THE GAMMA FUNCTION

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LaTeX Project

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Abstract

The main topic of this project is about Gamma function.The Gamma func- tion belongs to the category of Special transcendental functions,it was intro- duced first by the Swiss mathematician Leonhard Euler(1707-1783) in order to generalize the factorial to the non integer values.An elementry introducyion and basic properties are provided in this paper

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Contents

List of Figures

SOME NOTEABLE MATHEMATICIAN

(a) Karl Weierstrass

(b) Euler

(c) Daniel Bernouli

(d) Cornelius Lanczos

(e) Charles Hermite

Introduction

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In mathematics, the Gamma function (represented by Γ, the capital letter gamma from the Greek alphabet) is one commonly used extension of the factorial function to complex numbers. The gamma function is defined for all complex numbers except the non-positive integers. For any positive integer n,

$$\boxed{\Gamma(n = (n-1)!}$$

Derived by Daniel Bernoulli,for complex numbers with a positive real part,the gamma function is defined via a convergent improper integral:

$$\Gamma(z) = \int_0^\infty x^{z-1} e^{-x} dx, \quad \Re(z) > 0.$$

The gamma function then is defined as the analytic continuation of this integral function to a meromorphic function that is holomorphic in the whole complex plane except zero and the negative integers,where the function has simple poles.

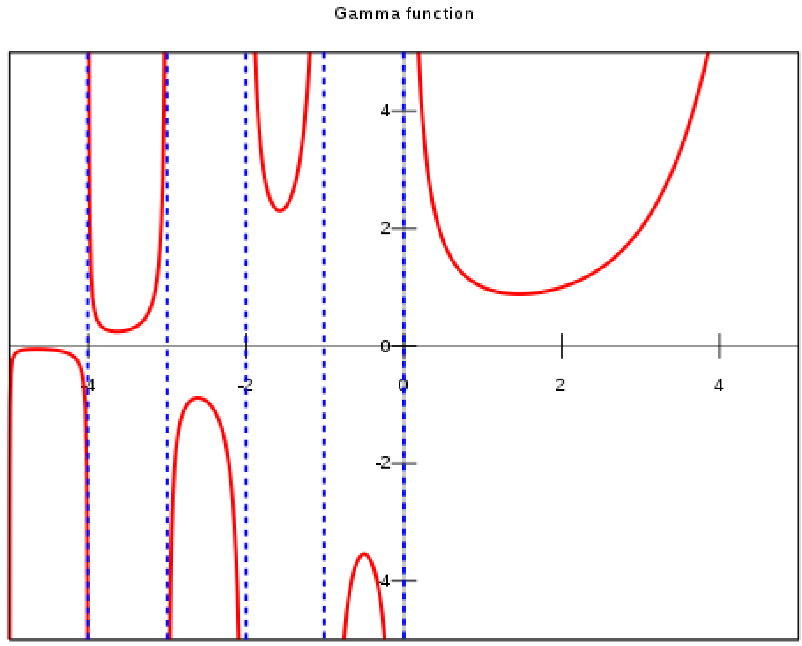

Figure 2: **The Gamma function**

The gamma function has no zeros,so the reciprocal gamma function $\frac{1}{\Gamma(z)}$ is an entire function.In fact,the gamma function corresponds to the Mellin transformation of the negative exponential function:

$$\Gamma(z) = \mathcal{M}e^{-x}(z)$$

1 Definition

1.1 Main definition

The notation $\Gamma(z)$ is due to Legendre.If thereal part of the complex number z is strictly positive $(\Re(z) > 0)$,then the integral

$$\Gamma(z) = \int_0^\infty x^{z-1} e^{-z} dx$$

converges absolutely,and is known as the **Euler integration of the second kind**.(Euler integration of the first kind is the beta function) [3] Using integration by parts one sees that:

$$\begin{aligned}\Gamma(z+1) &= \int_0^\infty x^z e^{-x} dx \\ &= \left[-x^z e^{-x}\right]_0^\infty + \int_0^\infty z x^{z-1} e^{-x} dx \\ &= \lim_{z\to\infty}\left(-x^z e^{-x}\right) - \left(-0^z e^{-0}\right) + z\int_0^\infty x^{z-1} e^{-x} dx.\end{aligned}$$

Recognizing that $-x^z e^- x \to 0$ as $x \to \infty$,

$$\begin{aligned}\Gamma(z+1) &= z\int_0^\infty x^{z-1} e^{-x} dx \\ &= z\Gamma(z).\end{aligned}$$

We can calculate $\Gamma(1)$:

$$\begin{aligned}\Gamma(1) &= \int_0^\infty x^{1-1} e^{-x} dx \\ &= \left[-e^{-x}\right]_0^\infty \\ &= \lim_{z\to\infty}\left(-e^{-x}\right) - \left(-e^0\right) \\ &= 0 - (-1) \\ &= 1.\end{aligned}$$

Given that$\Gamma(1) = 1$ and $\Gamma(n+1) = n\Gamma(n)$,

$\Gamma(n) = 1 \cdot 2 \cdot 3 \cdots (n-1) = (n-1)!$ for all positive integers n.This can be seen as an example of proof by induction.

