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| 1 | Number triangle (2 : 3) | 09xx\_Boethius\_DeInstitutioneArithmetica\_Mh\_1\_f\_4r\_detail\_02.jpg | Arithmetic diagram. The horizontal dimension “Latitudo” displays geometric progressions of factor 2, the diagonal dimension “Angularis” displays geometric progressions of factor 3. Therefore, the columns contain geometric progressions of factor 3/2. In other words, they form the continuous proportions of sequences of Pythagorean fifths.  The diagram shows the integer numbers of the format 2j×3k. |
| 2 | Number triangle (3 : 4) | 09xx\_Boethius\_DeInstitutioneArithmetica\_Mh\_1\_f\_4v\_detail\_01.jpg | “Latitudo”: geometric progressions of factor 3.  “Angularis”: geometric progressions of factor 4.  The columns contain continuous proportions for sequences of Pythagorean fourths. The diagram shows the integer numbers of the format 3j×4k. |
| 3 | Number triangle (4 : 5) | 09xx\_Boethius\_DeInstitutioneArithmetica\_Mh\_1\_f\_4v\_detail\_02.jpg | “Latitudo”: geometric progressions of factor 4.  “Angularis”: geometric progressions of factor 5.  The columns contain continuous proportions for sequences of Syntonic thirds. The diagram shows the integer numbers of the format 4j×5k. |
| 4 | Tetrahedron | 09xx\_Boethius\_DeInstitutioneArithmetica\_Mh\_1\_f\_8v\_detail\_01\_a.jpg |  |
| 5 | Tetrahedron (detail) | 09xx\_Boethius\_DeInstitutioneArithmetica\_Mh\_1\_f\_8v\_detail\_01\_b.jpg |  |
| 6 | Figurate numbers (1) | 09xx\_Boethius\_DeInstitutioneArithmetica\_Mh\_1\_f\_8v\_detail\_02\_a.jpg |  |
| 7 | Figurate numbers (2) | 09xx\_Boethius\_DeInstitutioneArithmetica\_Mh\_1\_f\_8v\_detail\_02\_b.jpg |  |
| 8 | Sequence of superparticular ratios | 09xx\_Boethius\_DeInstitutioneArithmetica\_Mh\_1\_f\_11r\_detail.jpg | Consecutive elements of the sequence 1-2-4-6-9-12-16-20-25-30-36 form the superparticular ratios 1:2, 2:3, 3:4, 4:5, 5:6, where each ratio occurs twice. The lower row is the sequence of the differences: 1-2-2-3-3-4-4-5-5-6. The sequence could be extended to infinity in order to contain all possible superparticular ratios. By ending at 5:6 “Sesquiquintus” the diagram is related to the syntonic tone system and Zarlino’s “senario”. |
| 9 | Ut queant laxis | 10xx\_\_Ut\_queant\_laxis\_\_Bayerische\_Staatsbibliothek\_Clm\_14965a\_fol\_30\_hexachords\_wikipedia.png | The assignment of the solmization syllables of the hexachord, ut-re-mi-fa-sol-la, to the musical scale. The relative solmization refers to ut = Gamma / C / F / G respectively. The B in the low octave is “B quadratum” the next octave contains both “b rotundum” (b-flat) and “b quadratum” (b). The spacing between the notes points out the distinction between tones and semitones.  **Ut** queant laxis / **re**sonare fibris / **mi**ra gestorum / **fa**muli tuorum / **sol**ve polluti / **la**bii reatum, / sancte iohannes.  This hymn exemplifies the solmization system of Guido of Arezzo. The first six musical phrases begin on successively higher notes of the hexachord ut-re-mi-fa-sol-la.  (Transcription and sound at https://en.wikipedia.org/wiki/Ut\_queant\_laxis  https://upload.wikimedia.org/wikipedia/commons/e/e9/Johannes.Hymnus.ogg). |
| 10 | Circles | 13xx\_odington\_walter\_CSI\_184\_1\_Circles.jpg |  |
| 11 | Number triangle (8 : 9) | 13xx\_odington\_walter\_CSI\_187\_4\_NumberTriangle\_8\_9.jpg | Number triangle with:  “Latitudo”: geometric progressions of factor 8.  “Angularis”: geometric progressions of factor 9.  The diagram contains the integer numbers of the format 8j×9k.  The columns form continuous proportions for sequences of Pythagorean whole tones. The ratio of the last to the first element of the last column 531,441:262,144 corresponds to six tones and leads immediately to the ratio of the Pythagorean comma, 531,441:524,288, if the first element is multiplied by 2. Therefore, six Pythagorean tones are a Pythagorean comma greater than an octave.  The same diagram already appears in a Boethius “De institutione musice” manuscript of the late 10th century, Staatsbibliothek Bamberg Msc.Class.9, fol. 91r.  http://bsbsbb.bsb.lrz.de/~db/0000/sbb00000080/images/index.html?id=00000080&seite=185&bibl=sbb (visited 30.9.2016) |
| 12 | Tetraktys square | 13xx\_odington\_walter\_CSI\_194\_1\_Tetraktys\_Square.jpg | The numbers 6, 8, 9, 12, the second tetraktys, are arranged on a square. This representation is a graph of four nodes in which all pairs of nodes are connected (kappa-4). The connections are labelled by the corresponding ratios. The natural representation of this configuration in three dimensions would be a tetrahedron. [(BOET\_1867) Boethius 1867, Inst. Mus. I, 10] |
| 13 | Size of Pythagorean Comma | 13xx\_odington\_walter\_CSI\_197\_SizeOfPythagoreanComma.jpg | The size of the Pythagorean comma is estimated by superparticular ratios. The diagram shows that the Pythagorean comma, 531,441:524,288, is greater than 75:74 and less than 74:73. It illustrates a proof given by Boethius [(BOET\_1867) Boethius 1867, Inst. Mus. III, 12]. |
| 14 | Greek Tetrachords | 13xx\_odington\_walter\_CSI\_207\_ToneSystem\_GreekTetrachords.jpg | The Pythagorean diatonic tone system with B and Bb over two octaves and a fifth. The proportion of the corresponding string lengths is indicated with numbers running from 1728 to 10368.  There are some mistakes in the numbers and the first p should be labelled q. The second p should be 2187 instead of 2538, the third p 2304 instead of 2302, the second i 4374 instead of 4324, d 6912 instead of 6962, and c should be 7776 (…XXVI instead of …XXVL).  Unfortunately, we could not find manuscript drawings of Odington’s diagrams. The arcs on the left and the specification of the semitones are rather unclear in this printed version of the diagram. |
| 15 | Hexachords | 13xx\_odington\_walter\_CSI\_216\_Hexachords.jpg | The seven hexachords defining Guido of Arezzo’s tone system. The use of B or Bb is indicated below the hexachords. The sixth note la in the highest “Quadrata”-hexachord misses (cut-off?). |
| 16 | Triangular grid of durations | 13xx\_torkesey\_\_Declaratio\_trianguli\_et\_scuti\_vatlib\_it\_mss\_reg\_lat\_1146\_55v\_triangle\_detail.jpg | The diagram corresponds to Boethius’s first number triangle [1]. It shows the integer numbers of the format 2j×3k. Multiplication by 2 is now vertically downwards. In addition to Boethius the diagonal connections top right to bottom left are explicitly labelled “Sesquitertia” and hold sequences of Pythagorean fourths, for example (9:12:16).  The diagrams displays the duration values of the the ars nova/ars subtilior-period, defining the smallest value as one time unit. Binary and ternary (perfect) division of durations is admitted on all five levels, so that the ratio of the longest duration (maxima) to the shortest duration (semi-minima) varies between 32:1 and 243:1. |
| 17 | Shield of the Trinity | 1210\_c\_PetrusPictaviensis\_CottonFaustinaBVII\_folio42v\_ScutumFidei\_early13thc\_\_wikipedia.jpg | A full graph with four nodes which are connected by pairs (kappa-4). Full graphs with more than four nodes cannot be drawn in the plane without intersecting connections. The diagram can be seen as a tetrahedron looked at from above – “Deus” is above “Pater”, “Filius” and “Spiritus sanctus”.  So far, we have not found an early diagram which uses this configuration to illustrate the musical tetraktys. |
| 18 | Diatonic Scale - 12 Semitones | 1498\_Heritius\_Erasmus\_\_MusicaSpeculativa\_7r\_HERSPE\_01GF\_diatonicScale.gif | Perfect and imperfect consonances shown as intervals within an octave of the diatonic scale. The equidistant horizontal lines seem to indicate a division of the octave into twelve equal semitones and a proper logarithmic understanding of pitch. |
| 19 | Lilium Musice Plane | 1506\_Kleinspeck\_michael\_\_Lilium\_musice\_plane\_musicologie\_org\_hexachords.jpg | The system of Guido of Arezzo’s seven hexachords. The use of b instead of b-flat is indicated by a natural sign. The tone names on the left are also bracketed by disjoint tetrachords having their semitone step at varying position. The text to the left possibly indicates an understanding of pitch depending on frequency (“soni vehementi percusso [?] aere excitent”). |
| 20 |  | 1510\_raffael\_schule\_von\_athen\_pythagoras.jpg | A detail of Raphael’s “School of Athens” (1510) showing Pythagoras. |
| 21 |  | 1510\_raffael\_schule\_von\_athen\_pythagoras\_2006\_Lauenstein\_017.jpg | A detail of Raphael’s “School of Athens” showing the tetraktys 6:8:9:12. Greek names of the ratios are given, however the ratio 8:9 is not labelled. It cannot be derived directly from the tetraktys as a figurate number I+II+III+IV=X as shown below. |
| 22 | Tetraktys Hammers | 1528\_agricola\_martin\_\_tetraktys\_hammers\_116.jpg | Four “Pythagorean hammers” of different weight illustrating the division of the octave in the proportion 6:8:9:12. In the lower arc diagram some of the ratios are given in their lowest terms. In reality, neither the ratio of the mass nor the ratio of the size of the hammers is in inverse proportion to the ratio of the characteristic frequencies as the legend claims.  https://en.wikipedia.org/wiki/Pythagorean\_hammers |
| 23 | Tetraktys Hammers | 1529\_Agricola\_Martin\_\_TetraktysHammers\_gouk\_138\_4\_29.jpg | See [22]. |
| 24 | Monochord: Hexachordum | 1529\_fogliano\_hexachordum.jpg | Acoustical experiments on the monochord. With four moveable bridges two sounds can be generated on a single string (the two middle bridges must be a bit higher). The current configuration would give a syntonic minor sixth (8 : 5). It is also shown how a syntonic major sixth (5 : 3) would be obtained. |
| 25 | Monochord: Tetraktys | 1529\_fogliano\_tetraktys.jpg | Demonstration of the multiple proportions of the tetraktys on a monochord with four moveable bridges. FD would be an octave (Diapason), FC a 12th (Diapasondiapente) and FA a double octave (Bisdiapason) below the reference tone BG. |
| 26 | Pythagorean Semitone and Pythagorean Commas | 1544\_Stifel\_\_Arithmetica\_integra\_E\_Rara\_ETHBib\_De\_musicis\_progressionibus\_Cap\_IX\_75v.jpg |  |
| 27 | Pythagorean Semitone and Pythagorean Commas (detail) | 1544\_stifel\_\_arithmetica\_integra\_e\_rara\_ethbib\_de\_musicis\_progressionibus\_cap\_ix\_75v\_detail.jpg | Estimation of the Pythagorean comma. Stifel shows that the Pythagorean semitone (256 : 243) is greater than three Pythagorean comas. The recisa can be calculated with the formula:  The numbers of the second recisum are not correct. Fortunately, their ratio is very close to the correct values. The step from the second to the third recisum is done correctly so that the ratio of Stifel’s numbers is again very close to the correct value. |
| 28 | Hexachords | 1547\_glareanus\_dodecachordon\_Lib\_1\_004\_034\_hexachords.jpg |  |
| 29 | Bisdiapason - Double Tetraktys | 1547\_glareanus\_dodecachordon\_Lib\_1\_025\_054\_Bisdiapason.jpg | Double tetraktys. Two octaves divided symmetrically, 6:8:9:12:16:18:24, defining the frame of the Greek tone system. The arrangement of the notes is in logarithmic fashion so that equal intervals correspond to equal semicircles. |
| 30 | Division of the Tone (8 : 9) | 1547\_glareanus\_dodecachordon\_Lib\_1\_028\_058\_tonus.jpg | Division of the Tonus (4096:4608 = 8:9) into smaller intervals. The arcs on the left and the “tone names” on the right suggest a kind of diatonic scale on the microtonal level. The numbers are obtained as follows:  f:a is a true diatonic semitone (Diesis): 4608:4374 = 256:243;  Diaschisma: 4608-4491 = 4491-4374=4330-4213 = 4213-4096 = 117;  Schisma: 4374-4352 = 4352-4330=22.  In other words, none of the ratios with the same names are equal ratios. For instance, the Diaschisma 4213:4096 is equal to 47.76 cent, whereas the Diaschisma 4608:4491 is only 44.52 cent. The comma is only 17.5 cent compared with the true Pythagorean comma of 23.46 cent. These numbers could not be used to show that the whole tone 9:8 is smaller than nine Pythagorean commas nor that the Pythagorean semitone is between three and four Pythagorean commas. Glareanus writes on the same page: “Semitonium minus non prorsus 4. haberre commata, sed tria superare.” [(GLAR\_1547) Glareanus 1547, 28]  A similar division of the tone was given by Johannes de Muris (1323) [494]. |
| 31 | Bisdiapason and Chromatic Genus | 1551\_Stapul\_Jac\_Fabrum\_\_Musica\_LibrisQuatuorDemonstrata\_32v.jpg | Division of the double octave in the proportion 2:3:4:6:8. Equal intervals have equal semicircles, however the fourth (Diatessaron) is too small compared with the fifth (Diapente), if horizontal distance is equated with interval size.  The lower diagram explains the Greek chromatic genus. The word “Monochordum” is used in the sense of division of the monochord leading to the corresponding scale.  The octave is divided into a tone (T) and two connected tetrachords. The tetrachords consists of a diatonic semitone (S), a chromatic semitone (A=Apotome) and a minor third (TR).  The use of the designation Trihemitonium (three semitones) suggests an interpretation of the fourths as consisting of five semitones. The underlying scale is g, a, b-flat, b, d, e-flat, e, g. |
| 32 | Senario Circle | 1562\_zarlino\_institutioni\_harmoniche\_0036\_p\_025\_senario\_Circle.jpg | Zarlino’s “senario”. The “Numeri Sonori overo Harmonici” 1, 2, 3, 4, 5, 6 are arranged on a circle. Each pair of opposite numbers is connected by two big circular arcs. Their cross sections contain the names of the corresponding intervals. The ratio 6:4 should be labelled Diapente instead of Diatessaron.  The senario is an extension of the tetraktys, where each pair of numbers defines a consonant interval. The problem with the senario is that it does not classify the minor sixth 8:5 as a consonant interval. The ottonario 1, 2, …, 8 however would also include the number 7 to the system of consonances. |
| 33 | Numeri Sonori | 1562\_zarlino\_institutioni\_harmoniche\_0037\_p\_026\_numeriSonori.jpg | “Numeri Sonori” between 1 and 36. The listed numbers contain only factors of 2, 3 and 5 in their prime number decomposition. Two numbers with this property, 27 and 32, are missing. The ratio 27:25 corresponds to the difference of a major tone 9:8 and a diatonic semitone 16:15. If added to the list, 27:25 were the only non-superparticular ratio between neighboured numbers. The number 32 however would have posed no problem: 30:32 = 15:16 and 36:32 = 9:8. |
| 34 | Tetraktys (6 : 8 : 9 : 12) | 1562\_zarlino\_institutioni\_harmoniche\_0070\_p\_059\_tetraktys\_6\_8\_9\_12.jpg | The tetraktys, 6:8:9:12 symbolized by equidistant parallel lines. They also stand for the four strings of a “*Cetera*, o Lira ritrovata da Mercurio” [(ZARL\_1562) Zarlino 1562, 58]. |
| 35 | Quinario | 1562\_zarlino\_institutioni\_harmoniche\_0072\_p\_061\_pentenario\_kappa\_5.jpg | The quinario {1, 2, 3, 4, 5} defines an extension of the tetraktys to five numbers. All ratios are labelled. There are two ratios 5:2 and 5:3 that are neither multiple nor superparticular. The diagram includes the major third (5:4) and the major sixth (5:3), but not the minor third (6:5) and minor sixth (8:5) among the consonant intervals. The graph is the full kappa-5. |
| 36 | Hexachords | 1562\_zarlino\_institutioni\_harmoniche\_0115\_p\_104\_hexachords.jpg | The hexachord system of Guido of Arezzo with the continuous proportion (of string lengths) for the Pythagorean tuning on the left. The constitutive Greek tetrachords are added to the left and the “tetrachord synemmenon” leading to the b-flat to the right; b and b-flat occur on different lines so that the congruence of the hexachords is visually obscured.  https://en.wikipedia.org/wiki/Musical\_system\_of\_ancient\_Greece |
| 37 | Syntonic Diatonic Scale | 1562\_zarlino\_institutioni\_harmoniche\_0133\_p\_122\_syntonicDiatonicScale.jpg | The syntonic diatonic scale grouped around the “Tetrachordo Diatonico sintono di Tolomeo”. The proportion 108:120:135:144 can be reduced to 36:40:45:48 (T-t-S).  There is no arc between 135 and 96, the tritone 45:32 (but an allusion to one at 96), and there is no arc for the problematic fifth 160:108 = 40:27 between the second and sixth degree of the scale. The scale is today’s usual diatonic scale of “just intonation”. |
| 38 | Distribution of the Comma | 1562\_zarlino\_institutioni\_harmoniche\_0141\_p\_130\_distributionOfTheComma.jpg | This diagram explains a temperament of the syntonic diatonic scale, where the whole tones are of equal size so that the major third is 1/7th of a syntonic comma smaller than the major third of the ratio 5:4. The major tones T are lowered by 4/7th, the minor tones t and the semitones S are increased by 3/7th of a syntonic comma. Hereby, the fifths are lowered by 2/7th of a syntonic comma. The deviations from the syntonic diatonic scale in multiples of 1/7th of a syntonic comma can be read from the boxes above the arc diagram. The only interval that is equal to an interval of the syntonic tone system is the chromatic semitone (b-flat, b) of the ratio 25:24. All the intervals of equal name are also of equal size, which is not true for the proper syntonic diatonic scale.  The scale (with b and b-flat) is generated by seven fifths of the size 695.810 cent. |
| 39 | Keyboard: Divided Keys | 1562\_zarlino\_institutioni\_harmoniche\_0152\_p\_141\_keyboard\_dividedKeys.jpg | Keyboard with 19 keys per octave. Each black key of the modern piano corresponds to a black and a white key. Furthermore, there are extra white keys between the semitones b-c and e-f. It is Zarlino’s intention to provide a tone system that combines the diatonic, chromatic and enharmonic genera within a syntonic context, where many major and minor thirds of the ratios 5 : 4 and 6 : 5 occur. The drawing itself does not directly explain the underlying tuning. It can be interpreted by comparing it with three diagrams from the previous pages.  Between the tone pairs c-d, f-g and g-a the order of the split keys is black/white, whereas between d-e and a-b it is white/black. This distribution seems to indicate that the black keys c-sharp, e-flat, f-sharp, g-sharp, b-flat lead to major diatonic semitones. In other words, leaving away the seven white extra keys d-flat, d-sharp, e-flat, g-flat, a-flat, a-sharp, b-sharp gives a chromatic scale with two lowered and three increased chromatic semitones c-c#-d-eb-e-f-f#-g-g#-a-bb-b.  There is no split key for the second degree of the scale, which probably means that the tuning is based on the temperament of the underlying diatonic scale where the syntonic comma is distributed over the seven pitches. [38] |
| 40 | Tetraktys (6 : 8 : 9 : 12) | 1571\_zarlino\_dimostrationi\_harmoniche\_112\_1\_Tetraktys\_6\_8\_9\_12.jpg | The tetraktys 12 : 9 : 8 : 6. The equidistant spacing of the numbers does neither reflect the size of the intervals nor string lengths at the monochord [21]. |
| 41 | Monochordo Regolare Diatonico | 1571\_zarlino\_dimostrationi\_harmoniche\_219\_MonochordoRegolareDiatonico.jpg | Two octaves of the syntonic diatonic scale beginning at the sixth degree (a) are shown on a monochord. The letters above the string are used in the text to explain the tuning. The notes are obtained in the following order:  a-b (T), b-e (4th), b-c (S), c-d (T), e-a (4th), e-f (S), f-g (T), … The minor tones (10:9), d-e and g-a, result from filling the fourths with a diatonic semitone (16:15) and a major tone (9:8). In other words, the major tone a-b is followed by two joint tetrachords of identical structure (sTt). |
| 42 | Nomi Moderni ed Antichi | 1571\_zarlino\_dimostrationi\_harmoniche\_278\_nomi\_moderni\_ed\_antichi.jpg | Five hexachords on Gamma, C, F, G and c are given together with the names of the notes in the Greek diatonic tone system. B (10) and Bb (17) are given on different lines. The number 17 indicates that the “Trite synemennon” is the last note added to the system. [36] |
| 43 | Temperamento del Monochordo Regolare Diatonico | 1571\_zarlino\_dimostrationi\_harmoniche\_284\_TemperamentoMonochordoRegolareDiatonico.jpg | Temperament of the syntonic diatonic scale [38]. This tuning favours the thirds over the fifths. The whole tones are of equal size so that the ratios of the major thirds are exactly 5:4. The boxes around the strings indicate by how many quarters of a syntonic comma the tones must be lowered or increased with respect to the regular syntonic diatonic scale. For example, D is lowered b 2/4th of a syntonic comma and F is increased by 3/4th of a syntonic comma.  The scale is generated by six fifths of size 696.578 Cent. The Pythagorean fifths (3:2 of 701.955 Cent) are lowered by a quarter of a syntonic comma (0.25×21.506 Cent = 5.377 Cent). |
| 45 | Diatonic Scale | 1577\_Salinas\_De\_musica\_113\_diatonicScale.jpg | Syntonic diatonic scale. The C-major scale running from E to e with an ambiguous second degree d. The corresponding string lengths below the tone names are given in their lowest terms. The eight pitch classes per octave admit three major triads (4:5:6) and three minor triads (10:12:15).  This scale is also given by Descartes, who called the syntonic comma (81:80) “Schisma”. [314] |
| 46 | Totius harmoniae vis hoc diagrammate fulget | 1577\_salinas\_De\_musica\_124\_chromaticScale.jpg | Chromatic scale of 24 pitch classes per octave. The indicated string lengths admit an interpretation in terms of Pythagorean fifths (3:2) and syntonic major thirds (5:4).  There are four different pitches for F#/Gb and four different pitches for A#/Bb. In the realisation in the 53-tet tuning the distance between neighboured pitches varies from 1 to 3 units. |
| 47 | Recentiorum hexachorda cum antiquorum tetrachordis | 1577\_Salinas\_De\_musica\_195\_RecentiorumHexachordaCumAntiqorumTetrachordis.jpg |  |
