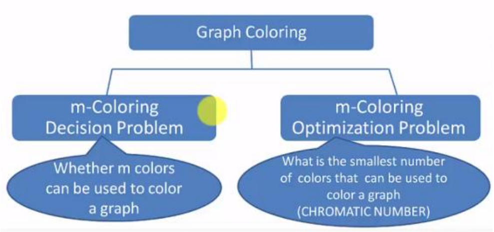
GRAPH COLORING ALGORITHM

Graph coloring is the procedure of assignment of colors to each vertex of a graph G such that **no adjacent vertices get same color.** The objective is to minimize the number of colors while coloring a graph. **The smallest number of colors required** to color a graph G is called its chromatic number of that graph. Graph coloring problem is a NP Complete problem.



Method to Color a Graph

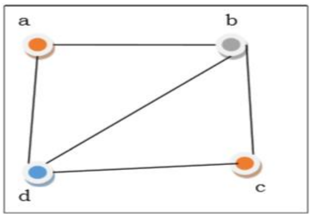
The steps required to color a graph G with n number of vertices are as follows −

Step 1 − Arrange the vertices of the graph in some order.

Step 2 − Choose the first vertex that has highest degree and color it with the first color.

Step 3 − Choose the next vertex and color it with the lowest numbered color that has not been colored on any vertices adjacent to it. If all the adjacent vertices are colored with this color, assign a new color to it. Repeat this step until all the vertices are colored.

Example



In the above figure, at first vertex a is colored red. As the adjacent vertices of vertex a are again adjacent, vertex b and vertex d are colored with different color, green and blue respectively. Then vertex c is colored as red as no adjacent vertex of c is colored red. Hence, we could color the graph by 3 colors. Hence, the chromatic number of the graph is 3.

Applications of Graph Coloring

Some applications of graph coloring include −

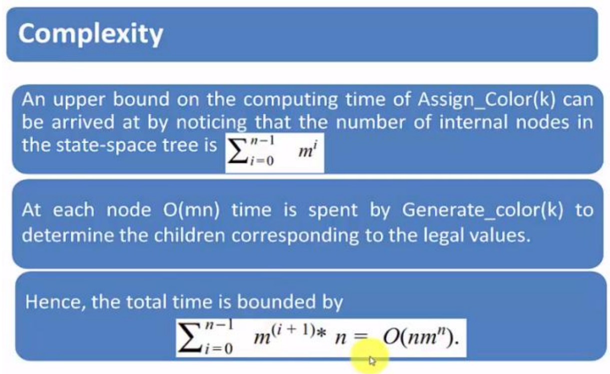
- Register Allocation
- Map Coloring
- Bipartite Graph Checking
- Mobile Radio Frequency Assignment
- Making time table, etc.

❖ **Vertex coloring** is the most commonly encountered graph coloring problem. The problem states that given m colors, determine a way of coloring the vertices of a graph such that no two adjacent vertices are assigned same color.

❖ Similarly, an edge coloring assigns a color to each edge so that no two adjacent edges are of the same color, and a **face coloring** of a planar graph assigns a color to each face or region so that no two faces that share a boundary have the same color.

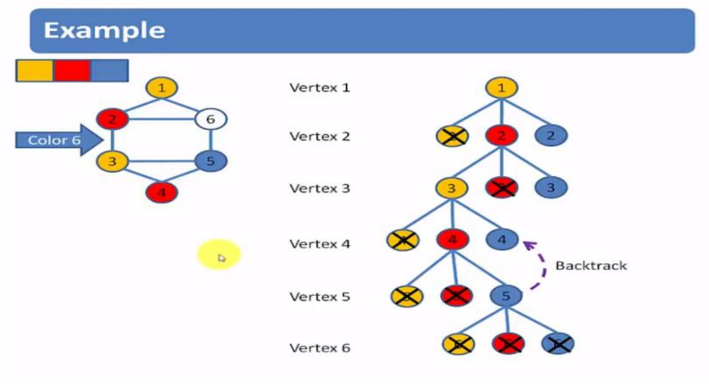
❖ The smallest number of colors needed to color a graph G is called its chromatic number.

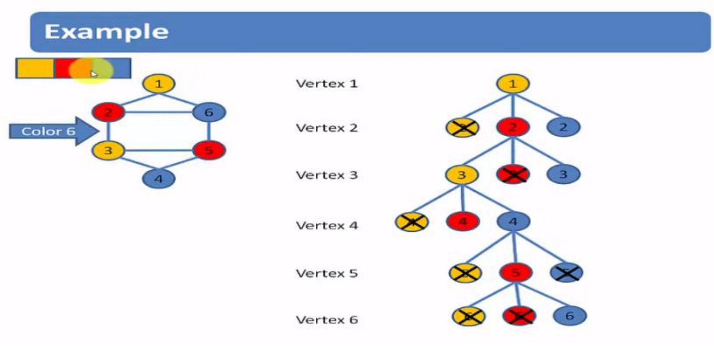
Since each node can be colored by using any of the m colors, the total numbers of possible color configurations are $\mathbf{m^v}$. The complexity is exponential which is very huge.