The identity $\Gamma(z) = \frac{\Gamma(z+1)}{z}$ can be used to uniquely extend the integral formulation for $\Gamma(z)$ to a meromorphic function defined for all complex numbers z,except integers less than or equal to zero.It is this extended version that id commonly reffered to as the gamma function. [3]

1.2 Alternative definitions

① **Euler's definition as an infinite product**

When seeking to approximate$z!$ for a complex number$z$,it is effective to first compute $n!$ for some large integer n.Use that to approximate a value for$(n+z)$,and then use the recursion relation $m! = m(m-1)!$backwards n times,to unwind it to an approximation for$z!$.Furthermore,this approximation is exact in the limit as n goes to infinity.

Specially,for a fixed integer m,it is the last case that

$$\boxed{\lim_{n\to\infty} \frac{n!(n+1)^m}{(n+m)!} = 1.}$$

If m is not an integer then it is not possible to say whether this equation is true because we have not yet(in this section)defined the factorial function for non-integers.However,we do get a unique extension of the factorial function to the non-integers by inserting that this equation continue to hold when the arbitrary integer **m** is replaced by an arbitrary complex number **z**.

$$\lim_{n\to\infty} \frac{n!(n+1)^z}{(n+z)! = 1.}$$

Multiplying both sides by$z!$ gives

$$\begin{aligned} z! &= \lim_{n\to\infty} n! \frac{z!}{(n+z)!}(n+1)^z \\ &= \lim_{n\to\infty} (1\cdots n)\frac{1}{(1+z)\cdots(n+z)}\left(\frac{2}{1}\cdot\frac{3}{2}\cdots\frac{n+1}{n}\right)^z \\ &= \prod_{n=1}^{\infty}\left[\frac{1}{1+\frac{z}{n}}\left(1+\frac{1}{n}\right)^z\right]. \end{aligned}$$

This **infinite product** converges for all complex numbers z except the negative integers,which fail because trying to use the recuesion relation $m! = m(m-1)!$ backwards through the value $m = 0$ involves a division by zero.

Similarly for the gamma function,the definition as an infinite product due to Euler is valid for all complex numbers z except the non-positive integers:

$$\Gamma(z) = \frac{1}{z}\prod_{n=1}^{\infty}\frac{\left(1+\frac{1}{n}\right)}{1+\frac{z}{n}}.$$

By this construction,the gamma function is the unique function that simultaneously satisfies $\Gamma(1) = 1$,$\Gamma(z+1) = z\Gamma(z)$ for all complex numbers z except the non positive integer ,and $\lim_{n\to\infty} \frac{\Gamma(n+z)}{\Gamma(n)n^z} = 1$ for all complex numbers z.

②Weierstrass's definiton

The definition for the gamm function due to Weierstrass is also valid for all complex numbers z except the non-positive integers:

$$\boxed{\Gamma(z) = \frac{e^{-\gamma z}}{z} \prod_{n=1}^{\infty} \left(1 + \frac{z}{n}\right)^{-1} e^{\frac{z}{n}}},$$

where $\gamma \approx 0.577216$ is the Euler-Mascheroni constantThis is the Hadamard Product of $\frac{1}{\Gamma(z)}$ in a rewritten form.Indeed,since $\frac{1}{\Gamma(z)}$ is entire of genus 1 with a simple zero at $z = 0$,we have the product rerpresentation

$$\frac{1}{\Gamma(z)} = ze^{Az+B} \prod_{p} \left(1 - \frac{z}{p}\right) e^{z/p},$$

where the product over the zeros$p \neq 0$ of $\frac{1}{\Gamma(z)}$.Since $\Gamma(z)$ has simple poles at the non-positive integers,it follows$\frac{1}{\Gamma(z)}$ has simple zeros at the non-positive integers,and so the equation above becomes Weierstrass's formula with$-Az - B$ in place of $-\gamma z$.The derivation of the constants $A = \gamma$ and $B = 0$ is somewhat technical,burt can be accomplished by using some identities involving the Riemann zeta function.

③In terms of generalized Laguerre polynomials

A representation of the incomplete gamma function in terms of generalized Laguerre polynomials is

$$\Gamma(z, x) = x^z e^{-x} \sum_{n=0}^{\infty} \frac{L_n^{(z)}}{n+1},$$

which converges for $\Re(z) > -1$ and $x > 0$.

2 Properties

2.1 General

Other important functionasl equations for the gamma function are Euler's reflection formula

$$\Gamma(1-z)\Gamma(z) = \frac{\pi}{\sin \pi z}, \quad z \notin \mathbb{Z}$$

which implies

$$\Gamma(z-n) = (-1)^{n-1} \frac{\Gamma(-z)\Gamma(1+z)}{\Gamma(n+1-z)}, \quad n \in \mathbb{Z}$$

and the Legendre duplication formula

$$\Gamma(z)\Gamma\left(z + \frac{1}{2}\right) = 2^{1-2z}\sqrt{\pi}\Gamma(2z).$$

The duplication formulae is a special case of the multiplication theorem

$$\prod_{k=0}^{m-1} \Gamma\left(z+\frac{k}{m}\right) = (2\pi)^{\frac{m-1}{2}} m^{\frac{1}{2}-mz}\Gamma(mz).$$

A simple but useful property,which can be seen from the limit definition,is:

$$\overline{\Gamma(z)} = \Gamma(\overline{z}) \Rightarrow \Gamma(z)\Gamma(\overline{z}) \in \mathbb{R}.$$

In particular,with $z = a + bi$,this product is

$$|\Gamma(a+bi)|^2 = |\Gamma(a)|^2 \prod_{k=0}^{\infty} \frac{1}{1+\frac{b^2}{(a+k)^2}}$$

