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Original Article

A Mathematical Optimal Control Approach for Reducing the Danger of Cultism in Higher Institution

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Abstract - *The work formulates an optimal control problem in mathematics for the dynamics of hidden cult operations in higher education institutions, taking into account multiple measures to mitigate the threat. A flow diagram for the model was provided, along with the results of both qualitative and quantitative analysis. The methods used by Momoh et al. (2021) were used to analyze the equilibrium points' local and global stability, respectively. It was discovered that the equilibrium points were unstable if the fundamental reproductive number was greater than one (R_{0c}>1) and locally asymptotically stable if it was less than one (R_{0c}<1). Sensitivity analysis was done on the model's parameters to determine each parameter's involvement in the fundamental reproduction number R_{0c}. Based on the results, the optimality system was created, taking into account three control measures: public enlightenment u_1, vigilant and moral instructions u_2, use of special force and legislation u_3. Furthermore, a numerical simulation was run using the RK4 technique to illustrate the consequences of the control schemes further. According to the research findings, option D which combines public education, vigilante and moral instruction, the use of special force and legislation is the most effective way to end cultism in the university system.*

Keywords - *Mathematical modeling, Cultism, Sensitivity index, Stability, Optimal control.*

1. Introduction

One of the most important and dehumanizing issues facing higher education institutions right now is the threat and hostility of cult members and cult-related activities. There has never been a greater chance of damaging lives and property on campuses, nor has it grown as swiftly and horribly as it has in the present day. Almost no institution is free from this threat, based on popular belief and understanding as of September 2003. According to Okwu (2006), violent confrontations between cult members on Nigerian campuses have claimed the lives of at least 5,000 students and faculty members.

A cult is defined sociologically as a tiny community of religious practices whose convictions are usually esoteric, individualistic, and secretive, according to the Oxford Concise Dictionary of Sociology (1996). Additionally, a secret cult is described by Lexican Webster's Dictionary as a collection of individuals who have a shared goal, whose meeting schedule and agenda are kept hidden from the general public, and rank-and-file initiation is usually done so in private. According to Ajiyi et al. (2010), cultism is defined as a custom performed by a group of individuals whose rules, regulations for admission, first action, and membership are all concealed and maintained that way, with their actions having detrimental consequences for members and non-members alike and impact of peer group, family background, cultural decadence, deterioration of educational standards, and militarization of Nigerian politics as some of the elements discussed as contributing to this menace.

The Pirates Confraternity, established in 1953 at the University of Ibadan by Nobel Laureate Wole Soyinka and others, is the source of cultism in Nigeria's post-secondary educational institutions. The National Association of Sea Dogs, a non-violent fraternity with the skull and crossbones as its emblem, fought against foreign conventions through intellectual and effective means, bringing back the age of chivalry and creating long-lasting solutions to issues of elitism and tribalism (Adewale, 2005). The organization did not engage in covert activities. Similarly, cultism has been a part of our post-secondary institutions for more than thirty years, according to Echekwube (1999). Even if they began more thoughtfully, they have since become aggressive and destructive, particularly in the 1980s. According to credible sources, Ekeanyanwu and Igbinoba (2007), there are 53 cult organizations in the Nigerian higher education system at the moment. These groups consist of Air Lord, Black-Axe, Black Beret Fraternity, Black Bra, Black Cats, Black Mamba, Buccaneers, Cappa Vendetta, Daughters of Jezebel, Eiye, KKK, Knight Cadet, King Cobra, Lucifer Knights, Mafians, Maphites, Mgba Mgba Brothers, Musketeers Fraternity, Oasis of the Silhouette, Panama, Pirates Confraternity, Ostrich Fraternity, Neo-Black Movement Sun Men, Soko, Red Sea Horses, Sea Dogs, Royal Queens, and Red Berets Eden Temple Fraternity, Among many others are the following fraternities: Mafioso, Scorpion, Soires, Ten Angels, Amazons, Apostles, Barracudas, Canary, Dragons, Frigates, Himalayas, Lynx, West End, the Vipers, Viqueens, Trojan Horse, Third Eye Fraternity, Vikings, and Walrus White Angels.

The prevalence of campus cultism in the Nigerian higher education institute system is placed within the larger context of student activism in higher education by Fayokun, KO (2011). He began by analyzing popular forms of student activism and drawing comparisons between traditional student activist organizations and the cults found in Nigeria's Higher Education Institutions. The study observes that cults pose grave risks to the safety of their adherents, institutions, and society as a whole, in addition to being unlike conventional student activist groups. In order to understand why and how cults draw in new members and keep them around, the research also looked further into the beginnings and growth of these organizations. In his conclusion, he stated that social, economic, political, and educational grievances are the root cause of campus cultism. He also added that cults draw students because of unmet needs that make them more likely to join activist groups and that in order to combat cultism effectively, these needs must be met.

In their study, Udoh and Chinwe (2015) found that cultism among university students is a real problem that requires immediate attention from the government and the institution as a whole in order to stop. They also found that in order to find a long-term solution, parents must collaborate closely with the institution's authorities. More significantly, counseling services in higher education should be considered seriously, and counselors are essential to the program's implementation in reducing the occurrence of cultism in higher education. They concluded that the following factors contributed to cultism in post-secondary education: parental involvement in covert cults, societal corruption, broken households, an innate sadistic character, and not punishing individuals found to be involved in cultist activities. And the following were the recommendations made by the authority of the school or institution regarding cultism: hold workshops to inform new students about the perils of cultism during their orientation and establish a department of guidance and counseling at every school to assist in guiding and counseling students.

Mathematical modeling techniques are applied to tackle real-world threats and complex societal concerns. By creating a model on domestic violence, Otoo et al.'s (2014) work illustrated this. They use a numerical method to model the spread of domestic abuse by using a continuous model. Similar to the susceptible, infectious, and recovered paradigm in epidemics, the spread of domestic violence was modeled as a system of differential equations including abusive, vulnerable, and violent victims. They utilized MATLAB software to evaluate the data that they gathered from Tamale's Domestic Violence and Victims Support Unit (DOVVSU). The population of victims of domestic violence is small, according to their study.

A mathematical representation of the mechanisms behind domestic abuse was also developed by Mohammed et al. (2019). It involved the use of six ordinary differential equations, one for each of the following compartments: potentially violent individuals, violent, recovered violent, susceptible victims, victims, and recovered victims. The Routh-Hurwitz criteria were used to conduct stability evaluations once the endemic and free from domestic violence equilibria were discovered. The basic reproduction number, which establishes whether or not domestic violence may be eliminated, was acquired by them. The analysis's findings showed that both the endemic and domestic violence-free equilibria are stable, indicating that violence can be eradicated and that society can continue to function even in its worst cases. Their numerical experiments' outcome concurs with the analytical one.

The fact that the cultism menace spread like a disease in higher education institutions is a reason for addressing this society's ill-using concept of epidemiology and optimal control approach to control the problem. Most of the research work carried out on the menace of cultism in higher education institutes was merely theoretical reviews from the social science background using questionnaires and others. Therefore, it is necessary to develop a mathematical idea with a useful scientific analysis to comprehend the threat and provide a significant remedy. This work focuses on bridging the gap in tackling the threat of cultism by employing an optimal control strategy and a sophisticated mathematical model to describe the dynamics in higher education while also profiling solutions to minimize and possibly eradicate the problem.