| 48 | Triangle over chromatic scale | 1577\_Salinas\_De\_musica\_230\_TriangleOverChromaticScale.jpg | Chromatic scale of 14 pitch classes per octave. There are two ambiguous pitches D and b. #E should be read as Eb: it is a chromatic semitone (25:24) lower than E. There are five regular chromatic semitones 25:24 and six regular diatonic semitones 16:15, the ratio of f#-g however is 27:25 instead.  The scale is a subset of Salina’s scale of 24 pitch classes [46].  The larger intervals of the scale are analysed in a triangle, which substitutes Boethius’s system of arcs and uses fewer lines to label the same number of relationships. The names of the intervals can be found directly below the point of intersection of the related oblique lines.  For example, the line from bottom left to top right beginning at E (2880) and the line from bottom right to top left beginning at G (2400) intersect over the parallelogram labelled “Semiditonum”, indicating a minor syntonic third with the ratio 6:5. The indicated intervals are minor tones, major tones, minor thirds, major third, fourths, fifths, minor sixths, major sixths and the octave. Since only one octave is given, the system is incomplete: The fourth a-d, for example, must be studied by its octave complement, the fifth D-a. |
| 49 | Chromatic Scale | 1614\_beekman\_JIB\_I\_Fol\_14\_r\_stevin\_chromaticScale.jpg | Chromatic scale of 12 pitch classes per octave. According to Beeckman it is inspired by Simon Stevin.  If this scale were an accurate approximation of the 12-tempered equal tuning, the differences between consecutive numbers would be monotone increasing. However, the ratios of the semitones vary between 1.0451 (=177.67/170) corresponding to 90.8 cent and 1.0702 (=122/114) corresponding to 117.4 cent. Therefore, the ratios of the whole tones vary from about 1.11 corresponding to 180.7 cent to 1.14 corresponding to 226.8 cent. In other words, their sizes vary by almost a quartertone.  A closer analysis of the scale reveals that the maximum deviation for tritoni is 118 cent, and for fifths and fourths 99 cent. |
| 50 | Caelum Trinitatis | 1617\_fludd\_uch\_1\_0028\_Tract\_I\_Lib\_I\_020\_CaelumTrinitatis.jpg | “Demonstratio Caelum trinitatis”. Conception of the universe with an equilateral triangle and concentric circles emphasizing trinity. |
| 51 | Double Triangle | 1617\_fludd\_uch\_1\_0029\_Tract\_I\_Lib\_I\_021\_doubleTriangle.jpg | As [50] with two joint equilateral triangles. |
| 52 | Double Cone | 1617\_fludd\_uch\_1\_0089\_Tract\_I\_Lib\_III\_081\_doubleCone.jpg | Musica Mundana, see [54]. |
| 53 | Double Tetraktys | 1617\_fludd\_uch\_1\_0092\_Tract\_I\_Lib\_III\_084\_doubleTetraktys.jpg | Conception of the universe. The tetraktys 1:2:3:4 appears twice in a system of concentric circles as “Proportiones Pyramidis formalis” and “Proportiones Pyramidis materialis”. The musical ratios are indicated with arcs and labelled. However, the tripla is not indicated. |
| 54 | Double Cone Tetraktys | 1617\_fludd\_uch\_1\_0097\_Tract\_I\_Lib\_III\_089\_doubleCone\_Tetraktys.jpg | Conception of the universe. A system of concentric circular sectors with the sun in the middle of the “Sphaera aequalitatis”. The four elements Terra, Aqua, Aer, Ignis are labelled. In contrast to [53] only the numbers 1, 2, 3 are given in the “pyramids” in order to emphasize trinity by ternary configuration. |
| 55 | Monochord | 1617\_fludd\_uch\_1\_0098\_Tract\_I\_Lib\_III\_090\_Monochord.jpg | Two octaves on a monochord, the “monochordum mundanum”. Unrealistically, the two octaves take up the full string instead of only three quarters. The distribution of tones and semitones is indicated at the shaded right side of the instrument. The lower octave between earth and sun – in the middle of the string – is a G-major scale, so F is F#. The second octave (“Diapason formalis”), however, with five consecutive whole tone steps is not diatonic at all: G-ab-bb, c, d, e, f#, gg (gg should be labelled g).  The interval structure of the “diapente formalis” is mirror symmetric to the structure of the “diapente materialis” (the circular arcs for the latter are misplaced; they should embrace C-G. Instead, A-G is incorrectly labelled “Sesquialtera” and the corresponding arc on the right is incomplete). The “Diatessaron formalis” and the “Diatessaron materials” are formed by equal tetrachords with the semitone on top. |
| 56 | Monochord - The Four Elements | 1617\_fludd\_uch\_1\_0105\_Tract\_I\_Lib\_III\_096\_097\_ad\_Monochord.jpg | Two octaves on a symbolic monochord. There are four diatonic tetrachords with the semitone on top, representing the four elements Terra, Aqua, Aeris, Ignis.  There is a whole tone step between Aqua and Aeris, which is shared by both octaves, so that the full range between Gamma and gg is only two octaves minus a whole tone. |
| 57 | Double Cone Tetraktys | 1617\_fludd\_uch\_1\_0107\_Tract\_I\_Lib\_III\_097\_doubleCone\_Tetraktys.jpg | The four elements combined with two pyramids, “Pyramis corpora” and “Pyramis Ignea”. The ratios of the tetraktys appear twice. As in [53] the tripla is not indicated. The configuration is the same as in [52] and [53], but here it refers to a smaller part of the universe. |
| 58 | Double Cone Tetraktys | 1617\_fludd\_uch\_1\_0110\_Tract\_I\_Lib\_III\_100\_doubleCone\_Tetraktys.jpg | As [57]. |
| 59 | Lambda Tetrakys | 1617\_fludd\_uch\_1\_0174\_Tract\_I\_Lib\_V\_164\_lambdaTetrakys.jpg | A “lamboma” a diagram in the shape of the Greek letter lambda. It contains the first three powers of 3 and 2. It can be viewed as a part of the full triangular diagram developed by Boethius [1, 62, 71]. |
| 60 | Templum musicae | 1624\_fludd\_eod\_uch\_1\_Tract\_II\_Part\_II\_Lib\_I\_161\_ad\_templum.jpg | The “Templum Musicae” is the central picture in Robert Fludd’s tract of the same name. The diagrams are explained in the text and serve as mnemonic aids. |
| 61 | Hexachords | 1624\_fludd\_eod\_uch\_1\_tract\_ii\_part\_ii\_lib\_i\_161\_ad\_templum\_detail\_hexachords.jpg | The system of Guido of Arezzo’s hexachords. Differently from Guido’s arrangement [9], however, the lowest octave has both B and Bb, so that the system is octave periodic. |
| 62 | Triangular diagrams | 1624\_fludd\_eod\_uch\_1\_Tract\_II\_Part\_II\_Lib\_I\_161\_ad\_templum\_detail\_triangles.jpg | Two triangular diagrams.  The upper triangle can be used to form consonant chords [516].  The lower diagram of durations corresponds to a diagram by Torkesey [16]. There are two mistakes, 16 instead of 12 and 24 instead of 27. There are no mistakes in the related diagram [71]. |
| 63 | Deriv. Diatonic Scale – Synopsis | 1624\_fludd\_uch\_1\_0359\_0361\_Tract\_II\_Part\_I\_Lib\_VI\_131\_133\_DerivDiatonicScale\_synopsis.jpg | This sequence of drawings explains the tuning of the monochord according to the Pythagorean diatonic scale. A whole tone is obtained by dividing the line segment between the fret of the lower note and the bridge into nine equal parts.  The lowest tetrachord is tuned with two consecutive whole tones and the fourth (division by 4) from the fundamental note Gamma. This gives the positions of a, b and c. Then D is constructed as a fifth (division by 3) over Gamma, and E and F are fourths over B and C. Finally, the octave G is obtained by dividing in half the open string. (Note the inconsistent use of lower case and upper case letters.) |
| 64 | Deriv. Diatonic Scale | 1624\_fludd\_uch\_1\_0359\_Tract\_II\_Part\_I\_Lib\_VI\_131\_derivDiatonicScale.jpg | [63] |
| 65 | Deriv. Diatonic Scale | 1624\_fludd\_uch\_1\_0360\_Tract\_II\_Part\_I\_Lib\_VI\_132\_derivDiatonicScale.jpg | [63] |
| 66 | Deriv. Diatonic Scale | 1624\_fludd\_uch\_1\_0361\_Tract\_II\_Part\_I\_Lib\_VI\_133\_derivDiatonicScale.jpg | [63] |
| 67 | Monochord | 1624\_fludd\_uch\_1\_0362\_Tract\_II\_Part\_I\_Lib\_VI\_134\_monochord.jpg | Four octaves on a monochord with accurately positioned frets. Each octave divides in half the remainder of the open string. A clear distinction between tones and semitones is made.  The interval G-ggg is wrongly labelled “Quinquies Diatessaron”. Actually, five fourths are a semitone greater than two octaves. So Quinquies Diatessaron would fit to the interval E-ff (or to B-cc).  This label, which appears in different drawings [69, 70] seems to imply that Fludd was aware of the fact that the Pythagorean diatonic scale can be constructed as a stack of fourths. In all these drawings, however, this label is misplaced.  A different explanation could refer to the construction of the two octaves of the Greek tone system with four tetrachords and the extra tetrachord (the tetrachord synemmenon), which adds the b-flat to the diatonic scale. [29, 36] In that case, however the arc would better embrace A-aa, and one would also expect the occurrence of b-flat. Interestingly, the B is redundantly highlighted to be a b-quadratum by natural signs. |
| 69 | Monochord | 1624\_fludd\_uch\_1\_0414\_Tract\_II\_Part\_II\_Lib\_III\_184\_monochord\_b.jpg | A monochord on a column. This drawing explains the monochord on the Templum musicae [60]. The chord on the stone column will not give a sound. The instrument is merely a tool for contemplation.  The ratios of the Pythagorean tetraktys are indicated on the left with arcs beginning at C. Selected interval names occur on the right. There is a major triad Gamma-B-D-G with the interval names Ditonus-Semiditonus-Diatessaron (major third-minor third-fourth). This is the only representation of a monochord by Fludd that names the thirds.  As in several other drawings ggg should be read as gg and gggg as ggg. Again, there is the irritating “Quinquies diatessaron” between G and ggg [67, 70]. |
| 70 | Monochord | 1624\_fludd\_uch\_1\_0417\_Tract\_II\_Part\_II\_Lib\_III\_187\_monochord.jpg | Three octaves of the diatonic scale on a monochord. The intervals of the Pythagorean tetractys, Diatessaron (fourth), Diapente (fifth), Diapason (octave), Diapason cum Diapente (octave + fifth) and Bis Diapsaon (double octave) are shown on the double octave C-cc. There are no arcs leading to A and E or their replica. Also B cannot be reached from the other notes. As in [67] and [69] the double octave G-ggg is incorrectly labelled “Quinquies diatessaron”. |
| 71 | Triangle of durations | 1624\_fludd\_uch\_1\_0434\_Tract\_II\_Part\_II\_Lib\_IV\_204\_lowerTriangle.jpg | Arithmetic triangle in the style of Boethius [1] representing numbers of the format 2j×3k. The diagram is used to explain the system of tone durations of binary and ternary division. Fludd has copied it from Torkesey [16]. The diagram occurs also on the Templum musicae (with two mistakes). As in Torkesey’s diagram the diagonal direction top right bottom left is labelled Sesquitertiae (3:4), an essential mathematical property of the diagram. |
| 72 | Trompette Marine | 1637\_mersenne\_\_Vol\_IV\_218\_TrompeteMarine.jpg | String instrument that imitates a trumpet. The sounds are produced as harmonics of the open string (flageolet). |
| 73 | Senario Triangle | 1650\_descartes\_Compendium\_13\_senario\_triangle.jpg | Zarlino’s senario. The ratios formed by number pairs from the set {1, 2, 3, 4, 5, 6} are represented in a triangular matrix. The arrangement is probably inspired by Ramon Llull’s third figure [74].  The related intervals are consonances of the syntonic diatonic tone system with major thirds 5:4 and minor thirds 6:5. The system, however, is incomplete, because it does not classify the minor sixth 8:5 as a consonance. An extension of the system to {1, 2, 3, 4, 5, 6, 7, 8} would include the minor sixth but also intervals with ratios containing the number 7, which are not viewed as consonances.  The first three rows of the triangle form the consonance system of the Pythagorean tetraktys {1, 2, 3, 4}.  Descartes inconsistently uses the traditional term “Ditonus” for the major third, but “Tertia minor” instead of “Semiditonus” for the minor third. It would have been clearer to use “Tertia maior” and “Tertia minor” in order to distinguish them clearly from the Pythagorean intervals. |
| 74 | Tertia Figura | 1651\_Raymundi\_Lullii\_Opera\_omnia\_30\_TertiaFigura.jpg | Ramon Llull’s tertia figura of the Ars brevis lists all pairs of letters from the set {B, C, D, E, F, G, H, I, K}. Equivalently, the letters can be arranged on a regular nonagon, in which each vertex is connected to all the other vertices, as Llull does in the primera figura of the Ars brevis.  The number of connections can be calculated with the formula 0.5n× (n-1), where n is the size of the alphabet. Therefore, Lull’s tertia figura has 0.5×9×8 = 36 entries and René Descartes’ figure of the senario 0.5×6×5 = 15 entries [73]. |
| 75 | Syntonic Chromatic Scale Measured by 12tet Semitones | 1665\_newton\_\_manus\_223\_105v\_detail\_01.jpg | Comparison of a syntonic chromatic scale with the 12-tempered equal tuning (12-tet). The first column of the table contains syntonic ratios, the last column tone names beginning at G. The second column contains string lengths on a string of length 720 units. They are integer numbers, because the lowest common multiple of the denominators in the first column is 720. The third column contains the base 10 logarithms of the values from the second column. The fourth column contains base 10 logarithms of the string lengths of the same string divided geometrically (12-tet). The values form an arithmetic sequence with common difference 0.025086. The fifth column contains the corresponding 12-tet string lengths. The sixth column shows the size of the syntonic intervals in terms of 12-tet semitones. These values are entirely correct. The six given decimal places result in a precision of 0.0001 Cent. |
| 76 | Geometric and Syntonic Division of the Octave | 1665\_newton\_\_manus\_225\_106v\_detail\_01.jpg | Comparison of the ratios of the 12-tempered equal tuning with the ratios of the syntonic chromatic scales. Some of the 12-tet ratios in the first column deviate slightly in the last digit from the correct values. The numbers at the right margin express the syntonic intervals in terms of 12-tet semitones and fractions of whole tones. So 1,058522 measures the semitone 16:15 as 1 12-tet semitone + 0.058522 12-tet tone, that is 1.117044 12-tet semitones. The correct value is 1.117313. The other values are correct to the indicated precision of two decimal places.  Newton was not sure about the ratio of the diminished fifth/tritone. He considers 25:18, 7:5, 45:32, 64:45, 10:7 and 36:25, two of them involving 7, which is not part of the syntonic system.  This page was probably written before 105v [75]. |
| 77 | Analysis of Newton's Chromatic Scale | 1665\_newton\_\_manus\_226\_107r\_detail\_01.jpg | This diagram analyses the intervals of Newton’s syntonic chromatic scale. The ratios given at the left of the diagram defined the scale as G-Ab-A-Bb-B-C-C#-D-Eb-E-F-F#, where the ratio of all the diatonic semitones is 16:15. With the exception of C# the interval structure of the scale is fully mirror symmetric. A similar but incomplete diagram [78] introduces an additional note Db with the ratio 64:45 for G-DB, completing the symmetry. The ratios determining the scale are most simple syntonic intervals with respect to the central tone G.  The rows in the grid list the intervals containing the same number of semitone steps. Since there are three different semitone steps, 16:15, 25:24 and 135:128, the size of the larger interval also varies.  If the interval A-D is to be examined, follow the diagonal dashed line from A at the top until it meets the vertical solid line from D. Following the horizontal line from that point to the right indicates that the interval is a fourth. The decoration with quarter circles at the point of intersection of the diagonal with the vertical line indicates that the interval has the exact ratio 4:3 indicated at the left.  The intersection point for the interval A-B has no pair of quarter circles on the diagonal line. Therefore, its ratio is not 9:8. An arrow pointing up left occurs at each minor tone 10:9, so A-B is a minor tone.  There is an arrow pointing down right at the diminished third F#-Ab, 256:225 (which is bigger than a major tone), but not at C#-Eb, maybe because of the hidden ambiguity of C#/Db.  Likewise, all the chromatic minor semitones (25:24 and 135:128) except C-C# have an arrow. Dotted quarter circles appear in connection with C#, their meaning is not entirely clear. [<WARD\_2013> Wardhaugh 2013, 96-97].  We calculated the sizes of the intervals with a computer program and found only three inconsistencies: There are dotted quarter circles at the diminished sixth C#-Ab and at the diminished octave C#-C, both greater than a fifth or major seventh, respectively. There are no quarter circles at the minor sixth C#-A. Otherwise, the pairs of quarter circles appear, if and only if, the ratio of the related interval agrees with the reference ratio indicated at the left of the diagram. |
| 78 | Analysis of Newton's Chromatic Scale | 1665\_newton\_\_manus\_227\_107v\_detail\_01.jpg |  |
| 79 | Division of the Octave into 53 and 612 Equal Parts | 1665\_newton\_\_manus\_229\_108v\_detail\_01.jpg | Approximations of Newton’s syntonic chromatic scale by equally tempered scales for different divisions of the octave.  The values for the interesting divisions into 53, 612 and 29 parts are entirely correct, i.e., Newton’s syntonic ratios are best expressed as multiples of the related unit interval with the indicated numbers. For example, the major whole tone G-A is 4.928/29 octaves, so the major whole tone in 29-tet measures 5 units. The minor third G-Bb measures 7.628/29 octaves, which is rounded to 8 units in 29-tet.  The divisions of the octave giving optimum approximations of the Pythagorean fifths are into 12, 41, 53, 306, 635, … equal parts. These numbers can be found with continued fractions [<SCHR\_1980> Schechter 1980, 41-42].  Since the Pythagorean comma and the syntonic comma are comparable in size and the 53rd part of an octave is between the two, 53 is an excellent choice for the syntonic scale. In other words in 53-tet, the Pythagorean comma is identified with the syntonic comma and taken as a unit interval, so that the intervals of the syntonic scale are very well expressed as integer multiples of the unit. The rms-deviation for 53-tet is 0.046 units corresponding to 1.04 Cent only.  It turns out that 306-tet does not provide good approximations for the non-Pythagorean intervals. They are almost in the middle between multiples of the related unit interval. For example the major third is 98.510/306 octave. Therefore, 612-tet (612 = 2×306) approximates both, the Pythagorean and the proper syntonic intervals, almost perfectly.  Using a computer program we calculated all n-tet approximations up to n = 2000 of the syntonic diatonic scale and evaluated their quality by relative rms-deviations. Among the numbers less than 118, the choice n = 53 is the best and n = 59 is the worst. However doubling 59 to n = 118 is better than n = 53. Since 59-tet gives good major thirds but bad fifths, Newton probably left the related column incomplete. Furthermore, in 59-tet the major tone (10 units) is equal to two diatonic semitones (5 units).  For n less than 1783, the division into 612 parts is the best choice with an rms-deviation of 0.017 units (corresponding to 0.03 Cent).  The smallest number n that allows to distinguish between the syntonic comma, chromatic semitones and diatonic semitones leading to a consistent scale is 29. In 29-tet the major tone measures 5 units, the minor tone 4 units, the diatonic semitone 3 units, the chromatic semitone 2 units and the syntonic/Pythagorean comma 1 unit. The octave is 6 major tones minus a comma (6×5–1 = 29). With an rms-deviation of 0.264 units corresponding to 10.91 Cent the difference between the true syntonic scale and its realisation in 29-tet becomes audible.  The values for 100-tet and 36-tet follow no obvious system. They deviate considerably from the closest approximations for the chromatic scale.  The second last column is equivalent to 36-tet. The best values in 36-tet are the multiples of 3, i.e., 12-tet.  The last column, equivalent to 120-tet, deviates at two places from the closest approximation. The minor third G-Bb measures 3.156/12 octave and the major sixth 8.844/12 octave, which Newton replaces by 3.1 and 8.9 units. This adjustment makes all the diatonic semitones equal to 1.1 units and generates a consistent diatonic scale, with an rms-deviation of 0.429 corresponding to 4.29 Cent. In other words, 120-tet is clearly inferior to 53-tet.  Newton’s selection of 53-tet, 612-tet, 59-tet, 29-tet and 120-tet reveals deep insight into algebraic techniques and logarithms. [<WARD\_2013> Wardhaugh 2013, 98] |
| 80 | Diatonic circles (1) | 1665\_newton\_\_manus\_230\_109r\_detail\_01.jpg | This diagram is an elaboration of Descartes’ circular hexachord diagram [316].  Where Descartes gives three hexachords, Newton gives five full diatonic scales, ranging from two sharps (the innermost) to two flats (the outermost). Note the enharmonic mistake in the tone names in the centre of the diagram  should be c#. The same mistake occurs already on 108v, the numbers there indicate that there is a diatonic semitone from this note to D [79].  The seventh note that Newton added to Descartes’ hexachord is a diatonic semitone above the highest note la of the hexachord, and not the leading note to ut. Therefore, ut of the middle scale corresponds to the fifth degree G of a C major scale and the solmisation refers to the mixolydian mode. All the scales have the same interval structure T-t-S-T-t-S-T. The transpositions of the scale by fifths generate four ambiguous notes A, B, D, E separated by syntonic commas. The central note ut is flanked by two major tones, so that the scales contain a Pythagorean third, F-A in the central scale.  The whole system contains the sequence of Pythagorean fifths F-C-G-D-A, so that the five scales share the note G (fa-fa-ut-sol-re) on top.  The numbers assigned to the pitches are consistent with the interval sizes in 53-tet and 120-tet given on fol. 108v [80].  53-tet: S = 5, t = 8, T = 9 (the outer set of numbers)  120-tet: S = 1.1, t = 1.9, T = 2. (the inner set of numbers)  The system does not cover the full range of the chromatic scale; G#/Ab is missing. In order to fill this gap Newton could have added more scales, with three sharps and/or three flats. However, the extension for three sharps could be done in two ways, either by starting with ut = E (40) or with ut = E (39). The former continues with a further transposition by a Pythagorean fifth the latter by an imperfect fifth 40:27.  It is interesting to note that by 12 transpositions by Pythagorean fifths a set of 24 pitches is reached that can be interpreted as two chromatic scales rotated by one unit. |
