom)

# Valid Encoder

Not all possible combination will be valid.

The encoder node  $E_i$  is invalid

- its in-degree is zero or
- all ancestor nodes of  $E_i$  are invalid encoder nodes

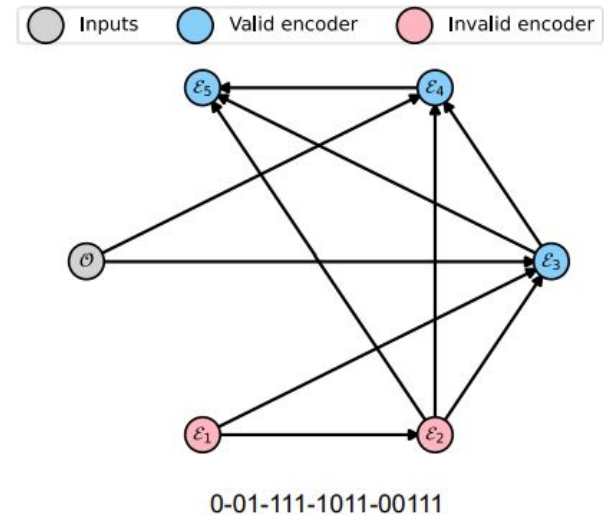
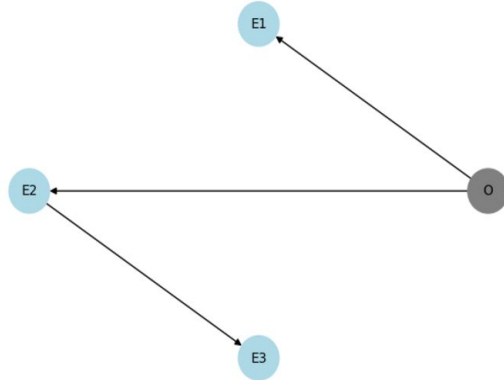


Fig - An example network topology of an MRESN with three valid encoders and two invalid encoders

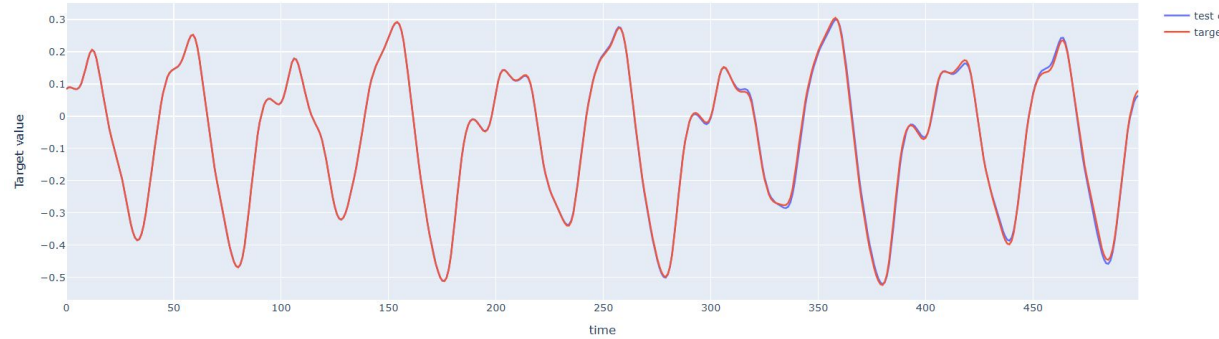
# Results:

Network Topology: 1-10-001



Most optimal connection from Genetic Algorithm

test output vs target output



MSE: 9.8e-06



## My Work

[https://github.com/Soumyadeepj/Genetic\\_ESN.git](https://github.com/Soumyadeepj/Genetic_ESN.git)





## Future Work:

- Effect of increment of the number of neurons and reservoir
  - Hyperparameter tuning for better MSE
  - Application of Genetic Algorithm in Liquid State Machine
-



# References

1. [https://github.com/tguidici/ODSC\\_2020\\_ESN/blob/master/teal\\_ODSC\\_2021\\_EAST\\_ESN.pdf](https://github.com/tguidici/ODSC_2020_ESN/blob/master/teal_ODSC_2021_EAST_ESN.pdf)
  2. Lukoševičius, M. (2012). A Practical Guide to Applying Echo State Networks. In: Montavon, G., Orr, G.B., Müller, KR. (eds) Neural Networks: Tricks of the Trade. Lecture Notes in Computer Science, vol 7700. Springer, Berlin, Heidelberg.  
[https://doi.org/10.1007/978-3-642-35289-8\\_36](https://doi.org/10.1007/978-3-642-35289-8_36)
  3. Claudio Gallicchio, Alessio Micheli, Luca Pedrelli, Deep reservoir computing: A critical experimental analysis, Neurocomputing Volume 268, 2017, Pages 87-99, ISSN 0925-2312, <https://doi.org/10.1016/j.neucom.2016.12.089>
  4. Z. Li, K. Fujiwara and G. Tanaka, "Designing Network Topologies of Multiple Reservoir Echo State Networks: A Genetic Algorithm Based Approach," 2024 *International Joint Conference on Neural Networks (IJCNN)*, Yokohama, Japan, 2024, pp. 1-9, doi: 10.1109/IJCNN60899.2024.10650945
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# Thank You

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