H.W. (Celebal
C.W. Tech.) week 4

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Bayes Theorem

Let, $\{\omega_1, \omega_2, \ldots, \omega_c\}$ be the finite set of 'c' states of nature. They are mutually exclusive and exhaustive events, with continuous random variable 'x'. (also called feature vector)

when the state of nature ω_j ;

$$P(\omega_j|x) = \frac{p(x|\omega_j) \cdot P(\omega_j)}{p(x)}$$

where, $P(\omega_j)$ = prior probability in that nature in state ω_j.

$P(\omega_j|x)$ = posterior probability can be

where, $p(x) = \sum_{j=1}^{c} p(x|\omega_j) \cdot P(\omega_j)$

ex

You are planning a trip. You are trying to decide whether to postpone due to rain. The chance of rain on any day is 15%. The morning of the ~~[illegible]~~ weather cloudy. The probability it being cloudy is 25%. and on days where it rains, it's cloudy in the morning 80% of the time.

Should I postpone the trip?

Ans:- $P(\text{rain}) = 0.15$

$P(\text{cloudy}) = 0.25$

$P(\text{cloudy}|\text{rain}) = 0.80$

now, bayes rule

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

∴ As per question,

$$P(\text{rain}|\text{cloudy}) = \frac{P(\text{cloudy}|\text{rain}) \cdot P(\text{rain})}{P(\text{cloudy})}$$

$$= \frac{0.80 \times 0.15}{0.25}$$

$$= \frac{0.12}{0.25}$$

$$= \frac{12}{25} = 0.48$$

∴ Answer = 48%

Because probability 48% rain, you not to be worry. You do not need postpone the trip.

Properties of Normal distribution

1) Bell shaped curve

- It has symmetric, bell shaped curve
- It is characterized by its mean (μ) and standard deviation (σ)
- The total area under the curve is always equal to 1.

2) It tends to Central Limit Theorem.

~~Applications~~

Applications of normal distribution

- statistical Analysis
- Quality control (manufacturing), process capability analysis.

Eigen values and Eigen vectors

Q) $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$

Ans:-

#1 $(A - \lambda I) \cdot x = 0$

$$\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix} - \lambda \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

#2 $|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix} - \lambda \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{vmatrix} = 0$$

#1

$(A - \lambda I) x = 0$

$(A - \lambda I) . x = 0$

#2

$|A - \lambda I| = 0$

#3

sum of diagonal

$\lambda^3 - \{\ \} \lambda^2 + [\ ] \lambda - |A| = 0$

sum of diagonal elements (given matrix)

#3

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$\cancel{\Rightarrow \lambda^2(\lambda-6)+}$

$\cancel{\Rightarrow \lambda^3 + 11\lambda - 6\lambda^2 - 6 = 0}$

$\cancel{\Rightarrow \lambda(\lambda^2+11) - 6(\lambda^2+1) = 0}$

-6
1
$1, 2, 3, 6$

$+1, -1, -2, +2, +3, -3, 6, -6$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$\cancel{(-1) \quad (-2)}$

$$(\lambda + (-1))(\lambda + (-2))(\lambda + (-3)) = 0$$

$$\Rightarrow (\lambda - 1)\cdot(\lambda - 2)\cdot(\lambda - 3) = 0$$

$\therefore$ roots are,

$$\boxed{\lambda = 1, 2, 3}$$

$\therefore$ Eigen values = 1, 2, 3 (Ans)

$$\begin{vmatrix} -3 & -2 \\ -4 & 1 \end{vmatrix} = -3-(+8) = -11$$

$$\begin{vmatrix} 8 & -2 \\ 3 & 1 \end{vmatrix} = 8-(-6) = 14$$

$$\begin{vmatrix} 8 & -8 \\ 4 & -3 \end{vmatrix} = -24-(-32) = -24+32 = 8$$

$\therefore 14 + 8 + (-11) = 11$

$$\det(A) = \det\begin{pmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{pmatrix}$$

$$\cancel{= (8)\begin{vmatrix} -3 & -2 \\ -4 & 1 \end{vmatrix} - (-8)\cdot\begin{vmatrix} 4 & -2 \end{vmatrix}}$$

$$= 8\cdot\begin{vmatrix} -3 & -2 \\ -4 & 1 \end{vmatrix} - (-8)\cdot\begin{vmatrix} 4 & -2 \\ 3 & 1 \end{vmatrix} + (-2)\cdot\begin{vmatrix} 4 & -3 \\ 3 & -4 \end{vmatrix}$$

$$-4\begin{vmatrix} -8 & -2 \\ -4 & 1 \end{vmatrix} + (-3)\cdot\begin{vmatrix} 8 & -2 \\ 3 & 1 \end{vmatrix} - (-2)\cdot\begin{vmatrix} 8 & - \\ 3 & - \end{vmatrix}$$

$$+3\cdot\begin{vmatrix} -8 & -2 \\ -3 & -2 \end{vmatrix} - (4)\cdot\begin{vmatrix} 8 & -2 \\ 3 & 1 \end{vmatrix} + 1\cdot\begin{vmatrix} 8 & -8 \\ 4 & -3 \end{vmatrix}$$

$$= 6 \rightarrow \det(A)$$

Finding Eigen vectors

$$\begin{bmatrix} 7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

crammer's rule

$\lambda = 1$

$$7x_1 - 8x_2 - 2x_3 = 0 \quad \text{—— (i)}$$

$$4x_1 - 4x_2 - 2x_3 = 0 \quad \text{—— (ii)}$$

$$\frac{x_1}{\begin{vmatrix} 8 & -2 \\ -4 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 7 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 7 & -8 \\ 4 & -4 \end{vmatrix}}$$

$$\Rightarrow \frac{x_1}{16-8} = \frac{-x_2}{-6} = \frac{x_3}{-28-(-32)}$$

$$\Rightarrow \frac{x_1}{8} = \frac{x_2}{6} = \frac{x_3}{4}$$

$$X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$$

$\lambda = 2$

$$\begin{bmatrix} 6 & -8 & -2 \\ 4 & -5 & -2 \\ 3 & -4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \frac{x_1}{\begin{vmatrix} -8 & -2 \\ -5 & -2 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 6 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 6 & -8 \\ 4 & -5 \end{vmatrix}}$$

$$\Rightarrow \frac{x_1}{16-10} = \frac{x_2}{-12-(-8)} = \frac{x_3}{-30-(-32)}$$

$$\Rightarrow \frac{x_1}{6} = \frac{x_2}{\cancel{20}\,4} = \frac{x_3}{2}$$

$$\therefore X_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$\lambda = 3$

$$\begin{bmatrix} 8-3 & -8 & -2 \\ 4 & -3-3 & -2 \\ 3 & -4 & 1-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 & -8 & -2 \\ 4 & -6 & -2 \\ 3 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$5x_1 - 8x_2 - 2x_3 = 0$

$4x_1 - 6x_2 - 2x_3 = 0$

$$\Rightarrow \frac{x_1}{\begin{vmatrix} -8 & -2 \\ -6 & -2 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 5 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & -8 \\ 4 & -6 \end{vmatrix}}$$

$$\Rightarrow \frac{x_1}{16-12} = \frac{x_2}{-10-(-8)} = \frac{x_3}{-30-32}$$

$$\Rightarrow \frac{x_1}{4} = \frac{x_2}{2} = \frac{x_3}{-62}$$

$$\therefore X_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -62 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -31 \end{bmatrix}$$

$$\therefore X_1 = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \quad X_3 = \begin{bmatrix} 2 \\ 1 \\ -31 \end{bmatrix}$$

(Ans)