| 81 | Diatonic circles (2) | 1665\_newton\_\_manus\_231\_109v\_detail\_01.jpg | A second version of five diatonic scales in circular arrangement. This time the interval structure of the scale is t-T-S-T-t-S-T. Compared with the diagram on fol. 109r [80] the first and second whole tone steps are exchanged, i.e., re is flattened by a syntonic comma. The seventh note of the scale is indicated with dotted lines and without solmisation. The three inner hexachords ut-re-mi-fa-sol-ut agree with Descartes’ hexachord diagram [316].  The diatonic scale contains no Pythagorean thirds and it is the syntonic diatonic scale given by Zarlino 1562 [37]. The second degree of the corresponding major scale is a major tone above the tonic. The second degree of Descartes’ diatonic scale, however, is either ambiguous [314] or on a minor tone from ut [315], so Descartes’ seven notes major scale is t-T-S-T-t-T-S and Newton’s major scale is T-t-S-T-t-T-S. This is because the seventh note that Descartes adds to the hexachord is the leading note to ut, so that ut agrees with the tonic of the major scale, where Newton’s seventh note is a whole tone below ut, so that ut corresponds to the fifth degree of the corresponding major scale [80].  The related 53-tet numbers surround the diagram. There are no 120-tet numbers and there are no note names added. |
| 82 | Diatonic Circles (detail) | 1665\_newton\_\_manus\_231\_109v\_detail\_02.jpg | Interpretation of the diatonic scales as part of a syntonic chromatic scale with 12 pitches. The diatonic semitone 16:15 is denoted by *a*, the chromatic semitone 135:128 by *b* and the chromatic semitone 25:24 by *c*. Therefore, *a+b* stands for a major tone 9:8 and *a+c* for a minor tone 10:9. The figure in the middle represents the diatonic scale shown on the same page [81] [<WARD\_2013> Wardhaugh 2013, 101].  The third figure represents a diatonic scale with flattened second degree. They are both contained in the same configuration of semitones, since *a.b+a* in the middle figure is replaced by *a+b.a* in the third at one place, the rest is the same.  The chromatic scale of the middle figure is C-Db-D+-Eb-E-F-F#-G-Ab-A-Bb-B-C, and in the third it is F-Gb-G-Ab-A-Bb-B-C-Db-D--Eb-E-F. The third figure has only flattened chromatic semitones. Indeed, the twelve pitches can be covered by two diatonic scales of the interval structure t-T-S-T-t-T-S a diatonic semitone apart, C major + Db major. Therefore, by contracting the semitones differently to whole tones in the same configuration a chromatic scale with only sharps can be obtained. It is (beginning on top) *b+a.c+a.b+a.a.c+a.b+a.a.*, which translates into G-G#-A-A#-B-C-C#-D-D#-E-F-F#-G, which can be interpreted as C major + C# major. The grouping *a+b.a+c.a.b+a.a+c.a.c+a.b+a.* translates into G-Ab-A-Bb-B-C-C#-D-Eb-E-F-F#-G. This is Newton’s chromatic scale, where the underlying (non-standard) major scale has the structure TtSTtStT [75, 76]. |
| 83 | Whirligig Octave Circle | 1672\_Salmon\_1213\_WhirligigOctaveCircle\_gouk\_130\_4\_16.jpg |  |
| 84 | Whirligig Octave Circle | 1672\_Salmon\_1213\_WhirligigOctaveCircle\_gouk\_130\_4\_16\_detail.jpg | The circular pitch diagram on the left gives the syntonic diatonic scale according to Zarlino 1562 [37] anticlockwise, with a major tone between C and D. The pitch classes are assigned to three octaves Base, Mean and Treble in the stave on the right with dotted lines. The angles in the circular diagram make no distinction between major and minor whole tones. They both measure 60° and the angles of the semitones measure 30°. These angles actually agree with the 12-tempered equal tuning, so that the scale appears to be mirror symmetric to the diameter of the circle through D.  The solmisation uses only mi, fa, sol, la, where mi-fa is a diatonic semitone (16:15), sol-la is a minor whole tone (10:9) and fa-sol as well as fa-mi are major whole tones (9:8). |
| 85 | Colour Spectrum and Musical Scale | 1675\_Newton\_ColourSpectrumAndMusicalScale\_gage\_1999\_140\_Fig\_060.jpg | See [86]. |
| 86 | Colour Spectrum and Musical Scale | 1675\_newton\_colourspectrumandmusicalscale\_gage\_1999\_140\_fig\_060\_detail.jpg | The spectre of a prism over a monochord string PZ with the midpoint T in a manuscript by Newton. The notes of the octave are given in a solmization with four syllables mi, fa, sol, la and correspond to a Dorian scale D-E-F-G-A-B-C-D. No interval ratios are given, cf. [92].  In this mapping between sound and light, increasing sound frequency is paired with decreasing light frequency [<MUZZ\_2012> Muzzulini 2012, 699-700]. |
| 87 | Spectre Monochord | 1675\_newton\_spectreMonochord\_NATP00287\_4.jpg | The spectre of a prism over a monochord string PZ with the midpoint T as [85]. Newton’s “Hypothesis explaining the properties of light” was published only in 1757. |
| 88 | Mysterium Aeternitatis | 1676\_mace\_thomas\_\_MusicalInfinity\_SpiralPoem\_gouk\_153\_4\_48.jpg | Magnum Mysterium Aeternitatis. Musical infinity – the octaves symbolize eternal recurrence. |
| 89 | Diatonic scales: F major and C major | 1680\_PhiloMus\_A\_B\_\_Synopsis\_of\_Vocal\_Musick\_1.jpg | The unidentified ‘A.B. Philomus’ included material about tuning theory in his 1680 Synopsis of Vocal Musick, including diagrams in which the octave was represented by a circle as in Descartes’ “Compendium musicae”. As in Descartes’ diagrams different scales were shown simultaneously and compared, and the discrepancies between their versions of the ‘same’ pitches were illustrated. A notable difference is that this author also superimposes a division of the octave into many small parts: here the just intonation was intended to be executed not exactly but according to an approximation. BeW  In this circular pitch diagram with a scale of 59 units the major whole tone measures 10 units, the minor whole tone 9 units and the diatonic semitone 5 units. There is only one minor whole tone ut-re. There are two diatonic scales C major (solid radiuses) and F major (dashed radiuses) shown with a seven notes solmisation ut-re-mi-fa-sol-ci. They generate the same pitches as Descartes’ three hexachords [316], however with only one ambiguous pitch G. Major tones allow no distinction between diatonic and chromatic semitones. The minor third A-C is divided into three equal semitones.  The given diatonic scale is not the closest 59-tet approximation. It is obtained when the fifth c-g of size 34.51/59 octave and the major sixth c-a of size 43.48/59 are replaced by 34 and 44 units respectively.  The 59-tet tuning has the highest rms-deviation of 0.368 units (corresponding to 7.50 Cent) for *n*-tet tunings with *n* less than 306. So 59 is the worst choice of n.  A reason for selecting 59 might have been the identification of the syntonic and Pythagorean comma as one unit. With major tones of size 10 the octave is equal to six major tones minus a comma, 6×10–1 = 59 units. [79]. In order to obtain the correct number of major and minor tones in the scale, *n* should be of the format (2k+1)×6-1. The syntonic comma measures 1/55.80 octave and the Pythagorean comma 1/51.15 octave. So 53 is the only number between 51.15 and 55.80 of the desired format with an rms-deviation of 0.046 units. The next candidate is 65-tet with major tones of 11, minor tones of 10 and diatonic semitones of 6 units, a scale with an rms deviation of 0.071 units. The next smaller candidate, 41-tet with major tones of 7, minor tones of 6 and diatonic semitones of 4 units has an rms-deviation of 0.158 units, which is considerably worse than 65. |
| 90 | Diatonic scale: microtonal graduation | 1680\_PhiloMus\_A\_B\_\_Synopsis\_of\_Vocal\_Musick\_2.jpg | Diatonic scale in 58-tet.  This second image from the Synopsis of Vocal Musick shows the sizes of various diatonic intervals in relation to its equal division of the octave: including intervals occurring in the just intonation like the greater and lesser tones and semitones. BeW  The major tone of 10, the minor tones of 9 and the diatonic semitones of 5 units lead to a Pythagorean comma of 2 units, which does not give very good fifths and an inaccurate major seventh of 53/58 instead of 52.60/58 octave. Differently from [89] this scale has the right number of major and minor whole tones. Read clockwise it corresponds to Descartes’ diatonic scale.  Possibly, Mersenne’s incorrect estimation of the octave as between 58 and 59 syntonic commas [210, 220], was the inspiration for this particular division of the octave and the division in [89]. |
| 91 | The Musicall Compass | 1684\_anonymous\_\_themusicallcompass\_herissone\_2000\_musictheory\_085.jpg | Three octaves Base, Mean, Treble of a diatonic scale arranged on a spiral. This is the earliest known diagram which makes a precise use of the spiral as a metaphor for the octave similarity. The angles correspond to 12-tet with semitones of 30° and whole tones of 60°. The additional sharps and flats seem to refer to the same pitch. This is consistent with 12-tet or at least with a fixed chromatic scale of 12 pitches per octave.  Since the radial spacing gets narrower with increasing pitch, it is somehow related to string length, but not exactly: the radius is not halved with each full rotation. |
| 92 | Monochord Spectre | 1704\_newton\_Opticks\_Book\_I\_Part\_II\_Plate\_1\_Fig\_4\_monochordSpectre.jpg | This diagram from Newton’s “Opticks” (1704) combines the diatonic scale on a monochord with the spectrum of a prism [85, 86, 87]. Here, the corresponding ratios of string length are indicated. However, the related spectral colours must be looked up in the legend. The colours are the same as in the other diagrams from left to right Purple, Indigo, Blew, Green, Yellow, Orange and Red. In other words, the assignment of the colours is inconsistent with the frequencies of sound and light. The incorrect order results from the observation that the violet part in the prismatic spectrum is broader than the red part.  It should be noted that Newton was against the wave theory of light, but acknowledged wave transmission from the retina to the brain. |
| 93 | Table des Progressions | 1726\_rameau\_\_NouveauSysteme\_24\_TableDesProgressions.jpg | Syntonic grid: two-dimensional arrangement of the pitch classes of the syntonic tuning with chains of fifths in the vertical and chains of thirds in the horizontal direction. The numbers are the corresponding powers of 3 and 5 of the frequency ratios with respect to ut = 1. They are of the format 3j×5k. The octave information (powers of 2) is not taken into account. This table is a number triangle in the style of Boethius. However, Boethius used only bases in superparticular ratio (*n* and *n*+1), although such a diagram can be constructed for any pair of numbers which are relatively prime.  This arrangement was later used by Leonhard Euler (1739) [94, 95] and Arthur von Oettingen (1866, 1917) and is very convenient for the study of harmonies and their ambiguities in the syntonic tone system (“just intonation”). |
| 94 | Chromatic Scale | 1739\_Euler\_\_Tentamen\_147\_ChromaticScale.jpg | Chromatic scale of twelve pitch classes in a grid. With F at the top, down left is the direction of fifths (V), down right is the direction of major thirds III), so that the horizontal direction indicates minor thirds down. The diagram uses German notation, where H stands for B and B for B-flat. The sharps Fis, Cis, Gis and Dis are written as Fs, Cs, Gs and Ds. Note the enharmonic mistake at the bottom; B should be A-sharp. In the syntonic tuning the compact arrangement with three rows of three fifths generates four different semitones: chromatic semitones 25:24 and 138:25, diatonic semitones 16:15 and 27:25.  Euler compares this syntonic chromatic scale (“Genus genuinum”) with 12-tet (“Genus aequabile”) in a table that gives the deviations as logarithmic parts of an octave [(EULE\_1739) Euler 1739, 149]. |
| 95 | Chromatic Scale Powers | 1739\_Euler\_\_Tentamen\_279\_ChromScalePowers.jpg | Chromatic scale of twelve pitch classes in a syntonic grid with fifths arranged horizontally and major thirds vertically down. The indicated powers of 3 and 5 in brackets make clear that the diagram refers to the syntonic tone system with fifths 3:2 and major thirds 5:4. Different values for *n* in the factor 2*n* give the tones of the same pitch class. The meaning of the constant factor 33 is not clear. It does not provide absolute frequencies in Hertz: substituting *n* = -2 would give 2^(-2)\*3^7=2187/4=546.75 for A, which would not be a reasonable Hertz value.  There are two enharmonic mistakes in the names of the notes: B (= B-flat) should be labelled A-sharp and F should be labelled E-sharp. [94] |
| 96 | Pitch Helix | 1834\_opelt\_SchraubenLinie.jpg | Two components of pitch illustrated by a helix (on the mantle of a cylinder). The circle at the bottom contains the 12 pitch classes ordered chromatically. The perpendicular direction is frequency. Intersection points of vertical lines with the helix represent notes of the same pitch class. The distance from the base doubles with each circulation.  The words “Dissonanz” and “Consonanz” make no sense; dissonance does not increase with the distance from the base. |
| 97 | Pitch Helix | 1852\_drobisch\_\_121\_skrews.jpg | One and several octaves on the pitch helix of [96]. |
| 200 | Diatonic Scale | 1200\_c\_d\_mu\_8\_cod\_375\_cim\_13\_p\_093\_\_guido\_micrologus\_087\_diatonicScale.jpg | Diatonic scale A-B-C-D-E-F-G-a-b-c over an octave and a minor third.  Tones and semitones are indicated between the names of the notes. Three fourths A-D, B-E and C-F are highlighted as well as the corresponding fifths D-a, E-b and F-c. There are no lines ending at G, although there is a fourth D-G. The meaning of the roman numerals is unclear. |
| 201 | Diatonic Scale | 1200\_c\_d\_mu\_8\_cod\_375\_cim\_13\_p\_094\_\_guido\_micrologus\_088\_diatonicScale.jpg | The diagram shows that the diatonic scale can be constructed as a chain of Pythagorean fifths by following the lines that connect the names of the notes: anticlockwise from the left F-C-G-d-a-E-b or from centre right F-c-G-D-a-e-b. This is realised on the scale C-D-E-F-G-a-b-c-d-e of an octave and a major third. The designations of the intervals are to be read in the direction of the text. The diagram also shows that the pitch classes of the Pythagorean diatonic scale are arranged symmetrically around D, the midpoint of the chain of fifths. |
| 202 | "duo semitonia tonum perficere non possunt" | 1498\_Heritius\_UB\_Muennchen\_4\_Cod\_ms\_752\_fol\_7r.jpg | « Dyapason constituitur ex quinque tonis, et duobus semitoniis. Constituitur enim ex Dyapenthe, et Dyathesseron, Nec etiam constituitur ex sex tonis, cum duo semitonia tonum perficere non possunt »  In other words, since two semitones are smaller than a tone, six tones are greater than an octave consisting of five tones and two semitones. |
| 203 | Perfect and Imperfect Consonances | 1498\_heritius\_ub\_muennchen\_4\_cod\_ms\_752\_fol\_7r\_detail.jpg | The octave of the diatonic scale ut-re-mi-fa-sol-la-fa-sol corresponding to G-A-B-C-D-E-F-G. On the left the Pythagorean consonances (“Consonantie perfectae”) the octave (“Dyapason”), consisting of a fifth (“Dyapenthe”) and a fourth (“Dyathesseron”) are shown. Unfortunately, the arcs of the fifth and fourth meet half a unit below sol.  On the right the other diatonic intervals (“Consonantiae imperfectae”) are listed. The tetrachord ut-re-mi-fa is labelled with “Semitono”, “Tono”, “Semiditono” (minor third) and “Ditonus” (major third). The major sixth ut-la is called “Tonus cum Dyapenthe” and the minor sixth re-fa is called “Semitonus cum Dyapenthe”. The tritone mi-fa and the seventh ut-fa are not indicated.  The equidistant horizontal lines suggest a division of the octave into twelve equal semitones. However, the text on the same page [202] makes clear that two semitones are smaller than a tone. Nevertheless, this diagram seems to be the first that sets the diatonic scale into a frame of twelve semitone steps per octave, and the horizontal lines express a proper logarithmic understanding of pitch, where each tone could be divided into two semitones. [18] |
| 204 | Monochord, Tetrachords | 1562\_zarlino\_institutioni\_harmoniche\_0145\_p\_134\_monochord\_tetrachords.jpg | Two octaves A-aa of a diatonic scale with b-flat (b rotundum) and b (natural, b quadratum) are shown on a monochord. The names of the Greek tetrachords of the structure s-T-T are added. These congruent tetrachords on B, E and a completely determine the pitches of the diatonic tone system. |
| 205 | Chromatic Tetrachord | 1562\_zarlino\_institutioni\_harmoniche\_0149\_p\_140\_chromaticTetrachord.jpg | In the chromatic tetrachord B-C-C#-E, the major semitone (16:15) is followed by a minor semitone (25:24) and a minor third, “Trihemituono” (6:5). Zarlino’s interpretation is based on syntonic intervals involving only the prime numbers 2, 3 and 5. |
| 206 | Enharmonic Tetrachord | 1562\_zarlino\_institutioni\_harmoniche\_0151\_p\_142\_enharmonicTetrachord.jpg | In the enharmonic tetrachord E-E#-F-a the diatonic semitone E-F (16:15) is divided into a minor semitone E-E# (25:24) and a diesis E#-F (128:125). The interval between the third and fourth note of the hexachord, the “Ditono”, is a major third (5:4). Zarlino’s interpretation of the enharmonic genus is based on syntonic intervals involving only the prime numbers 2, 3 and 5. |
| 207 | Table pour les facteurs d'instrumens | 1636\_mersenne\_HarmUniv\_PremierePreface\_noPages\_\_12tet\_chromaticScale.jpg | Approximation to the 12-tempered chromatic scale (12-tet) with integer values between 500 and 1000.  The correct sequence (rounded to the next integer) is:  1000, 944, 891, 841, 794, 749, 707, 667, 630, 595, 651, 530, 500  And the related sequence of the differences is:  56, 53, 50, 47, 45, 42, 40, 37, 35, 34, 31, 30  The purpose of these values is to find the position of the frets and their distances on string instruments, where 12-tempered tuning is accepted.  The two columns are inconsistent.  The values in the second column are monotone decreasing, as they should, but they do not agree with the numbers in the first column.  The correct differences of the first column are  56, 53, 49, 48, 44, 42, 40, 38, 31, **37**, 30, **32**  At two places they are not decreasing. 599 is obviously wrong. Probably it was 594 (= 630-36) in the manuscript and misread. The difference 28 fits with the next semitone 472 (= 500–28) not shown, and the difference 30 fits with 530. Maybe 530 / 32 was misread as 532 / 30.  These inconsistencies, in a preface that has many readers, are astonishing, since Mersenne gives very accurate six digit values in “Des Dissonances” [211]. |
| 208 | Syntonic and Pythagorean Diatonic Scale | 1636\_mersenne\_HarmUniv\_TraitezDesConsDiss\_Livre\_I\_DesConsonances\_004\_\_3octavesSyntPyth.jpg | Three octaves of the diatonic scales. The clefs added to the left indicate that the lowest note is C. The numbers 1 to 8 indicate frequency multiples. They highlight the first eight harmonics of the lowest note.  The first number column “Legitimes” is the frequency proportion of the syntonic diatonic scale with a major tone VT-RE (9:8). The next column “Pythago” contains the proportion of the Pythagorean diatonic scale for string lengths, so that the two tunings cannot be compared directly. There are many mistakes in this column, however the values from VT = 3888 to RE = 1728 are correct.  The next column “Notes” gives solmisation syllables. In the lowest octave Mersenne adds BI for the seventh degree of the scale to Guido of Arezzo’s six syllables ut, re, mi, fa, sol, la.  From RE of the second octave only four syllables RE, MI, FA, SOL are used. This octave reads:    Used in the given syntonic scale the syllables do not allow to distinguish major tones from minor tones. They would be consistent with a minor tone FA-RE [208]. In that case RE would indicate that the interval to the previous note of the scale (FA or SOL) is a minor tone (t) and the interval to next note (MI) is a major tone (T).  The next column “Consonan” lists the notes that are consonant with reference to the lowest note VT. The roman numeral IV stands for fourth and VIII for octave, etc. The Arabic numbers in the last column indicate the dissonances; the seconds, sevenths and their replica. |
| 209 | Termes radicaux des degrez de l'octave | 1636\_mersenne\_HarmUniv\_TraitezDesConsDiss\_Livre\_I\_DesConsonances\_050\_\_SyntonicDiatonicScaleSolmization.jpg | The syntonic diatonic scale with a minor tone C-D. The number 1 1/9 stands for 10/9. The solmisation syllables in column III are consistent with the syntonic intervals [208]. Column IV gives a different solmisation system, which would be easy to sing [515]. |
