

# The Tonnetz

Negative Harmony & Neo-Riemannian Transformations



*Emmanouil Karystinaios - Silvan David Peter*

# Organization

## FIRST PART

### Introduction:

1. Pitch class sets
2. Triads and Transpositions
3. PLR and Riemann

### The Tonnetz 1:

4. Configurations
5. Demonstration

### Python Implementation

## SECOND PART

### The Tonnetz 2:

6. Negative Harmony
7. Hamiltonian Paths
8. The Torus Model - Tension

### Bonus:

9. Infinite Tonnetze, Trajectories
10. Tonnetz Topologies

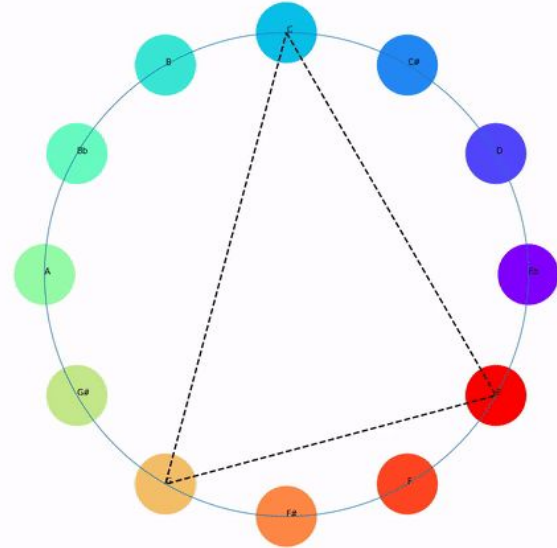
### Python Implementation

# Pitch Class Sets and Circle of Fifths

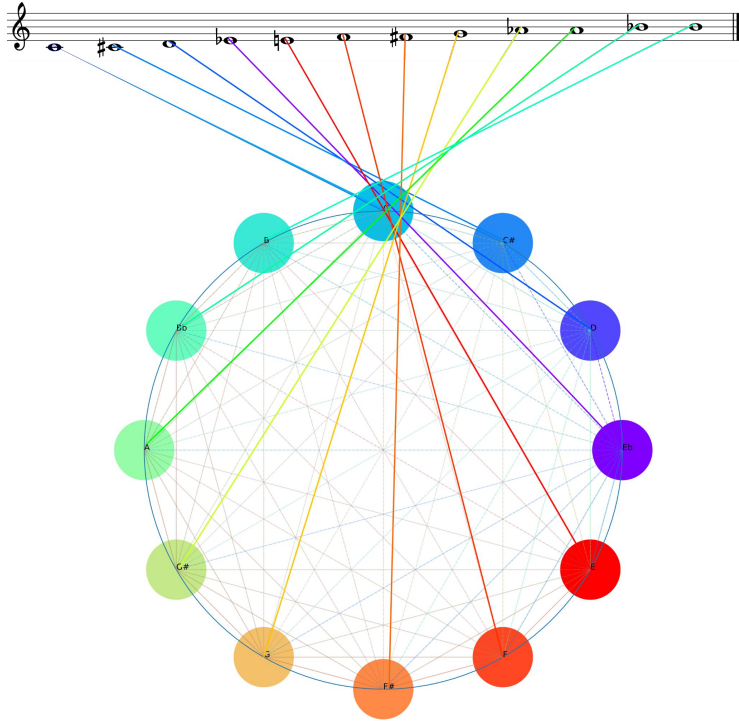
Equal temperament & division of the octave:

0	C	DO
1	C# or Db	
2	D	RE
3	D# or Eb	
4	E	MI
5	F	FA
6	F# or Gb	
7	G	SOL
8	G# or Ab	
9	A	LA
10	Bb	
11	B	SI

$$C_{maj} = \{0, 4, 7\}$$



# Circle Representation



A circle represents a perfect octave.

- Lines = Intervals
- Triangles = Triads
- Pentagons = Pentatonic Scales

Let's build mathematically, the Circle of Fifths by transforming this circle of PC notes.

