Masters Thesis

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January 2026

Chapter 1

Introduction

Quantum networks aim to interconnect spatially separated quantum processors through quantum channels, enabling distributed quantum communication, computation, and sensing. By allowing quantum information and entanglement to be shared across distant nodes, quantum networks form the backbone of technologies such as quantum key distribution, distributed quantum computing, and quantum-enhanced metrology. A comprehensive vision of quantum networks and their enabling technologies has been articulated in the seminal review by Wehner, Elkouss, and Hanson, which outlines both the opportunities and the fundamental challenges associated with building scalable and reliable quantum network infrastructures [**wehner2018quantum**].

At the heart of any quantum network lies entanglement, which serves as a fundamental resource for non-classical correlations between distant nodes. Reliable entanglement distribution is therefore a central task in quantum networking. However, direct transmission of entangled states over long distances is severely limited by decoherence, loss, and operational imperfections. Quantum repeaters address this challenge by enabling entanglement to be generated over short distances and subsequently extended across the network using local operations and classical communication.

Recently, quantum walks have emerged as a powerful framework for quantum information processing, offering both theoretical elegance and experimental feasibility. Their intrinsic connection to graph structures makes them particularly appealing for quantum network applications.

Motivated by these considerations, recent work has proposed entanglement distribution schemes based on quantum walks in arbitrary quantum networks. In particular, Chen *et al.* introduced a systematic framework in which local quantum walk operations are used as basic modules for distributing entangled states across a network [**chen2025entanglement**].

Scope of this thesis.

Building on this framework, the present M.Tech thesis focuses on the practical implementation and analysis of quantum-walk-based entanglement distribution protocols. The completed contributions include:

- Distribution of two-dimensional Bell and GHZ states using quantum walk dynamics.
- Distribution of two-qubit graph states that are locally unitary equivalent to Bell states but possess distinct stabilizer structures.
- Numerical analysis of decoherence effects, including T_1 relaxation and T_2 dephasing, on Bell-state fidelity.

Chapter 2

Background and Theoretical Framework

This chapter introduces the theoretical concepts that underpin the work presented in this thesis. We begin with an overview of quantum networks and their fundamental components, followed by a discussion of entanglement as a key resource for distributed quantum information processing. We then introduce quantum walks, both discrete and continuous, emphasizing their relevance to networked quantum systems. Finally, we review paradigms for entanglement distribution, with a focus on approaches based on local operations and quantum walk dynamics.

2.1 Quantum Networks

Quantum networks consist of spatially separated nodes capable of storing, processing, and transmitting quantum information, interconnected via quantum channels and coordinated using classical communication. Each node typically contains one or more quantum memories or processors, while quantum channels enable the transmission of quantum states or the distribution of entanglement between nodes.

The primary objective of a quantum network is not the direct transmission of arbitrary quantum states, but rather the reliable establishment of entanglement between distant nodes. Once shared entanglement is available, a wide range of distributed quantum protocols can be realized, including quantum teleportation, distributed quantum computation, clock synchronization, and secure communication.

The architecture of a quantum network is strongly constrained by physical considerations such as channel loss, decoherence, and imperfect operations. As a result, scalable quantum networks rely on modular designs in which entanglement is generated locally and extended across the network using entanglement swapping, purification, and feed-forward control. These principles form the basis of the quantum repeater paradigm.

2.2 Entanglement as a Resource

Entanglement is a uniquely quantum correlation that cannot be reproduced by classical systems. In the context of quantum networks, entanglement serves as a consumable resource that enables non-local operations between distant nodes. Bipartite entangled states, such as Bell states, are the fundamental building blocks for protocols like quantum teleportation and entanglement swapping, while multipartite entangled states, such as GHZ and graph states, enable more complex tasks including distributed sensing and networked quantum computation.

From a resource-theoretic perspective, entanglement cannot be generated using local operations and classical communication (LOCC) alone. Its distribution therefore requires coherent quantum dynamics across the network or the use of quantum channels. In practice, the quality of distributed entanglement is quantified using measures such as fidelity, purity, and entanglement monotones, all of which are sensitive to noise and decoherence.

In realistic network settings, entanglement distribution protocols must be robust against operational imperfections, including energy relaxation (T_1 processes), dephasing (T_2 processes), and measurement errors. Understanding how these noise sources affect entanglement generation and stabilization is essential for assessing the feasibility of any proposed protocol.

