What does a nonabelian group sound like?

Matthew Corley (Computer Science); William DeMeo (Math; mentor), Reg Bain (Music; co-mentor)

Background and prior work. Underlying many digital signal processing (DSP) algorithms, in particular those used for digital audio filters, is the convolution operation. This operation acts on a signal, $f(x)$, and can be viewed as a weighted sum of translations, $f(x-y)$. Most classical results of DSP are easily and elegantly derived if we define our functions on $\mathbb{Z}/n$, the abelian (or commutative) group of integers modulo n (see [6]). The term *abelian* here refers to the fact that the basic group operation is addition (modulo n) which is a commutative operation (i.e., $x+y=y+x$).

If we replace this "index set" (the set on which functions are defined) with a *nonabelian* group—where the group operation is now multiplication, xy—then instead of the usual translation, $f(x-y)$, we have a *generalized translation*, $f(xy^{-1})$. If we carry out convolution using this generalized translation, the resulting audio filters will naturally produce different effects than those obtained with ordinary (abelian group) convolution.

One of my faculty mentors, Dr. DeMeo, initiated research based on these ideas in 2004 and presented some preliminary findings at the International Symposium on Musical Acoustics (see [4], which received a "best paper" award). Similar ideas have been successfully applied to two dimensional image data as well as to other areas of engineering (see [1] and [7]). However, to date the application of nonabelian groups to audio signal processing seems relatively unexplored, and there are a number of fundamental open questions in this area that we hope to answer.

Research question. If the underlying index set of a digital audio filtering algorithm is modified to use various nonabelian groups (instead of the commonly used abelian group), how does this change the behavior of the filter and the resulting audio output?

Project goals and objectives. We propose to explore the idea of using the underlying finite group (i.e., the index set) as an adjustable parameter of a digital audio filter. By listening to samples produced using various nonabelian groups, we hope to get a sense of the "acoustical characters" of finite groups. We will attempt to associate these acoustical features with various mathematical properties of the groups, and develop a classification scheme that might be useful to practitioners in audio signal processing and computer music composition.

- *Goals:* Develop the mathematical theory necessary to provide sonic characterizations of nonabelian groups. Discover which mathematical features of a group can be used to describe how a given DSP algorithm based on that group will behave. Produce computer software that allows users to process and manipulate musical signals using nonabelian group filters.
- *Objective 1:* Develop an understanding of the basic math underlying signal processing algorithms in general and convolution in particular and show mathematically what effects the use of a nonabelian group index set will have on the convolution operation.
- *Objective 2:* Find a short list of nonabelian groups that are useful for nonabelian group audio filters and effects processors, prove their effectiveness both mathematically and experimentally, and document these discoveries.
- *Objective 3:* Implement a software program that takes an audio signal as input and allows the user to apply filters corresponding to specific nonabelian groups to achieve different effects.

Project significance. The proposed research introduces the novel concept of describing acoustical properties of mathematical groups. This may be interesting to some mathematicians, and we will present preliminary results of this work at the Mathematical Association of America's special session, "At the Intersection of Mathematics and the Arts." (Our abstract has already been accepted.) Of greater significance, however, will be the impact this research has on applications in digital audio engineering, especially if certain nonabelian groups are shown to produce interesting audio effects.

Methodology. We will conduct controlled experiments with very simple sound signals at first (sine waves and linear chirps), and filter these signals using standard convolution. Then we will filter the original signals using a generalized (nonabelian) convolution, substituting the underlying index set with various groups from the wide variety of nonabelian groups available in the SmallGroups library of GAP [5].

When we replace the index set $\mathbb{Z}/n$ with various finite nonabelian groups, in the beginning, the simplest examples of nonabelian groups (such as semidirect product groups), will be constructed "by hand" using GAP's `SemidirectProduct` function. Groups with a more complicated structure will be selected from GAP's vast SmallGroups library using various selection criteria. For each of the groups tested, we will implement the convolution function using the generalized (nonabelian) translation $f(xy^{-1})$ in place of ordinary translation $f(x-y)$ used in classical convolution.