# Circle of fifths



To build the circle of fifths we can think of the chromatic sequence as a Pitch Class Set :

**{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}**

Multiple the entire sequence x7:

**{0, 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77}**

Then apply modulo 12:

**{0, 7, 2, 9, 4, 11, 6, 1, 8, 3, 10, 5}**

Now rebuild the circle with these Pitch Classes.

# Circle of fifths

The circle of fifths has been used as a representation of closeness between scales, notes and chords.

The fifth is an interval of particular importance and consonance as it is the first to appear in the harmonic series after the octave.

Sidenote: Many instruments (mainly wind) that work on the harmonic series skip the first harmonic and jump to the 5th such as the trumpet other only use the the odd harmonic such as the Clarinet.

# Triads

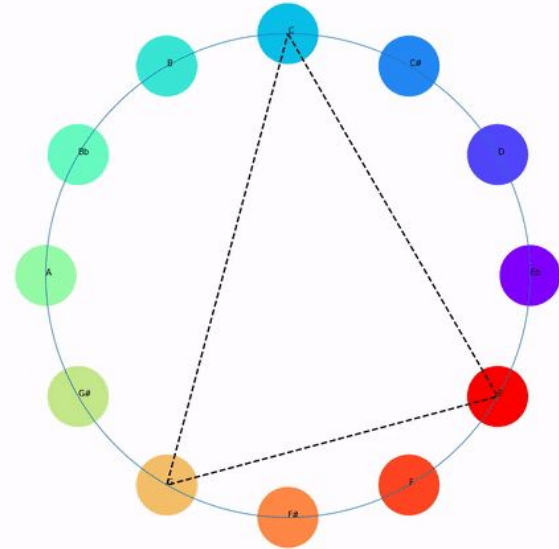
## Definition:

**Triad** is any combination of three pitch classes.

Transpositions of any group of triads can be seen as rotations in the Circle representation.

# Transposition of Triads

What other kinds of operations can we find on the circular representation???

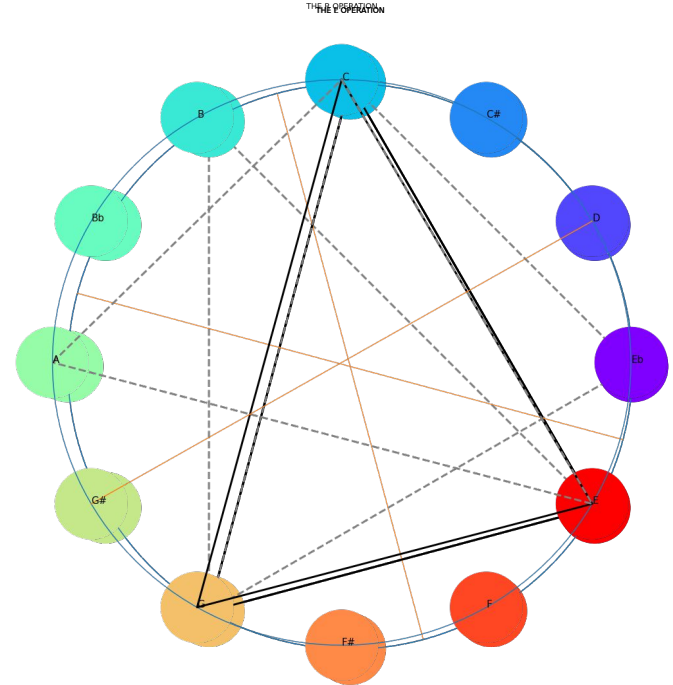




# PLR Operations - Mirror Symmetries

Let's look into Mirror symmetries on the  $C_{maj}$  chord:

- **Parallel** mirror along the axis of C-G
- **Leading Tone** along the axis of E-G
- **Relative** Along the axis of C-E



# PLR as Group Operations

The PLR group acts simply transitively on the set  $\{n_M, n = 0 \dots 11\} \cup \{n_m, n = 0 \dots 11\}$  of the 24 major and minor triads, where  $n_M$  (resp.  $n_m$ ) represents a major (resp. minor) triad with root  $n$  in the usual semi-tone encoding of pitch classes.

It is isomorphic to the dihedral group  $D_{24}$  of order 24, and is generated by the following two transformations.