2. Formulation of Cultism Model and Description

The model population is divided into four sections at any given time (t) The whole populace, as represented by, N(t) is split up into vulnerable cultists S, low profile cultists, L_c, high profile cultists H_c and school disciplinary committees on cultism D_c. Hence, the breakdown is as follows:

$$N(t) = S(t) + L_c(t) + H_c(t) + D_c(t)$$

Admission to the institution has the effect of recruiting susceptible cultists at a pace Λ. Students from this compartment depart based on their contact rate with either high-profile or low-profile cultists β_c and φ show the modification parameter that indicates that high-profile cultists have a greater influence than low-profile cultists, all of whom are depicted as $\frac{(L_c + \varphi H_c)\beta_c S}{N}$. Low-profile cultists denoted by L_c are those recently enlisted members who, for fear or other reasons, function covertly before rising to prominence as cultists at the rate ε_3 also, because of moral training and school watchfulness, some students move to the disciplinary compartment at the pace η. High-profile cultists H_c are those high-ranking cultists who perpetrate various undesirable acts at schools, such as rape, violent initiation, malpractice, and property destruction. They are brought before the disciplinary committee at the rate specified by school legislation and special force π. For every kid in the section of the school disciplinary committee, D_c . You are either dismissed from the school at the rate δ_3 or pardoned to go back to the vulnerable section at the ω_1. The death rate exists μ at every level, as well as expulsion from the school for participation in either covert cultism δ_1 or high-profile cultism δ_2. The model equations and schematic diagram for the cultism dynamics model are shown in Equation (1) and Figure 1.

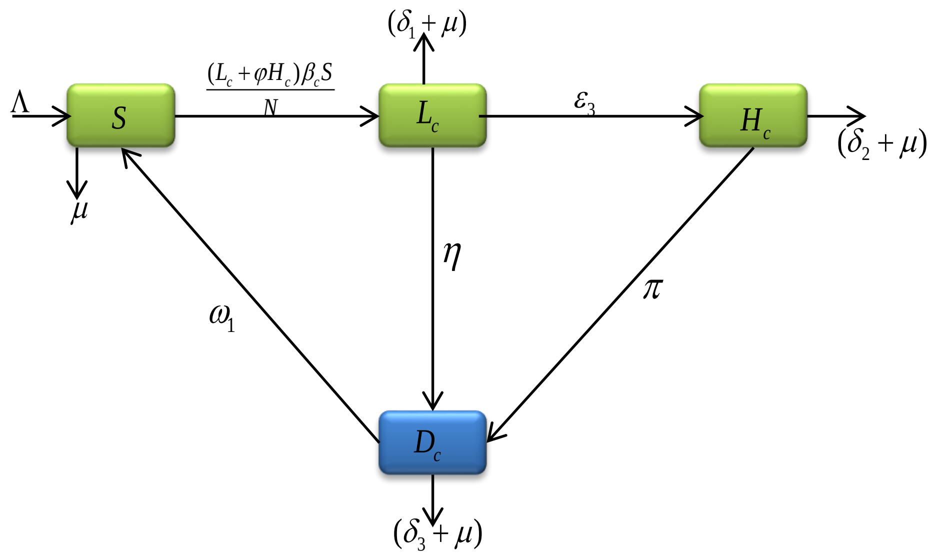


Fig. 1 The cultism dynamics model's schematic diagram

As a result, we provide the cultism model's equations,

$$\left.\begin{aligned}
\frac{dS}{dt} &= \Lambda - \frac{\left(L_c+\varphi H_c\right)\beta_c S}{N} + \omega_1 D_c - \mu S \\
\frac{dL_c}{dt} &= \frac{\left(L_c+\varphi H_c\right)\beta_c S}{N} - (\varepsilon_3+\eta+\delta_1+\mu)L_c \\
\frac{dH_c}{dt} &= \varepsilon_3 L_c - (\pi+\delta_2+\mu)H_c \\
\frac{dD_c}{dt} &= \pi H_c + \eta L_c - (\omega_1+\delta_3+\mu)D_c
\end{aligned}\right\} \tag{1}$$

Under the following initial circumstances:

$$S(0)\geq 0, L_c(0)\geq 0, H_c \geq 0, D_c \geq 0 \tag{2}$$

Table 1. Model parameter

Par / Variable	Description	Values	Reference
Λ	Rate of admission to universities	0.6	Assumed
φ	A modification parameter that indicates how much of an impact the well-known cultist has	0.3	Momoh et.al. (2021)
β_c	Initiation probability of cultism to susceptible students.	0.04	Momoh et al. (2021)
ε_3	Rate of change from low-profile to high-profile cultists	0.16	Momoh et al. (2021)
δ_1	Low-profile cultist-induced rate of departure or expulsion from academic institutions.	0.0035 per year	Momoh et al. (2021)

δ_2	High-profile cultist-instigated rate of expulsion and withdrawal from higher education.	0.014 per year	Momoh et al. (2021)
δ_3	Base disciplinary actions for expulsion/withdrawal from the university system committee measures.	0.018 per year	Assumed
π	Progression rate from high profile cultist to school disciplinary committee on cultism	0.4	Assumed
η	The rate at which a low-profile cultist becomes a member of the school disciplinary committee.	0.34	Assumed
μ	Natural mortality rate.	0.0004	Assumed
ω_1	The disciplinary committee on cultism granted repentant cultists another chance.	0.05	Assumed
S(t)	Susceptible students, at time t	50000	Assumed
L_c (t)	Low profile cultist at time t	30000	Assumed
H_c (t)	High profile cultist at time t	20000	Assumed
$D_c(t)$	School disciplinary committee on cultism at time t	10000	Assumed

3. Analysis of the Cultism Model

In this part, we build the equilibrium points, examine the stability of the model, and demonstrate that the solution to the cultism model equations is bounded.

3.1. Fundamental Characteristics of the Cult Model

3.1.1. Positivity and Solution Boundedness

It is crucial to establish the circumstances in which the system should have positive solutions. If every solution with positive initial conditions stays positive over time, the model would have biological significance.

Theorem 1

Let $S(0), L_c(0), H_c(0), D_c(0)$ be positive, to begin with, then the system (1) has a non-negative solution $S(t)>0, L_c(t)>0, H_c(t)>0, D_c(t)>0,$ for all instants $t>0$. In addition $\limsup_{t\to\infty} N(t)\le\frac{\Lambda}{\mu}$. Also, if $N(0)\le\frac{\Lambda}{\mu}$, then $N(t)\le\frac{\Lambda}{\mu}$. The practical area for model (1) $\Pi=\left\{(S,L_c,H_c,D_c)\in\mathbb{R}_+^4 : N\le\frac{\Lambda}{\mu}\right\}$ is attractive and positive invariant with regard to model (1)

Proof

Firstly, by taking into account the entire population at any particular time:

$N(t)=S(t)+L_c(t)+H_c(t)+D_c(t)$ and using the derivatives, we have obtained,

$$\frac{dN}{d.t}=\frac{dS}{d.t}+\frac{dL_c}{d.t}+\frac{H_c}{d.t}+\frac{D_c}{d.t}$$

Evaluating and considering the absence of cultism, the equations mentioned above become;

$$\frac{dN}{d.t} = \Lambda - \mu N \;\; \Rightarrow \frac{dN}{d.t} \leq \Lambda - \mu N$$

$N \rightarrow \frac{\Lambda}{\mu}$. Thus, the solutions' boundedness inside Π is here by established.

Now, by using the first equation of the model (1) yields,

$$\frac{dS}{dt} + \frac{\left(L_c + \varphi H_c\right)\beta_c S(t)}{N} + \mu S(t) \geq 0,$$

Integrating from $t = 0$ to $t = t$ yields,

$$\frac{d}{dt}\left[S(t)\exp\left\{\int_0^t \frac{\left[\left(L_c + \varphi H_c\right)\beta_c\right]}{N(t)}\varpi d\varpi + \mu t\right\}\right] \geq 0$$

It means that,

$$S(t) \geq \mathrm{S}(0)\exp\left\{-\left(\int_0^t \frac{\left[\left(L_c + \varphi H_c\right)\beta_c\right]}{N(t)}\varpi d\varpi + \mu t\right)\right\} > 0, \forall t > 0$$

To illustrate that $L_c(t)$, $H_c(t)$, $D_c(t)>0$ stay positive during the entire moment $t>0$ we used a similar strategy. As a result, the variable in the remaining equation is always non-negative $t>0$ We conclude that the solutions to model (1) are positively invariant and appealing within a certain region Π We inferred from the theorem that system (1) is both theoretically and physiologically possible in Π.