If the real part is an integer or a half-integer,this can be finitely expressed in closed form:

$$\begin{aligned}
|\Gamma(bi)|^2 &= \frac{\pi}{b\sin\pi b} \\
\left|\Gamma\left(\tfrac{1}{2}+bi\right)\right|^2 &= \frac{\pi}{\cos\pi b} \\
\left|\Gamma\left(1+bi\right)\right|^2 &= \frac{\pi b}{\sin\pi b} \\
|\Gamma(1+n+bi)|^2 &= \frac{\pi b}{\sin\pi b}\prod_{k=1}^{n}(k^2+b^2), \quad n\in\mathbb{N} \\
|\Gamma(-n+bi)|^2 &= \frac{\pi}{b\sinh\pi b}\prod_{k=1}^{n}(k^2+b^2)^{-1}, \quad n\in\mathbb{N} \\
\left|\Gamma\left(\tfrac{1}{2}\pm n+bi\right)\right|^2 &= \frac{\pi}{\cosh\pi b}\prod_{k=1}^{n}\left(\left(k-\tfrac{1}{2}\right)^2+b^2\right)^{\pm}, \quad n\in\mathbb{N}
\end{aligned}$$

Perhaps the best-known value of the gamma function at a non-integer argument is

$$\Gamma(\tfrac{1}{2}) = \sqrt{\pi}\ ,$$

which can be found by setting $z = \frac{1}{2}$ in the reflection or duplication formulas,by using the relation to the **beta function** given below with $x = y = \frac{1}{2}$, or simply by making the substitution $u = \sqrt{x}$ in the integral definition of the gamma function,resulting in a Gaussian integral.In general,for non-negative integer values of n we have:

$$\begin{aligned}
\Gamma\left(\tfrac{1}{2}+n\right) &= \frac{(2n)!}{4^n n!}\sqrt{\pi} = \frac{(2n-1)!!}{2^n}\sqrt{\pi} = \binom{n-\frac{1}{2}}{n} n!\sqrt{\pi} \\
\Gamma\left(\tfrac{1}{2}-n\right) &= \frac{(-4)^n n!}{(2n)!}\sqrt{\pi} = \frac{(-2)^n}{(2n-1)!!}\sqrt{\pi} = \frac{\sqrt{\pi}}{\binom{-1/2}{n} n!}
\end{aligned}$$

where the **double factorial** $(2n-1)!! = (2n-1)(2n-3)\cdots(3)(1)$.

It might be tempting to generalize the result that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$ by looking for a formula for other individual values $\Gamma(r)$ where r is rational,especially because according to **Gauss's digamma theorem**,it is possible to do so for the closely related digamma function at every rational value.However,these numbers$\Gamma(r)$ are not known to be expressible by themselves in terms of elementary functions.It has been proved that $\Gamma(n+r)$ is **transcendental number** and **algebrically independent** of π for any integer n and each of the fractions $r = \frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$. In general,when computing values of the gamma function,we must settle for numerical appeoximations.

The derivatives of the gamma function are described in terms of the **polygamma function**.For example:

$$\Gamma'(z) = \Gamma(z)\psi_0(z).$$

For a positive integer m the derivative of the gamma function can be calculated as follows(here γ is the **Euler-Mascheroni constant**):

$$\boxed{\Gamma'(m+1) = m!\left(-\gamma + \sum_{k=1}^{m} \frac{1}{k}\right)}$$

For $\Re(x) > 0$ the nth derivative of the gamma function is:

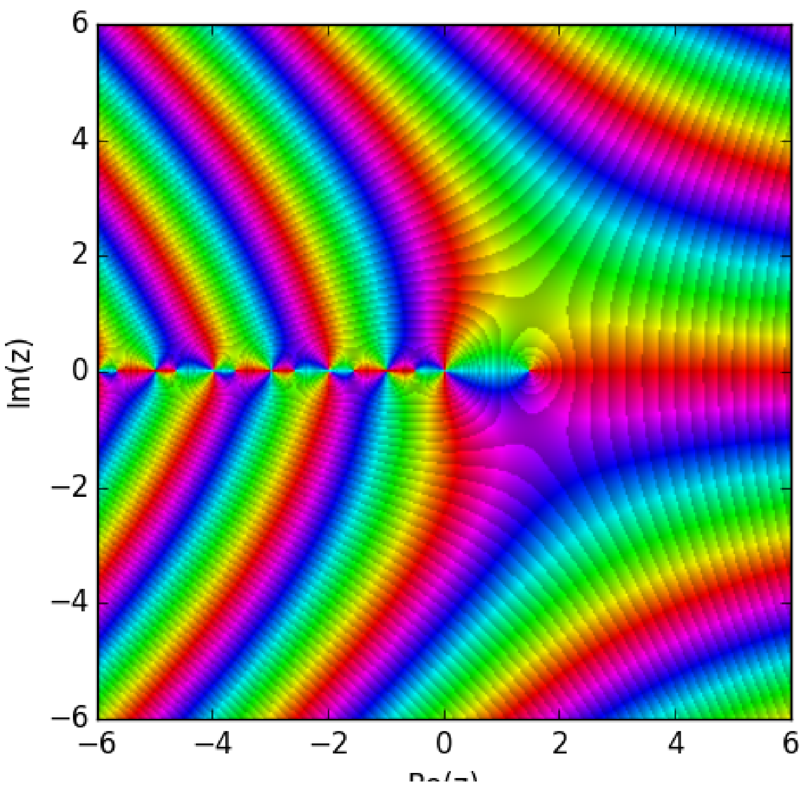

Figure 3: **Derivative of the function $\Gamma(z)$**

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$$\frac{d^n}{dx^n}\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}(\ln t)^n dt.$$