After completing these controlled experiments, we will analyze the results to compare the effects of the choice of group on the resulting convolution filter. Finally, we will attempt to make a connection between the mathematical properties of the group and the acoustical properties of the resulting convolution.

Both GAP and Matlab will be used for much of the initial prototyping and testing. Matlab provides easy methods for constructing wav files "from scratch" with its `wavwrite()` function. Additionally, Myoung An (a colleague of Dr. DeMeo) has provided us with the Matlab code that she and Richard Tolimieri developed for their work in image processing, where they applied nonabelian group filters to the processing of 2D digital images. This code will be a valuable resource as we seek to apply similar ideas to audio signal processing.

As the project progresses, we will likely use the JavaSound library and implement our generalized DSP algorithms in Java. JavaSound provides methods for reading and altering wav files frequencies and sound intensity levels, which will prove useful when we apply our generalized DSP algorithms to more complex sounds.

Project timeline.
October 2013–December 2013: Become more familiar with music analysis/synthesis and DSP algorithms, and gain further knowledge of group theory and its role in classical DSP implementations.

January 2014–April 2014: Write code to implement algorithms for general nonabelian group DSP. Identify specific characteristics of groups that make them more (or less) useful as an index set on which to define DSP operations like convolution.

May 2014–October 2014: Gather and analyze results, and write up reports. Submit manuscript to an academic journal. Prepare for and attend conferences.

Anticipated results, final products, and dissemination. By the end of the Spring 2014 semester, I expect to have written Matlab programs to test the results of the modified DSP implementations described above. I also expect to have developed a Java software program which allows easy application of nonabelian group filters through a graphical user interface. I hope that the results will prove interesting and have practical applications for computer music composition.

The abstract for this project has already been accepted for presentation at the Joint Mathematics Meetings in Baltimore in 2014. In addition, I will submit the work to the International Computer Music Conference (ICMC), the International Symposium on Musical Acoustics (ISMA), and the 14th International Conference on New Interfaces for Musical Expression (NIME). Previous work on topics related to this proposal by my faculty mentors have been accepted at both ICMC and ISMA, so we have high expectations for this project. I will write up a formal article describing the research and results and submit the manuscript to at least one scholarly journal in mathematics or music. Finally, if my project proposal is accepted and I become a Magellan Scholar, I will be honored to present the work at Discovery Day 2014.

Personal statement. I have never had considerable ability with music, but I have always been fascinated by its intersection with my favorite subject, mathematics. This project piqued my interest because it allows me the rare opportunity to contribute to both fields. I believe that I have developed the necessary skills to succeed in this project through my past mathematical research projects and my current internship developing Java applications at a local software company. A Magellan Grant would allow me the fiscal freedom to dedicate time to engaging in research and possibly traveling to share my findings with international audiences.

References

[1] Gregory S. Chirikjian and Alexander B. Kyatkin. *Engineering Applications of Noncommutative Harmonic Analysis: With Emphasis on Rotation and Motion Groups*. CRC Press, 2002.

[2] J. H. Conway, R. T. Curtis, R. A. Wilson, S. P. Norton, and R. A. Parker. *ATLAS of Finite Groups*. Oxford University Press, 1986.

[3] William DeMeo. Characterizing musical signals with Wigner-Ville interferences. In *Proceedings of the International Computer Music Conference*. ICMC, 2002. Available from: `http://math.hawaii.edu/~williamdemeo/ICMC2002.pdf`.

[4] William DeMeo. Topics in nonabelian harmonic analysis and DSP applications. In *Proceedings of the International Symposium on Musical Acoustics*. ISMA, 2004. Available from: `http://math.hawaii.edu/~williamdemeo/ISMA2004.pdf`.

[5] The GAP Group. *GAP – Groups, Algorithms, and Programming, Ver. 4.4.12*, 2008. Available from: `http://www.gap-system.org`.