The transformation  $L : \mathbb{Z}_{24} \rightarrow \mathbb{Z}_{24}$  is called the leading-tone operation, and is such that:

$$L(n_M) = (n + 4)_m \quad \text{and the complementary} \quad L(n_m) = (n + 8)_M$$

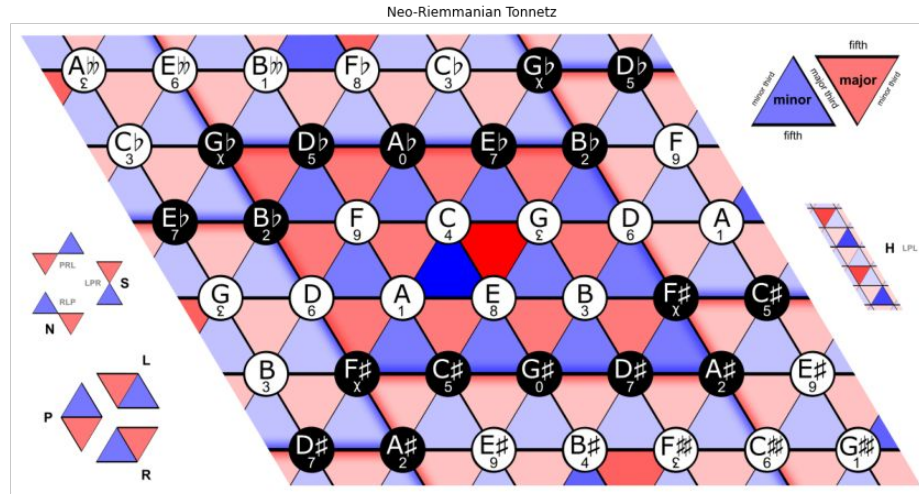
The transformation  $R : \mathbb{Z}_{24} \rightarrow \mathbb{Z}_{24}$  is called the relative operation, and is such that:

$$R(n_M) = (n + 9)_m \quad \text{and the similarly the complementary}$$

Though not a generator, the operation  $P = (RL)3R$ , called the parallel operation, is often considered, and is such that  $P(n_M) = n_m$ .

# The Tonnetz

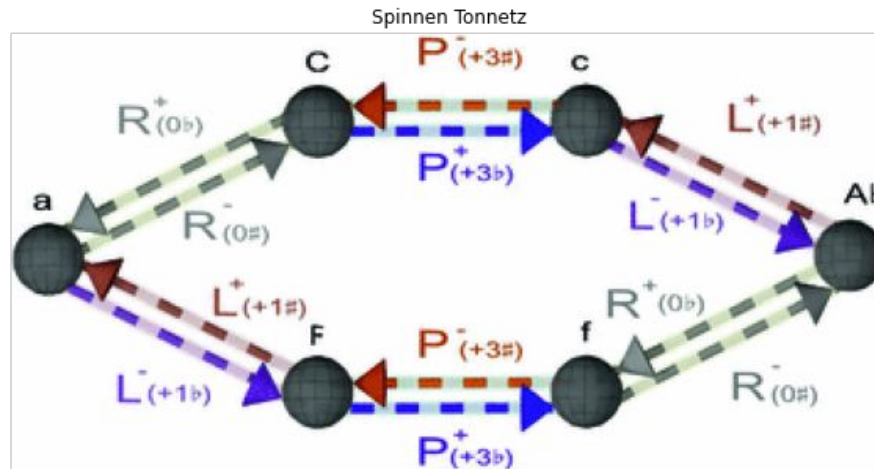
Stacking PLR Operation results to the Tonnetz space:



Vertices in this graph are Notes. It's Called the Neo-Riemmanian Tonnetz.

# The Tonnetz

Stacking PLR Operation results to the Tonnetz space:



Vertices in this graph are Chords. It's Called the Spinnen Tonnetz.