Using the identity

$$\Gamma^{(n)}(1) = (-1)^n n! \sum_{\pi\vdash n} \prod_{i=1}^{r} \frac{\zeta^*(a_i)}{k_i! \cdot a_i} \qquad \zeta^*(x) := \begin{cases} \zeta(x) & x \neq 1 \\ \gamma & x = 1 \end{cases}$$

where $\zeta(z)$ is the **Riemann zeta function**,and π is a partition of n given by

$$\pi = \underbrace{a_1 + \cdots + a_1}_{k_1 terms} + \cdots + \underbrace{a_r + \cdots + a_r}_{k_2 terms}$$

we have in particuler

$$\Gamma(z) = \frac{1}{z} - \gamma + \frac{1}{2}\left(\gamma^2 + \frac{\pi^2}{6}\right) z - \frac{1}{6}\left(\gamma^3 + \frac{\gamma\pi^2}{2} + 2\zeta(3)\right) z^2 + O(z^3)$$

2.2 Inequalities

When restricted to the positive real numbers,the gamma function is a strictly Logarithmic convex function.This property may be stated in any of the following three equivalent ways:

- For any two positive real numbers x_1 and x_2,and for any $t \in [0, 1]$,

$$\Gamma(tx_1 + (1-t)x_2) \leq \Gamma(x_1)^t \Gamma(x_2)^{1-t}$$

- For any two positive real numbers x and y with $y > x$,

$$\left(\frac{\Gamma(y)}{\Gamma(x)}\right)^{\frac{1}{y-z}} > \exp\left(\frac{\Gamma'(x)}{\Gamma(x)}\right).$$

•For any positive real number x,

$$\Gamma''(x)\Gamma(x) > \Gamma'(x)^2.$$

The last of these statements is,essentially by definition,the same as the statement that $\psi^{(1)}(x) > 0$,where $\psi^{(1)}$ is the polygamma function of order 1.To prove the logarithmic convexity of the gamma function,it therefore suffices to observe that $\psi^{(1)}$ has a series representation which,for positive real x,consists of only positive terms.

Logarithmic convexity and Jensen's inequality together imply,for any positive real numbers $x_1, ..., x_n$ and $a_1, ..., a_n$,

$$\Gamma\left(\frac{a_1x_1 + \cdots + a_nx_n}{a_1 + \cdots + a_n}\right) \leq (\Gamma(x_1)^{a_1} \cdots \Gamma(x_n)^{a_n})^{\frac{1}{a_1+\cdots+a_n}}.$$

There are also bounds on ratios of gamma functions.The best-known **Gautschi's inequality**,which says that for any positive real number x and any $s \in (0, 1)$,

$$x^{1-s} < \frac{\Gamma(x+1)}{\Gamma(x+s)} < (x+1)^{1-s}.$$

2.3 Stirling's formula

The behavior of $\Gamma(x)$ for an inceasing positive real variable is given by Stirling's formula

$$\Gamma(x+1) \sim \sqrt{2\pi x}\left(\frac{x}{e}\right)^{x},$$

where the symbol$\sim$ means asymptotic convergence;the ratio of the two sides converges to 1 in the limit$x \to +\infty$.This growth is faster than exponential,$\exp(\beta x)$,for any fixed value of β.