3.2. Cultism Equilibrium States

There are two equilibrium states in the cultism model: the cultism present stable condition and the cultism free stable condition. Moreover, they are obtained as follows:

3.2.1. Cultism Free Equilibrium State

The cultism free stable condition $\left(E_C^*\right)$ for the system (1) is given by (CFE). The CFE denoted by,

$$E_c^* = \left(S^*, L_c^*, H_c^*, D_c^*\right) \text{is given by,}$$

$$E_c^* = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right) \tag{3}$$

3.2.2. Existence of Cultism Present Stable Condition

For the cultism present stable condition, we let $E_c^{**}=(S^{**}, L_c^{**}, H_c^{**}, D_c^{**})$ the cultism present stable condition of the model (1). Further, we let the force of cultism be,

$\xi^{**} = \dfrac{\left(L_c^{**} + \varphi H_c^{**}\right)\beta_c}{N}$ and $N^{**} = S^{**} + L_c^{**} + H_c^{**} + D_c^{**}$ be the total population, and we solve system (1) at a steady state to get,

$$\left.\begin{aligned}
S^{**} &= \frac{k_1 k_2 H_c^{**}}{\xi^{**}} \\
L_c^{**} &= \frac{k_2 H_c^{**}}{\varepsilon_3} \\
H_c^{**} &= \frac{\xi^{**}\Lambda\varepsilon_3 k_3}{(\xi^{**}+\mu)k_1k_2k_3 - \xi^{**}\omega_1(\varepsilon_3\pi + \eta k_2)} \\
D_c^{**} &= \frac{\left(\varepsilon_3\pi + \eta k_2\right)H_c^{**}}{\varepsilon_3 k_3}
\end{aligned}\right\} \tag{4}$$

Where,

$$\mathrm{k}_1 = (\varepsilon_3 + \eta + \delta_1 + \mu),\ k_2 = (\pi + \delta_2 + \mu),\ k_3 = (\omega_1 + \delta_3 + \mu)$$

3.3. Basic Reproduction Number

The fundamental reproduction number, in a completely sensitive student population, is the average number of secondary cultists recruited by one cultist R_{0c}. In order to determine the system's fundamental reproduction number (1), we used the procedure in (Diekmann, Heesterbeek, and Roberts, 2010) to get,

$$F = \begin{pmatrix} \dfrac{\beta_c S^*}{N^*} & \dfrac{\varphi\beta_c S^*}{N^*} \\ 0 & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} \varepsilon_3 + \eta + \delta_1 + \mu & 0 \\ -\varepsilon_3 & \pi + \delta_2 + \mu \end{pmatrix}$$

Thus, the basic reproduction number of the paradigm of cultism (1) is represented by R0c= (FV^{-1}) is given by,

$$R_{0c} = \frac{\beta_c(\varphi\varepsilon_3 + \pi + \mu + \delta_2)}{(\varepsilon_3 + \eta + \delta_1 + \mu)(\pi + \delta_2 + \mu)} \tag{5}$$

3.4. Local Stability of Cultism-Free Equilibrium State

Theorem 2

The cultism free stable condition of the model Equation (1) is locally and asymptotically stable (LAS) if $R_0<1$ and unstable if $R_0>1$.

Proof

The Jacobian matrix of the system (1) at cultism free stable condition $J(E_C^*)$ is given by,

$$\begin{bmatrix} -\mu & -\beta_c & -\varphi\beta_c & \omega_1 \\ 0 & -r_1 & \varphi\beta_c & 0 \\ 0 & \varepsilon_3 & -r_2 & 0 \\ 0 & \eta & \pi & -r_3 \end{bmatrix} \tag{6}$$

Where,

$$\left. \begin{array}{l} r_1 = (\varepsilon_3 + \eta + \delta_1 + \eta) - \beta_c,\ r_2 = \pi + \delta_2 + \mu, \\ r_3 = \omega_1 + \delta_3 + \mu \end{array} \right\}$$

The polynomial features for $J(E^0)$ in (6) are hereby defined as follows:

$$A_0\lambda^4 + A_1\lambda^3 + A_2\lambda^2 + A_3\lambda + A_4 \tag{7}$$

Where the coefficients are given by:

$$\begin{array}{l} A_0 = 1,\ A_1 = \mu + r_1 + r_2 + r_3, \qquad A_2 = r_2(\mu + \mathrm{r}_1) + \mu\,\mathrm{r}_1 + r_3(\mu + \mathrm{r}_1 + \mathrm{r}_2) - \varphi\varepsilon_3\beta_c), \\ A_3 = (r_3(\mathrm{r}_2(\mu + \mathrm{r}_1) + \mu\,\mathrm{r}_1 - \varphi\varepsilon_3\beta_c) + \mu\,\mathrm{r}_1\,r_2 - \varphi\varepsilon_3\beta_c(\mu + \mathrm{r}_1) + \varphi\varepsilon_3\beta_c r_1), \\ A_4 = r_3(\mu\,\mathrm{r}_1\mathrm{r}_2 - \varphi\varepsilon_3\beta_c(\mu + \mathrm{r}_1) + \varphi\varepsilon_3\beta_c\,\mathrm{r}_1) \end{array}$$

Applying the Routh-Hurwitz criterion, which states that if the matrix's determinant and coefficients are positive, then all roots of the polynomial (7) have a negative real component $H_i>0$, for $i = 1,\ldots,4,$ it is clear that $A_i>0$ for $i = 0,\ldots,4$ are positive and when $R_0<0$.

The Routh-Hurwitz array matrix setup allowed us to get the following:

$$H = \begin{vmatrix} A_1 & A_0 & 0 & 0 \\ A_3 & A_2 & A_1 & A_2 \\ 0 & A_4 & A_3 & A_2 \\ 0 & 0 & 0 & A_4 \end{vmatrix} \tag{8}$$

H is called the Hurwitz matrix, and the principal minors are,

$$H_1 = A_1 > 0$$

$$H_1 = \mu + r_1 + r_2 + r_3 > 0$$

$$H_2 = \begin{vmatrix} A_1 & A_0 \\ A_3 & A_2 \end{vmatrix} = A_1A_2 - A_0A_3 > 0$$

$$\Rightarrow A_1A_2 > A_0A_3$$

$$H_3 = \begin{vmatrix} A_1 & A_0 & 0 \\ A_3 & A_2 & A_1 \\ 0 & A_4 & A_3 \end{vmatrix}$$

$$H_3 = A_1\begin{vmatrix} A_2 & A_1 \\ A_4 & A_3 \end{vmatrix} - A_0\begin{vmatrix} A_1 & 0 \\ 0 & A_3 \end{vmatrix} + 0\begin{vmatrix} A_1 & A_0 \\ A_3 & A_2 \end{vmatrix}$$

$$= A_1A_2A_3 - A_1^2A_4 - A_0A_1A_3$$

$A_1A_2A_3 - A_1^2A_4 - A_1A_3 > 0$ if and only if $A_1A_2A_3 > (A_1^2A_4 + A_1A_3)$

$$H_4 = \begin{vmatrix} A_1 & A_0 & 0 & 0 \\ A_3 & A_2 & A_1 & A_0 \\ A_5 & A_4 & A_3 & A_2 \\ 0 & 0 & A_5 & A_4 \end{vmatrix} = A_1\begin{vmatrix} A_2 & A_1 & A_0 \\ A_4 & A_3 & A_2 \\ 0 & 0 & A_4 \end{vmatrix} - A_0\begin{vmatrix} A_1 & 0 & 0 \\ 0 & A_3 & A_2 \\ 0 & 0 & A_4 \end{vmatrix}$$

$$= A_1A_2\begin{vmatrix} A_3 & A_2 \\ 0 & A_4 \end{vmatrix} - A_1^2\begin{vmatrix} A_2 & A_0 \\ 0 & A_4 \end{vmatrix} + A_1A_0\begin{vmatrix} A_2 & A_1 \\ A_4 & A_3 \end{vmatrix} - A_0A_1\begin{vmatrix} A_3 & A_2 \\ 0 & A_4 \end{vmatrix}$$

The equation is greater than 0 if and only if,

$$= A_1A_2A_3A_4 + A_1A_2A_3 > (A_1^2A_2A_4 + A_1^2A_4 + A_1A_3A_4)$$

As a result, the polynomial (7) 's eigenvalues are all negative in real parts, suggesting that $\lambda_1,\lambda_2,\lambda_3,\lambda_4<0$. Since all the values of $\lambda_i=1,2,3,4$. when $R_{0c}<1$ We get to the conclusion that there is a locally asymptotically stable cultism free equilibrium point.