2.3 Quantum Walks

Quantum walks provide a quantum analogue of classical random walks and describe the coherent evolution of a quantum particle on a graph. The structure of the underlying graph directly influences the dynamics of the walk, making quantum walks a natural framework for modeling transport and information flow in networked systems.

2.3.1 Discrete-Time Quantum Walks

In discrete-time quantum walks, the evolution is governed by repeated applications of a coin operation and a conditional shift operation. The coin acts on an internal degree of freedom, while the shift propagates the walker across the graph conditioned on the coin state. Discrete-time quantum walks have been extensively studied in quantum algorithms, where they exhibit speedups over classical counterparts in search and sampling tasks.

2.3.2 Continuous-Time Quantum Walks

Continuous-time quantum walks dispense with the coin degree of freedom and are generated directly by a Hamiltonian proportional to the graph Laplacian or adjacency matrix. The system evolves continuously under Schrödinger dynamics, and the connectivity of the graph determines the structure of the Hamiltonian.

Continuous-time quantum walks are particularly well-suited for physical implementations and for modeling quantum dynamics on fixed network topologies. Their Hamiltonian formulation also makes them naturally compatible with adiabatic quantum computation and variational quantum algorithms.

2.4 Entanglement Distribution Paradigms

Traditional entanglement distribution protocols rely on direct transmission of entangled pairs or on entanglement swapping using Bell-state measurements. While these approaches are conceptually simple, they often require non-local measurements or high-fidelity two-qubit operations across distant nodes, which can be experimentally challenging.

An alternative paradigm is to use local coherent dynamics to distribute entanglement across a network. In this approach, entanglement is generated and propagated through the network using unitary evolution, measurements on intermediate nodes, and classical feed-forward operations. Quantum walks provide a natural mechanism for realizing such protocols, as their dynamics intrinsically depend on the network topology and can generate multipartite entanglement through interference effects.

Recent work has demonstrated that quantum-walk-based protocols can distribute Bell states, GHZ states, and higher-dimensional entangled states using only local operations and measurements, making them compatible with quantum repeater architectures. These protocols avoid the need for global measurements and offer a flexible framework for entanglement distribution in arbitrary network topologies.

The concepts introduced in this chapter provide the theoretical foundation for the implementation and analysis presented in the subsequent chapters. In particular, they motivate the use of quantum walks as structured and physically motivated tools for entanglement distribution in quantum networks.

2.5 Current Status and Future Directions

The work presented in this thesis is structured as a progressive study of quantum-walk-based entanglement distribution protocols, beginning with small-scale implementations and extending toward more general and scalable network scenarios.

Completed Work

At the current stage, the following components have been successfully implemented and analyzed:

- Implementation of quantum-walk-based protocols for distributing two-qubit Bell states in small quantum networks, following the framework proposed in the literature.
- Extension of the protocol to distribute multipartite GHZ states in two-dimensional network configurations.
- Distribution of two-qubit graph states that are locally unitary (LU) equivalent to Bell states but characterized by distinct stabilizer generators, highlighting the flexibility of the quantum-walk-based approach.
- Numerical investigation of decoherence effects on Bell-state distribution, incorporating energy relaxation (T_1) and dephasing (T_2) noise models, and analyzing their impact on state fidelity and purity.

These results establish the correctness of the implementation and provide a baseline understanding of how noise affects quantum-walk-based entanglement distribution in small networks.

Planned Work

Building on the completed results, the remaining part of the thesis will focus on the following extensions:

- Generalization of the entanglement distribution protocols to three-qubit and larger graph states, including linear chains and two-dimensional graph-state structures relevant for quantum networking and measurement-based computation.
- Systematic study of multipartite entanglement generation using quantum walks, with emphasis on stabilizer structure and scalability.
- Exploration of variational approaches to quantum-walk-based entanglement distribution, where parameters in the coin or walk operators are optimized to enhance robustness against noise or to maximize target-state fidelity.
- Comparative analysis of fixed versus variational quantum walk protocols under realistic noise models, with the aim of identifying potential advantages of adaptive or learned walk dynamics.

Together, these planned investigations aim to advance the understanding of quantum walks as versatile and noise-resilient tools for entanglement distribution in quantum networks, and to assess their potential role in scalable quantum networking architectures.