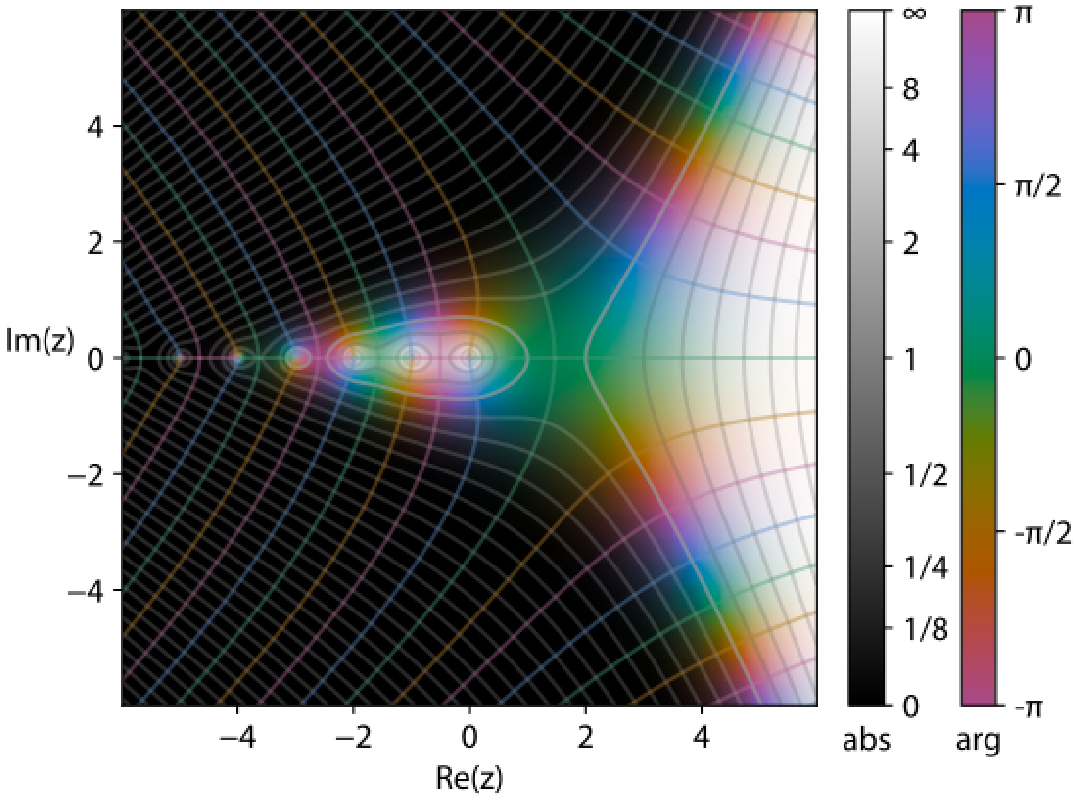

Figure 4: **Representation of the gamma function in complex plane.Each point z is colored according to the argument of $\Gamma(z)$.The contour plot of the modulus $|\Gamma(z)|$ is also displayed**

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Another useful limit for asymptotic approximation for $x \to \infty$ is:

$$\Gamma(x+\alpha) \sim \Gamma(x)x^{\alpha}, \quad \alpha \in \mathbb{C}.$$

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2.4 Residues

The behavior for non-positive z is more intricate.Euler's integral does not converge for $z \leq 0$,but the function it defines in the positive complex half-plane has a unique analytic contribution to use Euler's integral for positive arguments and extend the domain to negative numbers by repeated application of the recurrence formula,

$$\boxed{\Gamma(z) = \frac{\Gamma(z+n+1)}{z(z+1)\cdots(z+n)}}$$

choosing n such that $z+n$ is positive.The product in the denominator is zero when z equals any of the integers $0, 1, 2, \ldots$.Thus the gamma function must be undefined at those points to avoid division by zero;it is a meromorphic function with simple poles at the non-positive integers.

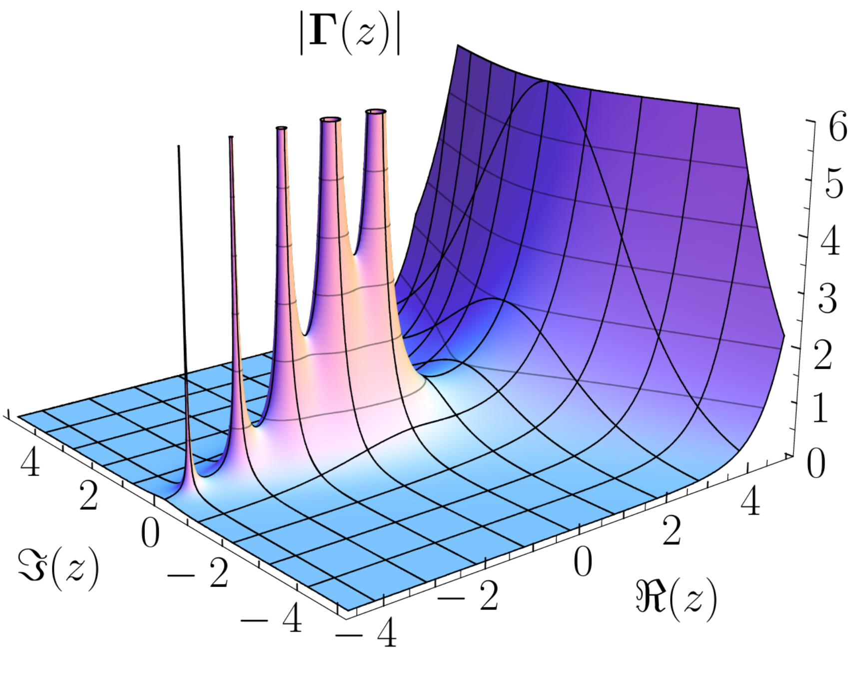

Figure 5: **3-dimensional plot of the absolute value of the complex gamma function**

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For a function f of a complex variable z,at a simple pole c the residue of f is given by:

$$\mathrm{Res}(f, c) = \lim_{z \to c}(z-c)f(z).$$

For the simple pole $z = -n$,we rewrite recurrence formula as:

$$(z+n)\Gamma(z) = \frac{\Gamma(z+n+1)}{z(z+1)\cdots(z+n-1)}.$$

and the denominator

$$z(z+1)\cdots(z+n-1) = -n(1-n)\cdots(n-1-n) = (-1)^n n!.$$

2.5 Fourier series expansion

The logarithm of the gamma function has following Fourier series for $0 < z < 1$:

$$\boxed{\ln\Gamma(z) = \left(\frac{1}{2} - z\right)(\gamma + \ln 2) + (1-z)\ln\pi - \frac{1}{2}\ln\sin(\pi z) + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\ln n}{n}\sin(2\pi n z)}$$

which was for a long tome attribute to Ernst Kummer,who derived iy in 1847. [4] [1] However,Laroslav Blagouchine discover that Carl Johan Malmsten proved this series in 1842. [5] [6]

2.6 Raabe's formula

In 1840 Joseph Ludwig Raabe proved that

$$\int_a^{a+1} \ln\Gamma(z)dz = \frac{1}{2}\ln 2\pi + a\ln a - a, \quad a > 0.$$

In paruculer,if $a = 0$ then

$$\int_0^1 \ln\Gamma(z)dz = \frac{1}{2}\ln 2\pi$$

The latter can be derived taking the ologarithm in the above multiplication formual,which an expression for thw Riemann sum of the integered.Takink the limit for $a \to \infty$.