3.5. Global Stability of the Cultism -Free, Stable Condition

Utilizing the technique developed by Castillo et al. (2002), the global asymptotic stability of the equilibrium free of cultism is identified. Using this approach, two prerequisites that ensure the cult-free state's worldwide stability were taken into account. Therefore, our system of Equation (1) is re-written in the following form:

$$\left.\begin{aligned} \frac{dX}{dt} &= F\left(X,Z\right) \\ \frac{dZ}{dt} &= G\left(X,Z\right), G\left(X,0\right) = 0 \end{aligned}\right\} \tag{9}$$

Where, $X=\left(S,D_c\right)$ represent the student populations that are not cultist $X\in\Re^2$, while $Z=\left(L_c,H_c\right)$ represent the student populations that are cultist $Z\in\Re^2$. We denote the cultism-free state by $E^0=\left(X^0,0\right)$. The following two conditions H_1 and H_2 must be met to guarantee a global asymptotic stability:

$$H_1:\frac{dX}{dt}=F\left(X^0,0\right), X^0 \text{ is Globally Asymptotically Stable (GAS).}$$

$$H_2:G\left(X,Z\right)=CZ-G\left(X,Z\right), \text{where } G\left(X,Z\right)\geq 0,\ for\left(X,Z\right)\in\Omega$$

Where, $C=D_ZG\left(X^0,0\right)$ is a $M-matrix$ (the off-diagonal of C are non-negative) and Ω represents the biologically viable area.

Lemma 1

The point $K^0=\left(X^0,0\right)$ is called stable global asymptotic stable point, if in addition $R_{0m}<1$ and the conditions H_1 & H_2 hold. The following theorem is formed:

Theorem 3

The cultism free, stable condition point for the cultism model is globally asymptotically stable when $R_{0m}<1$ and unstable when $R_{0m}>1$.

Proof

For condition H_1, let $X=(S,D_c)$, as the none cultist student state variables (compartments), and all the cultism variables $\left(L_c,D_c\right)$ are zero. The Jacobian matrix of $F(X,0)$ is given as,

Where, $X^0=(\frac{\Lambda}{\mu},0)$

$$X\in\square\Rightarrow$$

$$\left.\begin{aligned}\frac{dS}{dt}&=\Lambda-\frac{\left(L_c+\varphi H_c\right)\beta_c S}{N}+\omega_1 D_c-\mu S\\ \frac{dD_c}{dt}&=\pi H_c+\eta L_c-(\omega_1+\delta_3+\mu)D_c\end{aligned}\right\}\tag{10}$$

Assessing (10) based on the condition of the matrix $F(X,0)$, we have

$$F(X,0)=\begin{bmatrix}\Lambda+\omega_1 D_c-\mu S\\ -(\omega_1+\delta_3+\mu)D_c\end{bmatrix}\tag{11}$$

Assessing the Jacobean matrix of Equation (11) at cultism free, stable conditions gives

$$J_{F(X,0)} = \begin{bmatrix} -\mu & \omega_1 \\ 0 & -(\omega_1 + \delta_3 + \mu) \end{bmatrix} \tag{12}$$

It is evident that, according to Hurwitz criteria, Equation (12) has negative real roots for each of the Eigen-values. Thus, $H_1 : \frac{dX}{dt} = F(X^0, 0)$ is globally asymptotically stable since the eigenvalues are negative.

For condition H_2: Taking the cultism compartment of the system (1), we have

$$\left.\begin{aligned} \frac{dL_c}{dt} &= \frac{(L_c + \varphi H_c)\beta_c S}{N} - (\varepsilon_3 + \eta + \delta_1 + \mu)L_c \\ \frac{dH_c}{dt} &= \varepsilon_3 L_c - (\pi + \delta_2 + \mu)H_c \end{aligned}\right\} \tag{13}$$

Evaluating the Jacobian matrix of Equation (13) at cultism-free equilibrium, we have

$$C = \begin{bmatrix} \beta_c - (\varepsilon_3 + \eta + \delta_1 + \mu) & \varphi\beta_c \\ \varepsilon_3 & -(\pi + \delta_2 + \mu) \end{bmatrix} \begin{bmatrix} L_c \\ H_c \end{bmatrix} \text{ and}$$

$$G(X, Z) = \begin{bmatrix} G_1(X, Z) \\ G_2(X, Z) \end{bmatrix}$$

$$= \begin{bmatrix} (L_c + \varphi H_c)\beta_c (1 - \frac{S}{N}) \\ 0 \end{bmatrix} \tag{14}$$

From (14), it is clear that $\hat{G}(X, Z) \geq 0$. and hence the two conditions hold by lemma 1. We get to the conclusion that system (1) 's cultism-free equilibrium is globally asymptotically stable.

3.6. Sensitivity Analysis of the Cultism Model

The effect of Equation (5) 's model parameters on the basic reproduction number R_{0C} is discussed in this subsection. We look into how these criteria affect the threat of cultism. To determine the impact, we partially differentiate R_{0C} w.r.t the model parameters in Equation (5) in an attempt to compute the sensitivity index. We used the formula as presented in (Legesse and Sheferaw 2020), $\varphi_{\chi}^{R_{0C}} = \frac{\partial R_{0C}}{\partial \chi} \times \frac{\chi}{R_{0C}}$, where χ is the parameter being tested.

The sensitivity index of R_{0C} with respect to $\beta_c, \varphi, \varepsilon_3, \delta_1, \pi, \mu, \delta_2$ and η, are given as follows:

$$\varphi_b^{R_{0C}} = \frac{\partial R_{0C}}{\partial \beta_c} \times \frac{\beta_c}{R_{0C}}$$

Table 2. Sensitivity indices for R_{0c}

Par.	Baseline Values	References	Remarks
φ	0.3	Momoh et.al. (2021)	Positive
ε_3	0.16	Momoh et.al., (2021)	Negative
β_c	0.04	Momoh et al. (2021)	Positive
π	0.06	Assumed	Positive
μ	0.00004	Momoh et.al., (2021)	Negative
δ_1	0.0035 per year	Momoh et.al., (2021)	Negative
δ_2	0.014 per year	Momoh et.al., (2021)	Negative
η	0.34	Assumed	Negative

4. Optimal Control of the Cultism Model

Three control measures were added to the cultism model Equation (1) based on the results of the sensitivity analysis, which were derived in subsection (3.6) and displayed in Table 2, public enlightenment u_1, vigilant and moral instructions u_2, use of special force and legislation u_3.

