$$\sum_{i=1}^{k} P_{Gi} - P_D - P_L = 0 \tag{7.19}$$

where

k = total number of generating plants

P_{Gi} = generation of ith plant

P_D = sum of load demand at all buses (system load demand)

P_L = total system transmission loss

To solve the problem, we write the Lagrangian as

$$\mathcal{L} = \sum_{i=1}^{k} C_i(P_{Gi}) - \lambda\left[\sum_{i=1}^{k} P_{Gi} - P_D - P_L\right] \tag{7.20}$$

It will be shown later in this section that, if the power factor of load at each bus is assumed to remain constant, the system loss P_L can be shown to be a function of active power generation at each plant, i.e.

$$P_L = P_L(P_{G1}, P_{G2}, ..., P_{Gk}) \tag{7.21}$$

Thus in the optimization problem posed above, P_{Gi} $(i = 1, 2, ..., k)$ are the only control variables.

For optimum real power dispatch,

$$\frac{\partial \mathcal{L}}{\partial P_{Gi}} = \frac{dC_i}{dP_{Gi}} - \lambda + \lambda\frac{\partial P_L}{\partial P_{Gi}} = 0, \quad i = 1, 2, ..., k \tag{7.22}$$

Rearranging Eq. (7.22) and recognizing that changing the output of only one plant can affect the cost at only that plant, we have

$$\frac{\dfrac{dC_i}{dP_{Gi}}}{\left(1 - \dfrac{\partial P_L}{\partial P_{Gi}}\right)} = \lambda \text{ or } \frac{dC_i}{dP_{Gi}} L_i = \lambda, \; i = 1, 2, ..., k \tag{7.23}$$

where

$$L_i = \frac{1}{(1 - \partial P_L / \partial P_{Gi})} \tag{7.24}$$

is called the *penalty factor* of the ith plant.

The Lagrangian multiplier λ is in rupees per megawatt-hour, when fuel cost is in rupees per hour. Equation (7.23) implies that minimum fuel cost is obtained, when the incremental fuel cost of each plant multiplied by its penalty factor is the same for all the plants.

The $(k + 1)$ variables $(P_{G1}, P_{G2}, ..., P_{Gk}, \lambda)$ can be obtained from k optimal dispatch Eq. (7.23) together with the power balance Eq. (7.19). The partial derivative $\partial P_L/\partial P_{Gi}$ is referred to as the *incremental transmission loss (ITL)$_i$*, associated with the ith

Equation (7.23) can also be written in the alternative form

$$(IC)_i = \lambda[1-(ITL)_i]\ i = 1, 2, \ldots, k \tag{7.25}$$

This equation is referred to as the *exact coordination equation*.

Thus it is clear that to solve the optimum load scheduling problem, it is necessary to compute ITL for each plant, and therefore we must determine the functional dependence of transmission loss on real powers of generating plants. There are several methods, approximate and exact, for developing a transmission loss model. A full treatment of these is beyond the scope of this book. One of the most important, simple but approximate, methods of expressing transmission loss as a function of generator powers is through *B*-coefficients. This method is reasonably adequate for treatment of loss coordination in economic scheduling of load between plants. The general form of the loss formula (derived later in this section) using *B*-coefficients is

$$P_L = \sum_{m=1}^{k}\sum_{n=1}^{k} P_{Gm}B_{mn}P_{Gn} \tag{7.26}$$

where

P_{Gm}, P_{Gn} = real power generation at m, nth plants

B_{mn} = loss coefficients which are constants under certain assumed operating conditions

If P_Gs are in megawatts, B_{mn} are in reciprocal of megawatts*. Computations, of course, may be carried out in per unit. Also, $B_{mn} = B_{nm}$.

Equation (7.26) for transmission loss may be written in the matrix form as

$$P_L = P_G^T B P_G \tag{7.27}$$

where

$$P_G = \begin{bmatrix} P_{G1} \\ P_{G2} \\ \vdots \\ P_{Gk} \end{bmatrix} \text{ and } B = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{lk} \\ B_{21} & B_{22} & \cdots & B_{2k} \\ \vdots & & & \vdots \\ B_{k} & B_{k2} & \cdots & B_{kk} \end{bmatrix}$$

It may be noted that B is a symmetric matrix.

For a three plant system, we can write the expression for loss as

$$P_L = B_{11}P_{G1}^2 + B_{22}P_{G2}^2 + B_{33}P_{G3}^2 + 2B_{12}P_{G1}P_{G2} + 2B_{23}P_{G2}P_{G3} + 2B_{31}P_{G3}P_{G1} \tag{7.28}$$

With the system power loss model as per Eq. (7.26), we can now write

$$\frac{\partial P_L}{\partial P_{Gi}} = \frac{\partial}{\partial P_{Gi}}\left[\sum_{m=1}^{k}\sum_{n=1}^{k} P_{Gm}B_{mn}P_{Gn}\right]$$

*B_{mn} (in pu) = B_{mn} (in MW^{-1}) × Base MVA

$$= \frac{\partial}{\partial P_{Gi}} \left[\sum_{\substack{n=1 \\ n \neq i}}^{k} P_{Gi} B_{in} P_{Gn} + \sum_{\substack{m=1 \\ m \neq i}}^{k} P_{Gm} B_{mi} P_{Gi} + P_{Gi} B_{ii} P_{Gi} \right] \tag{7.29}$$

It may be noted that in the above expression other terms are independent of P_{Gi} and are, therefore, left out.