| 210 | 9 Syntonic commas | 1636\_mersenne\_HarmUniv\_TraitezDesConsDiss\_Livre\_II\_DesDissonances\_123\_\_9SyntComas.jpg | The first 9 powers of 80 and of 81. The table is used to express the size of the minor and the major tones as multiples of the syntonic comma (cs). Mersenne shows that t < 9 cs < T < 10 cs.  From this he concludes correctly that the octave contains more than 52 and less than 59 syntonic commas. He wrongly claims that the octave is between 58.5 and 59 syntonic commas.  From the approximate values t = 8.5 cs, T = 9.5 cs and S = 5 cs a much better estimation for the octave by 55.5 cs could be given. The estimations can be found even without using large numbers. The correct value for the octave is 55.798 cs. |
| 211 | 12-tet and Syntonic Chromatic Scale | 1636\_mersenne\_HarmUniv\_TraitezDesConsDiss\_Livre\_II\_DesDissonances\_132\_\_ChromaticScale\_12tetSyntonic.jpg | Comparison of the 12-tempered chromatic scale with a syntonic chromatic scale of 12 notes.  The 12-tet values are very accurate.  Rows 2, 3, 4 and 10 are entirely correct. Rows 5, 7, 8, 9 deviate by 1, row 6 deviates by 3 (correct: 133484), row 12 deviates by 4 (correct: 188775) and row 11 deviates by 8 units (correct: 178180).  Demiton majeur: 16:15  Demiton moyen: 135:128  Demiton mineur: 25:24  The value in row 7, column II is incorrect. It must be 142222 in order to fit to the semitones, giving a tritone C-F# of the ratio 45:32.  The resulting scale is C-Db-D-Eb-E-F-F#-G-Ab-A-Bb-B-C. It has the same structure as the scale given by Newton centred about G [77]. |
| 212 | Syntonic Chromatic Scale | 1636\_mersenne\_HarmUniv\_TraitezDesConsDiss\_Livre\_II\_DesDissonances\_136\_\_chromaticCircleKappa12.jpg | Syntonic chromatic scale of twelve notes arranged on a circle. The structure of the scale is different from [210]. It has four different semitones:  Chromatic semitones: 25:24 and 135:128  Diatonic semitones: 16:15 and 27:25.  The arrangement of the notes on the circle reflects a logarithmic understanding of pitch. However, the angles are neither equal to 30° (12-tet) nor do they express the four different sizes of the semitones.  The resulting scale is C-C#-D-Eb-E-F-F#-G-G#-A-Bb-B-C. Mersenne uses only sharps as alteration signs. The underlying C-major scale has a flat second degree and is of the form t-T-S-T-t-T-S, so that the solmisation agrees with Descartes‘ solmisation [316, 208, 209].  A solmisation syllable is given for both Bb (B-FA) and B (MI).  The diagram is a complete analysis of the scale, where for each pair of notes (except for the semitones) both possible ratios are indicated on the connecting line. For example, B-D (MI-RE) has the ratios 32:27 and 27:16 corresponding to a Pythagorean minor third and a Pythagorean major sixth.  This diagram was probably inspired by a similar diagram for the syntonic diatonic scale by Johannes Lippius (1612) [515]. Mersenne’s estimation of the octave as a multiple of the syntonic comma [210] has also its predecessor in Lippius: “Octava comprehendit Commata ultrà quinquaginta” [(LIPP\_1612) Lippius 1612, fol. C7r]. |
| 213 | Six Divisions de l'Octave | 1636\_mersenne\_HarmUniv\_TraitezDesConsDiss\_Livre\_II\_DesDissonances\_140\_\_SixDivisionsDeLOctave.jpg | The six consonant divisions of the octave correspond to the perfect major triad and its inversions (I, II, III) and to the perfect minor triad and its inversions (IV, V, VI) if the numbers are taken as frequency proportions. |
| 215 | Non-Standard Division of the Octave | 1702\_Salmon\_ThePracticalTheory\_BritishLibrary\_Add\_MS\_4919\_fol\_6r\_01\_NonStandardDivOfOctave.jpg | Systematic division of the octave into superparticular ratios. The musically non-standard intervals and ratios are highlighted with dots.  The last row shows “The four Chromatic Hemitones being the accidentall flats & sharps”, which are obtained by the arithmetical division of the major and the minor tones into consecutive superparticular ratios. Together with the ordinary diatonic semitone (16:15) Salmon’s chromatic scale has five different semitones [217, 218]. |
| 216 | Syntonic Diatonic Scale | 1702\_Salmon\_ThePracticalTheory\_BritishLibrary\_Add\_MS\_4919\_fol\_6r\_02\_SyntonicDiatonicScale.jpg | Derivation of the syntonic diatonic scale. The non-arithmetic divisions of the minor third and the fourth are indicated with dots. The second degree D is a major tone above the “Key” C.  The solmisation by four syllables fa, sol, la, mi agrees with the distribution of major and minor tones. |
| 217 | Syntonic Diatonic and Chromatic Scale | 1705\_Salmon\_TheTheoryOfMusickReduced\_2041\_RoyalSocietyOfLondon\_SyntonicDiatonicAndChromaticScale.jpg | Syntonic diatonic scale embedded into chromatic scales that use the semitones 16:15, 17:16, 18:17, 19:18 and 20:19. The division of the diatonic tones, by sharps only, have the smaller intervals below, so that 18:17 and 20:19 stand for chromatic semitones, and 17:16 and 19:18 for diatonic semitones. The chromatic semitone 18:17 is greater than the diatonic semitone 19:18. Since C-D# of the ratio 45:38 (= (9/8) × (20/19)) is different from a minor third 6:5 (= (9/8) × (16/15)), a second chromatic scale beginning at A, with the same set of pitch classes, is given. The two chromatic scales are chromatic scales for C-major and A-minor. The order of the major and minor tones in C-D-E for the A-minor diatonic scale should be reversed in order to obtain perfect minor triads on A, D and E.  Salmon’s chromatic scale has five different semitones, but the difference between the greatest and the smallest semitone is smaller than in the ordinary syntonic chromatic scales based on Pythagorean fifths and syntonic thirds only. This means that Salmon’s scale is closer to 12-tet than these other scales. |
| 218 | Lute Tuning | 1705\_Salmon\_TheTheoryOfMusickReduced\_Foldout\_RoyalSocietyOfLondon\_LuteTuning.jpg | Tuning of the lute with Salmon’s chromatic scale. [217] |
| 219 | Mercator: 53-tet | 1731\_Holder\_William\_\_A\_Treatise\_of\_the\_natural\_grounds\_079\_080\_mercator\_53tet.jpg |  |
| 220 | "aberrat Mersennus" | 1731\_Holder\_William\_\_A\_Treatise\_of\_the\_natural\_grounds\_079\_aberratMersennus.jpg | Mercator has noticed Mersenne’s inaccurate estimation of the octave with syntonic commas. Actually, Mersenne claimed that the octave is between 58.5 and 59 commas. The true value 55.80 is closer to 56 than to 55. Since Mercator has used logarithms, it is astonishing that he did not get a more accurate result. [210] |
| 221 | 53-tet Intervals | 1731\_Holder\_William\_\_A\_Treatise\_of\_the\_natural\_grounds\_080\_53tet\_intervals.jpg | Sizes of the syntonic intervals expressed in terms of Mercator’s “artificial comma” of 1/53 octave. A diesis is the difference between the octave and three major thirds and has the ratio 128:125. |
| 222 | Chromatic Scale | 1731\_Holder\_William\_\_A\_Treatise\_of\_the\_natural\_grounds\_ad\_118\_ChromaticScale.jpg | I: Syntonic diatonic scale with a minor tone C-D together with the surrounding chromatic scale, C-C#-D-Eb-E-F-F#-G-G#-A-Bb-B-C. The chromatic semitones are all the same ratio 25:24, so that there are three diatonic semitones 27:25 and four diatonic semitones 16:15 in the scale.  II-IV: It is shown how the diatonic scale is distorted if the same set of pitch classes is used in a different key. The D-minor scale (II) has a big semitone A-Bb (27:25) and two consecutive minor tones Bb-C-D. The D-major scale (III) has a big semitone F#-G. The Cb-major scale, however, has many inaccurate intervals. |
| 301 | Consonance Circle | 1628\_beeckman\_\_consonanceCircle\_\_MS\_Middelburg\_167r.jpg | It is clearly visible that the lines bounding the intervals do not all meet at a single point (as they were intended to). It is also quite clear that the diagram is seriously inaccurate in other respects. BeW |
| 302 | Diatonic Scale 1 | 1628\_beeckman\_\_diatonicScale\_1\_\_MS\_Middelburg\_171\_r.jpg | The straight line connecting 228 with 405, the tritone, does not go through the centre of the circle. The maior tone 405-360 is much smaller than the minor tone 360-324. The two semitones are not equal and the “Schisma” (syntonic comma) is about half the size of the semitones.  Nevertheless, the diagram is almost symmetric about the vertical diameter, which clearly indicates the logarithmic understanding of pitch. |
| 303 | Diatonic Scale 2 | 1628\_beeckman\_\_diatonicScale\_2\_\_MS\_Middelburg\_171r.jpg | The fifth D-A is almost on a straight line. The minor tone C-D is much larger than the major tones D-E and F-G. |
| 304 | Hexachords | 1628\_beeckman\_\_hexachords\_\_MS\_Middelburg\_172r.jpg | At first sight, this diagram seems to be the most accurate of the Middelburg manuscript. However, the fifths E-B is on the vertical diameter of the circle and the semitone B-C is much larger than the semitones E-F and A-Bb. |
| 305 | Consonance Circle | 1635\_descartes\_\_consonanceCircle\_\_MS\_Leiden\_ublwhs\_hug\_29\_a\_f042v.jpg |  |
| 306 | Diatonic Scale 1 | 1635\_descartes\_\_diatonicScale\_1\_\_MS\_Leiden\_ublwhs\_hug\_29\_a\_f047v.jpg | This diagram shows the pitch symmetry in the syntonic diatonic scale, which corresponds to the symmetry in the names of the intervals.  However, the minor tone on top (360:324) has the same size (60°) as the adjacent major tones, so that the tritone (405:288 = 45:32) – on the horizontal diameter – measures 180° instead of 177°. The semitones are about half the size of the major tones. Therefore and because of the “Schisma” (the syntonic comma) the two minor tones at the bottom are clearly smaller than the major tones.  Possibly, the diagram has been constructed based on the knowledge that the octave is close to six major tones of about 60° (correct: 61.2°), or five major tones and two semitones of about 30° (correct: 33.5°), so that the point of reference are the easily constructible angles.  The size of the syntonic comma could have been estimated through the comparable size of the Pythagorean comma: Boethius knew that the size of the major tone is between 7 and 9 Pythagorean commas. The Schisma in the diagram, however, is much bigger than a seventh of a major tone. Otherwise, it would have been difficult to add the designation of the interval to the drawing… |
| 307 | Diatonic Scale 2 | 1635\_descartes\_\_diatonicscale\_2\_\_MS\_Leiden\_ublwhs\_hug\_29\_a\_f047v.jpg | This symmetric diagram, too, makes no distinction between the angles of the minor and major tones (as [306]), and the semitones are all about 15°. Again, the tritones E-Bb and F-B are on diameters. A straight edge compass construction seems to have been begun at the bottom as the dark dots on the circle line seem to indicate.  The symmetry in the diagrams of the diatonic scales is also present in the Ms. Groningen [310, 311] and, with the exception of the 1653 English edition [322, 323, 326, 327, 328], in all the printed editions of Descartes’ “Compendium musicae”. |
| 308 | Hexachords | 1635\_descartes\_\_hexachords\_\_MS\_Leiden\_ublwhs\_hug\_29\_a\_f048v.jpg |  |
| 309 | Consonance Circle | 1640\_descartes\_\_consonanceCircle\_\_\_MS\_Groningen.jpg |  |
| 310 | Diatonic Scale 1 | 1640\_descartes\_\_diatonicScale\_1\_\_MS\_Groningen.jpg | The size of the “Schisma” (syntonic comma) is very accurate. No distinction is made between major and minor tones; they are all about 60°. The minor tone 432-405 is little smaller than the minor tone 288-540. |
| 311 | Diatonic Scale 2 | 1640\_descartes\_\_diatonicScale\_2\_\_MS\_Groningen.jpg | The diagram makes no distinction between major and minor tones. There are some visible traces from a compasses construction: The angles of the tones measure 60° and the angles of the semitones 30°. |
| 312 | Hexachords | 1640\_descartes\_\_hexachords\_\_MS\_Groningen.jpg | This diagram necessarily distinguishes the major from the minor tones. |
| 313 |  | 1650\_descartes\_\_\_consonanceCircle\_\_19\_BnF.jpg |  |
| 314 | Diatonic Scale 1 | 1650\_descartes\_\_diatonicScale\_1\_\_32\_BnF.jpg | The minor tone 360-324 is equal to the adjacent major tones (60°). The other minor tones are smaller. The angle of the “Schisma” is too large, so that it can hold the text… |
| 315 | Diatonic Scale 2 | 1650\_descartes\_\_diatonicScale\_2\_\_32\_BnF.jpg | The minor tone G-A and C-D are smaller than the major tones D-E and F-G. The semitones measure about 15°. |
| 316 | Hexachords | 1650\_descartes\_\_hexachords\_\_35\_BnF.jpg |  |
| 321 | Consonance Circle | 1653\_descartes\_\_consonanceCircle\_\_en\_017.jpg | Both the fifth and the fourth measure 180° and both thirds, the major and the minor, measure 90°. The circle defined by the numbers 2, 3 (, 4) represents half a monochord string of length 4. Likewise, 4, 5, 6 (,8) indicate half a monochord string of length 8. |
| 322 | Diatonic Scale 1 | 1653\_descartes\_\_diatonicScale\_1\_\_en\_032.jpg | The circumference corresponds to one halve of a string of length 576. The string is attached at E = 576, its middle is at E = 288, the position of the bridge corresponds to 0 (not shown).  The angles of the semitone E-F and the major tone D-E are almost equal, since they are nearly an octave apart.  The angles of the two semitones are equal, which is inconsistent with the idea of the circular string.  The names of the pitches are not given in the other editions and the manuscripts. |
| 323 | Diatonic Scale 2 | 1653\_descartes\_\_diatonicScale\_2\_compendium\_\_en\_032.jpg |  |
| 324 | Hexachords | 1653\_descartes\_\_hexachords\_compendium\_\_en\_035.jpg |  |
| 325 | Consonance Circle | 1653\_descartes\_brouncker\_\_consonanceCircle\_\_animadv\_070.jpg |  |
| 326 | Diatonic Scale 1 | 1653\_descartes\_brouncker\_\_diatonicScale\_1\_A\_\_animadv\_074.jpg |  |
| 327 | Diatonic Scale 1 | 1653\_descartes\_brouncker\_\_diatonicScale\_1\_B\_\_animadv\_075.jpg |  |
| 328 | Diatonic Scale 1 | 1653\_descartes\_brouncker\_\_diatonicScale\_1\_C\_\_animadv\_076.jpg |  |
| 329 | Consonance Circle | 1656\_descartes\_\_consonanceCircle\_\_brockt\_1978\_20.jpg |  |
| 330 | Diatonic Scale 1 | 1656\_descartes\_\_diatonicScale\_1\_\_brockt\_1978\_36.jpg |  |
| 331 | Diatonic Scale 2 | 1656\_descartes\_\_diatonicScale\_2\_\_brockt\_1978\_36.jpg |  |
| 332 | Hexachords | 1656\_descartes\_\_hexachords\_\_brockt\_1978\_40.jpg |  |
| 333 | Consonance Circle | 1668\_descartes\_\_consonanceCircle\_\_buzon\_2012\_078.jpg |  |
| 334 | Diatonic Scale 1 | 1668\_descartes\_\_diatonicScale\_1\_\_buzon\_2012\_100\_A.jpg | In contrast to the 1650-Utrecht edition [314], the angles of the minor tones are consistently smaller than the angles of the major tones. |
| 335 | Diatonic Scale 2 | 1668\_descartes\_\_diatonicScale\_2\_\_buzon\_2012\_100\_B.jpg | Differently from [334], the angles of the minor tones measure 60°, and they are larger than the major tones. The angle of the semitone E-F is far too small. |
| 336 | Hexachords | 1668\_descartes\_\_hexachords\_\_buzon\_2012\_104.jpg | This diagram is an accurate copy of the corresponding diagram of the Utrecht edition [316]. |
| 337 | Consonance Circle | 1683\_descartes\_\_consonanceCircle\_\_17\_pdf\_172.jpg |  |
| 338 | Diatonic Scale 1 | 1683\_descartes\_\_diatonicScale\_1\_\_28\_pdf\_174.jpg |  |
| 339 | Diatonic Scale 2 | 1683\_descartes\_\_diatonicScale\_2\_\_28\_pdf\_174.jpg |  |
| 340 | Hexachords | 1683\_descartes\_\_hexachords\_\_30\_pdf\_165.jpg | The note mi (B) of the inner ring has been forgotten. |
| 401 | Greek Tetrachords | 1495\_de\_Podio\_Guillermus\_Ars\_musicorum\_Liber\_II\_XII\_greekTetrachords.jpg | The three Greek tetrachords, enharmonic, chromatic and diatonic. In all three cases, the notes are labelled b-c-d-e, so that c and d have a variable position and stand for different pitches. The minor third in the chromatic genus is called “Trihemitonus” and not “Semiditonus”, which shows that the fourth consists of seven (not equally sized) semitones.  It is unclear whether the two diesis’ in the enharmonic genus are the same ratio. If they were, their ratio would be the square root of 256:243.  The diatonic genus is given with solmisation.  One could think that the spacing of the notes in the x-direction represents interval size. However the arc of the “Tonus” c-d is wider than the arc of d-e. Since all the other diagrams seem to indicate positions on the monochord [402, 403, 404], it is more likely that this diagram is of the same kind. |
| 402 | Diatonic Scale, Two Octaves | 1495\_de\_Podio\_Guillermus\_Ars\_musicorum\_Liber\_III\_XX\_diatonicScaleTwoOctaves.jpg | Diatonic scale over two octaves. The horizontal spacing of the notes is that of a monochord. The A is positioned at the beginning, in the middle and at three quarters of the entire string symbolized by an arc without a label. As an interval this arc represents infinity...  The second octave has both Bb and B. The minor third A-C is divided into a minor plus a major plus a minor semitone. The minor semitone (s) in the Pythagorean tuning is 256:243, and the major semitone (S) is 2187:2048. The scale contains two diatonic tetrachords A-Bb-C-D and B-C-D-E of the structure sTT. |
| 403 | Tetraktys | 1495\_de\_Podio\_Guillermus\_Ars\_musicorum\_Liber\_III\_XXI\_tetractys.jpg | The consonances of the Pythagorean tetraktys in ascending order giving the notes Gamma-C-D-G-d-g over a monochord string. The spacing is not very accurate. |
| 404 | Tritonus | 1495\_de\_Podio\_Guillermus\_Ars\_musicorum\_Liber\_III\_XXVIII\_tritonus.jpg | Tritonus. Three Pythagorean tones in a row forming a tritone. The corresponding notes would be Bb-C-D-E. An arc forming a fourth ends at E, so that the lowest tone is divided by B into a major and a minor semitone. The indicated labels “semitonus maj.” and “Coma” are misleading. It is not true that T = cP+ S + s, but T = S + s = cP + s + s. The size of the comma is depicted inaccurately large.  The sizes of the arcs of the tones are approximately in the proportion 40 : 37 : 32. The diagram seems to represent a part of a monochord. |
| 405 | Comparison of the Pythagorean comma with the Pythagorean semitone | 1551\_Stapul\_Jac\_Fabrum\_\_Musica\_LibrisQuatuorDemonstrata\_pdf\_050.jpg | The table of numbers compares the Pythagorean comma with the Pythagorean semitone.  A/b is the Pythagorean semitone and c/d the Pythagorean comma. The numbers from e to r occur in groups of three numbers of a similar size.  The correct values of the last group would be:  p: 263453980612401802360312389697536 = 2^76 × 3^20  q: 278128389443693511257285776231761 = 3^68  r: 324518553658426726783156020576256 = 2^108  q/p is the ratio of 4 Pythagorean commas, r/p the ratio of 4 Pythagorean semitones and r/q the ratio of 4 Pythagorean semitones minus 4 Pythagorean commas, a ratio that is close to the Pythagorean minor third 32:27.  Faber Stapulensis concludes the second book of his “Elementa musicalia” by stating that the tone (9 : 8) is greater than seven commas, because it consist of two semitones (that are each greater than three commas) and a comma [(FABE\_1551) Faber Stapulensis 1551, §36]: .  Remarkably, he does not give the upper bound , although he says – what the correct table would prove – that four commas are greater than a semitone. The correct value for the tone (9 : 8) is . |
| 406 | Geometric division of musical intervals | 1551\_Stapul\_Jac\_Fabrum\_\_Musica\_LibrisQuatuorDemonstrata\_pdf\_069.jpg | The construction is used to illustrate that any ratio can be divided geometrically, so that the given musical interval is exactly halved.  It makes use of Euclid’s altitude theorem. If bc – and not Ab – is viewed as the full string, this works well: bk, the height in the right triangle Afk, is the geometric mean of bf and bA = bc, so that bk is half an octave higher than bc. Likewise, bi is half a Pythagorean fifth higher than bc, and bh is half a syntonic minor third higher than bc. [503] |
| 407 | Circular Pitches, Three Octaves | 1581\_Galilei\_Vincenzo\_DialogoDellaMusicaAnticaEDellaModerna\_067\_circularPitchesThreeOctaves.jpg | Circular arrangement of the octave species according to Ptolemy. This is an early use of the circle for representing pitch. The full circle comprises three octaves. In this respect the diagram is similar to the transposition wheel by Robert Fludd [516]. |
| 494 | Comparison of five tones with two fourths | 1323\_De\_Muris\_Johannes\_\_Musica\_speculativa\_fol\_099r\_QuinqueToniBisDiatesseron.jpg | Two fourths (“Bis Diatesseron”) are smaller than five tones (“quinque toni Continui”). The two proportions begin with the same number 32768 and end with 58254 2/9 and 59049.  Equal horizontal spacing is used for equal musical intervals in both sub-diagrams. |
| 495 | Division of the tone into small intervals | 1323\_De\_Muris\_Johannes\_\_Musica\_speculativa\_fol\_107v\_TonusMicrodivision.jpg | Division of the tone (4374 : 3888 = 9 : 8) into six small intervals. These six intervals are all different. The proportion reads 4374 : 4270 : 4166 : 4131 : 4096 : 3992 : 3888. The symmetric sequence of the differences is 108, 108, 35, 35, 108, 108. There is a diatonic semitone (4096 : 3888 = 256 : 243) labelled “Semitonium minus” at the right side. The corresponding interval 4374 : 4166 labelled “Lymma” (which is usually used for the semitone 256 : 243) is a smaller musical interval. Therefore, the “Comma” (4166 : 4096) in the middle is greater than a true Pythagorean comma.  The terms of the proportion are placed equidistantly, so that the representation is neither that of a monochord nor that of (logarithmic) pitch.  A similar division of the tone is later given by Glareanus (1547) [30]. |
| 496 | Arithmetic, geometric and harmonic mean | 1492\_Gaffurio\_\_Theorica\_musicae\_fol\_037r\_arithmeticGeometricHarmonicMean.jpg | The three means used in music theory.  Arithmetic mean:  Geometric mean:  Harmonic mean:  The application of the formulas to the three examples gives      .  In the three terms of a harmonic proportion the ratio of the inner differences is equal to the ratio of the outer terms.  If three numbers are in harmonic proportion as 3 : 4 : 6 their reciprocals are in arithmetic proportion:  and vice versa. In other words harmonic division of a string length ratio corresponds to arithmetic division of a frequency ratio. The division of the octave into a fifth and a fourth is called harmonic division since the related string lengths are in harmonic proportion 6 : 4 : 3.  We are more used to arithmetic division of frequency ratios and avoid the term harmonic division. |