$$\left.\begin{aligned}
\frac{dS}{dt} &= \Lambda-(1-u_1)\frac{\left(L_c+\varphi H_c\right)\beta_c S}{N}+\omega_1 D_c-\mu S\\
\frac{dL_c}{dt} &= (1-u_1)\frac{\left(L_c+\varphi H_c\right)\beta_c S}{N}-(\varepsilon_3+u_2\eta+\delta_1+\mu)L_c\\
\frac{dH_c}{dt} &= \varepsilon_3 L_c-(u_3\pi+\delta_2+\mu)H_c\\
\frac{dD_c}{dt} &= u_3\pi H_c+u_2\eta L_c-(\omega_1+\delta_3+\mu)D_c
\end{aligned}\right\} \tag{15}$$

with starting parameters $S(0)=S_0,\ \ L_c(0)=L_{c0},\ H_c(0)=H_{c0},\ D_c(0)=D_{c0},$

The purpose of this part is to investigate the optimal workload required to mitigate the threat of cultism in higher education. In order to achieve this, we develop an objective function,

$$J=\min_{u_1,u_2,u_3}\int_0^{t_f}\left(B_1L_c+B_2H_c+\frac{C_1u_1^2}{2}+\frac{C_2u_2^2}{2}+\frac{C_3u_3^2}{2}\right)dt \tag{16}$$

Since reducing the number of cultists in higher education institutions at the lowest possible cost of control is our goal $u_1(t)$, $u_2(t)$, and $u_3(t)$. we seek optimal control u_1^*,u_2^* and u_3^* that

$$J\left(u_1^*,u_2^*,u_3^*\right)=\underset{u_1,u_2,u_3\in\Pi}{MIN}\ J\left(u_1,u_2,u_3\right) \tag{17}$$

Where, Π is the collection of quantifiable functions derived from [0 t f] onto [0 1] Pontryagin's Maximum Principle, Pontryagin, Boltyanskii, Gemkrelidze, and Mishchenko (1962) offer the necessary conditions that an ideal control needs to satisfy. Equation (15) shows that an optimal control exists based on the state variables' adjoint variable. This principle converts (15) into a problem of minimizing pointwise a Hamiltonian H, with respect to (u_1, u_2, u_3). The Hamiltonian is given as

$$\left.\begin{aligned}
H =& \left(Q_1 L_c + Q_2 H_c + \frac{C_1 u_1^2}{2} + \frac{C_2 u_2^2}{2} + \frac{C_3 u_3^2}{2} \right) \\
&+ \lambda_S \left(\Lambda - (1-u_1)\frac{(L_c + \varphi H_c)\beta_c S}{N} + \omega_1 D_c - \mu S \right) \\
&+ \lambda_{L_c} \left((1-u_1)\frac{(L_c + \varphi H_c)\beta_c S}{N} - (\varepsilon_3 + u_2\eta + \delta_1 + \mu) L_c \right) \\
&+ \lambda_{H_c} \left(\varepsilon_3 L_c - (u_3\pi + \delta_2 + \mu) H_c \right) \\
&+ \lambda_{D_c} \left(u_3 \pi H_c + u_2 \eta L_c - (\omega_1 + \delta_3 + \mu) D_c \right)
\end{aligned}\right\} \tag{18}$$

Where, $\lambda_S, \lambda_{L_c}, \lambda_{H_c}, \lambda_{D_c}$ are the adjoint variable or co-state variable and

$$N = S + L_c + H_c + D_c$$

Theorem 4

Given an optimal control $u_1^*,\ u_2^*,\ u_3^*$ and S, L_c, H_c, D_c of the corresponding state systems (15) that minimize $J(u_1, u_2, u_3)$ over Π. Then, there exists adjoin variables $\lambda_S, \lambda_{L_c}, \lambda_{H_c}, \lambda_{D_c}$ Satisfying,

$$-\frac{\partial \lambda_j}{\partial t} = \frac{\partial H}{\partial j}$$

Where, λ_j for $i = S, L_c, H_c, D_c$ are the adjoint variables and the controls u_1^*, u_2^*, u_3^* obey the optimality conditions. Such that,

$$\left.\begin{aligned}
u_1^* &= \max\left\{0,\ \min\left[1, \frac{\left(\frac{(L_c^* + \varphi H_c^*)\beta_c S^*}{N^*}\right)(\lambda_S - \lambda_{L_c})}{C_1}\right]\right\} \\
u_2^* &= \max\left\{0,\ \min\left[1, \frac{\eta L_c^*(\lambda_{L_c} - \lambda_{D_c})}{C_2}\right]\right\} \\
u_3^* &= \max\left\{0,\ \min\left[1, \frac{\pi H_c(\lambda_{H_c} - \lambda_{D_c})}{C_3}\right]\right\}
\end{aligned}\right\} \tag{19}$$

Proof

Differentiating the Hamiltonian function evaluated at the optimal control yields the differentiable equations controlling the adjoint variables. Thus, the adjoint system is:

$$\left.\begin{aligned}
-\frac{H_\tau}{dS} &= \left((1-u_1)\frac{(L_c+\varphi H_c)\beta_c}{N}\right)\left(\lambda_S-\lambda_{L_c}\right)+\mu\lambda_S \\
-\frac{H_\tau}{dL_c} &= -B_1+\left((1-u_1)\frac{\beta_c S}{N}\right)\left(\lambda_S-\lambda_{L_c}\right)+\varepsilon_3\left(\lambda_{L_c}-\lambda_{H_c}\right)+u_2\eta\left(\lambda_{L_c}-\lambda_{D_c}\right)+\left(\delta_1+\mu\right)\lambda_{L_c} \\
-\frac{H_\tau}{dH_c} &= -B_2+\left((1-u_1)\frac{\varphi\beta_c S}{N}\right)\left(\lambda_S-\lambda_{L_c}\right)+u_3\pi\left(\lambda_{H_c}-\lambda_{D_c}\right)+(\delta_2+\mu)\lambda_{H_c} \\
-\frac{H_\tau}{dD_c} &= \omega_1(\lambda_{D_c}-\lambda_S)+(\delta_3+\mu)\lambda_{D_c}
\end{aligned}\right\} \tag{20}$$

with transversality conditions:

$$\lambda_S(t_f)=\lambda_{L_c}(t_f)=\lambda_{H_c}(t_f)=\lambda_{D_c}(t_f)=0.$$

Hence, solving $\frac{\partial H_\tau}{\partial u_1},\frac{\partial H_\tau}{\partial u_2}$ and $\frac{\partial H_\tau}{\partial u_3}$ and equating the result to zero gives the characterization of controls

$$\left.\begin{aligned}
u_1^* &= \max\left\{0,\ \min\left[1,\frac{\left(\frac{(L_c^*+\varphi H_c^*)\beta_c S^*}{N^*}\right)(\lambda_S-\lambda_{L_c})}{C_1}\right]\right\} \\
u_2^* &= \max\left\{0,\ \min\left[1,\frac{\eta L_c^*(\lambda_{L_c}-\lambda_{D_c})}{C_2}\right]\right\} \\
u_3^* &= \max\left\{0,\ \min\left[1,\frac{\pi H_c(\lambda_{H_c}-\lambda_{D_c})}{C_3}\right]\right\}
\end{aligned}\right\} \tag{21}$$

Because the state system (15) and the adjoint system (20) were apriori bounded, the optimality system's uniqueness was established. We limit the duration of the time interval $[0, t_f]$ to ensure the uniqueness of the optimality system (Michael, M., Libin, M. and Weimin, H, 2012) and (Okosun, K.O., Ouifki Rachid, and Nazir Marcus, 2011). We conclude that by standard control arguments involving the bounds on the controls (Suzanne, M.L., John, T.W. 2007).

$$u_i^* = \begin{Bmatrix} 0 & 0 & u_i^c \le 0 \\ u_1^c & if & 0 < u_i^c < 1 \\ 0 & if & u_i^c \ge 0 \end{Bmatrix},\ for\, i=1,2,3.$$

Where,

$$u_1^* = \frac{\left(\frac{(L_c^* + \varphi H_c^*)\beta_c S^*}{N^*}\right)(\lambda_S - \lambda_{L_c})}{C_1}$$

$$u_2^* = \frac{\eta L_c^* (\lambda_{L_c} - \lambda_{D_c})}{C_2}$$

$$u_3^* = \frac{\pi H_c (\lambda_{H_c} - \lambda_{D_c})}{C_3}$$

5. Numerical Simulations

To demonstrate the value of the controls taken into consideration, computational models of the suggested cultism model with controls (15) and without controls (1) were examined. To determine the model's solution, a numerical method called the RK4 is applied. Table 1 provides the parameter values used in the simulation process. Years are used as the time level in the graphical representation. Both the weight and the balancing constants are regarded as $B_1 = 0.01,\ B_2 = 0.05, C_1 = 2.0, C_2 = 1.5,\ C_3 = 2.7$. The dashed blue curves in Figures 2-5 illustrate the dynamics of the various population classes in the absence of control measures, while the bold red curves depict the population behavior in the presence of control variables.