# Tonnetz - A Beehive

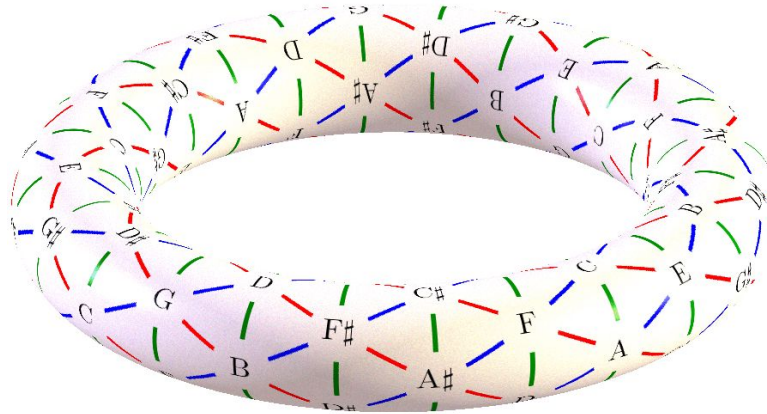
The Tonnetz Creates a beehive grid (hexagonal). So it results to three axes:

- The axis of third minors
- The axis of third majors
- The axis of fifths

Therefore it is also called the Tonnetz  $T(3, 4, 5)$  for minor third, major third and fourth the complementary of fifth correspondingly.

# Tonnetz a mathematical Playground

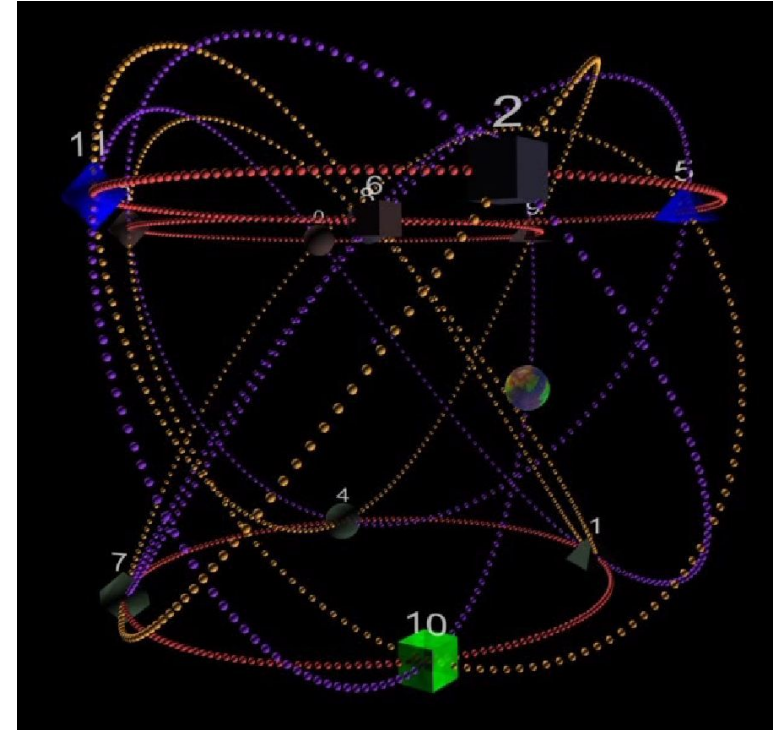
The Neo-Riemannian Tonnetz creates two main loops along its axes. Therefore, topologically it is equivalent to the torus :



It didn't take long until people started to play around with different less conventional and less tonal configurations of the Tonnetz