Simplifying Eq. (7.29) and recognizing that $B_{ij} = B_{ji}$, we can write

$$\frac{\partial P_L}{\partial P_{Gi}} = \sum_{j=1}^{k} 2 B_{ij} P_{Gj} \tag{7.30a}$$

Assuming quadratic plant cost curves as

$$C_i(P_{Gi}) = \frac{1}{2} a_i P_{Gi}^2 + b_i P_{Gi} + d_i$$

We obtain the incremental cost as

$$\frac{dC_i}{dP_{Gi}} = a_i P_{Gi} + b_i \tag{7.30b}$$

Substituting $\partial P_L / \partial P_{Gi}$ and dC_i / dP_{Gi} from above in the coordination Eq. (7.22), we have

$$a_i P_{Gi} + b_i + \lambda \sum_{j=1}^{k} 2 B_{ij} P_{G_j} = \lambda \tag{7.31}$$

Collecting all terms of P_{Gi} and solving for P_{Gi}, we obtain

$$(a_i + 2\lambda B_{ii}) P_{Gi} = - \lambda \sum_{\substack{j=1 \\ j \neq i}}^{k} 2 B_{ij} P_{Gj} - b_i + \lambda$$

$$P_{Gi} = \frac{1 - \dfrac{b_i}{\lambda} - \displaystyle\sum_{\substack{j=1 \\ j \neq i}}^{k} 2 B_{ij} P_{Gj}}{\dfrac{a_i}{\lambda} + 2 B_{ii}}; \; i = 1, 2, \ldots, k \tag{7.32}$$

For any particular value of λ, Eq. (7.32) can be solved iteratively by assuming initial values of P_{Gi}s (a convenient choice is $P_{Gi} = 0$; $i = 1, 2, \ldots, k$). Iterations are stopped when P_{Gi}s converge within specified accuracy.

Equation (7.32) along with the power balance Eq. (7.19) for a particular load demand P_D are solved iteratively on the following lines:

1. Initially choose $\lambda = \lambda_0$.
2. Assume $P_{Gi} = 0$; $i = 1, 2, \ldots, k$.
3. Solve Eq. (7.32) iteratively for P_{Gi}s.

4. Calculate $P_L = \sum_{i=1}^{k}\sum_{j=1}^{k} P_{Gi}B_{ij}P_{Gj}$.

5. Check if power balance equation (7.19) is satisfied, i.e.

$$\left|\sum_{n\neq 1}^{k} P_{Gi} - P_D - P_L\right| < \varepsilon \quad \text{(a specified value)}$$

If yes, stop. Otherwise, go to step 6.

6. Increase λ by $\Delta\lambda$ (a suitable step size); if $\left(\sum_{i=1}^{k} P_{Gi} - P_D - P_L\right) < 0$ or

decrease λ by $\Delta\lambda$ (a suitable step size); if $\left(\sum_{i=1}^{k} P_{Gi} - P_D - P_L\right) > 0$,

repeat from step 3.

Example 7.4

A two-bus system is shown in Fig. 7.8. If 100 MW is transmitted from plant 1 to the load, a transmission loss of 10 MW is incurred. Find the required generation for each plant and the power received by load when the system λ is Rs 25/MWh.

The incremental fuel costs of the two plants are given below:

$$\frac{dC_1}{dP_{G1}} = 0.02P_{G1} + 16.0 \text{ Rs/MWh}$$

$$\frac{dC_2}{dP_{G2}} = 0.04P_{G2} + 20.0 \text{ Rs/MWh}$$

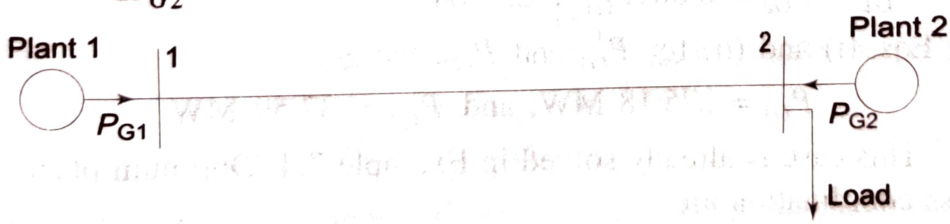

Fig. 7.8 A two-bus system for Example 7.4

Solution Since the load is at bus 2 alone, P_{G2} will not have any effect on P_L. Therefore

$$B_{22} = 0 \text{ and } B_{12} = 0 = B_{21}$$

Hence

$$P_L = B_{11}P_{G1}^2 \tag{i}$$

For $P_{G1} = 100$ MW, $P_L = 10$ MW, i.e.