In the sections that follow, we examined four distinct approaches to eliminating cultism in the university system, each utilizing a different set of the three control above variables. A graphic representation shows the impact and efficacy of each instance on the elimination of the threat. For the cultism dynamics state variables, cultism aversion was also computed, with the results shown in Tables 3 to 6.

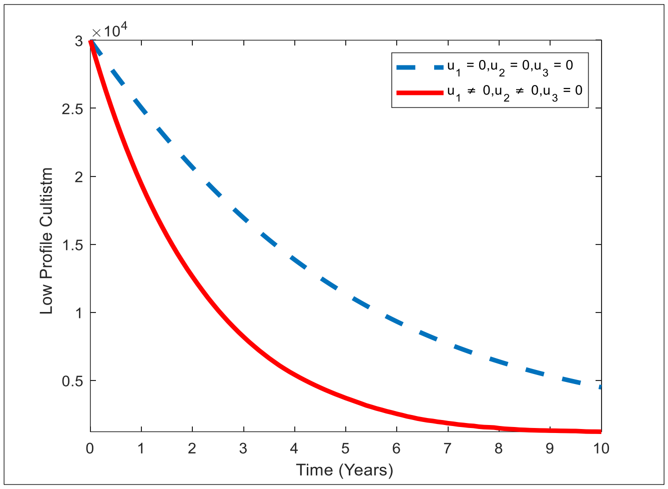
5.1. Control Strategies A: Public Enlightenment, Vigilant and Moral Instruction

The first strategy we try to implement here is by making the control $u_3 = 0$ (i. e., use of specific force and legislation) while public enlightenment, vigilant and moral instruction are implemented ($u_1 \neq 0$, $u_2 \neq 0$, $u_3 \neq 0$). The aversion table and the graphic results are displayed below. It is demonstrated graphically how the first strategy affects the dynamics of cultism in Figure 2's sub-graphs (a) and (b). Table 3 further presents this numerically, demonstrating that the low profile and persons under the high profile class fell dramatically when using this set of controls. The matching control profile for the current strategy is displayed in Figure 2(c). The graphical results of this case reveal that the combined application of u_1 and u_2 is practical and effective in reducing the amount of cultism that escapes from the university system.

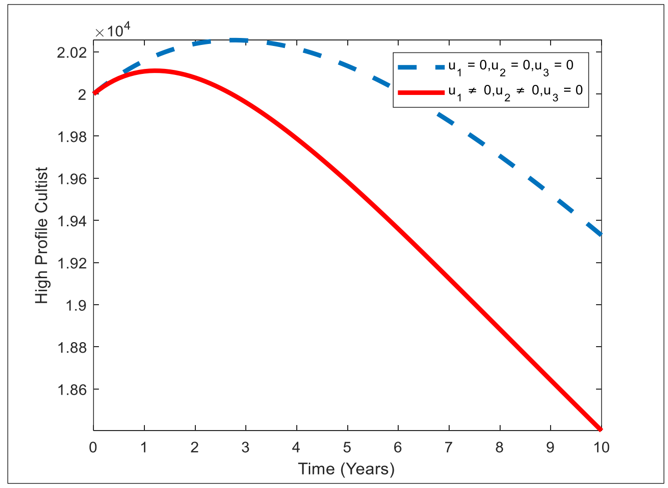
The outcomes of the cultism spread variables in the system with and without controls are displayed in Table 3. The initiation aversion that arises from the application of the controls on cultism-public enlightenment, vigilante, and moral instruction is also displayed. When put into practice, this plan might eliminate roughly 4193.3 potential cultists from the student body at universities.

Table 3. Cultism aversion when public enlightenment and vigilant and moral instruction are executed

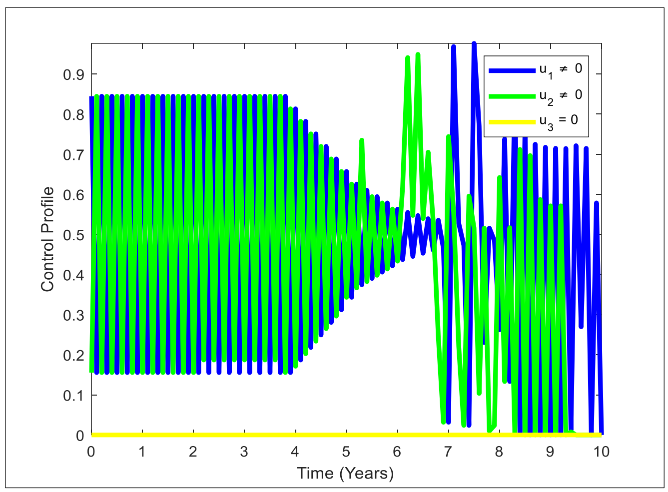
Variables	Without Control	With Control Strategy A	No. of Cultists Averted
L_c	4490.4	1224.1	3266.3
H_c	19329	18402	927
TIA			4193.3



(a)



(b)



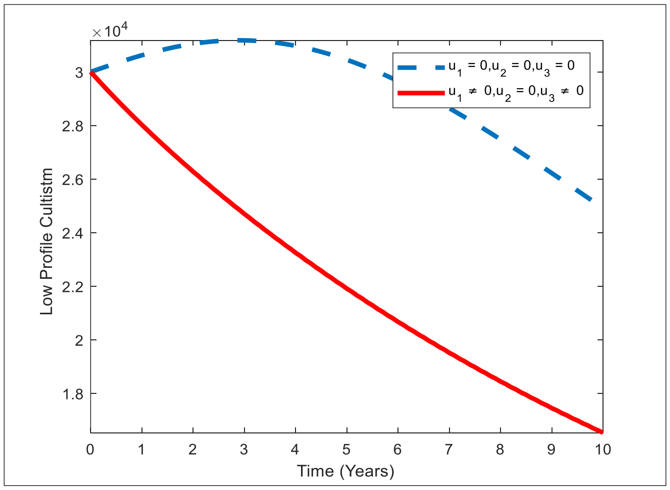
(c)

Fig. 2 Public enlightenment and vigilant and moral instruction

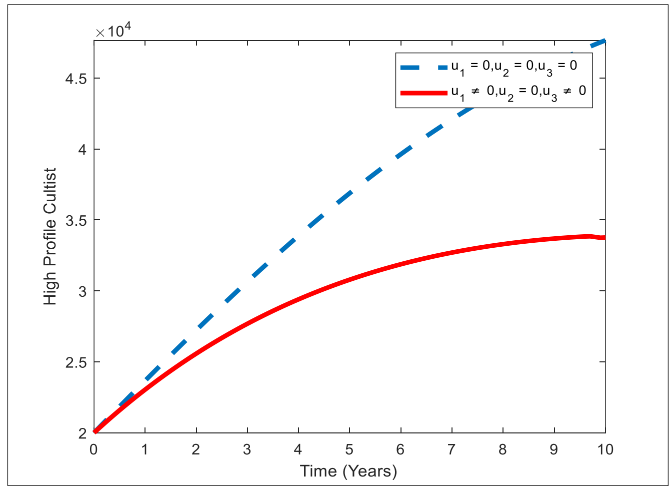
5.2. Control Strategy B: Vigilant and Moral Instruction, Use of Specific Force and Legislation

This strategy demonstrates the impact of the execution of public enlightenment and the use of specific force and legislation, which is denoted by u_1 and u_3 , respectively, the dynamics of cultism. For this purpose, the vigilant and moral instruction control is taken zero ($u_3 = 0$) in the control model (15), and the desired numerical results graphically are given in Figure 3.