| 497 | Diatonic Scale | 1492\_Gaffurio\_\_Theorica\_musicae\_fol\_056v\_diatonicScale.jpg | The diatonic tone system is shown on a monochord. As derived from Boethius’ seven hexachords the lowest octave has only b (natural) the higher octaves have both B and Bb (forming a chromatic semitone or “apotome”). |
| 498 | Two semitones | 1514\_cochlaeus\_Tetrachordum\_musices\_tract\_I\_fol\_A4v\_TwoSemitones.jpg | Two Pythagorean diatonic semitones (semitonium minus = limma, 256:243) are less than a whole tone (9:8). The Pythagorean chromatic semitone (semitonium maius = apotome, 2187:2048) is by a Pythagorean comma (531,441:524,288) greater than the Pythagorean diatonic semitone.  The diagram illustrates a statement by Erasmus Heritius (1498): “duo semitonia tonum perficere non possunt” [202].  In the syntonic tone system, however, the diatonic semitone (16:15) is larger than the chromatic semitones (135:128 and 25:24). |
| 499 | Greek tetrachord genera | 1514\_cochlaeus\_Tetrachordum\_musices\_tract\_I\_fol\_A5v\_GreekTetrachords.jpg | The three Greek tetrachord genera diatonic, chromatic and enharmonic are displayed synoptically. No difference is made between the semitones.  One could get the impression that “tria hemitonia” are equal to three semitones, leading to a fourth of seven equal semitones and that the semitone is divided into equal quarter tones in the enharmonic genus.  In contrast to this representation, the Greek tetrachords are usually constructed in reverse order with the small intervals at the bottom. |
| 500 | Comparison of six tones with the octave | 1518\_Gaffurio\_\_De\_harmonia\_musicorum\_instrumentorum\_opus\_Lib\_II\_fol\_60v\_SexTonisDiapason.jpg | Comparison of the octave with six tones (9:8). The spacing in the vertical dimension is that of a monochord. The octave “Nete hyperbolaion” above the “Mese” in the middle is indicated at a quarter from the top. The horizontal lines get closer and closer with increasing pitch. The ratio between the tone and the Pythagorean comma is accurate and the comma on top of the octave is represented half as big as the comma at the lower end of the string. |
| 501 | Small syntonic intervals and their differences | 1529\_fogliano\_Musica\_theorica\_fol\_31v\_smallIntervalsAndTheirDifferences.jpg | The diagram analyses the smallest intervals within the syntonic diatonic and the chromatic scale [502]. The ratios of all the intervallic differences are determined and written into the related arcs. Unfortunately, these ratios are not given in their simplest terms. It is not obvious that the difference of the “semitonium maius” and the “semitonium minus” is a syntonic comma, 405:400 = 81:80, and that the difference of the “Tonus minor” and the “Semitonium minimum” is a “semitonium minus”, 225:240 = 16:15.  The three semitones are called “minus” (25:24), “maius” (16:15) and “maximus” (27:25) by later authors.  Since all possible connections of nodes are given, the graph is the full kappa-6. |
| 502 | Syntonic diatonic and chromatic scale | 1529\_fogliano\_Musica\_theorica\_fol\_34v\_syntonicChromaticAndDiatonicScale.jpg | The syntonic diatonic scale (including B and Bb) is indicated by the arcs above the chromatic scale. The arcs below indicate how the other notes of the chromatic scale can be reached from the notes of the diatonic scale. The full chromatic scale has 14 pitches, because D and Bb are ambiguous. The ambiguous pitches are separated by a syntonic comma.  The scale is identical to a scale given by Salinas 1571 [48]. |
| 503 | Bisection of the syntonic comma | 1529\_fogliano\_Musica\_theorica\_fol\_36r\_bisectionOfSyntonicComma.jpg | By using Euclid’s altitude theorem the geometric mean of the line segments of lengths 80 and 81 is constructed. In other words, the syntonic comma is halved geometrically. This can be used for tuning the mean-tone temperament. |
| 504 | Decomposition of the minor sixth | 1529\_fogliano\_Musica\_theorica\_fol\_36v\_syntonicConsonances.jpg | The syntonic minor sixth (120:75 = 8:5) is divided into smaller intervals.  We cannot decipher nor interpret the arc between the two semitones. |
| 505 | Tuning of the monochord with the syntonic chromatic scale | 1529\_fogliano\_Musica\_theorica\_fol\_38r\_tuningSyntonicChromaticScale.jpg | The numbering added to the ratios seems to indicate the order by which the positions on the monochord of the notes of the chromatic scale [502] can be found. This process begins with the note B (natural). However, this works only, if the whole string is divided into 30 equal parts in advance, which also determines the position of the midpoint c. |
| 506 | Pythagorean and syntonic comma | 1581\_galilei\_vincenzo\_dialogodellamusicaanticaedellamoderna\_045.jpg | See [508, 507]. |
| 507 | Comparison of the Pythagorean with the syntonic comma | 1581\_Galilei\_Vincenzo\_DialogoDellaMusicaAnticaEDellaModerna\_045\_\_WrongPythagoreanAndSyntonicComma.jpg | From his incorrect calculation of the Pythagorean comma (521,441 instead of 531,441) Galilei concludes that the Pythagorean comma is smaller than the syntonic comma. (The “correct” number at the bottom right would be 42,236,7**2**1.)  The incorrect value of the Pythagorean comma is consistently used by Galilei also on the following pages. |
| 508 | Calculation of the Pythagorean Comma | 1581\_Galilei\_Vincenzo\_DialogoDellaMusicaAnticaEDellaModerna\_045\_\_WrongPythagoreanComma.jpg | Galilei determines the ratio of the Pythagorean comma by repeatedly subtracting Pythagorean fifths from seven octaves. A mistake occurs in the very last multiplication, which should give 531,441. The incorrect result would imply that six major tones are smaller than an octave instead of greater than an octave by a Pythagorean comma. |
| 509 | Dimostratione de' tredici Tuoni, secondo la mente d'Aristosseno | 1581\_Galilei\_Vincenzo\_DialogoDellaMusicaAnticaEDellaModerna\_052\_\_13Tuoni\_Aristosseno.jpg | Aristoxenus’ “tonoi”. The entire diatonic system (two octaves A-Aa) is transposed progressively by semitones. The vertical spacing of the notes is logarithmic, so that equal vertical distances correspond to equal musical intervals.  If the notes on horizontal lines are indeed the same pitch, the system of these tonoi defines a division of the octave into twelve equal semitones.  Transpositions are translations in the pitch domain, so that the natural visualisation of this system of scales is logarithmic.  Aristoxenus’ system of scales as described by Galilei anticipates the system of twelve major scales in the 12-tempered equal tuning and is essentially different from the system of “church modes”, which uses the same names and where the modes have different characteristic intervallic structures with respect to their tonic. |
| 510 | Dimostratione degli otto Tuoni, secondo la mente di Boethio | 1581\_Galilei\_Vincenzo\_DialogoDellaMusicaAnticaEDellaModerna\_058\_\_8Tuoni\_Boethius.jpg | According to Galilei the system of Boethius is a subset of Aristoxenus’ tonoi [509]. The transpositions follow the steps of an underlying diatonic scale (TTsTTsT). This is also highlighted by the names of the notes in the summary below. The horizontal spacing of the notes defining the octave genera makes no difference between semitones and tones. Since the radiuses of the semicircles are the same, this is still a kind of logarithmic representation as in a musical staff. |
| 511 | The eight "church modes" | 1581\_Galilei\_Vincenzo\_DialogoDellaMusicaAnticaEDellaModerna\_071\_\_8TuoniEcclesiastici.jpg | The eight church modes are given by their tonic and ambitus. The representation does not indicate the individual differences regarding the *repercussio* (“dominant”) and alterations (B, Bb). |
| 512 | Major triad with octave replicas | 1581\_Galilei\_Vincenzo\_DialogoDellaMusicaAnticaEDellaModerna\_073\_\_majorTriad.jpg | A major triad with two octave replicas forming the proportion 60 : 48 : 40 : 30 : 24 : 20 :15 : 12 : 10 of string lengths on a monochord. These ratios are expressed twice, in the distances of the horizontal lines from the apex of the triangle as well as in the lengths of the parallel horizontal line segments. This follows from the triangle proportionality theorems.  The corresponding proportion of frequencies is 4 : 5 : 6 : 8 : 10 : 12 : 16 : 20 : 24. |
| 513 | Twelve modes according to Glareanus | 1581\_Galilei\_Vincenzo\_DialogoDellaMusicaAnticaEDellaModerna\_078\_\_12Tuoni\_Glareano.jpg | The twelve modes according to Glareanus are given by their tonic and ambitus. |
| 514 | Superparticular ratios and their squares | 1581\_Galilei\_Vincenzo\_DialogoDellaMusicaAnticaEDellaModerna\_134\_\_2d\_intervals.jpg | Doubling an interval corresponds to squaring its ratio. Therefore, if an interval is given as a ratio of lengths the doubled interval is visible as a ratio of areas. In the drawings the doubling of the ratios 2 : 1 (for three different shapes: circle, square and equilateral triangle), 3 : 2, 4 : 3, 5 : 4 and 6 : 5 are shown. |
| 515 | Syntonic diatonic scale on a circle | 1612\_Lippius\_Synopsis\_musicae\_novae\_fol\_F3r\_DiatonicCircle.jpg | Syntonic diatonic scale on a pitch circle. This seems to be the first representation of this kind. The numbers around the circle indicate string lengths. The scale is ascending in anti-clockwise direction.  Possibly Descartes (1618) [322] and probably Mersenne (1636) [212] were inspired by this diagram. Mersenne also connects all pairs of notes and he is familiar with the solmisation bo-ce-di-ga-lo-ma-ni-bo [209]. This solmisation method was devised by Hubert Waelrant (about 1574) [http://www.mu-sig.de/Theorie/Notation/Notation09.htm ; https://en.wikipedia.org/wiki/Hubert\_Waelrant]. |
| 516 | Transposition wheel for the lute | 1624\_fludd\_uch\_1\_0462\_Tract\_II\_Part\_II\_Lib\_VI\_232\_transpositionWheel.jpg | Mechanical tool used for transposition on the lute. The full circle comprises three octaves. The radial as well as the angular direction are organised logarithmically so that equal distance corresponds to equal musical intervals. Vincenzo Galilei also used three octaves for the full circles in order to describe the modal system of Ptolemy [407]. |
| 550 | Dodeci Semituoni di proportione Sesquidecimasettima non adempiano perfettamente la Diapason | 1588\_zarlino\_SopplimentiMusicali\_lib\_IV\_202.jpg | Twelve semitones of the ratio 18:17 are smaller than an octave. The geometric progression is shown on a monochord. |
| 551 | Dodeci Semituoni di proportione Sesquidecimasettima non fanno una Diapason perfetta | 1588\_zarlino\_SopplimentiMusicali\_lib\_IV\_205.jpg | As [550]. Twelve semitones of the ratio 18:17 are smaller than an octave. The values of 18^n and 17^n are shown. The difference is 12.5 cent, which is 53% of a Pythagorean comma. The tuning was proposed by Vincenzo Galilei [(GALV\_1581) Galilei 1581, 49]. |
| 552 | Consonantiae Diapason in Duodecim Partes Aequalis Divisio | 1588\_zarlino\_SopplimentiMusicali\_lib\_IV\_209.jpg | Using a mesolabio in order to construct the positions of the frets on a lute in 12-tet tuning [553, 555]. |
| 553 | Consonantiae Diapason in Duodecim Semitonia aequalis divisio | 1588\_zarlino\_SopplimentiMusicali\_lib\_IV\_211.jpg | Using a mesolabio in order to construct the positions of the frets on a lute in 12-tet tuning [552, 555]. |
| 554 | Triangle with Consonant Chords | 1624\_fludd\_uch\_1\_0417\_Tract\_II\_Part\_II\_Lib\_V\_217\_upperTriangle.jpg | This matrix can be used to read off the notes of the consonant triads (3-5-8 and 3-6-8) and their inversions in the C major scale. |
| 555 | Mesolabio | 1637\_Descartes\_\_Geometry\_318\_GeometricProgressionTool.jpg | Mesolabio. Mechanical tool for constructing geometric sequences. It can be used to divide a musical interval ratio into any number of equal parts. [<BARB\_1996> Barbieri 1996, 202-207] |
| 556 | Dissonant four-note chords | 1722\_rameau\_traite\_38\_accords.jpg | Dissonant chords of four notes and their proportions in the syntonic diatonic scale. |
| 557 | Overtones | 1722\_RameauTraiteStringsDivision.jpg | Overtones – the harmonics of a string. The problematic number 7 is absent. |
| 558 | Syntonic tone system with 53 pitch classes | 1917\_oettingen\_\_Grundlagen\_der\_Musikwissenschaft\_176\_chi53.jpg | Syntonic tone system of 53 pitch classes per octave. The configuration is symmetric around D. Oettingen divided the octave into 1000 equal parts. Therefore, cent values are obtained by multiplying the numbers by 1.2. The structure of the scale is identical to the one we used in a program to compare the various syntonic chromatic scales. It has three smallest steps 81:80, 2048:2025 and 3125:3072, which measure 21.5 cent, 19.55 cent and 29.6 cent respectively. |
| 559 | Syntonic chromatic scale with 12 pitches | 1619\_Kepler\_Lib\_III\_47\_chromaticScale\_stave.jpg | The ratios defining this syntonic chromatic scale of 12 pitches per octave are: Semitonium 16:15, Limma 135:128, Diesis 25:24. We found no earlier syntonic chromatic scale with 12 pitch classes. The syntonic chromatic scales suggested by Fogliano [502] and Salinas [46, 48] had at least 14 pitches. |
| 560 | Comparison of Vincenzo Galilei's tempered chromatic scale (s = 18/17) with Kepler's syntonic chromatic scale | 1619\_Kepler\_Lib\_III\_49\_v\_galilei\_chromaticScale.jpg | Comparison of Vincenzo Galilei’s tuning based on semitones of the ratio 18:17 with Kepler’s syntonic chromatic scale [559, 550, 551]. |
| 601 | Hexachords of plainchant | 1492\_Duran\_Domingo\_Marcos\_\_Lux\_Bella\_p\_01\_The\_Hexachords\_of\_Plainchant.jpg | The seven hexachords of Guido of Arezzo. Since the first hexachord begins at Gamma, there is no Bb in the lowest octave. Therefore, the system of pitches is not fully octave-periodic. |
| 602 | Hexachords of musica ficta | 1492\_Duran\_Domingo\_Marcos\_\_Lux\_Bella\_p\_08\_The\_Hexcachords\_of\_Musica\_Ficta.jpg | These fourteen hexachords complete the traditional seven hexachords [601] to a tone system with twelve pitches per octave in a range of three octaves beginning with F below Gamma. The five hexachords on top are wrapped around and completed with notes in the lowest octave. [<VOGE\_1982> Vogel 1982] |
| 603 | Musical wheel | 1492\_Duran\_Domingo\_Marcos\_\_Lux\_Bella\_p\_10\_The\_Musical\_Wheel\_\_3octaves.jpg | Extension of Guido of Arezzo’s hexachord system. Three octaves of a diatonic scale G-A-Bb-C-D-Eb-F are arranged anticlockwise around a circle. From each of these pitches a hexachord ut-re-mi-fa-sol-la (with a semitone between mi and fa) is built. This procedure creates a set of twelve pitch classes C, C#, D, Eb, E, F, F#, G, Ab, A, Bb, B defining the accidentals of the musica ficta [<VOGE\_1982> Vogel 1982]. The diagram combines the hexachords of [601] and [602]. [https://en.wikipedia.org/wiki/Musica\_ficta].  This seems to be the earliest diagram using the circle line to represent a pitch range of three octaves. A circular diagram with three octaves appears in Galilei (1588) [407] and a very similar diagram in Fludd (1618) [516]. |
| 604 | Nine hexachords | 1509\_Duran\_Domingo\_Marcos\_\_Lux\_Bella\_p\_02\_Nine\_Hexachords.jpg | This diagram extends the traditional system of seven hexachords of the first edition of Durán’s Lux Bella [601] to nine hexachords, where the two additional hexachords from the ‘sobre agudas’ (superacutae), c and f, are wrapped around and completed in the ‘graves’, resulting in an octave-periodic diatonic system with B and Bb. This extension of the hexachord system is also shown on Robert Fludd’s Templum musicae [60, 61]. |
| 605 | Conjunctae (hexachords) | 1509\_Duran\_Domingo\_Marcos\_\_Lux\_Bella\_p\_09\_The\_Conjunctae.jpg | The ‘conjunctae’ are those hexachords beginning at A, Bb, D, Eb leading to the accidentals C#, Eb, F# and Ab of ‘musica ficta’. This diagram together with [604] defines a tone system of twelve pitch classes per octave covering a range of diatonic scales from three flats to two sharps – in Pythagorean tuning. The entire system is shown in the circular diagram [603]. |
| 2000 | Trinity - Tetragrammaton | 11xx\_Alfonsi\_\_DialogiContraIudaeos\_MS\_E\_4\_fol\_153v.jpg | The tetragrammaton IEVE on a triangle. The diagram is a forerunner of the trinity shield, cf. [2027]. The central Ieve is later one taken as a fourth entity to form a full graph with four nodes and six edges. |
| 2001 | Philosophia et septem artes liberales | 1167\_1185\_Herrad\_of\_Landsberg\_\_Hortus\_Deliciarum\_\_Die\_Philosophie\_mit\_den\_sieben\_freien\_Künsten.JPG | The seven liberal arts arranged on a circle:  Grammatica – Rethorica – Dialettica (Trivium)  Musica – Arithmetica – Geometria – Astronomia (Quadrivium). |
| 2002 | Tetrachords defining the diatonic scale | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_005\_\_1\_2r\_\_tetrachords\_diatonicScale\_matrix.jpg | Tetrachords of the structure s-T-T (semitone-tone-tone) defining the diatonic scale, as [2025]. |
| 2003 | Hic sunt omnes voces … | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_006\_\_2\_2v\_\_Hic\_sunt\_omnes\_voces\_per\_quadratum.jpg | The Pythagorean tone system between Gamma and dd together with the related solmization syllables, the middle and upper octave have both, b-fla and b, cf. [2004] containing the same information. |
| 2004 | Solmisation of the diatonic scale | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_007\_\_3\_3r\_\_diatonicScale\_solmization.jpg | Solmization of the diatonic tone system between Gamma and dd (two octaves and a fifth). Guido of Arezzo’s syllables, ut-re-mi-fa-sol-la, define a relative system of solmization based on hexachords, so that the same note can get different syllables. For example c can be fa, ut or sol, and f can be fa or ut depending on the reference ut, which can be c, f or g. The right half of the table has b-flat in the middle and upper octave, but b in the lower octave. The tone system is not yet fully octave periodic, since the lowest hexachord begins at gamma, cf. [604, 60, 61] |
| 2005 | Division of the tripla | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_024\_\_20\_11v\_Joh\_Cotto\_Afflig\_mus\_superparticularRatios.jpg | There seem to be two subgroups of the seven columns 1, 3, 5, 7 and 2, 4, 6, both dividing the twelfth (tripla) into subintervals octave + fourth + tone versus fifth + octave. A different interpretation of the diagram places the numbers 1, 2, 3, 4, 8 and 9 between the columns, so that the ratios between neighboured numbers correspond to the names of the ratios covering the two numbers. This second reading is supported by a similar diagram, where these numbers are added and the three columns in the middle look like stems of palm trees [2008]. With the number 6 the system of Pythagorean numbers between 1 and 9 would be complete. As it is, only powers of 2 and 3 occur, i.e., numbers that are part of the lambdoma, cf. [2014]. |
| 2006 | Intervals in the hexachord with solmisation | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_025\_\_21\_12r\_Joh\_Cotto\_Afflig\_mus\_intervals\_solmization.jpg | The intervals between semitone and major sixth given with solmization syllables with the lower note ut if possible. Semitone, minor third and minor sixth have mi as the lower note. The mi in the minor sixth refers to ut = g and the fa to ut = c. |
| 2007 | Fourths and fifths defining the diatonic scale | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_026\_\_22\_12v\_Joh\_Cotto\_Afflig\_mus\_diatonicScale\_circleOfForths.jpg | This diagram visualising the construction of the diatonic scale by fifths and fourths also occurs in Guido of Arezzo’s “Micrologus”, see [201]. |
| 2008 | Division of the tripla | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_028\_\_22\_13r\_Joh\_Cotto\_Afflig\_mus\_superparticularRatios.jpg | This diagram is equivalent to [2005] and is pasted on a small sheet (fol. 13) between the pages 22 and 23 of the same text. |
| 2009 | Authentic and plagal modes | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_034\_\_28\_16v\_Joh\_Cotto\_Afflig\_mus\_authenticPlagalModes.jpg | These forerunners of set diagrams explain the system of the eight modal scales very clearly. |
| 2010 | Diatonic tetrachords | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_037\_\_31\_18r\_Joh\_Cotto\_Afflig\_mus\_greek\_tetrachodrs.jpg | The tetrachords of the diatonic scale separated by the columns of an arcade. |
| 2011 | Authentic and plagal modes | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_044\_\_38\_21v\_Joh\_Cotto\_Afflig\_mus\_modes\_circle.jpg | The ambits of the eight medieval modes arranged on a circle. The authentic modes are on the right, the plagal modes on the left, so that the modes with the same finalis are opposite:  I / II: D (dorian / hypodorian)  III / IIII: E (phrygian / hypophrygian)  V / VI: F (lydian / hypolydian)  VII / VIII: G (mixolydian / hypomixolydian)  Whereas the modes I, V and VII include the note below the finalis, the lowest note in mode IV is the finalis F, and not E. |
| 2012 | Counterpoint in two parts | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_069\_\_63\_34r\_Joh\_Cotto\_Afflig\_mus\_Counterpoint2Voices.jpg |  |
| 2013 | Archimedes: ordo planetarum | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_084\_\_78\_41v\_Archimedes\_ordinem\_planetarum\_cum\_distancia.jpg | Harmony of the planets. The numbers of the lambdoma [2014] are assigned to the “seven planets”:  Luna 1, Sol 2, Venus 3, Mercuri 4, Mars 9, Jupiter 8, Saturnus 27  In the given order the powers of 2 and 3 alternate. |