[6] Richard Tolimieri and Myoung An. *Time-Frequency Representations*. Birkhäuser, Boston, 1998.

[7] Richard Tolimieri and Myoung An. *Group Filters and Image Processing*. Kluwer Acad., 2004. Available from: `http://prometheus-us.com/asi/algebra2003/papers/tolimieri.pdf`.

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Magellan Scholar BUDGET FORM

Student's Name: Matthew Corley

Student salary	Hours Estimated number of hours student will work	Rate Enter the hourly wage	Subtotal
Research hours during semesters when enrolled in classes	120	10.00	$1,200.00
Research hours during semesters when NOT enrolled in classes			$0.00
Fringe: Student salary * student fringe rate (What is fringe? See budget instructions or guidebook)			
Enrolled in classes	$1,200.00	0.55%	$6.60
NOT enrolled in classes	$0.00	.0829	0
Materials/Supplies		Enter sub-total from below:	$100.00
Travel		Enter sub-total from below:	$2,350.00

TOTAL:	$3,650.00
Amount requested for MGS award:	$3,000.00

NOTE: Magellan Scholar awards are processed through "E" funds. All expenditures MUST remain compliant with E fund procurement requirements. All budgets must be reviewed by department business managers prior to submission.

Budget Justification/Description

Student Salary: Indicate estimated number of student research hours per week and hourly rate separated by semesters when student is enrolled in classes or not enrolled in classes (generally fall or spring vs summer semesters).

Student will work an estimated 8 hours per week for the Spring semester, beginning to log hours on January 13, 2014 and ending the first week of May. Not including spring break, this is approximately 15 weeks. 15 * 8 = 120 hours. Any student work outside this period will be unpaid.

Materials/Supplies: Indicate items, quantity, and estimated price. *Be sure to include taxes on all purchases*.

Matlab student version for student's laptop - $100

Travel: Indicate location, purpose of travel, estimate itemized costs (transportation, lodging, registration, etc).

International Symposium on Musical Accoustics
Le Mans, France. July 7-12, 2014

Transportation: $1600 flight
Lodging: $750 for 5 nights in a hotel $1600 + $750 = $2350

If we are not accepted to this conference, we will (pending acceptance) attend one of the other international conferences to which we apply (such as the 2014 International Computer Music Conference and the 14th International Conference for New Interfaces for Musical Expression). The student will apply for a travel grant through the Honors College to cover the remaining travel costs.

Transcript Data

STUDENT INFORMATION

Name : Matthew J. Corley

Birth Date: 08-NOV

Student Type: Continuing

Curriculum Information

Current Program

Bachelor Sci in Computer Sci

College: College of Engr & Computing

Campus: USC Columbia

Major: Computer Science

***Transcript type:ADVS Advising is NOT Official ***

TRANSFER CREDIT ACCEPTED BY INSTITUTION -Top-

: Advanced Placement Exam

Subject	Course	Title	Grade	Credit Hours	Quality Points	R
MATH	141	CALCULUSAB	CR	4.000	0.00	

	Attempt Hours	Passed Hours	Earned Hours	GPA Hours	Quality Points	GPA
Current Term:	0.000	0.000	4.000	0.000	0.00	0.000

Unofficial Transcript

: Advanced Placement Exam

Subject	Course	Title	Grade	Credit Hours	Quality Points	R

CSCE	145	COMPUTSCIA	CR	4.000			0.00

	Attempt Hours	Passed Hours	Earned Hours	GPA Hours	Quality Points	GPA	
Current Term:	0.000	0.000	4.000	0.000	0.00		0.000

Unofficial Transcript

: International Baccalaureate Ex

Subject	Course	Title	Grade	Credit Hours	Quality Points	R
CHEM	111	HL CHEMISTRY	CR	4.000		0.00
CHEM	112	HL CHEMISTRY	CR	4.000		0.00
ENGL	101	HL ENGLISH A1	CR	3.000		0.00
ENGL	102	HL ENGLISH A1	CR	3.000		0.00
HIST	102	HL HIST AMERICAS	CR	3.000		0.00
HIST	112	HL HIST AMERICAS	CR	3.000		0.00

