What does a nonabelian group sound like?

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Background and prior work. In digital signal processing, the convolution function can be described as a sum of weighted translations. For normal DSP algorithms, this underlying function is defined on the “index set” of the integers modulo n, with the group operation addition.

My faculty mentor, Dr. William DeMeo (Mathematics), initiated research in this direction in 2004 and presented his initial findings at the International Symposium on Musical Acoustics. (His paper [2] appeared in the proceedings and won the conference's “best paper award.”)

Similarly, the idea of indexing signals with nonabelian groups has been applied to two dimensional image data with successful results (see [4]).

See also [3].

TODO: add more stuff here!

Research question. If the underlying index set of a digital signal processing algorithm is modified to various nonabelian groups instead of the ordinary abelian groups, what effects will this have on the audio output?

Project goals and objectives.

- *Goal:* Develop mathematical theory to give sonic characterizations to nonabelian groups.
- *Objective:* Find a short list of nonabelian groups that are useful for nonabelian group audio filters and effects processors, prove their effectiveness both mathematically and experimentally, and document the results.
- *Objective:* Implement a software program that takes audio as input and allows user to apply filters corresponding to specific nonabelian groups to acheive different effects.

Project significance. The proposed research introduces the novel concept of giving sonic characteristics to mathematical groups. There is also a possibility of applications in digital audio engineering, especially if certain nonabelian groups are shown to produce interesting audio effects.

Methodology. We will replace the index set $\mathbb{Z}/n$ with various finite nonabelian groups, using the GAP library's SmallGroups and IsAbelian functions to find appropriate groups to test. For each of the groups tested, we implement the convolution function using the generalized (nonabelian) translation $f(xy^{-1})$ in place of oridinary translation $f(x-y)$ used in classical convolution. (Here, the group multiplication is replacing the traditional addition modulo n as the group operation. We are simply generalizing the underlying translation operation, and the result is a generalized convolution.

Matlab will be used for much of the initial testing and prototyping, as it provides easy methods for constructing WAV files ”from scratch” with its wavwrite() function. Additionally, Myoung An (a colleague of Dr. DeMeo) has made available to us the Matlab code she has developed for her work in image processing, in which she and Richard Tolimieri applied nonabelian group filters to the processing of digital images. This code will be a valuable resource as we seek to apply similar ideas to audio signal processing.

As the project progresses, we will likely use the JavaSound library and implement our generalized DSP algorithms in Java. JavaSound provides methods for reading and altering wav files frequencies and sound intensity levels, which will prove useful when we apply our generalized DSP algorithms to more complex sounds.

TODO: need more specifics here!

Project timeline.

October 2013–December 2013: Become more familiar with current DSP algorithms and gain further knowledge of group theory, and its role in classical DSP implementations.

December 2013–March 2014: Write code to implement DSP algorithms with a selection of nonabelian groups and group operations.

January 2014–May 2014: Identify specific characteristics of groups that make them more (or less) useful as an index set on which to define DSP operations like convolution. Gather and analyze results, and write up reports.

June 2014–October 2014: Submit manuscript to an academic journal. Prepare for and attend conferences.

Anticipated results/Final Products and Dissemination. By the end of the Spring 2014 semester, I expect to have written Matlab programs to test the results of the modified DSP implementations described above. I also expect to have developed a Java software program which allows easy application of nonabelian group filters through a graphical user interface. I hope that the results will prove interesting and have practical applications for computer music composition.

The abstract for this project has already been accepted for presentation at the Joint Mathematics Meetings in Baltimore in 2014. In addition, I will submit our work to the International Computer Music Conference (ICMC) and International Symposium on Musical Accoustics (ISMA), as well as the 14th International Conference on New Interfaces for Musical Expression (NIME). Previous, similar works by my faculty mentor Dr. DeMeo have been accepted at both ICMC and ISMA (see [2] and [1]), so we have high expectations for this project. Furthermore, I will send in the formal paper to several Mathematical and Music scholarly journals. Of course, I will also present the results of our work at Discovery Day 2014.

TODO: add something about Bain's interest/collaboration!

Personal statement. I have never had considerable ability with music, but I have always been fascinated by its intersection with my favorite subject, mathematics. This project piqued my interest because it allows me the rare opportunity to contribute to both fields. I believe that I have developed the necessary skills to succeed in this project through my past mathematical research projects and my current internship developing Java applications at a local software company. A Magellan Grant would allow me the fiscal freedom to dedicate time to engaging in research and possibly traveling to share my findings with international audiences.

References

[1] William DeMeo. Characterizing musical signals with Wigner-Ville interferences. In *Proceedings of the International Computer Music Conference*. ICMC, 2002. Available from: `http://math.hawaii.edu/~williamdemeo/ICMC2002.pdf`.

[2] William DeMeo. Topics in nonabelian harmonic analysis and DSP applications. In *Proceedings of the International Symposium on Musical Acoustics*. ISMA, 2004. Available from: `http://math.hawaii.edu/~williamdemeo/ISMA2004.pdf`.

[3] Richard Tolimieri and Myoung An. *Time-Frequency Representations*. Birkhäuser, Boston, 1998.

[4] Richard Tolimieri and Myoung An. *Group Filters and Image Processing*. Kluwer Acad., 2004. Available from: `http://prometheus-us.com/asi/algebra2003/papers/tolimieri.pdf`.