# Tonnetz as a Hypersphere

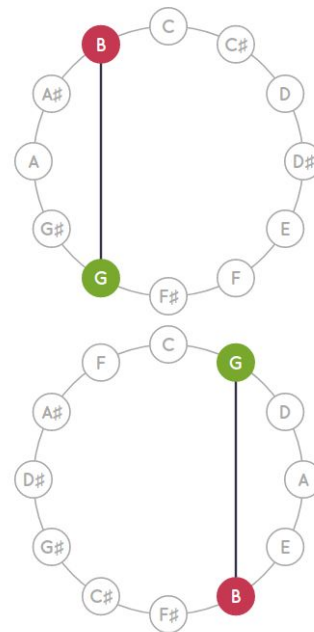
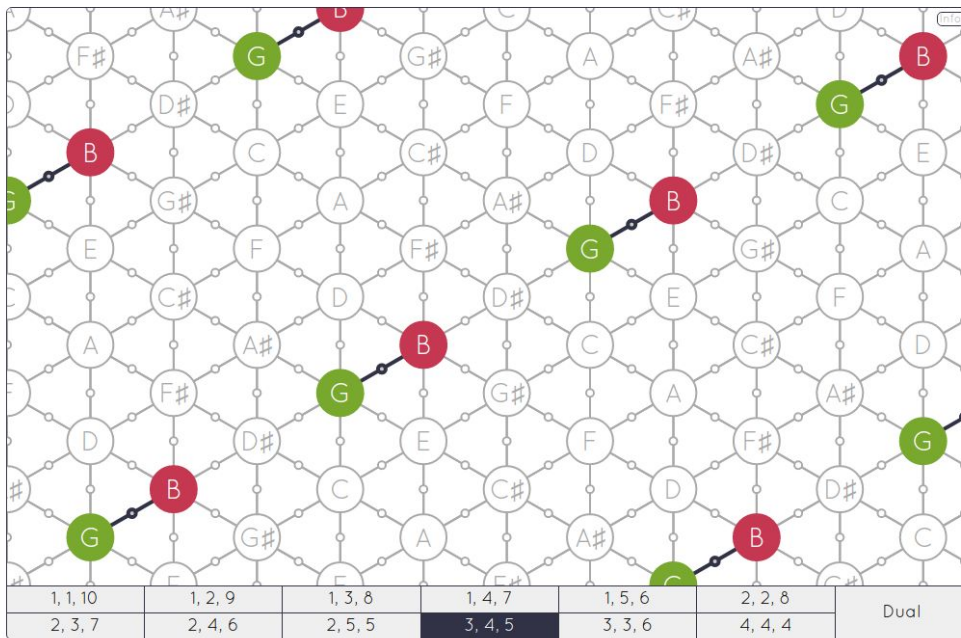
Another representation of the Tonnetz is one in 4 dimensions that manages to close the infinite grid in a compact representation.