	Attempt Hours	Passed Hours	Earned Hours	GPA Hours	Quality Points	GPA	
Current Term:	0.000	0.000	20.000	0.000	0.00		0.000

Unofficial Transcript

INSTITUTION CREDIT -Top-

Term: Fall 2010

College: Sumter Campus

Major: No Major

Student Type: High School, Concurrent

Academic Standing: Good Standing

Subject	Course	Campus	Level	Title	Grade	Credit Hours	Quality Points	Start and End Dates	R	CEU Contact Hours
MATH	142	USC Sumter	UG	CALCULUS II	A	4.000	16.00			

	Attempt Hours	Passed Hours	Earned Hours	GPA Hours	Quality Points	GPA	
Current Term:	4.000	4.000	4.000	4.000	16.00		4.000
Cumulative:	4.000	4.000	4.000	4.000	16.00		4.000

Unofficial Transcript

Term: Fall 2011

College: College of Engr & Computing

Major: Computer Science

Student Type: New Freshman

Academic Standing: Good Standing

Additional Standing: Dean's List

Subject	Course	Campus	Level	Title	Grade	Credit Hours	Quality Points	Start and End Dates	R	CEU Contact Hours
CSCE	146	USC Columbia	UG	ALGORITHMIC DESIGN II	A	4.000	16.00			
MATH	241	USC Columbia	UG	VECTOR CALCULUS	A	3.000	12.00			
PHYS	211	USC Columbia	UG	ESSENTIALS OF PHYSICS I HONORS	B	3.000	9.00			
PHYS	211L	USC Columbia	UG	ESSENTIALS PHYSICS I LAB HONORS	A	1.000	4.00			
POLI	300	USC Columbia	UG	SOCIAL&POLITCL PHILOSPHY	B+	3.000	10.50			
SPAN	210	USC Columbia	UG	INTERMEDIATE SPAN II	A	3.000	12.00			

	Attempt Hours	Passed Hours	Earned Hours	GPA Hours	Quality Points	GPA	
Current Term:	17.000	17.000	17.000	17.000	63.50		3.735

Cumulative:						21.000	21.000	21.000	21.000	79.50	3.786

Term: Spring 2012

College: College of Engr & Computing

Major: Computer Science

Student Type: Continuing

Academic Standing: Good Standing

Subject	Course	Campus	Level	Title	Grade	Credit Hours	Quality Points	Start and End Dates	R	CEU Contact Hours
CSCE	190	USC Columbia	UG	COMPUTING IN MODRN WRLD	A	1.000	4.00			
CSCE	215	USC Columbia	UG	UNIX/LINUX FUNDAMENTALS	A	1.000	4.00			
HIST	109	USC Columbia	UG	LATIN-AMER CIVILIZATION HONORS	B	3.000	9.00			
MATH	574	USC Columbia	UG	DISCRETE MATH I	A	3.000	12.00			
SCHC	158	USC Columbia	UG	RHETORIC HONORS	C+	3.000	7.50			
SCHC	392B	USC Columbia	UG	PROSEM:INITIATG RES EXP HONORS	C+	1.000	2.50			
SPAN	309	USC Columbia	UG	ADV SPAN LANGUAGE I	B+	3.000	10.50			

	Attempt Hours	Passed Hours	Earned Hours	GPA Hours	Quality Points	GPA
Current Term:	15.000	15.000	15.000	15.000	49.50	3.300
Cumulative:	36.000	36.000	36.000	36.000	129.00	3.583