2.7 Pi function

An alternative notaton which was originally introduce by Gauss and which was sometime used is the Π-function, which in terms of the gamma functiom is

$$\boxed{\Pi(z) = \Gamma(z+1) = z\Gamma(z) = \int_0^{\infty} e^{-t}t^z dt,}$$

so that $\Pi(n) = n!$ for every non-negative integer n.

Using the function the reflection formula takes on the form

$$\Pi(z)\Pi(-z) = \frac{\pi z}{\sin(\pi z)} = \frac{1}{\sin(z)}$$

where $\sin c$ is the normalized sinc function,while teh multiplication theorem takes on the form

$$\Pi\left(\frac{z}{m}\right)\Pi\left(\frac{z-1}{m}\right)\cdots\Pi\left(\frac{z-m+1}{m}\right) = (2\pi)^{\frac{m-1}{2}} m^{-z-\frac{1}{2}}\Pi(z).$$

We also sometimes find

$$\pi(z) = \frac{1}{\Pi z},$$

which is an entire function,defined for every complex number,just like the **reciprocal gamma function** .

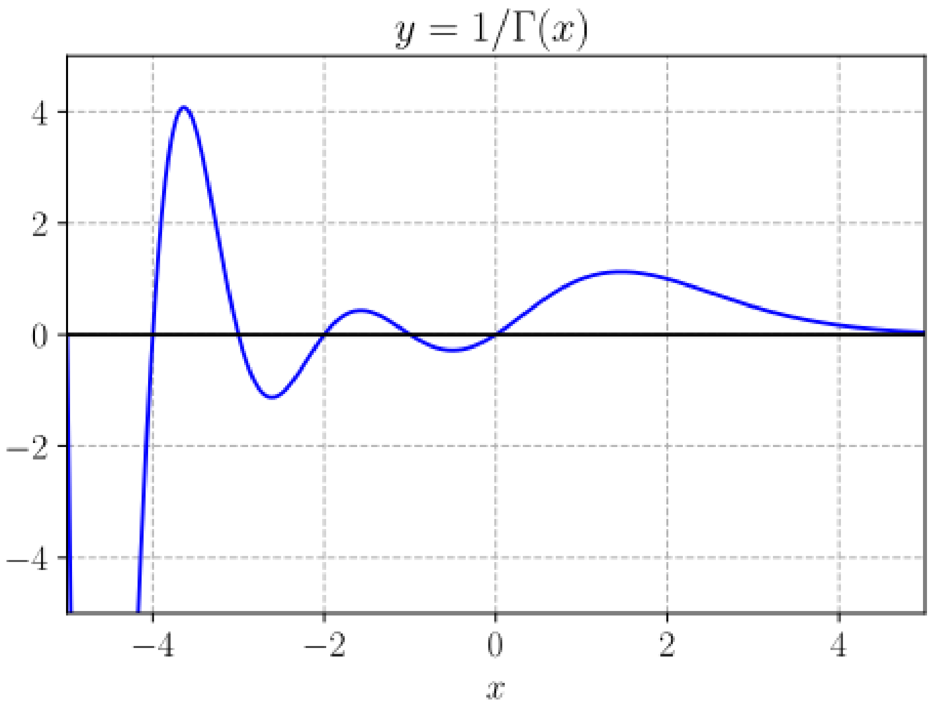

Figure 6: **Plot of $\frac{1}{\Gamma(x)}$ along the real axis**

That $\pi(z)$ is entire entails it has no poles,so $\Pi(z)$,like$\Gamma(z)$,has no zeros.

The volume of an n-ellipsoid with radii $r_1, ..., r_n$ can be expressed as

$$\boxed{V_n(r_1, ..., r_n) = \frac{\pi^{\frac{n}{2}}}{\Pi\left(\frac{n}{2}\right)} \prod_{k=1}^{n} r_k.}$$

3 Relation to other functions

• In the first integral above,which defines the gamma function,the limits of integration are fixed.The upper and lower incomplete gamma functions are the functions obtained by allowing the lower or upper(respectively)limit of integration to vary.

• The gamma function is related to beta function by the formula

$$\mathrm{B}(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}dt=\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}.$$

• The logarithmic derivative of the gamma function is called the digamma function;higher derivatives are the polygamma functions.

• The analog of the gamma function over a finite field or a finite ring is the Gaussian sums,a type of exponential sum.