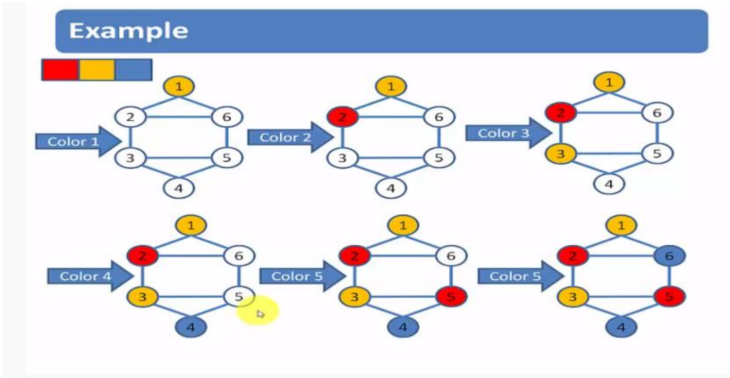


Using Backtracking:

- By using the backtracking method, the main idea is to assign colors one by one to different vertices right from the first vertex (vertex 0).
- Before color assignment, check if the adjacent vertices have same or different color by considering already assigned colors to the adjacent vertices.
    - If the color assignment does not violate any constraints, then we mark that color as part of the result. If color assignment is not possible then backtrack and return false.



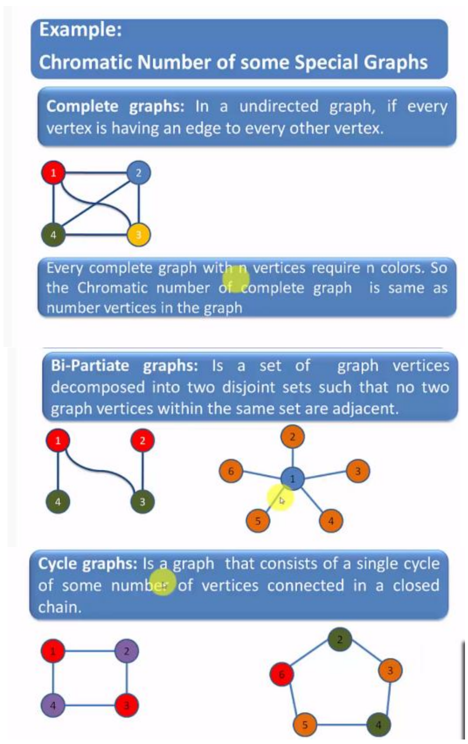




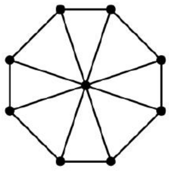
Chromatic number:

The following table gives the chromatic numbers for some named classes of graphs.

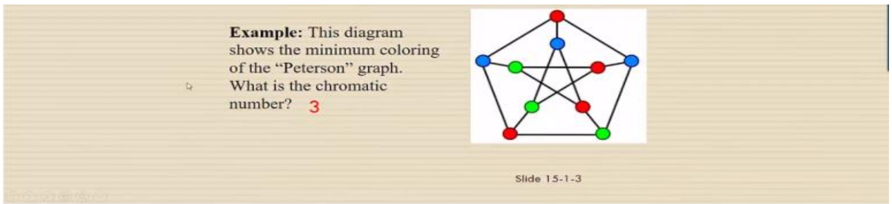
graph G	$\gamma(G)$
complete graph K_n	n
cycle graph $C_n, n > 1$	$\begin{cases} 3 & \text{for n odd} \\ 2 & \text{for n even} \end{cases}$
star graph $S_n, n > 1$	2
wheel graph $W_n, n > 2$	$\begin{cases} 3 & \text{for n odd} \\ 4 & \text{for n even} \end{cases}$



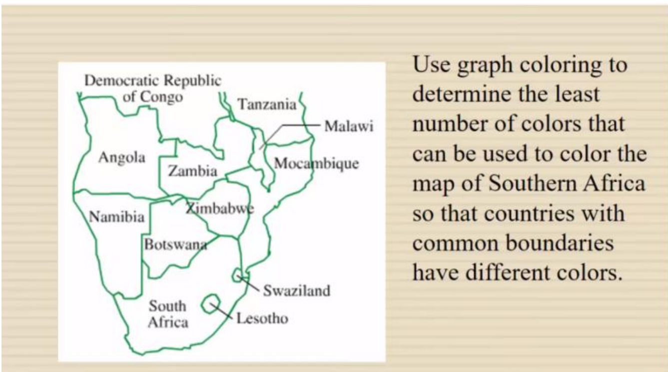
*In the mathematical discipline of **graph** theory, a **wheel graph** is a **graph** formed by connecting a single universal vertex to all vertices of a cycle.*