Term: Fall 2012

College: College of Engr & Computing

Major: Computer Science

Student Type: Continuing

Academic Standing: Good Standing

Additional Standing: Dean's List

Subject	Course	Campus	Level	Title	Grade	Credit Hours	Quality Points	Start and End Dates	R	CEU Contact Hours
CSCE	211	USC Columbia	UG	DIGITAL LOGIC DESIGN HONORS	B+	3.000	10.50			
CSCE	240	USC Columbia	UG	INTRO TO SOFTWARE ENGR	B+	3.000	10.50			
MATH	554	USC Columbia	UG	ANALYSIS I HONORS	B	3.000	9.00			
MATH	587	USC Columbia	UG	INTRO TO CRYPTOGRAPHY	A	3.000	12.00			
SCHC	285B	USC Columbia	UG	PROSEM:NATURL HIST OF SC HONORS	A	4.000	16.00			

	Attempt Hours	Passed Hours	Earned Hours	GPA Hours	Quality Points	GPA
Current Term:	16.000	16.000	16.000	16.000	58.00	3.625
Cumulative:	52.000	52.000	52.000	52.000	187.00	3.596

Unofficial Transcript

Term: Spring 2013

College: College of Engr & Computing

Major: Computer Science

Student Type: Continuing

Academic Standing: Good Standing

Additional Standing: Dean's List

Last Academic Standing: Good Standing

Subject	Course	Campus	Level	Title	Grade	Credit Hours	Quality Points	Start and End Dates	R	CEU Contact Hours
CSCE	212	USC Columbia	UG	INTRO COMPUTER ARCHITECT	A	3.000	12.00			
LING	505R	USC Columbia	UG	TOPIC/MATH LINGUISTICS HONORS	B+	3.000	10.50			
MATH	526	USC Columbia	UG	NUMERICAL LINEAR ALGEBRA	B+	4.000	14.00			
MATH	555	USC Columbia	UG	ANALYSIS II HONORS	B+	3.000	10.50			
STAT	509	USC Columbia	UG	STATISTICS FOR ENGINEERS	A	3.000	12.00			

	Attempt Hours	Passed Hours	Earned Hours	GPA Hours	Quality Points	GPA
Current Term:	16.000	16.000	16.000	16.000	59.00	3.688
Cumulative:	68.000	68.000	68.000	68.000	246.00	3.618

Unofficial Transcript

TRANSCRIPT TOTALS (UNDERGRADUATE) -Top-

	Attempt Hours	Passed Hours	Earned Hours	GPA Hours	Quality Points	GPA
Total Institution:	68.000	68.000	68.000	68.000	246.00	3.618
Total Transfer:	0.000	0.000	28.000	0.000	0.00	0.000
Overall:	68.000	68.000	96.000	68.000	246.00	3.618

Unofficial Transcript

COURSES IN PROGRESS -Top-

Term: Fall 2013

College:				College of Engr & Computing		
Major:				Computer Science		
Student Type:				Continuing		
Subject	**Course**	**Campus**	**Level**	**Title**	**Credit Hours**	**Start and End Dates**
CSCE	311	USC Columbia	UG	Operating Systems	3.000	
CSCE	330	USC Columbia	UG	Programming Language Structures	3.000	
CSCE	350	USC Columbia	UG	CSCE 350: Data Structures and Algorithms (Fall 2013)	3.000	
CSCE	390	USC Columbia	UG	Professional Issues in Computer Science and Engineering	1.000	
ENGL	462	USC Columbia	UG	Technical Writing	3.000	
MATH	546	USC Columbia	UG	HNRS:Algebraic Structures I	3.000	

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Magellan Scholar Mentor Collaboration Form

Student name: Matthew Corley How long have you known student? 1 year

Section I: Please comment briefly on the strengths and weaknesses of the applicant. Please include such factors as intellectual ability, research and writing ability, analytical skills, initiative and maturity, and level of independence (novice through advanced).

Matthew Corley is among the brightest and most talented undergraduates I have met. Since meeting him last year, when he was a sophomore and served as my grader for an upper level math class, it has been a pleasure to work with him.