| 2014 | Archimedes: ordo planetarum – lambdoma | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_085\_\_79\_42r\_Archimedes\_ordinem\_planetarum\_cum\_distancia\_lambda.jpg | The upper part of the picture is determined by a central lambdoma, 1, 2, 4, 8, powers of two on the left and 1, 3, 9, 27, powers of 3 on the right hand side. The numbers within the arcs read  8, 9, 12 – 16, 18, 24 – 32, 36, 48 on the left and  9, 12, 18 – 27, 36, 54 – 81, 108, 162 on the right.  Together with the 6 on top the three octaves on the left are filled by a sequence of fourths (Diatessaron) and major seconds (Tonus):  Diat-T-Diat-Diat-T-Diat-Diat-T-Diat. With 6 = C this gives the notes:  **C**-F-G-**c**-f-g-**c**'-f'-g'-**c**''.  Correspondingly, the three twelfths on the right are filled by a sequence of fifths (Diapente) and fourths:  Diap-Diat-Diap-Diap-Diat-Diap-Diap-Diat-Diap, translating into  **C**-G-c-**g**-d'-g'-**d**''-a''-d'''-**a**'''.  In the upper half of the lower part the proportion 192:216:243:256 divides the fourth into two tones (9:8) and a semitone (265:243) defining the Pythagorean diatonic tetrachord. The proportion in the lower part 288:334:354:384, however, does not work out properly. |
| 2015 | Tetraktys – superparticular ratios | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_098\_\_92\_48v\_GuidoDiArezzo\_Micrologus\_superparticularRatios.jpg | The numbers 1, 2, 3, 4 are arranged equidistantly. The ratio of the outer terms 4:1, the double octave is divided into an octave (Diapason, 2:1) a fifth (Diapente, 3:2) and a fourth (Diatessaron, 4:3). Together with the major second (Tonus, 9:8) all the superparticular ratios of the Pythagorean tone system are shown. |
| 2016 | Micrologus: fol. 50r | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_101\_\_95\_50r\_GuidoDiArezzo\_Micrologus.jpg |  |
| 2017 | Arsis – thesis: musica motus vocum | 1200\_c\_d\_mu\_8\_cod\_375\_cim\_13\_page\_101\_a\_\_95\_50r\_GuidoDiArezzo\_Micrologus\_arsisThesis.jpg |  |
| 2018 | Diatonic scales and vowels | 1200\_c\_d\_mu\_8\_cod\_375\_cim\_13\_page\_101\_b\_\_95\_50r\_GuidoDiArezzo\_Micrologus\_diatonicScale\_vowels.jpg | The five vowels a, e, i, o, u are assigned to the notes of the diatonic scale, repeatedly from Gamma to D, from E to b and from c to g. This seems to describe a non-octave-periodic solmization system. |
| 2019 | Vowels and pitch – two parts counterpoint | 1200\_c\_d\_mu\_8\_cod\_375\_cim\_13\_page\_101\_c\_\_95\_50r\_GuidoDiArezzo\_Micrologus\_twoPartsCounterpoint.jpg | The vowels a, e, i, o, u are assigned to the pitches C, D, E, F, G, cf. [2018]. A short musical example is given, in which the vowels of the text are sung on the related pitch. In order to be consistent, “as” and “neque” in the lower part should be on C and D instead of D and E. What looks like a two part vocal counterpoint seem to be two lines of a continuous text with the meaning: Holy John your merits and not your wealth should be celebrated. |
| 2020 | Tetraktys – logarithmic pitch | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_107\_\_101\_53r\_GuidoDiArezzo\_Micrologus\_margin\_Tetraktys\_6\_8\_9\_12.jpg | This diagram in the margin from a different hand visualizes the tetraktys 6:8:9:12 explained in the text with the notes A, D, E, a without numbers. The symmetrical arrangement seems to express a logarithmic understanding of pitch, although the ratio of the whole tone to the fourth is not expressed in the distances of the nodes of the graph. The position of the numbers is not compatible with string lengths: the distance between 6 and 8 is even little greater than between 9 and 12. |
| 2021 | Constitutiones – combinatorics | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_114\_\_108\_56v\_GuidoDiArezzo\_Micrologus\_constitutiones\_columns.jpg | Combinatorics: a classification of the subsets of tetrachords and pentachords. |
| 2022 | Matrix of tetrachords and fifths | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_119\_\_113\_59r\_GuidoDiArezzo\_Micrologus\_tetrachords\_fifths\_matrix.jpg | The diatonic scale from Gamma to aa is given in a matrix arrangement. The columns are tetrachords on Gamma, D, a and e. The tones and semitones are indicated. The rows form sequences of fifths. There are diminished fifths between F and c in the second row and between b and f in the third row. The numbers from I to VIII are the numbers of the modal scales counted from d (dorian I/II) and from a. |
| 2023 | « tu patris sempiternus » - diatonic transpositions | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_126\_\_120\_62v\_GuidoDiArezzo\_Micrologus\_transpositions\_tu\_patris\_sempiternus.jpg | Tonal transposition of a melody “tu patris sempiternus es filius” (you are the eternal son of the father). The regular positions of the note names and the parallel lines indicating the flow of the melody blur the distinction between tones and semitones. |
| 2024 | Ratios – columns | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_143\_\_137\_71r\_GuidoDiArezzo\_Micrologus\_ratios\_columns.jpg | Between the five columns of an arcade, the tones of the diatonic scale are arranged in tetrachords. This drawing corresponds to the matrix arrangement in [2022]. It is unclear why the numbers III and VIII are added to a and c in the third interspace. |
| 2025 | Tetrachords defining the diatonic scale | 1200\_c\_D\_Mu\_8\_Cod\_375\_Cim\_13\_Page\_150\_\_144\_77v\_GuidoDiArezzo\_Micrologus\_tetrachords\_diatonicScale\_matrix.jpg | The matrix gives the Greek diatonic tetrachords beginning with the semitone on B, E, b, e, bb, which define the diatonic scale. |
| 2026 | Tetragrammaton – triangle | 1210\_c\_Alfonsi\_\_DialogiContraIudaeos\_MS\_D\_11\_fol\_38v.jpg | The tetragrammaton IEVE distributed over the vertices of triangle.  <https://en.wikipedia.org/wiki/Tetragrammaton> |
| 2027 | Scutum fidei – trinity tetrahedron (kappa-4) | 1231\_before\_Grosseteste\_Robert\_\_ScutumFidei\_TrinityTriangle\_gage\_1999\_125.jpg | The Scutum fidei ,trinity shield, showing a diagram with the nodes “Pater”, “Spiritus sanctus”, “Filius homo” and “Divina essential” in the centre. It is given as a full directed graph (kappa-4), where all pairs of nodes are connected in both directions. The labels on the outer edges are “non est” the labels on the central edges are “est”. Interpreted mathematically, the relation “est” is reflexive, but not transitive, which makes the diagram enigmatic.  The arrangement can be viewed as a tetrahedron seen from above, so that there are no overlapping edges. As far as we know this configuration was not used to represent the Pythagorean tetraktys. An equivalent representation of the second tetraktys (6:8:9:12) with overlapping edges is given by Walter Odington [12]. As a colour model the tetrahedron was used by Johann Heinrich Lambert (1772) [5049]. |
| 2028 | Vices and virtues – trinity shield | 1260\_c\_\_1024px\_Peraldus\_Vices\_and\_Virtues\_\_wikipedia.jpg | A knight with sword, spear and trinity shield fighting against the seven sins. |
| 2029 | Knight – trinity shield | 1260\_c\_428px\_Peraldus\_Knight\_wikipedia.jpg | Detail of [2028]. |
| 2030 | Trinity shield | 1260\_c\_Trinity\_knight\_shield\_wikipedia.jpg | Detail of [2028]. |
| 2031 | Eye music – circular time | 1390\_c\_cordier\_baude\_EyeMusic.jpg | The rondeau “Belle, Bonne, Sage” and the circular canon “Tout par compas suy composés” are examples of eye music.  These early examples from the ars subtilior period are contained in the Chantilly Codex (Chantilly, Musee Conde MS 564) and use red note notation.  The circle expresses the circular nature of the canon in an explicit way other than repeat marks. |
| 2032 | Scutum fidei Christianae | 140x\_Jerome\_of\_Prague\_\_Scutum\_fidei\_Christianae.jpg | Fivefold trinity shield, the classical “Deus: Pater, Filius, Spiritus Sanctus” on top, cf. [2027]. |
| 2033 | Destructio sive eradicatio totius Arboris Porphirii | 1503\_Augustinus\_Triumphus\_\_DestructioSiveEradicatioTotiusArborisPorphirii.jpg | Augustinus Triumphus contested the validity of the Porphyrian tree, cf. https://en.wikipedia.org/wiki/Porphyrian\_tree |
| 2034 | Typus Arithmeticae | 1503\_Reisch\_Georg\_\_Margarita\_philosophica\_\_TypusArithmeticae.jpg | The lower part of the dress of the women displays a lambdoma, the sequence 1, 3, 9, 27 over her right and 1, 2, 4, 8 over her left leg. Boethius is dealing with number of ratios. The sum of an octave (2:1) and a fifth (3:2) gives (2\*3):(1\*2) = 6 : 2 = 3 : 1, the ratio of the twelfth. Pythagoras is busy with a set of “calculi”, small pebbles used for counting – and not with Pythagorean number triplets, which are probably considered as belonging to geometry and not to arithmetic. |
| 2035 | Typus Musices | 1512\_Reisch\_Georg\_\_Margarita\_philosophica\_\_TypusMusices.jpg | This picture mainly presents music as “musica instrumentalis”. Although there is poet, there are no singers. The man at the bottom right, Pythagoras, measures the weight of hammers, above him a blacksmith, who was, according to the legend, responsible for Pythagoras’ theory of consonance. |
| 2036 | Music of the spheres | 1518\_Gaffurio\_\_De\_harmonia\_musicorum\_instrumentorum\_opus\_Lib\_III\_fol\_94v\_The\_music\_of\_the\_spheres\_wikipedia.jpg | The Three Graces Euphrosine, Aglaia and Thalia flanking Apollo, and eight muses on the left side of a snake with three heads, which also serve as the root of a tree. Thalia, the Muse of comedy and idyllic poetry, occurs a second time at the bottom. The circles of the muses are assigned the names of the notes of the Greek tone system on the left branches of the tree. The right branches contain the names of the eight medieval modes over a diatonic scale. The eight circles on the right show the seven planets and the sphere of the heavenly bodies. At the bottom of tree, the four elements Terra, Aqua, Aer, Ignis are visualised by concentric hemispheres.  The structure of the diagram resembles a Porphyrian tree [2033]. |
| 2037 | Arithmetica integra: fol. 249v | 1544\_stifel\_Arithmetica\_integra\_pdf\_264.jpg |  |
| 2038 | Integer powers of 2 | 1544\_stifel\_arithmetica\_integra\_pdf\_264\_detail\_geomProgr.jpg | Stifel introduced negative and fractional powers, which is generally considered a mathematical breakthrough that led to logarithms. Here the powers of 2 for indices between -3 and 6 are shown in tabular form. |
| 2039 | “Pascal’s triangle” | 1554\_Rudolff\_Stifel\_45\_PascalTriangle.jpg | This triangular arrangement of numbers containing the so-called binomial coefficients is known under the name Pascal’s triangle. However, it was known much earlier. [<EDWA\_2013> Edwards 2013], see also <https://en.wikipedia.org/wiki/Pascal%27s_triangle>  Possibly, it was used by Jost Bürgi (1620 in order to calculate his logarithmic table [2112], [<MUZZ\_2015> Muzzulini 2015, 209-210]. Binomial coefficients can also be used to approximately calculate integer powers of superparticular ratios [<MUZZ\_2015> Muzzulini 2015, 211-212]. |
| 2040 | Le istitutioni harmoniche: title page | 1562\_zarlino\_institutioni\_harmoniche\_0001\_title.jpg |  |
| 2041 | Musica tree | 1562\_zarlino\_institutioni\_harmoniche\_0022\_p\_011\_musica\_tree.jpg |  |
| 2042 | Terra-Acqua-Aria-Fuoco: 8-12-18-27 | 1562\_zarlino\_institutioni\_harmoniche\_0025\_p\_014.jpg | The four elements are given numbers of the geometric progression 8 : 12 : 18 : 27 of Pythagorean fifths.  Opposite numbers Terra – Aria (8 : 18) and Acqua – Fuoco (12 : 27) form dissonant major ninths, neighbours form consonant fifths (2:3) and Terra – Fuoco (8:27) corresponds to an octave plus a Pythagorean major sixth. The circular arrangement suggests a closed universe, although the geometric progression could be continued arbitrarily with suitable numbers. These numbers can be read from Boethius’ number triangles [1]. |
| 2043 | Diapasondiapente | 1562\_zarlino\_institutioni\_harmoniche\_0028\_p\_017\_diapasonDiapente.jpg | Symmetrical division of the Diapasondiapente, the ratio 3:1, into Pythagorean consonances. The numbers are arranged equidistantly on a straight line. The arcs form a complete kappa-4 graph.  The symmetrical structure is also revealed in a circular arrangement as in [2042] with opposite octaves 12:6 and 18:9.  Zarlino gives a similar diagram with the five numbers {1, 2, 3, 4, 5} on a straight line, the quinario [35], which serves to justify the syntonic consonances. The senario {1, 2, 3, 4, 5, 6}, however, is represented by Zarlino in circular form [32]. |
| 2044 | Diagonal of the square | 1562\_zarlino\_institutioni\_harmoniche\_0043\_p\_032\_squareDiagonal.jpg | The ratio of the diagonal to the side in a square ( sqrt(2) : 2) is irrational and corresponds two a tritonus in twelve tempered tuning. Zarlino in his early writings opposed twelve tempered tuning and propagated mean tone temperaments [38, 43]. |
| 2045 | Multiple ratios in circular arrangement | 1562\_zarlino\_institutioni\_harmoniche\_0045\_p\_034\_mulipla\_Circle.jpg | Multiple ratios from 2:1 to 10:1 arranged on circle with 1 as the midpoint.  The closing of the circle at 10 is arbitrary. |
| 2046 | Circular arrangement of superparticular ratios | 1562\_zarlino\_institutioni\_harmoniche\_0047\_p\_036\_superparticolari\_Circle.jpg | The superparticular ratios from 3:2 to 10:9 arranged on a circle. There is an arc between 2 and 10, which is not labelled. |
| 2047 | Three types of ratios | 1562\_zarlino\_institutioni\_harmoniche\_0048\_p\_037.jpg | Three types of ratios obtained from arithmetic sequences {3, 5, 7}, {4, 7, 10} and {5, 9, 13} and their Latin names. The numbers 5 and 7 occur twice in the diagram. |
| 2048 | Three types of ratios | 1562\_zarlino\_institutioni\_harmoniche\_0049\_p\_038.jpg | Three types of ratios and their Latin names:  2 : (2\*2+1=5), 2 : (3\*2+1=7), 2 : (4\*2+1=9)  3 : (2\*3+1=7), 3 : (3\*3+1=10), 3 : (4\*3+1=13)  4 : (2\*4+1=9), 4 : (3\*4+1=13), 4 : (4\*4+1=17) |
| 2049 | Three types of ratios | 1562\_zarlino\_institutioni\_harmoniche\_0050\_p\_039.jpg | Three types of ratios and their Latin names:  3 : (2\*3+2=8), 3 : (3\*3+2=11), 3 : (4\*3+2=14)  4 : (2\*4+3=11), 4 : (3\*4+3=15), 4 : (4\*4+3=19)  5 : (2\*5+4=14), 5 : (3\*5+4=19), 5 : (4\*5+4=24) |
| 2050 | Division of the Tripla (3:1) by superparticular ratios | 1562\_zarlino\_institutioni\_harmoniche\_0055\_p\_044.jpg | Composition of the Tripla 3:1 as a product of superparticular ratios:  (3:2)(4:3)(5:4)(6:5)=6:2=3:1. The corresponding notes form a major triad in just intonation, 2:3:4:5:6, with the fifth in the bass. The right part of the diagram contains the same information as the left part. |
| 2051 | Tripla: arithmetic division (1 : 2 : 3) | 1562\_zarlino\_institutioni\_harmoniche\_0064\_p\_053\_01.jpg | The Tripa (3:1) divided into a Dupla (2:1) and a Sesquialtera (3:2). The numbers 1, 2, 3 seem to represent frequencies. |
| 2052 | Tripla: harmonic division (2 : 3 : 6) | 1562\_zarlino\_institutioni\_harmoniche\_0064\_p\_053\_02.jpg | The Tripa (3:1) divided into a Dupla (2:1) and a Sesquialtera (3:2). The numbers 2, 3, 6 indicate string lengths at the monochord. If the string is stopped at C and plucked between C and B the octave of the open string AB is produced. If it is stopped at D and plucked between D and B the twelfth of AB is produced. |
| 2053 | Consonances within the fifth (Diapente) | 1562\_zarlino\_institutioni\_harmoniche\_0102\_p\_091.jpg | The syntonic (superparticular) consonances are shown on a monochord. The line segments form the following proportion ac:cd:de:ef:fb = 10:5:3:2:10, resulting in:  ab:cb = 30:20 = 3: 2 = Diapente (fifth)  cb:db = 20:15 = 4:3 = Diatessaron (fourth)  db:eb = 15:12 = 5:4 = Ditono (major third)  eb:fb = 12 : 10 = 6:5 = Semiditono (minor third)  with the bridge at b.  Playing the note c on the open string ab and then stopping it successively at c, d, e, f generates the major triad c-g-c'-e'-g' as part of the overtone sequence with respect to C of a string twice as long as ab. |
| 2054 | Mesolabio | 1562\_zarlino\_institutioni\_harmoniche\_0107\_p\_096.jpg | The mesolabio was invented by Eratosthenes. It is a mechanical tool by which one or more intermediate proportional numbers between two given numbers can be found (a = aq^0 : aq^1 : ^… : aq^n = b for n-1 intermediate proportional numbers between a and b. In other words, the elements of a finite geometrical sequence between a and b are sought).  Zarlino explains how a mesolabio can be used to divide the octave ab:cb = 2:1 into two equal intervals (12-tet tritones). There are two movable congruent rectangles oqpn and sutr with the diagonals nq and ru. The segment lm is equal to ac and ux is equal to cb. The rectangles are adjusted horizontally so that xm and ru intersect on pq. Then qy is the mean proportional between ux and lm.  (Reason: The lines mx and ou intersect at z. Because of the parallel line segments uy and qm the dilation mapping zxu onto zyq also maps zyq onto zml. Therefore zx:zy = zy:zm and ux:qy=qy:lm.)  Dividing a musical interval into two equal parts can also be done using Euclid’s altitude theorem, i.e., with straightedge and compasses only. Zarlino explains this construction for the octave on the preceding pages [(ZARL\_1562) Zarlino 1562, 93-94].  By using more moveable rectangles, any musical interval can be divided into more equal parts [(ZARL\_1562) Zarlino 1562, 95-96].  Dividing the octave into three equal parts (12-tet major thirds) is equivalent with the “doubling the cube” problem: 1 : q : q^2 : q^3 = 2, the motivation for Eratosthenes’ invention. https://en.wikipedia.org/wiki/Doubling\_the\_cube |
| 2055 | Monochordo della prima specie del Genere diatonico | 1562\_zarlino\_institutioni\_harmoniche\_0111\_p\_100.jpg | The construction of the Pythagorean diatonic tone system with congruent tetrachords is shown on a monochord. The diatonic tetrachords have two whole tones 9:8 over a Pythagorean semitone 256:243. The notes on the left generate two octaves of the Pythagorean seven notes scale corresponding to the C major scale, if the “Mese (4608)” is identified with the note a. The extra tetrachord “Tetrachordo Synemenon” on the right adds a Bb (“Trite synemenon” 4374) to the upper octave, so that there is a chromatic semitone between Bb and B, a major semitone of the ratio 2187:2048, in the complete Pythagorean diatonic tone system. |
| 2056 | Diatonic scale and harmony of the spheres | 1562\_zarlino\_institutioni\_harmoniche\_0113\_p\_102.jpg | The Pythagorean diatonic tone system is assigned to the nine muses and to the planets. Calliope is not assigned a planet.  On the left side, the octave, A-B-C-D-E-F-G-a, is assigned to the seven classical planets Luna, Mercurius, Venus, Sol, Mars, Jupiter, Saturnus, and to the “celum stellatum” (the sphere of the fixed stars) with the lowest note on the innermost circle. On the right side, however, the octave D-E-F-G-a-b-flat- c-d is assigned to the same planets with the highest note on the innermost circle. Even the positions of the semitones do not correspond. The arcs indicating fifths, fourths and octaves are arranged symmetrically so that they do not highlight the Greek tetrachords.  A comparable less ambiguous correspondence was given by Gaffurio (1518) [2036]. |
| 2057 | Diatonico molle | 1562\_zarlino\_institutioni\_harmoniche\_0117\_p\_106.jpg | Diatonico molle. A division of the defining tetrachords (Tetrachordo Hypaton and Tetrachordo Meson) into a huge tone 8:7, a minor tone 10:9 and a semitone 21:20. This generates a diatonic scale with superparticular steps only. Involving the prime numbers 2, 3, 5 and 7, a three-dimensional grid would be needed in order to describe the ratios of the pitch classes geometrically in a way comparable to the two-dimensional grid of the syntonic tone system. [2058, 2059] |
| 2058 | Diatonico toniaco | 1562\_zarlino\_institutioni\_harmoniche\_0118\_p\_107.jpg | Diatonico toniaco. A division of the defining tetrachords (Tetrachordo Hypaton and Tetrachordo Meson) into a major tone 9:8, a huge tone 8:7 and a semitone 28:27. This generates a diatonic scale with superparticular steps only. Involving the prime numbers 2, 3 and 7, a two-dimensional grid could be used in order to describe the ratios of the pitch classes geometrically in a way comparable to the two-dimensional grid of the syntonic tone system. [2057, 2059] |
| 2059 | Diatonico equale | 1562\_zarlino\_institutioni\_harmoniche\_0119\_p\_108.jpg | Diatonico equale. A division of the defining tetrachords (Tetrachordo Hypaton and Tetrachordo Meson) into a minor tone 10:9 and two smaller intervals minor tone 11:12 and a semitone 12:11. This generates a diatonic scale with superparticular steps only, which can be viewed as an approximation of the whole tone scale. Involving the prime numbers 2, 3, 5 and 11, a three-dimensional grid would be needed in order to describe the ratios of the pitch classes geometrically in a way comparable to the two-dimensional grid of the syntonic tone system. [2057, 2058] |
| 2060 | Harmonic and arithmetic division of the octave | 1562\_zarlino\_institutioni\_harmoniche\_0320\_p\_309\_harmonicAndArithmeticDivision.jpg | In arithmetic division the middle term is the average value of the outer terms, i.e., the differences between consecutive numbers are equal: 4-3 = 3-2.  In harmonic division, the ratio of the outer terms 6:3 = 2:1 is equal to the ratio of the differences (6-4):(4-3) = 2:1. Harmonic division is obtained from arithmetic division by a reflection in the pitch domain. Furthermore, harmonic ratios of string lengths correspond to arithmetic ratios of frequencies, and arithmetic ratios of string lengths correspond to harmonic ratios of frequencies. |
| 2061 | Superparticular intervals over the proportion 60 : 50 : 48 : 45 : 40 | 1571\_zarlino\_068\_IntervalliSuperparticolari\_60\_50\_48\_45\_40.jpg |  |