<https://guichaoua.gitlab.io/web-hexachord/>

## THE TONNETZ

ONE KEY - MANY REPRESENTATIONS

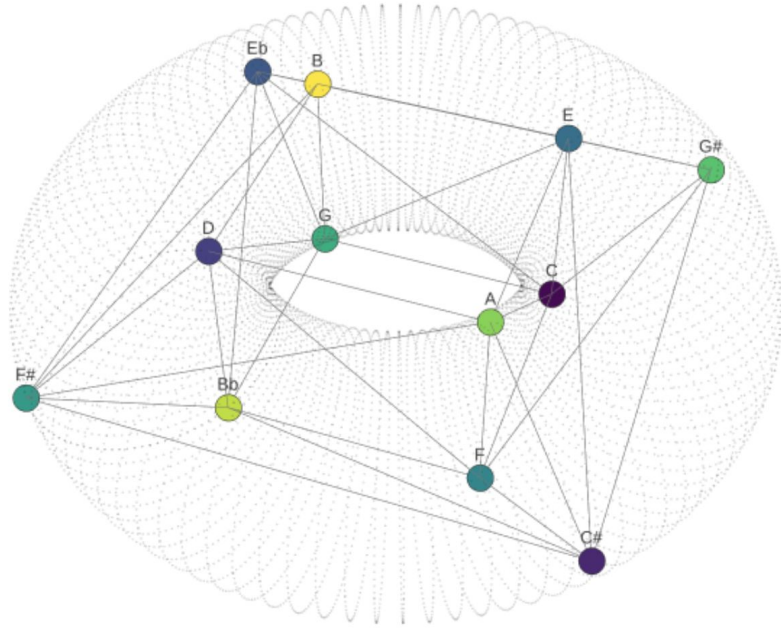




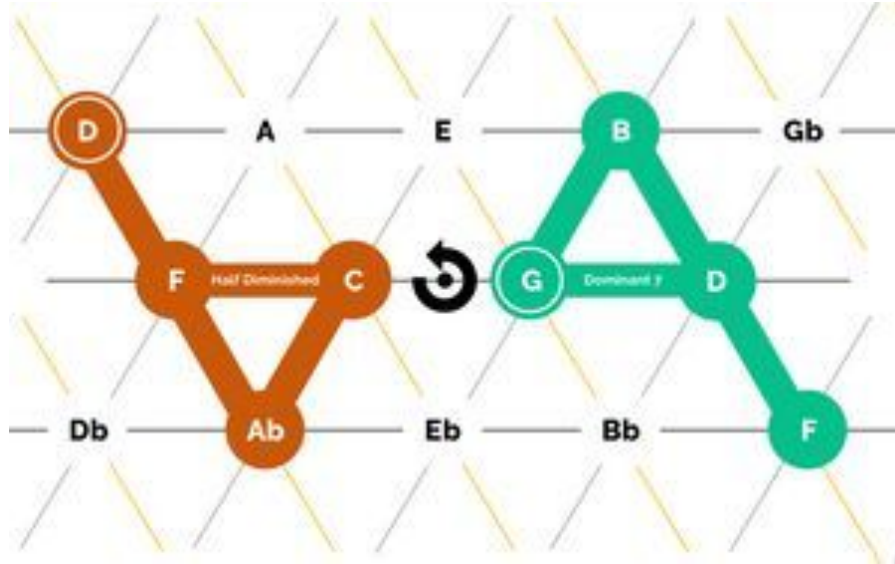
# Exercise

1. Parse the MusicXML File and output a list of Chords.
2. Code a *bijjective* function/object that takes a list of notes and returns their respective pitch classes.
3. Code a function that performs transposition on triads.
4. Code the PLR operations.

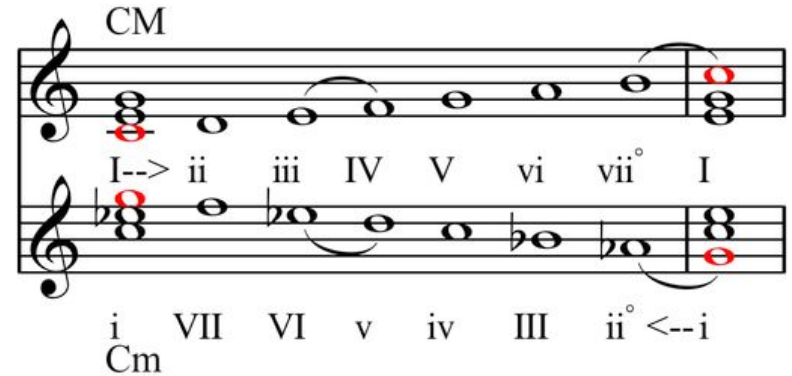
# More Tonnetz



# Negative Harmony



Jacob Collier recently set anew the trend of Negative Harmony. Negative harmony can be viewed as a Rotation of a Chord's representation in the Tonnetz along the 5ths axis.



# The Hamiltonian Song

## Definition

A **Hamiltonian path** on a graph is a walk on the graph that visits each vertex only once.

A **Hamiltonian circle** on a graph is a hamiltonian path that returns on the starting vertex.



# Tonal Tension

## Definition:

**Tonal Tension** refers to the feeling of stability and instability in music. It is specified as the specific sense created by melodic and harmonic motion.

## Tension and Release Examples:

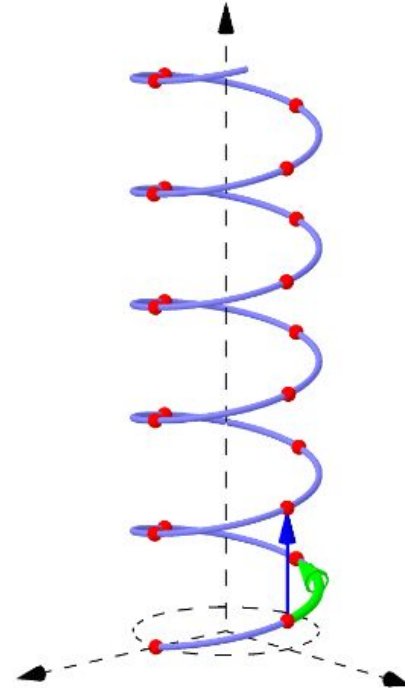
- Moving from the Dominant 7 to the Tonic;
- Moving from the Major Tonic to a distant key chord.