Matt and I worked together on a project over the summer and we developed a very efficient and productive project workflow using robust tools like the Git revision control system. This enables us to contribute simultaneously on a project and keep a detailed history of all contributions. We have set up a GitHub team for the proposed project, and the associated website can be found at http://soundmath.github.io/GroupSound/

Matt has already taken advance mathematics classes, and has built a solid foundation, but perhaps more important for a project like this is his exceptionally high aptitude for technical subjects. I can explain challenging, even deep mathematical ideas to Matt and he understands surprisingly quickly. I have not achieved this level of communication with any other undergraduate, and only rarely with graduate students.

Matt is very motivated and ambitious, and seems eager to undertake research at a high level. He is an independent thinker and shows strong potential for making original contributions to mathematics and computer science. I think Matt is ideally suited for the research outlined in his proposal, and it will be my pleasure to be his mentor on this project.

I have no doubt in Matt's ability to successfully complete this project and reach the goals and objectives he describes in the project proposal. I expect him to make many interesting new discoveries in this relatively unexplored area.

Section II: Briefly describe how you will be working with the student during this project. Please address the concrete ways you will help the applicant develop the skills and techniques needed to complete project.

Matt and I will meet regularly, once every two weeks or so, to discuss his progress. At each meeting, especially at the start, I will teach him more about group theory and the basic mathematical principles underlying signal processing and harmonic analysis. Matt, in turn, will continue to teach me about software development, and we will discuss progress he has made with both the mathematical and software development aspects of the project. We have already met a few times to discuss some of the mathematical background and, as expected, Matt has quickly picked up some of the most the important principles needed for research in this area.

Matt will also meet with his co-mentor, Professor Reginald Bain, in the Music Department to learn the basic principles of musical acoustics, musical analysis and synthesis, and computer music composition---subjects that are integral to the proposed project.

I will coach Matt as he prepares to present his results at Discovery Day as well as other international mathematics and music conferences. Finally, with the help of Dr. Bain, I will advise Matt about prospects for publishing his results in major academic journals in the field of computer music, musical acoustics, or applied math.

1) **Based on your knowledge of the applicant's abilities and personal characteristics, please rate your recommendation of this candidate for a Magellan Scholar award.**

☑ Very Highly Recommend ☐ Highly Recommend ☐ Recommend With Reservations ☐ Do Not Recommend

2) ✔ *(please check)* **By submitting this form, I confirm that the student applicant was the primary author and the text created is largely his/her own work.**

Mentor Name: William DeMeo

Please do not use this form if you cannot save PDFs.

Magellan Scholar Co-mentor Collaboration Form

Student name: Matthew Corley How long have you known student? _____

Section I: Please comment briefly on your background and expertise as related to this project.
I was asked to serve as a co-mentor for this project because it involves synthesis. I know how to program in a wide variety of digital synthesis programming languages (including Csound, Max/MSP, and SuperCollider), and these or similar tools will be required for the project. I can also help evaluate the results of the project from a musical point of view, that is, help lead the way toward musical applications that may be a natural outcome of this line of research. As an electronic composer with a deep interest in mathematical ideas that lay at the foundation of music, I couldn't be more excited about this opportunity for interdisciplinary collaboration between the USC School of Music and Mathematics Department.

Reginald Bain, Professor
Composition & Theory
Director of the Experimental Music Studio (xMUSE)
USC School of Music

Section II: Briefly describe how you will be working with the student during this project. Please address the concrete ways you will help the applicant develop the skills and techniques needed to complete project.
I will assist Matthew with the musical side of this research. The three of us (Dr. DeMeo, Matthew and myself) will meet in person as necessary and use on-line collaboration tools (like GitHub) to track the progress of the project. I will provide Matthew with appropriate readings from the synthesis theory literature, and advise him regarding software, documentation, tutorials, and audio tools that will be required to render the final audio product.

Based on your knowledge of the applicant's abilities and personal characteristics, please rate your recommendation of this candidate for a Magellan Scholar award.

☐ Very Highly Recommend ☐ Highly Recommend ☐ Recommend With Reservations ☐ Do Not Recommend

Co-mentor Name: _____

Department or Affiliation: _____