| 2062 | Harmonic and counter-harmonic mean | 1571\_zarlino\_107\_HarmonicaMediocrita\_ProContrHarmonica\_table.jpg | 4 is the harmonic mean between 6 and 3, since 6:3 = (6-4):(4-3).  5 is the counter-harmonic mean between 6 and 3, since 6:3 = (5-3):(6-4).  In other words, Zarlino’s senario {1, 2, 3, 4, 5, 6} is generated by harmonic and counter-harmonic division of the octave.  The arrangement of the letters and numbers in matrix form is not very convincing (maybe one of the vertical dividers right to a/2 should be removed) to indicate the harmonic division.  The 15 pairs of numbers from the senario generate only 11 different intervals, since 2:1 = 4:2 = 6:3, 3:2 = 6:4 and 3:1 = 6:2. Each of these 11 intervals is listed once with corresponding letters, their distribution in the table seems to be arbitrary. |
| 2063 | Progressione arithmetica | 1571\_zarlino\_111\_Ragionamento\_ProgrArithmetica\_TutteLeConsonanze\_table.jpg |  |
| 2064 | Cubo – corpo perfetto | 1571\_zarlino\_112\_2\_CVBO.jpg | “Et volevano, che questa loro Massima & Perfetta harmonia havesse grande forza nella Musica,& nelle speculationi delle cose naturali: & che non si potesse ritrovar cosa alcuna piu perfetta di questa medietà: & che contenendosi tra tre intervalli, havesse presa la natura della Sostanza di un Corpo perfetto: il quale consta simigliantemente di tre intervalli: cioè lunghezza, larghezza & profondità, overo altezza: indutti dall'harmonia, che si trova tra le qualità del corpo Cubo: il quale essendo composto di Dodici lati, Otto angoli, & Sei superficie [...]” [(ZARL\_1571) Zarlino 1571, 111]  The cube is compared with harmonic division, because it has 12 edges, 8 vertices and 6 faces and then with the tetractys 12:9:8:6 [40], which also includes the dissonant tone 9:8 and combines harmonic division 12:8:6 with arithmetic division 12:9:6 of the octave. |
| 2065 | Consonanze della Musica nel Quadrato (12x12) | 1571\_zarlino\_116\_ConsonanzeNelQuadrato\_12x12.jpg | A system of five parallel strings of equal length is tuned to the same note. The oblique divider generates the Pythagorean consonances in the proportion 2:3:4: 6 above the divider. In the lower part the tone (9:8) and the syntonic thirds (10:8 and 12:10) are generated as complements to 12. An extended version of this diagram, Helicona deae, is given by Salinas [2070]. |
| 2066 | Construction of the geometric mean | 1571\_zarlino\_161\_MezanaProportionale\_construction.jpg | The geometric mean ch = ck of the two segments, dc and cg, on the hypotenuse is constructed by use of Euclid’s altitude theorem. |
| 2067 | Mesolabio | 1571\_zarlino\_163\_Mesolabio.jpg | The division of an arbitrary interval into three equal parts is discussed: “Sia adunque la chorda ab, sopra la quale sia accomodato qual si voglia intervallo tra ab & cb: & sia dibisogno di partirlo in tre intervalli.” [(ZARL\_1571) Zarlino 1571, 163].  However, the mesolabio shown divides the interval of the ratio ac : cb, which is close to a twelfth (3:1), into two equal intervals: fg:kr = kr:os, where fg = ac and os = cb. The mesolabio in the diagram [2054] divides the octave in the same way. |
| 2068 | Division of the tone (16 : 17 : 18) | 1571\_zarlino\_166\_Prop\_16\_17\_18.jpg | The whole tone 9:8 cannot be divided into two equal semitones of a rational ratio. The arithmetic division 18:17:16 generates two slightly different semitones 18:17 and 17:16. The exact mean proportional value would be the square root of 16\*18, which is approximately equal to 16.97. |
| 2069 | Senario triangle | 1577\_Salinas\_062\_BisdiapasonCumDiapente\_Senario.jpg | The intervals of Zarlino’s senario {1, 2, 3, 4, 5, 6} are analysed with a triangular diagram. The names of the intervals can be found below the intersection of the related diagonal lines, e.g., 5 : 3, Hexachordum maius, the major sixth, consists of a Diatessaron or fourth of 4 : 3 and a Ditonum or major third of 5 . 4. As a proportion of string lengths 6:5:4:3:2:1 the senario generates a minor triad A-C-E-a-e-a-e. |
| 2070 | Consonances within the square – helicona | 1577\_Salinas\_076\_Helicona.jpg | “Pandite nunc helicona deae, cantusque movete.” (Now, spread the Helicona of the goddess and move the songs (?)).  A system of six strings of the same length 24 and tuned to the same pitch is arranged on a rectangle. The vertical line of length 12 is divided at 4, 6, 8 and 9. As a divider of the strings the line AL generates Pythagorean intervals between the parts of the strings within the triangle ALF:  Fourth: 12:9 = 4:3  Tone: 9:8  Fourth: 8:6 = 4:3  Fifth: 6:4 = 3:2  The other parts of the strings generate syntonic intervals within the trapezium ALRM:  Major third: 15:12 = 5:4  Semitone: 16:15  Major tone: 18:16 = 9:8  Minor tone: 20:18 = 10:9  Minor third: 24:20 = 6:5  The instrument presents the syntonic tone system as a natural extension of the Pythagorean system. |
| 2071 | De Musica: p. 78 | 1577\_Salinas\_078.jpg |  |
| 2072 | De Musica: p. 79 | 1577\_Salinas\_079.jpg |  |
| 2073 | Limma and Apotome – Tritonus | 1577\_Salinas\_079\_limmaAndApotome.jpg | The diatonic semitone, Limma (90.2 cent), and the chromatic semitone, Apotome (113.7 cent) are explained with a diatonic tetrachord, and a tritone. The numbers 2187-2048-1944-1728-1536 could be labelled Bb-B-C-D-E. |
| 2074 | Hierarchical classification of the syntonic intervals | 1577\_Salinas\_083\_intervalClassificationTree.jpg | These hierarchical classifications of the syntonic intervals do not include the sixths. |
| 2075 | Divisio Consonantiarum/Intervallorum minorum | 1577\_Salinas\_084\_DivisioIntervallorumInNumeris.jpg |  |
| 2076 | Division of the fifth (80 : 81 : 108 : 120) | 1577\_Salinas\_087\_DiapenteDivision.jpg |  |
| 2077 | Table of intervals | 1577\_Salinas\_095\_IntervallTreeNumbers.jpg |  |
| 2078 | Syntonic chromatic scale with 15 pitches | 1577\_Salinas\_117\_IntervalsOfChromaticScale.jpg |  |
| 2079 | Analysis of the chromatic scale with 15 pitches | 1577\_Salinas\_119\_chromaticScale\_connections.jpg |  |
| 2080 | Division of the minor tone (360 : 375 : 384 : 400) | 1577\_Salinas\_121\_DivisionTonusMinor.jpg |  |
| 2081 | Syntonic chromatic scale with 24 pitches per octave | 1577\_Salinas\_122\_TypiPraemissiExpositio.jpg |  |
| 2082 | Major and minor triads on D, E, F, G, A, C within the chromatic scale | 1577\_Salinas\_128\_majorAndMinorTriads.jpg |  |
| 2083 | Major and minor triads on C#, Eb, F#, G#, Bb, B within the chromatic scale | 1577\_Salinas\_129\_majorTriads\_alteredBase.jpg |  |
| 2084 | Augmented triad (64 : 80 : 100) | 1577\_Salinas\_130\_AugmentedTriad.jpg |  |
| 2085 | Syntonic diatonic scale | 1577\_Salinas\_137\_1\_IntervallorumDiatonicorumDispositio.jpg |  |
| 2086 | Sexdecim soni generi Chromatici | 1577\_salinas\_137\_2\_SexdecimSoniGenerisChromatici.jpg |  |
| 2087 | Intervalla Diapason in genere Chromatico | 1577\_salinas\_137\_3\_IntervallaDiapasonInGenereChromatico.jpg |  |
| 2088 | Six major tones versus six minor tones | 1577\_salinas\_142\_1\_SexToniMaioresVsMinores.jpg |  |
| 2089 | Comparison of the major semitone (16 : 15) with the diesis (128 : 125) | 1577\_salinas\_142\_2\_semitoniumMaius\_Commata.jpg |  |
| 2090 | Tritonus | 1577\_Salinas\_144\_Tritonum\_Triangle.jpg |  |
| 2091 | Tempered chromatic scale | 1577\_Salinas\_146\_temperedEnharmonicChormaticScale.jpg |  |
| 2092 | Tempered chromatic scale | 1577\_Salinas\_153\_temperedEnharmonicChormaticScale.jpg |  |
| 2093 | Bisection of the major third (5 : 4) | 1577\_Salinas\_158\_MajorThirdHalved\_construction.jpg |  |
| 2094 | Bisection of the syntonic comma (81 : 80) | 1577\_Salinas\_159\_SintonicCommaHalved\_construction.jpg |  |
| 2095 | Mean tone temperament of the diatonic scale | 1577\_Salinas\_160\_diatonicScale\_HalvedMajorThird.jpg |  |
| 2096 | Mean tone temperament of the chromatic scale | 1577\_Salinas\_162\_chromaticScale\_MeanTemperament.jpg |  |
| 2097 | Temperament of the Lyra and Viola | 1577\_Salinas\_170\_Temp\_ViolaLyraCymbala.jpg |  |
| 2098 | Right-angled triangle | 1577\_Salinas\_173\_rightTriangle.jpg |  |
| 2099 | Tetrachords in the syntonic diatonic scale | 1577\_Salinas\_181\_Tetrachords.jpg |  |
| 2100 | Tetrachords in the syntonic diatonic scale | 1577\_Salinas\_183\_Tetrachords.jpg |  |
| 2101 | Arithmetic and harmonic division of the consonances (from fifth to double octave) | 1577\_Salinas\_186\_ArithmeticAndHarmonicDivisionOfConsonances.jpg |  |
| 2102 | Musical polygons | 1592\_Ptolemy\_\_Musical\_Polygons\_\_fauvel\_flood\_wilson\_032\_2003\_032\_.jpg |  |
| 2103 | The five Aristotelian predicables in two different arrangements of kappa-5 | 1597\_Praedicabilia\_AristotelisOrganon.jpg |  |
| 2104 | Octave replica | 1610\_xx\_campion\_thomas\_\_of\_counterpoint\_octaves.jpg |  |
| 2105 | Movement of a plucked string – frequency | 1614\_15\_beeckman\_Fol24r\_55\_Fig31\_stringMovement.jpg |  |
| 2106 | Geometric derivation of fifths and fourths according to Stevin | 1618\_ beeckman\_geometricDivision\_JIB\_I\_233dpf.jpg |  |
| 2107 | Temple of the rosy cross | 1618\_rosy\_cross\_temple.jpg | The fantasy architecture of the “Collegium fraternitatis” seems to be a relative of Robert Fludd’s “Templum musicae” [60]. Actually, Fludd was strongly interested in the Rosicrucians and defended them, but was not a member of the secret fraternity, cf. https://en.wikipedia.org/wiki/Robert\_Fludd |
| 2108 | Derivation of the syntonic consonances | 1619\_Kepler\_Lib\_III\_27\_derivationConsonances.jpg | Hierarchy of the syntonic consonances.  The next generation, e.g. from 1/3, is got as follows: Add numerator and denominator to obtain the new denominator 1+3 = 4, the old numerator and denominator become the new numerators:  1/3 🡪 1/(1+3) = 1/4; 3/(1+3) = 3/4.  The quotient of the two ratios under the same bracket is always equal to the ratio left to that bracket. For example, (2/5):(3/5) = 2/3.  The numbers on the right would be the denominators of the next generation, if the procedure were continued. |
| 2109 | Universal Tabel: classification of organ sounds | 1619\_praetorius\_vol\_II\_ad127\_UniversalTabel.jpg | Classification of the sounds of contemporary organs. |
| 2110 | Overtones (1 : 2 : 3 : 4 : 5 : 6) | 1619\_praetorius\_vol\_III\_010\_overtones.jpg | If the numbers to the right are read as frequencies, the tones form a sequence of overtones: G, g, d, g', b', d''. Praetorius is familiar with the phenomenon of overtones. [<MUZZ\_2006> Muzzulini 2006, 115-119] |
| 2111 | Undertones (6 : 5 : 4 : 3 : 2 : 1) | 1619\_praetorius\_vol\_III\_010\_undertones.jpg | If the numbers to the right are read as string lengths, the tones form a sequence of “undertones”, the tones having the highest tone as a common overtone. The notes are not very clearly assigned to the staff (because of the text) and correspond to d'', d', g, d, B, G. To be consistent B should be replaced by Bb and the labels “Tertia minor” and “Tertia major” interchanged. |
| 2112 | Arithmetische und Geometrische Progress Tabulen | 1620\_buergi\_jost\_\_logCircle\_colored.jpg | Title page of Jost Bürgi’s logarithmic tables.  The red numbers form an arithmetic progression, and the black number form a geometric progression of factor 1.0001:  1.0001500 = 1.05126847  1.00011000 = 1.10516539  …  1.000123027.0022 = 9.999999997 = 10  The diagram (or the complete tables) can be used to calculate the angles in Descartes’ diagrams. Red numbers correspond to pitch and black numbers to frequency or string length. |
| 2113 | Theatrum instrumentorum XXI: Trumscheidt – trumpet marine | 1620\_Praetorius\_II\_TheatrumInstrumentorum\_XXI\_trummsch.jpg |  |
| 2114 | Theatrum instrumentorum XXIII: drums | 1620\_Praetorius\_II\_TheatrumInstrumentorum\_XXIII.jpg |  |
| 2115 | Theatrum instrumentorum: timbre – snare | 1620\_Praetorius\_II\_TheatrumInstrumentorum\_XXIII\_detail.jpg |  |
| 2116 | Tempered fourth | 1624\_beeckman\_tempered\_fourthsStevin\_JIB\_II\_194r\_pdf\_325.jpg |  |
| 2117 | Plucked string – octave. Letter to Mersenne | 1629\_Beeckman\_a\_Mersenne\_stringOctave.jpg |  |
| 2118 | Circular slide rule | 1632\_Oughtred\_CircularSlideRule.jpg |  |
| 2119 | Logarithmic spiral | 1636\_mersenne\_vol\_II\_119\_bpt6k5471093v\_spiral\_detail.jpg |  |
| 2120 | Coincidence theory of consonance | 1638\_galileo\_dialoghi\_fig13\_80.jpg |  |
| 2121 | Descartes’s diatonic scale and Newton’s colour circle | 1650\_1704\_descartesNewton\_kreise.jpg |  |
| 2122 | Correspondence between Pythagorean musical intervals and colours | 1650\_MarinCureau\_1650\_detail\_1.jpg |  |
| 2123 | Correspondence between Pythagorean musical intervals and colours | 1650\_marincureau\_1650\_detail\_2.jpg |  |
| 2124 | Correspondence between Pythagorean musical intervals and colours | 1650\_marincureau\_1650\_detail\_3\_AllRatios.jpg |  |
| 2125 | Triad: Intellectus – Sapientia – Virtus | 1652\_Kircher\_Oedipus\_Aegyptiacus\_Vol\_I\_Trinity\_IntellectusSapientiaVirtus\_Horus.jpg | A modified trinity shield with no label in the centre of the triangle and an appendix of concentric circles Horus.  We are unable to decipher the diagram completely.  Horus the Egyptian god of the sky is a son of Isis and Osiris. Amun is a major Ancient Egyptian deity and the patron god of Thebes. |
| 2126 | Pan and the music of the spheres | 1653\_Kircher\_Oedipus\_Aegyptiacus\_Vol\_II\_204\_Pan.jpg | Pan, the god of woods, fields and flocks. The pan flute in his hands moves the system of planets causing the harmony of the spheres: The legend to K reads “Harmonia 7. Planetarum”. |
| 2127 | Speculum Cabalae mysticae | 1653\_Kircher\_Oedipus\_Aegyptiacus\_Vol\_II\_Fol\_287\_Speculum\_Cabalae\_mysticae.jpg | This mystic diagram combines trees and concentric circles. In the outer ring 72 names of god, all consisiting of four letters are given. The tree at the bottom left refers to the magic number Seven: “Arbor Mystica 7 planetas, membra corporis et praesides Angelos continens”. The pomegranate tree to the right refers to the number Twelve: “Arbor malorum punic. 12 signas zodiaci, 12 tribus Israel et 12 nominis Dei evolutions continens”. |
| 2128 | Nomina Dei | 1653\_Kircher\_Oedipus\_Aegyptiacus\_Vol\_II\_NominaDei.jpg | Circular diagram containing names and symbols, in its appearance similar to Ramon Llull’s combinatorial circles and to Fludd’s transposition wheel for the lute [516]. |
| 2129 | Monochords | 1657\_schott\_acustica\_pars\_ii\_p\_279\_iconismus\_x\_fig\_1\_4\_monochord.jpg |  |
| 2130 | Senario | 1657\_schott\_acustica\_pars\_ii\_p\_279\_iconismus\_x\_fig\_5\_Senario.jpg | This diagram is a copy of Zarlino’s senario diagram [32], showing the intervals generated by number pairs from the set {1, 2, 3, 4, 5, 6}. The mistake in Zarlino’s diagram for the fifths 6:4 is corrected, however the interval of 6:1 is wrongly labelled “Disdiapason” (two octaves) instead of “Disdiapason cum diapente”. Compared with the original, Schott’s diagram is rotated clockwise by 30°, so that the octaves 2:1 and 6:3 are aligned horizontally, and the octave 4:2 vertically. |
| 2131 | Helicone Ptolemaei | 1657\_schott\_acustica\_pars\_ii\_p\_289\_iconismus\_xi\_fig\_1\_2\_HeliconePtolemaei.jpg | The “Helicone” on the left can be viewed as a system of equally tuned strings of equal lengths 16. The divider AE generates superparticular ratios between the corresponding parts of neighboured strings. The highest pitch of FO is four octaves above the pitch of the open string AC.  Fig. II justifies the syntonic tone system as a natural extension of the Pythagorean system, see [2070]. |
| 2132 | Instrumento Chordotomo: proportional compasses | 1657\_schott\_acustica\_pars\_ii\_p\_289\_iconismus\_xi\_fig\_3\_InstrumentoChordotomo.jpg |  |
| 2133 | Graphical score | 1657\_schott\_Acustica\_Pars\_II\_p\_313\_Iconismus\_XVI\_GraphicalScore.jpg |  |
| 2134 | Magic organ | 1657\_schott\_Acustica\_Pars\_II\_p\_323\_Iconismus\_XVII\_MagicOrgan.jpg |  |
| 2135 | Magic organ | 1657\_schott\_Acustica\_Pars\_II\_p\_329\_Iconismus\_XVIII\_MagicOrgan.jpg |  |
| 2136 | Vasorum Echeorum dispositio | 1657\_schott\_Acustica\_Pars\_II\_p\_365\_Iconismus\_XXIII\_Vasorum\_Echeorum\_dispositio.jpg |  |
| 2137 | Musically gifted donkeys | 1657\_schott\_acustica\_pars\_ii\_p\_373\_iconismus\_xxiv\_fig\_1\_donkeys.jpg |  |
| 2138 | Cats’ piano | 1657\_schott\_acustica\_pars\_ii\_p\_373\_iconismus\_xxiv\_fig\_2\_catsPiano.jpg |  |
| 2139 | Modes and triads | 1657\_schott\_acustica\_pars\_ii\_p\_393\_iconismus\_xxvi\_fig\_1\_2\_modesAndTriads.jpg |  |
| 2140 | Progression of modes and triads | 1657\_schott\_acustica\_pars\_ii\_p\_393\_iconismus\_xxvi\_fig\_3\_5\_progressionsModesAndTriads.jpg |  |
| 2141 | Abacus Melotheticus contractus | 1657\_schott\_acustica\_pars\_ii\_p\_397\_iconismus\_xxvii\_fig\_1\_Abacus\_Melotheticus\_contractus.jpg |  |
| 2142 | Columna heptaedra Melothetica | 1657\_schott\_acustica\_pars\_ii\_p\_397\_iconismus\_xxvii\_fig\_2\_Columna\_heptaedra\_Melothetica.jpg |  |
| 2143 | Mensa Tonographica | 1657\_schott\_Acustica\_Pars\_II\_p\_413\_Iconismus\_XXIX\_Mensa\_Tonographica.jpg |  |
| 2144 | Typus Sceptrologia Musurgica | 1657\_schott\_Acustica\_Pars\_II\_p\_427\_Iconismus\_XXXI\_Typus\_Sceptrologia\_Musurgica.jpg |  |
| 2145 | Enneagram – magic square | 1665\_Kircher\_arithmologia\_Eneagramm.jpg |  |
| 2146 | Astrological chart – trinitas | 1667\_Simpson\_\_Christopher\_25\_\_AstrologicalChart\_Trinitas\_\_gouk\_152\_4\_47.jpg |  |
| 2147 | Megaphone – wave propagation | 1672\_morland\_acousticCircularWavesPropagation.jpg |  |
| 2148 | Visualisation of the coincidence theory of consonance | 1677\_North\_Francis\_coincidenceTheory.jpg |  |
| 2149 | Circle of fifths | 1679\_Diletsky\_Nikolay\_\_Idea\_grammatiki\_musikiyskoy\_wikipedia\_circle\_of\_fifths.jpg |  |
| 2150 | Solmisation – pitch circles | 1683\_anonymous\_nouvelle\_methode\_tres\_seure\_\_Explication\_des\_4\_cercles\_1\_4.jpg |  |
| 2151 | Chromatic scale with superparticular semitone ratios | 1698\_wallis\_253\_chromatic\_scale\_detail.jpg |  |
| 2152 | Chromatic scale | 1698\_wallis\_chromaticScale.jpg |  |
| 2153 | Pitch grid | 1710\_Henfling\_Conrad\_\_Epistola\_de\_novo\_suo\_Systemate\_Musico\_Fig\_66.jpg |  |
| 2154 | Musicalischer Circul – circle of fifths | 1711\_Heinichen\_Johann\_David\_\_Neu\_erfundene\_und\_gruendliche\_Anweisung\_261\_musicalischer\_circul.jpg |  |
| 2155 | Musicalischer Circul – circle of fifths | 1711\_Heinichen\_Johann\_David\_\_Neu\_erfundene\_und\_gruendliche\_Anweisung\_musicalischer\_circul.jpg |  |
| 2156 | Colour mixing in Newton’s colour diagram | 1719\_taylor\_\_New\_principles\_Fig\_25\_colourWheel\_adNewton.jpg |  |
| 2157 | Coincidence theory of consonance | 1725\_euler\_adversariamathematica\_\_scan\_from\_2010\_mathesisgraphe\_077.jpg |  |
| 2158 | Coincidence theory: intervals | 1725\_euler\_adversariamathematica\_\_scan\_from\_2010\_mathesisgraphe\_077\_detail\_01.jpg |  |
| 2159 | Coincidence theory: triads | 1725\_euler\_adversariamathematica\_\_scan\_from\_2010\_mathesisgraphe\_077\_detail\_02.jpg |  |
| 2160 | Coincidence theory: major triad | 1725\_euler\_adversariamathematica\_\_scan\_from\_2010\_mathesisgraphe\_077\_detail\_02b.jpg |  |
| 2161 | Méthode de plain chant | 1728\_La\_Salle\_\_MethodeDePlainChant\_p6\_p21.jpg |  |
| 2162 | Powers and roots of ratios | 1731\_Holder\_William\_\_A\_Treatise\_of\_the\_natural\_grounds\_078.jpg |  |
| 2163 | Cacophony of a minor triad with overtones | 1737\_RameauCacophonie.jpg |  |
| 2164 | Table des couleurs & des tons de la Musique | 1738\_voltaire\_CouleurTon\_MonochordSpectrum\_adNewton.jpg |  |
| 2165 | Space notation | 1741\_lebeuf\_jean\_\_traite\_historique\_et\_pratique\_154.jpg |  |
| 2166 | Space notation | 1741\_lebeuf\_jean\_\_traite\_historique\_et\_pratique\_156.jpg |  |
| 2167 | Vibrating string | 1748\_euler\_StringPeriodicallyContinued.jpg |  |
| 2168 | Table des mots | 1749\_rousseuTableDesMots.jpg |  |
| 2169 | Table des mots: tymbre | 1749\_rousseuTableDesMots\_tymbre.jpg |  |
| 2170 | Vibrating string and a cycloid pendulum | 1749\_smith\_stringCycloid.jpg |  |
| 2171 | Circular diagrams – combination tones | 1754\_Tartini\_\_Trattato\_di\_musica\_Figures.jpg |  |
| 2172 | Waveform of sound | 1765\_euler\_IQK\_1.jpg |  |
| 2173 | Waveform of sound | 1765\_euler\_IQK\_2.jpg |  |
| 2174 | Noeuds et ventre : vibrating string | 1768\_rousseauDict\_String\_noeudsVentres\_1768.jpg |  |
| 2175 | Musique Oculaire | 1770\_Guyot\_MusiqueOculaire\_gage\_186.jpg |  |
| 2176 | Overtones on a vibrating string | 1771\_74\_sulzer\_superposString.jpg |  |
| 2177 | Speaking machine | 1791\_kempelen\_maschine\_p\_428\_Tab\_XXII\_0501.jpg |  |
| 2178 | Speaking machine | 1791\_kempelen\_maschine\_p\_432\_Tab\_XXIII\_0507.jpg |  |
| 2179 | Speaking machine | 1791\_kempelen\_maschine\_p\_434\_Tab\_XXIV\_0511.jpg |  |
| 2180 | Speaking machine | 1791\_kempelen\_maschine\_p\_439\_Tab\_XXV\_0518.jpg |  |
| 2181 | Speaking machine | 1791\_kempelen\_maschine\_p\_444\_Tab\_XXVI\_0525.jpg |  |
| 2182 | Kempelen’s speaking machine | 1791\_kempelen\_sprechapparat\_NZZ\_131014.jpg |  |
| 2183 | Young 1800: Plate V | 1800\_Young\_plate\_V\_29to40.jpg |  |
| 2184 | Superposition of triangular vibrations | 1800\_Young\_plate\_V\_33to34\_triangularVibration\_trapezoid.jpg |  |
| 2185 | Triangular vibration | 1800\_Young\_plate\_V\_33to38.jpg |  |
| 2186 | Beats between triangular vibrations | 1800\_Young\_plate\_V\_36to38\_triangularBeats.jpg |  |
| 2187 | Triangular and sinusoidal vibration | 1800\_Young\_plate\_V\_39to40\_triangularVibration.jpg |  |
| 2188 | Young 1800: Plate VI | 1800\_Young\_plate\_VI\_41to51.jpg |  |
| 2189 | Beats between sinusoids | 1800\_Young\_plate\_VI\_41to42.jpg |  |
| 2190 | Beats | 1800\_Young\_plate\_VI\_43\_beats.jpg |  |
| 2191 | Lissajous Figures | 1800\_Young\_plate\_VI\_44to46\_LissajousFigures.jpg |  |
| 2192 | Plucked vibrating string | 1800\_Young\_plate\_VI\_47to48\_VibratingPluckedString.jpg |  |