• The gamma function also shows up an important relation with the zeta function,$\zeta(z)$.

$$\pi^{-\frac{z}{2}}\Gamma\left(\frac{z}{2}\right)\zeta(z)=\pi^{-\frac{1-z}{2}}\Gamma\left(\frac{1-z}{2}\right)\zeta(1-z).$$

It also appears in the following formula:

$$\zeta(z)\Gamma(z)=\int_0^\infty \frac{u^z}{e^u-1}\frac{du}{u},$$

• The gamma function is related to the Streched exponential function.For instance ,the moments that function are

$$\langle\tau^n\rangle\equiv\int_0^\infty dt\ t^{n-1}e^{-\left(\frac{t}{\tau}\right)^\beta}=\frac{\tau^n}{\beta}\Gamma\left(\frac{n}{\beta}\right).$$

4 Particuler values

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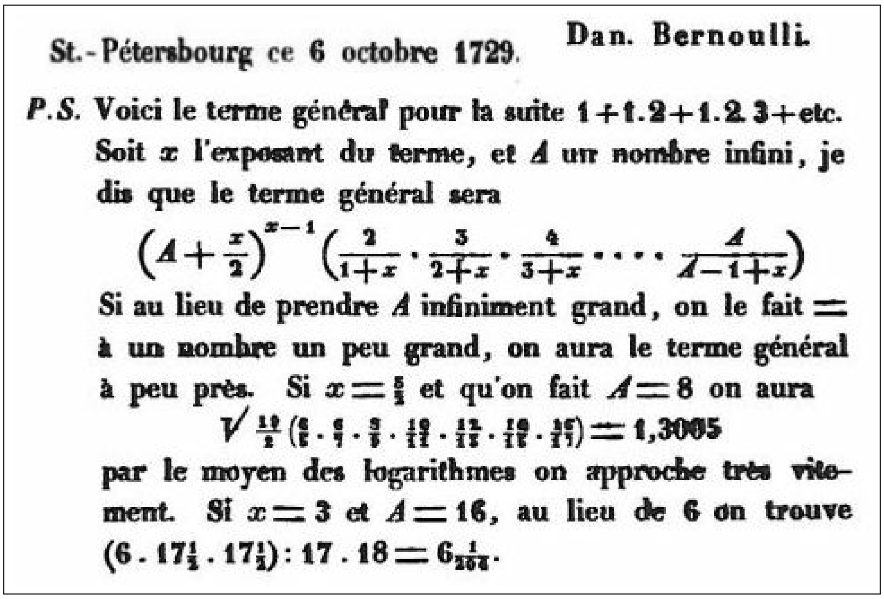
Function	value in π	value
$\Gamma\left(-\frac{3}{2}\right)$	$\frac{a\sqrt{\pi}}{3}$	$\approx +2.3632718012073547030 6$
$\Gamma\left(-\frac{1}{2}\right)$	$-2\sqrt{\pi}$	$\approx -3.5449077018110320545 9$
$\Gamma\left(\frac{1}{2}\right)$	$\sqrt{\pi}$	$\approx +1.7724538509055160279 1$
$\Gamma(1)$	$0!$	$\approx +1$
$\Gamma\left(\frac{3}{2}\right)$	$\frac{\pi}{2}$	$\approx +0.8862269254527580136 4$
$\Gamma(2)$	$1!$	$\approx +1$
$\Gamma\left(\frac{5}{2}\right)$	$\frac{3\sqrt{\pi}}{4}$	$\approx +1.3293403881791370204 7$
$\Gamma(3)$	$2!$	$\approx +2$
$\Gamma\left(\frac{7}{2}\right)$	$\frac{15\sqrt{\pi}}{8}$	$\approx +3.3233509704478425511 8$
$\Gamma(4)$	$3!$	$\approx +6$

.

The complex-valued gamma function is undefined for non-positive integers,but in these cases the value can be defined in the Riemann sphere as ∞.the reciprocal gamma function is well defined and analytic at these values

$$\frac{1}{\Gamma(-3)} = \frac{1}{\Gamma(-2)} = \frac{1}{\Gamma(-1)} = \frac{1}{\Gamma(0)} = 0$$

5 Some important figures



St.-Pétersbourg ce 6 octobre 1729. Dan. Bernoulli.

P.S. Voici le terme général pour la suite $1+1.2+1.2.3+$etc. Soit x l'exposant du terme, et A un nombre infini, je dis que le terme général sera

$$\left(A+\frac{x}{2}\right)^{x-1}\left(\frac{2}{1+x}\cdot\frac{3}{2+x}\cdot\frac{4}{3+x}\cdots\frac{A}{A-1+x}\right)$$

Si au lieu de prendre A infiniment grand, on le fait $=$ à un nombre un peu grand, on aura le terme général à peu près. Si $x=\frac{3}{2}$ et qu'on fait $A=8$ on aura

$$\sqrt{\frac{19}{2}}\left(\frac{4}{5}\cdot\frac{6}{7}\cdot\frac{8}{9}\cdot\frac{10}{11}\cdot\frac{12}{13}\cdot\frac{14}{15}\cdot\frac{16}{17}\right)=1,3005$$

par le moyen des logarithmes on approche très vitement. Si $x=3$ et $A=16$, au lieu de 6 on trouve $(6\,.\,17\tfrac{1}{2}\,.\,17\tfrac{1}{2}):17\,.\,18=6\frac{1}{204}$.