## How to measure Tonal Tension?

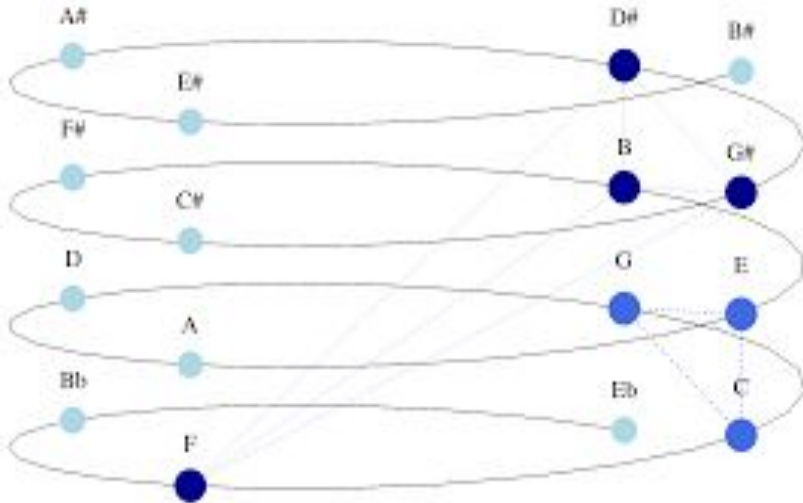
There are elaborate music theories on how to compute tension which are always distant from the computational aspect.

# Tonal Tension from the Tonnetz

The spiral array. Two consecutive pitch classes lying on the helix are a perfect fifth apart (considering the orientation of the curved arrow), while the vertical arrow connects two pitch classes a major third far from each other.

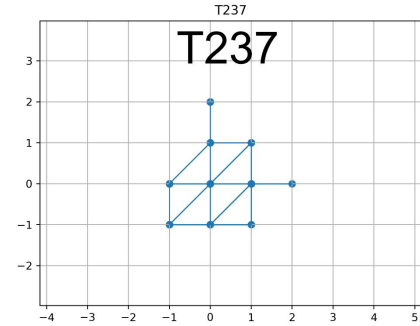
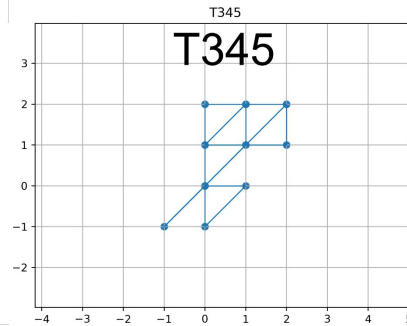
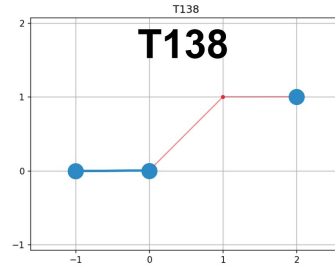
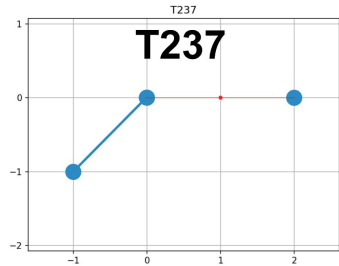
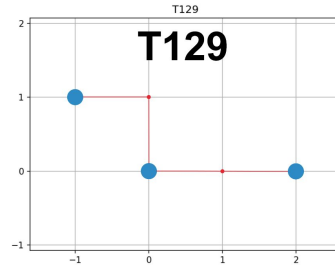
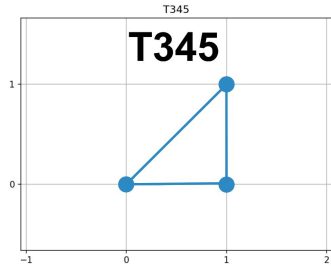


# Chew's Helical Model



Inspired by the Torus configuration of the Tonnetz the tension of two chords is computed by considering the barycenter of each chord manifold in the spiral array.

# Other Tonnetz Configurations



A C major Chord

Jazz excerpt on different Tonnetze



# Assignment

## 1. Hamiltonian Paths

Find all Hamiltonian Paths in the Tonnetz given a starting triad and the Vocabulary of triads (i.e. Given by the space).

## 2. Negative Harmony

Given a Piece and a reference point Rotate everything in the Tonnetz 180 Degrees and return the midi and the audio.