| 2193 | Plucked and bowed vibrating string | 1800\_Young\_plate\_VI\_49to51.jpg |  |
| 2194 | Comparison of different systems of temperament | 1800\_young\_plate\_VII\_53\_temperamentCircles\_2013\_Pesic\_28.jpg |  |
| 2195 | Vibrating string | 1802\_koch1.jpg |  |
| 2196 | Overtones on a vibrating string | 1802\_koch2.jpg |  |
| 2197 | Combination tones | 1832\_haellstroem\_444.jpg |  |
| 2198 | Combination tones | 1832\_haellstroem\_445.jpg |  |
| 2199 | Reed pipe | 1832\_willis\_Tafel\_IV\_Fig\_1\_Zunge.jpg |  |
| 2200 | Sequence of vowels | 1832\_willis\_Tafel\_IV\_Fig\_2\_Vokalreihe.jpg |  |
| 2201 | Reed pipe with changeable length | 1832\_willis\_Tafel\_IV\_Fig\_8\_9\_Zungenpfeifen.jpg |  |
| 2202 | Vibration patterns and simple tones | 1844\_a\_seebeckSchwingungsformen\_354.jpg |  |
| 2203 | Overtones | 1855\_brandtNotenSpektren\_333.jpg |  |
| 2204 | Siren disc | 1863\_helmholtz\_023\_Lochsirene\_Quinte.jpg |  |
| 2205 | Harmonic motion | 1863\_helmholtz\_035\_SinusKreis.jpg |  |
| 2206 | String sounds | 1863\_helmholtz\_135\_Saitentöne.jpg |  |
| 2207 | Synthesizer | 1863\_helmholtz\_184\_Synthesizer.jpg |  |
| 2208 | Synthesizer: generator | 1863\_helmholtz\_186\_Synthesizer.jpg |  |
| 2209 | Vibration pattern synthesizer | 1881\_a\_Koenig\_347\_Wellensirene.jpg |  |
| 2210 | Shear transformation of a vibration pattern | 1881\_a\_koenig\_348\_SirenentonGeschert.jpg |  |
| 2211 | Quasi-periodic oscillation | 1881\_b\_Koenig\_372\_QuasiperiodSignal.jpg |  |
| 2212 | Vibration patterns | 1881\_b\_koenig\_380\_Schwingungsformen.jpg |  |
| 2213 | Vibration patterns | 1881\_b\_koenig\_381\_Schwingungsformen.jpg |  |
| 2214 | Vibration patterns | 1881\_b\_koenig\_383\_Schwingungsformen.jpg |  |
| 2215 | Synthesizer | 1881\_b\_Koenig\_386\_Sirenensynthesizer.jpg |  |
| 2216 | Schema eines nach dem Principe von Hering gebauten Registrirapparates | 1891\_langendorff\_027\_015\_Hering.jpg |  |
| 2217 | König‘sche Kapsel mit empfindlicher Flamme | 1891\_langendorff\_083\_062.jpg |  |
| 2218 | Rotirender Spiegel und manometrische Flamme | 1891\_langendorff\_083\_063.jpg |  |
| 2219 | Vocalklangcurven | 1891\_langendorff\_084\_064.jpg |  |
| 2220 | Projection der Schwingungen einer Lissajous‘schen Stimmgabel | 1891\_langendorff\_084\_065.jpg |  |
| 2221 | Trautonium | 1931\_MusikuStrom3\_Anzeige\_1931\_141e45bf10\_08363f05a2\_trautonium.jpg |  |
| 2222 | Aufstellung harmonischer Regionen | 1939\_1948\_Schoenberg\_StructuralFunctionsOfHarmony\_HarmonischeRegionen.jpg |  |
| 2223 | Schema der Toneigenschaften | 1939\_albersheim\_118\_Fig\_01.jpg |  |
| 2224 | Spezifische Vokalreihe | 1939\_albersheim\_249\_Fig\_02.jpg |  |
| 2225 | Helligkeitsgrade einer Vokalfarbe | 1939\_albersheim\_250\_Fig\_03.jpg |  |
| 2226 | Akustisches Farbendreieck | 1939\_albersheim\_252\_Fig\_04.jpg |  |
| 2227 | Klangverdunkelung bei Tonhöhensteigerung | 1939\_albersheim\_257\_Fig\_05.jpg |  |
| 2228 | Schema der Klangeigenschaften | 1939\_albersheim\_268\_Fig\_06.jpg |  |
| 2229 | Farbenviereck - Farbenkreis - Vokaldreieck | 1939\_albersheim\_270\_Fig\_07\_08\_09.jpg |  |
| 2230 | Vokaldreieck nach Hellwag-Stumpf | 1939\_albersheim\_272\_Fig\_10.jpg |  |
| 2231 | Trennung von spezifischer Helligkeit und spezifischer Qualität | 1939\_albersheim\_274\_Fig\_11.jpg |  |
| 2232 | Auffassung der Dimensionen des Vokaldreiecks im Sinne von Sättigung und Helligkeitsgrad | 1939\_albersheim\_276\_Fig\_12.jpg |  |
| 2233 | Auffassung der Dimensionen des Vokaldreiecks im Sinne von Qualität und Helligkeitsgrad | 1939\_albersheim\_280\_Fig\_13.jpg |  |
| 2234 | "Ähnlichkeitssystem der Hauptvokale" nach Huber | 1939\_albersheim\_283\_Fig\_14.jpg |  |
| 2235 | Vokalschema nach Trendelenburg | 1939\_albersheim\_295\_Fig\_15.jpg |  |
| 2236 | Darstellung des Vokaldreiecks im 2-dimensionalen Koordinatensystem | 1939\_albersheim\_299\_Fig\_16.jpg |  |
| 2237 | Vokaldreieck unter Berücksichtigung der Sättigungsdiskontinuität | 1939\_albersheim\_305\_Fig\_17.jpg |  |
| 2238 | Hubersches Vokaldreieck, abgeändert | 1939\_albersheim\_306\_Fig\_18.jpg |  |
| 2239 | Einordnung der offenen Vokale in das Vokaldreieck | 1939\_albersheim\_318\_Fig\_19.jpg |  |
| 2240 | Akustischer Farben-Halbkreis | 1939\_albersheim\_350\_Fig\_20\_FarbenHalbkreis.jpg |  |
| 2241 | Zwei Schnitte durch den Vokalkörper | 1939\_albersheim\_352\_Fig\_21\_22\_FarbenkoerperSchnitte.jpg |  |
| 2242 | Farbenkörper | 1939\_albersheim\_353\_Fig\_23\_AkustFarbenkoerper.jpg |  |
| 2243 | Response patterns for periodic impulse | 1940\_Schouten\_FilterBank.jpg |  |
| 2244 | Sensory-response patterns for the components of a modulated wave | 1947\_matthesMiller.jpg |  |
| 2245 | Half-clipped sinusoids | 1962\_craigJeffres.jpg |  |
| 2246 | Vibration patterns | 1969\_plompSteneken.jpg |  |
| 2247 | Spectral dynamics | 1975\_grey\_139\_Spektraldynamik\_3.jpg |  |
| 2248 | Timbre space | 1978\_Grey\_Gordon\_1496\_Fig\_2\_timbreSpace.jpg |  |
| 2249 | Timbre space | 1978\_Grey\_Gordon\_1496\_Fig\_3\_timbreSpace.jpg |  |
| 2250 | Spectral transformation | 1978\_Grey\_Gordon\_1497\_Fig\_4\_spectralTrafo.jpg |  |
| 5056 | Caricature of Louis-Bertrand Castel's Ocular Harpsichord | 17xx\_CharlesGermainDeSaintAubin\_\_a\_caricature\_of\_louis\_bertrand\_castels\_ocular\_organ.jpg | “Que n’ont ils tous Employés leurs tems à la même Machine. Le Père Castel tapote des Sons et des Couleurs” – Have they not spent their whole time for the same machine? Père Castel plunking around with sounds and colours. |
| 5046 | Newton's Crucial Experiment | 175x\_nollet\_newtonExpCrucis\_mollon.jpg |  |
| 5003 | Triangular Prism | 1593\_dellaPorta\_GiovanniBattista\_triangularPrism\_gage\_1999\_126\_Fig\_052.jpg |  |
| 5004 | Colours on a sphere | 1611\_Forsius\_Siegfrid\_zollinger\_71.jpg |  |
| 5005 | Five primitive colours (kappa-5) | 1613\_D\_Aguilon\_gage\_183.jpg | The five basic Aristotelean colours white, yellow, red, blue, black on a horizontal line act as nodes of a graph in which each pair of nodes is connected by an arc. The three central arcs labelled orange, violet and green are interpreted as mixed colours. The remaining seven combinations are not labelled.  In edition from 1597, the five Aristotelian predicables are shown in the same arrangement [2103].  Zarlino (1562) uses an equivalent diagram representing the quinario, the set {1, 2, 3, 4, 5}, to explain the consonances of the syntonic tone system [35], and he gives a similar graph with only four nodes in order to analyse the musical interval of the twelfth [2043]. |
| 5007 | Colorum Annulus | 1629\_Fludd\_ColorumAnnulus\_gage\_009.jpg | In a kind of brightness scale, Robert Fludd arranges white, yellow, orange, red, green, blue, black around a circle, so that the greyscale black and white meet. The seven colours are explained as combinations of the basic qualities whiteness (“albedine”), redness (“rubedine”) and blackness (“nigredine”). |
| 5008 | Colours of urine | 1629\_fludd\_gage\_133\_FluddUrinfarben.jpg | “Urinarum cum Borealium tum Australium scala”. Two intersecting cones are indicated, a motif that occurs frequently in Fludd’s drawings of a mystical background. |
| 5010 | Refraction: tennis ball | 1658\_Descartes\_\_Discours\_de\_la\_méthode\_La\_Dioptrique\_011\_refractionTennisBall.jpg | A tennis-ball when entering water changes its direction. The metaphor is used to explain the refraction of light. However, light is refracted in the opposite direction. |
| 5011 | Refraction: ratios of sines | 1658\_Descartes\_\_Discours\_de\_la\_méthode\_La\_Dioptrique\_022\_refractionSine.jpg | Visualisation of the law of refraction. When light enters a denser medium, the proportion AH: GI = KM:NO between the sines of incidence and the sines of refraction is stated in the surrounding text. The formula cannot be derived directly from the drawing alone. |
| 5012 | Reflector telescope (1637) | 1658\_Descartes\_\_Discours\_de\_la\_méthode\_La\_Dioptrique\_132\_reflectorTelescope.jpg | An early reflector telescope (the drawing is present in the 1637 edition of the Discours de la méthode). |
| 5013 | Explanation of the rainbows | 1658\_Descartes\_\_Discours\_de\_la\_méthode\_Les\_meteores\_251\_rainbow.jpg | Explanation of the two rainbows. In a sphere (in each raindrop) the light is either reflected twice generating the colours of the inner and stronger rainbow, or it is reflected three times generating the outer fainter rainbow. The explanation was picked up by Newton in 1704 [5037]. |
| 5014 | Triangular Prism | 1658\_Descartes\_\_Discours\_de\_la\_méthode\_Les\_meteores\_255\_prism.jpg | Triangular prism. According to this drawing, refraction only takes place when the light leaves the prism. Usually one tries to make the incoming light a thin parallel beam. Here the outgoing light is controlled by a slit in the supporting beam. The drawing does not seem to explain the colour spectre. |
| 5015 | Spinning light particles | 1658\_Descartes\_\_Discours\_de\_la\_méthode\_Les\_meteores\_258\_spinningLightParticles.jpg | Colours are explained by Descartes as spinning light particles of different speed and direction of rotation - within a universe filled up completely with particles. |
| 5016 | Multiple reflection in a sphere | 1658\_Descartes\_\_Discours\_de\_la\_méthode\_Les\_meteores\_263\_multipleReflection.jpg | Multiple reflections within a sphere and refraction of a beam of light when it enters and leaves a sphere of a higher density. |
| 5017 | Inverted rainbow | 1658\_Descartes\_\_Discours\_de\_la\_méthode\_Les\_meteores\_268\_invertedRainbow.jpg | An inverted rainbow is explained by light that is first reflected on the water surface before it is reflected in the usual way by the water-drops. |
| 5018 | Newton's crucial experiment | 1666\_newton\_crucialExperiment\_BAL\_191592.jpg | An early sketch of Newton’s crucial experiment. At the second prism behind a hole in the canvas the entering beam of light is refracted, but it does not change its (red) colour any more. This shows that monochromatic light is elementary. Other than the entering sunlight that is split into various colours at the first prism. |
| 5019 | Frontispiece (copperplate) | 1671\_kircher\_ArsMagna\_LucisEtUmbrae\_000\_TitelKupfer.jpg |  |
| 5020 | Sun earth and moon - shadows | 1671\_kircher\_ArsMagna\_LucisEtUmbrae\_006\_Sonne\_Erde\_Mond\_Schatten.jpg |  |
| 5021 | De colore apparente - multiple reflection | 1671\_kircher\_ArsMagna\_LucisEtUmbrae\_024\_KugelMehrfachreflexion\_as\_Descartes.jpg | Multiple reflection and refraction in a sphere of glass as in Descartes’ explanation of the two rainbows [5013, 5016]. |
| 5022 | Analogia rerum cum coloribus - five primitive colours | 1671\_kircher\_ArsMagna\_LucisEtUmbrae\_049\_farbgraph\_kappa\_5.jpg | The system of colours by Aguilonius [5005] is transferred to various systems of five related notions. Differently from Aguilonius, all combinations (except black/white) are assigned colour names. |
| 5023 | Colours from triangular prism | 1671\_kircher\_ArsMagna\_LucisEtUmbrae\_055\_prisma.jpg | Colours at a triangular prism. The drawing does not show the different angles of refraction for the different spectral colours. The incoming light from the sun (top right) seems to be compound of parallel rays of different colours in the order of the rainbow. The rays leaving the prism are still parallel. The only difference that could account for the different colours is the different distance covered within the prism. The light with the longest path is blue (violet) the light with the shortest path is red. This view obviously disagrees with Newton theory of “rays differently refrangible” developed at about the same time.  Kircher’s drawing is very similar to the prism by Giovanni Battista della Porta [5003]. |
| 5024 | Chameleon | 1671\_kircher\_ArsMagna\_LucisEtUmbrae\_063\_Chamaeleon.jpg |  |
| 5025 | Projector system with three light sources | 1671\_kircher\_ArsMagna\_LucisEtUmbrae\_091\_projektorMehrereLichtquellen.jpg |  |
| 5031 | Newton: Opticks, Book I, Part II, Plate I | 1704\_newton\_Opticks\_Book\_I\_Part\_II\_Plate\_I.jpg |  |
| 5032 | Newton: Opticks, Book I, Part II, Plate II | 1704\_newton\_Opticks\_Book\_I\_Part\_II\_Plate\_II.jpg |  |
| 5035 | Experiment with prisms, comb, lense and mirror | 1704\_newton\_Opticks\_Book\_I\_Part\_II\_Plate\_II\_Fig\_6\_Comb.jpg |  |
| 5033 | Newton: Opticks, Book I, Plate III | 1704\_newton\_Opticks\_Book\_I\_Part\_II\_Plate\_III.jpg |  |
| 5034 | Colour wheel | 1704\_newton\_Opticks\_Book\_I\_Part\_II\_Plate\_III\_Fig\_11\_ColourWheel.jpg | In its abstraction and speculative nature this diagram is unique in Newton’s “Opticks”. None of the other figures is about perception. The similarity of this diagram with a circular pitch diagram by Descartes [314] might obfuscate the fact that Newton gives the interior of the circle a meaning in terms of colour. Descartes’ diagram shows the pitch classes of the syntonic diatonic scale in a circular arrangement, whereas Newton’s diagram represents a two-dimensional perceptual colour order with the dimensions hue (angle) and saturation (radius): point z is a weakly saturated orange close to red. At the same time, the radiuses separating the colours define the pitch classes of the diatonic scale.  If the diagram is compared with [92] and [86], where a monochord is compared with a prismatic spectre it turns out the assignment of colours and pitches is inconsistent. In the circular diagram, increasing pitch correspond to increasing frequency of light, and in the spectre/monochord drawing the increasing pitch corresponds to decreasing frequency of light. [<MUZZ\_2012> Muzzulini 2012, 699-701] |
| 5036 | Newton: Opticks, Book I, Plate IV | 1704\_newton\_Opticks\_Book\_I\_Part\_II\_Plate\_IV.jpg |  |
| 5037 | Explanation of the rainbows | 1704\_newton\_Opticks\_Book\_I\_Part\_II\_Plate\_IV\_Fig\_15\_Rainbow.jpg | Explanation of the two rainbows. In the raindrops of the upper rainbow the light is reflected three times before it reaches the eye of the observer. In the raindrops of the lower rainbow it is reflected only twice. Compared with Descartes’ drawing [5013], this drawing does not only explain the fact that there are two rainbows, it also explains the different colours and the opposite order of the spectral colours in the two rainbows with the varying angles of refraction. |
| 5038 | Colour experiment with three prisms and a lense | 1704\_newton\_Opticks\_Book\_I\_Part\_II\_Plate\_IV\_Fig\_16\_3prisms.jpg |  |
| 5027 | Newton: Opticks, Book II, Plate I | 1704\_newton\_Opticks\_Book\_II\_Plate\_I.jpg |  |
| 5028 | Newton's rings | 1704\_newton\_Opticks\_Book\_II\_Plate\_I\_Fig\_1\_2\_newtonsRings.jpg |  |
| 5029 | Complementary colours | 1704\_newton\_Opticks\_Book\_II\_Plate\_I\_Fig\_3\_ComplementaryColors.jpg |  |
| 5030 | Complementary colours | 1704\_newton\_Opticks\_Book\_II\_Plate\_I\_Fig\_4\_complementaryColors.jpg |  |
| 5039 | Trois Couleurs Primitives | 1708\_anonymous\_boutet\_\_3couleurprimitives\_2\_mollon.jpg |  |
| 5040 | Trois Couleurs Primitives | 1708\_anonymous\_boutet\_\_3CouleurPrimitives\_mollon.jpg |  |
| 5041 | Colour circles | 1708\_anonymous\_boutet\_\_traite\_de\_la\_peinture\_en\_mignature\_ad\_154\_155\_ColourCircles.jpg |  |
| 5042 | Coloritto - the Art of Mixing Colours | 1725\_le\_blon\_Coloritto\_3Primaries\_mollon.jpg |  |
| 5043 | Explanation of the rainbows | 1734\_scheuchzer\_rainbow.jpg |  |
| 5044 | Three colour printing - Cardinal Fleury | 1738\_LeBlon\_Fleury.jpg |  |
| 5045 | Spectre de Newton | 1752\_d\_agoti\_newtonSpektrFarbig\_mollon.jpg |  |
| 5047 | Scheme of Colours | 1770\_1772\_harris\_scheme\_of\_colours.jpg |  |
| 5048 | Scheme of Colours - Three Primitives | 1770\_1772\_harris\_scheme\_of\_colours\_prismatic.jpg |  |
| 5049 | Tetrahedron of Colours | 1772\_lambert\_Farbenpyramide\_Appendix.jpg |  |
| 5050 | Ordnung der Farbenclasse - Die Blühenden Farben | 1772\_schiffermueller\_\_Versuch\_eines\_Farbensystems\_015\_ad.jpg |  |
| 5051 | Colour Triangle | 1775\_Mayer\_Tobias\_\_color\_triangle\_Lichtenberg.jpg |  |
| 5052 | Colour Vision - Three Receptors Theory | 1777\_palmer\_3rezeptortheorie\_facs\_Princ\_mollon.jpg |  |
| 5053 | Scale of Colours | 1786\_Elliot\_anonym\_IR\_UV\_quot\_mollon.jpg |  |
| 5054 | Colour Perception - Light Rays versus Circumstances | 1789\_monge\_CoulNatRel\_quot\_mollon.jpg |  |
| 5055 | Schiller/Goethe: Temperamentrose - Circle of Colours and Temperaments | 1799\_Schiller\_Goethe\_\_\_temperamentrose\_gage\_160.jpg |  |
| 5057 | Light and Heat - Thermometer | 1800\_herschel\_IR\_Thermometer\_mollon.jpg |  |
| 5058 | Light and Heat | 1800\_herschel\_waermeLichtChar\_mollon.jpg |  |
| 5059 | Wavelengths of light | 1802\_Young\_\_The\_Bakerian\_lecture\_on\_the\_theory\_of\_light\_and\_colours\_039\_wavelengthOfLight.jpg |  |
| 5061 | Colour Circle | 1809\_goethe\_\_farbenkreis\_goethe\_wikipedia\_.jpg |  |
| 5060 | Ideales / Reales | 1809\_Runge\_ColourCircleOfIdealAndReal\_gage\_1999\_188\_Fig\_090.jpg |  |
| 5069 | Farbenkugel - Sphere of Colours | 1810\_runge\_farbenkugel\_appendix.jpg |  |
| 5062 | Colour Triangle | 1810\_Runge\_Farbenkugel\_Fig\_01\_p\_04\_Triangle.jpg |  |
| 5063 | Colour Triangle | 1810\_Runge\_Farbenkugel\_Fig\_02\_p\_05\_Triangle.jpg |  |
| 5064 | Colour Triangle | 1810\_Runge\_Farbenkugel\_Fig\_03\_p\_06\_Triangle.jpg |  |
| 5065 | Double Cone of Colours | 1810\_Runge\_Farbenkugel\_Fig\_04\_p\_08\_DoubleCone.jpg |  |
| 5066 | Hexagon and Concentric Triangles of Colours | 1810\_Runge\_Farbenkugel\_Fig\_05\_p\_08\_CircleHexagonAndConcentricEquilateralTriangles.jpg |  |
| 5067 | Circle - Complementary Colours | 1810\_Runge\_Farbenkugel\_Fig\_06\_07\_p\_11\_CircleComplementaryColours.jpg |  |
| 5068 | Colour Sphere | 1810\_Runge\_Farbenkugel\_Fig\_08\_p\_13\_Sphere.jpg |  |
| 5070 | Colour Circle | 1824\_1828\_Turner\_Colour\_Circle\_No\_1\_\_D17149\_10.jpg |  |
| 5071 | Colour Circle | 1824\_1828\_Turner\_Colour\_Circle\_No\_2\_\_D17150\_10.jpg |  |
| 5072 | Triangle of Colours | 1830\_Delacroix\_Farbdreieck\_gage\_136.jpg |  |
| 5073 | Hexagon of Colours | 1830\_merimee\_gage\_176\_.jpg |  |
| 5074 | Definitive or Fundamental Scale of Colours | 1835\_fields\_gage\_175.jpg |  |
| 5075 | Light and Colour - Goethe's Theory | 1843\_Turner\_William\_\_Light\_and\_Colour\_Goethe\_s\_Theory.jpg |  |
| 5076 | Analagous Scale Of Sounds And Colours | 1845\_Field\_George\_Chromatics\_AnalagousScaleOfSoundsAndColours.jpg |  |
| 5077 | Colours and Sounds | 1845\_Field\_George\_Chromatics\_Plate\_VII\_ex\_XXI\_XXII.jpg |  |
| 5078 | Colours and Sounds | 1845\_Field\_George\_SoundsAndColours\_gage\_1999\_264\_Fig\_134.jpg |  |
| 5079 | Maxwell with his colour top | 1855\_Maxwell\_farbkreisel.jpg |  |
| 5081 | Compound Colours | 1860\_maxwell\_\_On\_the\_Theory\_of\_Compound\_Colours\_Plate\_I.jpg |  |
| 5082 | Compound Colours | 1860\_maxwell\_\_On\_the\_Theory\_of\_Compound\_Colours\_Plate\_II.jpg |  |
| 5080 | Maxwell's Colour Top | 1860\_maxwell\_FarbkreiselFoto\_mollon.jpg |  |
| 5083 | Colour topology according to Newton | 1867\_Helmholtz\_PhysiologischeOptik\_282\_Farbkreis.jpg |  |
| 5084 | Hering's System of Complementary Colours | 1878\_hering\_coloursystem.jpg |  |
| 5085 | Coloured Vowels | 1883\_galton\_inquiries\_into\_human\_faculty\_ad\_106\_coloured\_vowels.jpg |  |
| 5086 | Effects of Colours on Muscular Activity | 1887\_Feres\_Charles\_EffectOfColourOnMuscularActivity\_gage\_1999\_266\_Fig\_136.jpg |  |
| 5087 | Receptor Sensitivity in Colour Vision | 1892\_Koenig\_Dieterici\_RezKurven\_mollon.jpg |  |
| 5088 | Wavelengths of Spectral Colours | 1896\_helmholtz\_287\_SonnenLichtSpektrumMessbar.jpg |  |
| 5089 | Pitch Colour Correspondence | 1896\_helmholtz\_288\_TonFarbe\_1To1.jpg |  |
| 5090 | Colour Pitch Correspondence according to Drobisch | 1896\_helmholtz\_309\_TonFarbe\_Drobisch.jpg |  |
| 5091 | Yellow/Blue-Top Generating White | 1896\_helmholtz\_315\_GelbBlauKreisel.jpg |  |
| 5092 | Complementary colours | 1896\_helmholtz\_317\_1\_Gegenfarben.jpg |  |
| 5093 | Complementary colours | 1896\_helmholtz\_317\_2\_Gegenfarben.jpg |  |
| 5094 | Table for mixing spectral colours | 1896\_helmholtz\_321\_FarbmischtafelSpektralfarben.jpg |  |
| 5095 | Colour Cone | 1896\_helmholtz\_326\_Farbkegel.jpg |  |
| 5096 | Physiological Colour Model - Hue and Saturation | 1896\_helmholtz\_332\_Schuhsohle.jpg |  |
| 5097 | Limited range of three spectral primary colours | 1896\_helmholtz\_334\_SchuhsohleEingeschr3Ecke.jpg |  |
| 5098 | Virtual primary colours | 1896\_helmholtz\_340\_SchuhsohleUmschl3Ecke.jpg |  |
| 5099 | Sensitivity of the Colour Receptors | 1896\_helmholtz\_346\_RGV\_Resonanzkurven.jpg |  |
| 5100 | Additive Colour Mixing | 1896\_helmholtz\_350\_AdditiveFarbmischungGlasplatte.jpg |  |
| 5101 | Three colour printing | 1904\_meyers\_dreifarbendruck.jpg |  |
| 5102 | Newton's Wheel | 1910\_c\_Kupka\_Frantisek\_\_Newton\_sWheel\_\_gage\_1999\_143\_Fig\_062.jpg |  |
| 5103 | Table of Polarities | 1911\_12\_Kandinsky\_\_TableOfPolarities\_gage\_1999\_194\_Fig\_092.jpg |  |
| 5105 | Sound Colour Circle for Itten | 1919\_20\_hauer\_\_Farbenkreis\_fuer\_Itten\_\_maur\_k\_1985\_067\_Fig\_077.jpg |  |
| 5106 | Sound Colour Circle for Itten | 1919\_Hauer\_to\_Itten\_farbTonKreis\_hauer\_1919\_10\_05\_www\_musikzeit\_at\_2009.jpg |  |
| 5104 | Circle of Musical Keys, Temperaments and Colours | 1918\_hauer\_ColorCircle.jpg |  |
| 5107 | Konstruktive Komposition mit Dreiklang Gelb-Rot-Blau | 1925\_Buchheister\_carl\_\_KonstruktiveKomposMit3Klang\_Maur\_1985\_148.jpg |  |
| 5108 | MacAdam Ellipses | 1942\_MacAdam\_\_Visual\_Sensitivities\_to\_Color\_Differences\_in\_Daylight\_271.jpg |  |
| 5109 | Dimension of Hue | 2013\_briggs\_david\_\_dimensionOfHue.jpg |  |
|  |  |  |  |
|  | Triplicis uisus, directi, reflexi, & refracti, de quo optica disputat, argumenta | 1572\_Alhazen\_\_Opticae\_thesaurus\_GC5\_R4947\_572i\_HoughtonLibraryHarvardUniversity.jpg |  |