Figure 7: **Daniel Bernouli's letter to Christian Goldbach,october6,1729**

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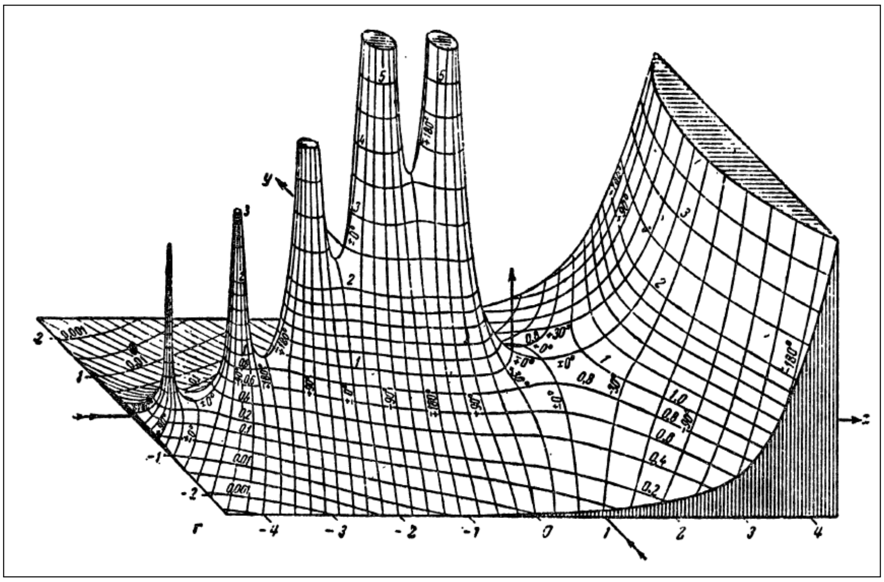

Figure 8: **A hand drawn graph of the absolute value of the complex gamma function**

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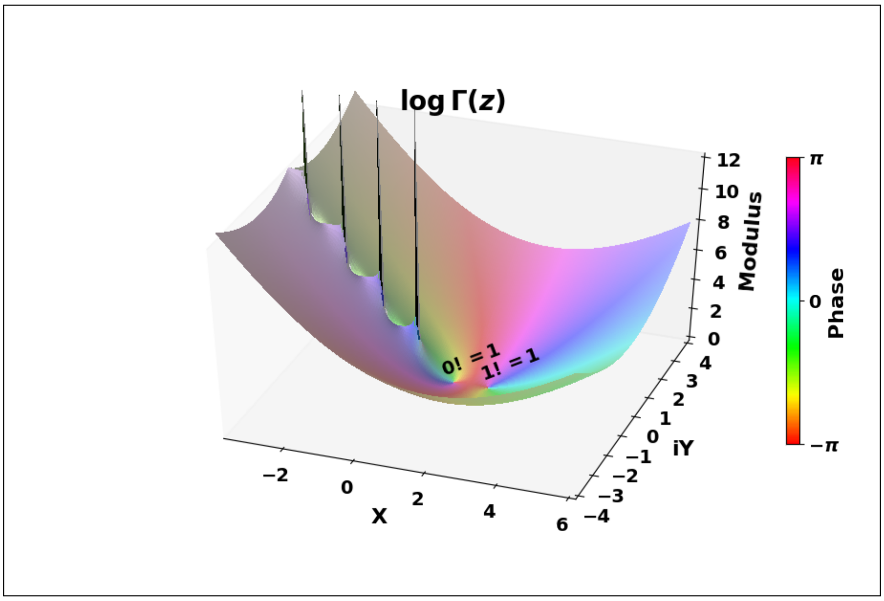

Figure 9: **Log gamma function** [7]

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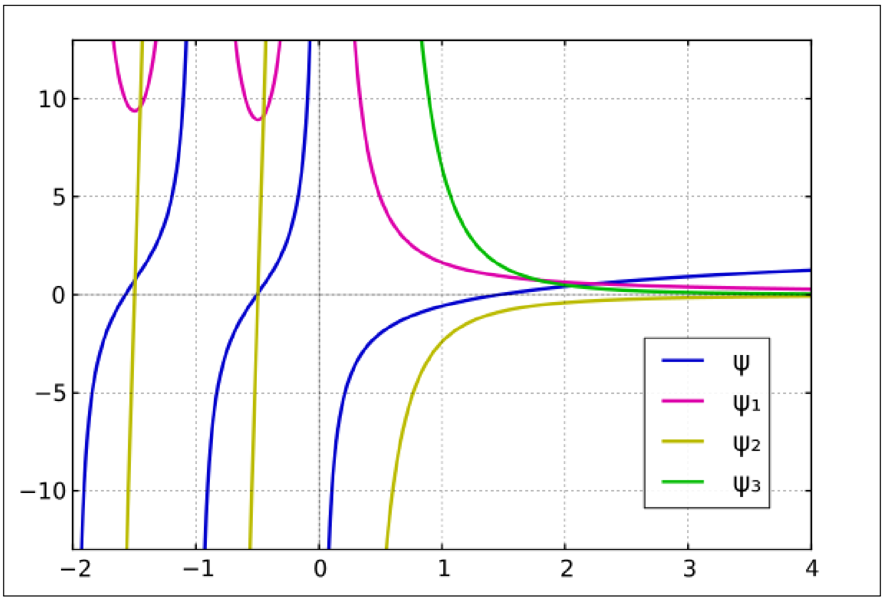

Figure 10: **polygamma function**

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The problem of extending the factorial to non-integer arguments was apparently first considered by Daniel Bernoulli and Christian Goldbach in the 1720s,and was solved at the end of the same decade by Leonhard Euler.

6 Conclusion

In this project i have learned many things about the Gamma function:

• The integral of the Gamma function from $-\infty$ to any finite,positive number converges.

• The critical points of the function on the negative real line migrate towards the asymptotes.

• The Gamma function flattens out as we move to more negative values.

I have also learned how to use LaTeX,commands.I also came to know the difference between LaTeXand ms word.

Where LaTeXis WYSIWYM type of typesetting language and MS WORD in other hand,is a WYSIWYG type of document editor.

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