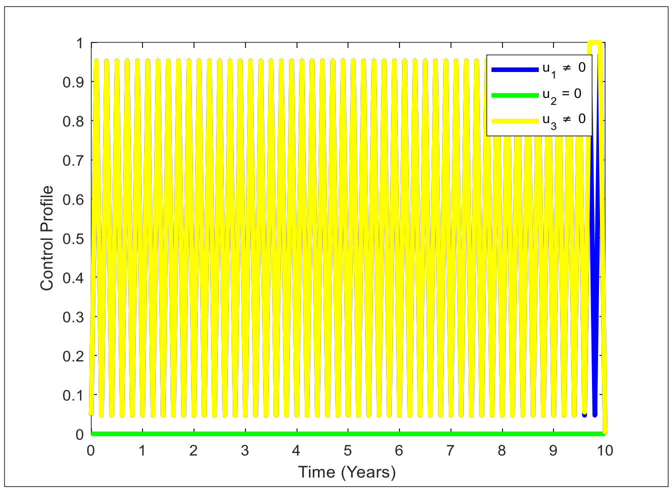
Figure 3, with sub-plots (a-b), graphically depicts the effects of public education, the use of specific force, and legislation control on various population classes. It is evident that, overall, strategy B appears to be the most effective strategy when compared to strategy A. The number of cases of cultism further supports this spread shown in Table 4, which indicates that, on average, 11049 cases of cultism spread were prevented by the implementation of strategy B. Figure 3(c) presents the corresponding control profile.



(a)



(b)



(c)

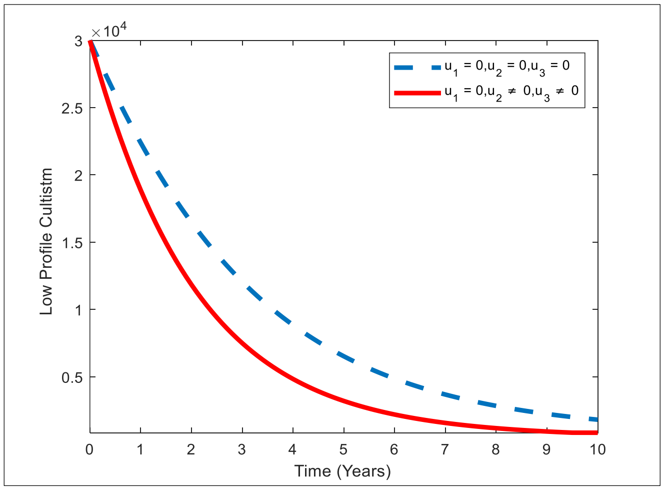
Fig. 3 The public enlightenment and use of specific force and legislation

Table 4. Aversion when public enlightenment and use of specific force and legislation is implemented

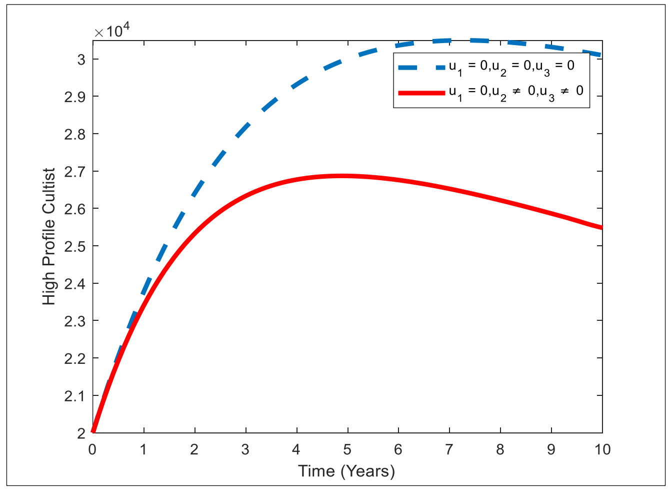
Variables	Without Control	With Control Strategy A	No. of Cultists Averted
L_c	22775	13747	9028
H_c	2777.4	755.8	2021.6
TIA			**11049.6**

5.3. Control Strategy C: Vigilant and Moral Instruction and Use of Specific Force and Legislation

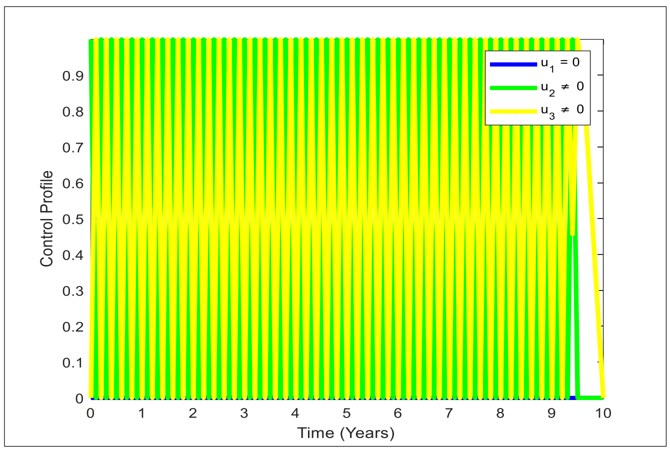
In this case, we put the public enlightenment control equal to zero, i.e., u_1=0 to understand the dynamics of the cultism dynamic, take into consideration the control over moral and vigilant training, the use of specific force, and the control over laws. The impact of this method is graphically displayed in Figures 4(a) and (b), whereas the matching control profile of this example is shown in sub-plot Figure 4(c). When compared to earlier tactics, it is evident that poor performance occurs in the absence of public awareness, as demonstrated by Table 5's list of cultism aversion. The strategy's ability to stop the spread of cultism is demonstrated by the approximately 5583 cultists who were prevented from entering the university system as a result of its implementation.



(a)



(b)



(c)