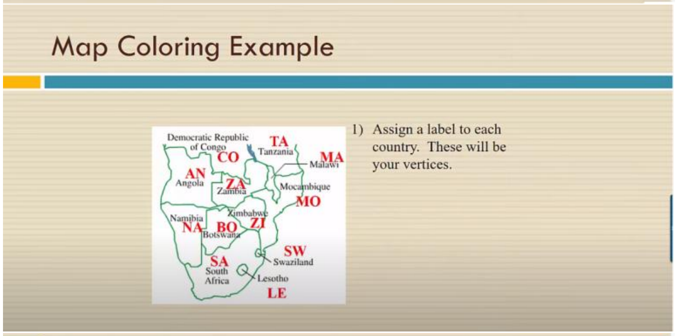


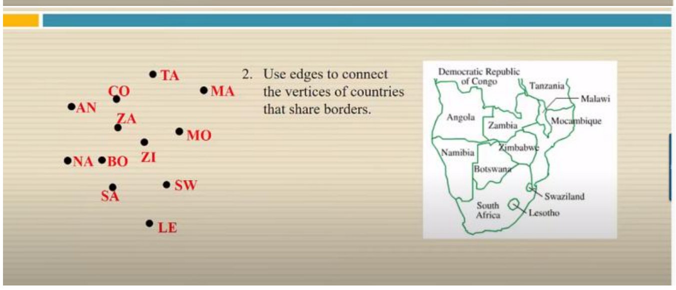
Another approach for coloring a graph:

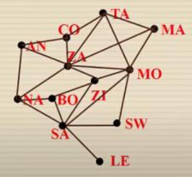


MAP coloring:

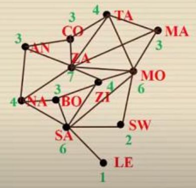








3) Use the procedure for coloring a graph.



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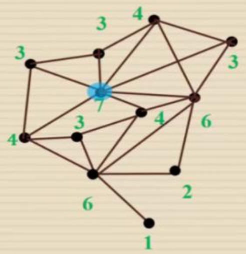
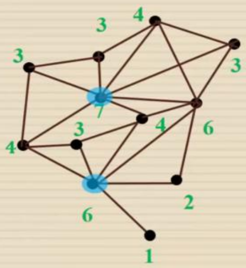
- Label Degrees



3) Use the procedure for coloring a graph.
- Label Degrees
- Start coloring with highest degree vertex



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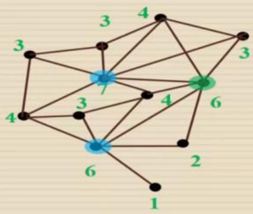

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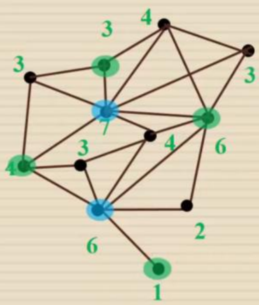

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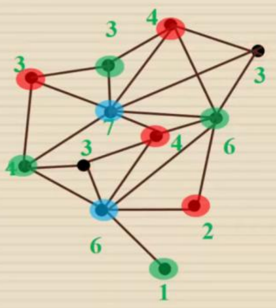

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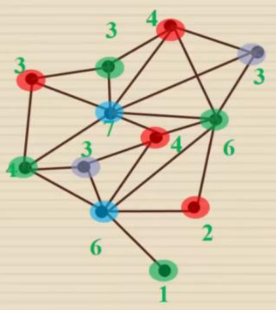

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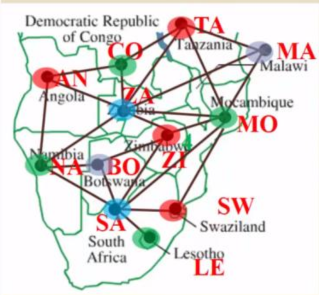

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The result is a 4-color graph. *It turns out that 4-colors are necessary for this map although other configurations are possible.*

The Four Color Problem

- Historically, mapmakers found that 4 colors sufficed for any map.
- Around 1850, mathematicians started to consider whether this was true for all possible maps.
- In 1976, Kenneth Appel and Wolfgang Haken used computers to prove the Four Color Theorem.
- Because no human can confirm the result, some mathematicians do not consider it proven.