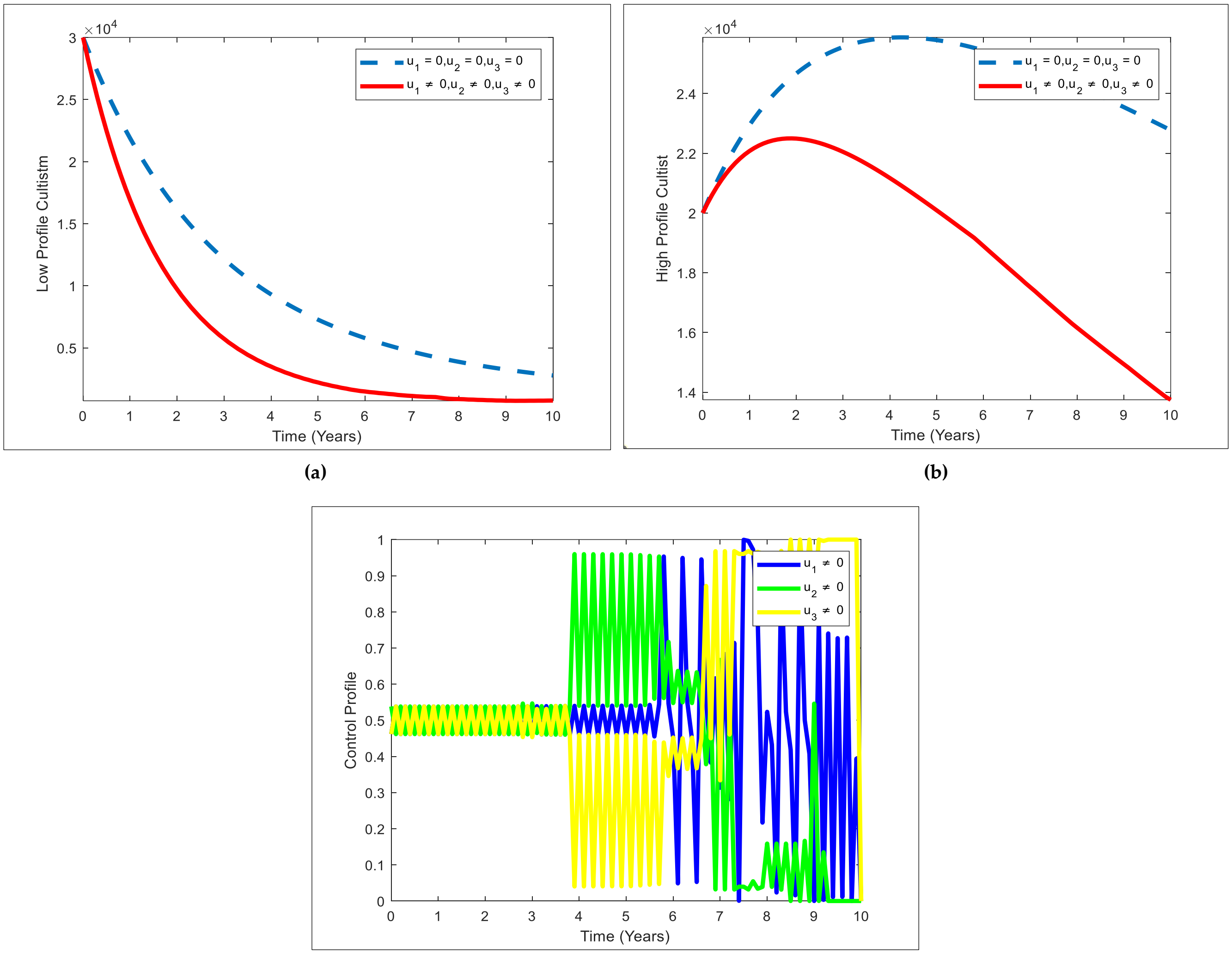
Fig. 4 The vigilant and moral instruction and use of specific force and legislation

Table 5. Aversion when public enlightenment and vigilant and moral instruction are implemented

Variables	Without Control	With Control Strategy A	No. of Cultists Averted
L_c	1818.5	849.4838	969.0162
H_c	30093	25479	4614
TIA			**5583.0162**

5.4. Control Strategy D: Public Enlightenment, Vigilant and Moral Instruction and Use of Specific Force and Legislation

Using all three of the controls public education, vigilante and moral training, and the deployment of special force and legislation this tactic enforced the dynamics of cultism spread. The impact of this method is graphically depicted in Figures 5(a) and (b), whereas the matching control profile of this example is shown in sub-plot Figure 5(c). When compared to earlier tactics, it is evident that ineffective condom use leads to poor performance, as seen by the infection aversion chart in chart 5. By using this tactic, about 22278 incidences of cultism were avoided. It is clear from the graphical interpretations of all four sets of controls that were described that the second technique is the most effective one for getting rid of cultism in the university system. This is further demonstrated at a glance by the graphs and the aversion table shown in Table 6. When the control method was put into practice, it was found to have prevented roughly 22278 incidents of cultism from spreading.



(a) (b)

(c)

Fig. 5 Public enlightenment, vigilant and moral instruction and specific force legislation

Table 6. Aversion of public enlightenment, vigilant and moral instruction and use of specific force and legislation are implemented

Variables	Without Control	With Control Strategy A	No. of Cultists Averted
L_c	24914	16533	8381
H_c	47654	33757	13897
TIA			**22278**

6. Conclusion

An optimal control mathematical model for the dynamics of cultism in higher education was created in this study. The fundamental reproduction number, which calculated the rate of fresh secondary initiation into cultism, was computed in order to track the cultism initiation process. The cultism-free equilibrium point was also explored and shown to be asymptotically stable both locally and globally. Sensitivity analysis was performed, and the findings indicate how each parameter affects the reproduction number. The interpretation is that increasing a parameter with a positive index raises the possibility that cultism will escalate in a higher education setting while increasing a parameter with a negative index lowers the number of cult cases. When the control strategies were put into place, the spread of cultism was slowed down, according to numerical simulation. The combination of public education, moral instruction, and the use of targeted force and law is known as strategy D, and it is the most effective approach to take in order to end cultism in the university system.

References

[1] Otoo D., Sebil C., and Amponsah S.K., "Mathematical Modeling of Domestic Violence and its Trends, Case Study Tamale Metropolis, Ghana," *Journal of Asian Scientific Research*, vol. 4, no. 8, pp. 436-447, 2014. [Google Scholar] [Publisher Link]

[2] Ibrahim Adamu Mohammed, and Samuel Musa, "Mathematical Model on the Dynamics of Domestic Violence," *Abacus (Mathematics Science Series)*, vol. 44, no. 1, pp. 427-447, 2019. [Google Scholar] [Publisher Link]

[3] O.J. Okwu, "A Critique of Students' Vices and the Effect on the Quality of Graduates of Nigerian Tertiary Institutions," *Journal of Social Sciences*, vol. 12, no. 3, pp. 193-198, 2006. [Google Scholar]

[4] I.A. Ajayi, Haastrup T. Ekundayo, and F.M. Osalusi, "Menace of Cultism in Nigerian Tertiary Institutions: The Way Out," *The Anthropologist*, vol. 12, no. 3, pp. 155-160, 2010. [CrossRef] [Google Scholar] [Publisher Link]

[5] A.O. Echekwube, *Contemporary Ethics: History, Theories and Issues, Lagos*, Spero Book Ltd., Lagos, Nigeria, 1999. [Google Scholar]

[6] Adelola I.O.A., *Secret Cults in Nigerian Institutions of Learning*, A Periscopal Appraisal, Reading on Campus Secret Cults, pp. 51-69, 1997. [Google Scholar]

[7] Ekeanyanwu, Nnamdi Tobechukwu, and Angela Igbinoba A.O., "The Media and Cultism in Nigerian Higher Institutions of Learning: A Study of Coverage, Treatment and Relevance," *International Journal of Communication*, no. 6, pp. 370-388, 2007. [Google Scholar]

[8] Edwin Chukwuaguzie Elujekwute, Shehu Habib, and Olusola Regina Ogundipe, "Assessment of Information and Communication Technology (ICT) Facilities in the Management of Secondary Schools in North-Central States of Nigeria," *International Journal of Management, Social Sciences, Peace and Conflict Studies (IJMSSPCS)*, vol. 4, no. 1, pp. 153-167, 2021. [Publisher Link]

[9] O. Diekmann, J.A.P. Heesterbeek, and M.G. Roberts, "The Construction of Next-Generation Matrices for Compartmental Epidemic Models," *Journal of the Royal Society Interface*, vol. 7, no. 47, pp. 873-885, 2010. [CrossRef] [Google Scholar] [Publisher Link]

[10] Castillo-Chavez C., Feng Z., and Huang W., *On the Computation of R0 and its Role on Global Stability*, Mathematical Approaches for Emerging and Reemerging Infectious Diseases An Introduction, pp. 229-250, 2002. [Publisher Link]

[11] Michael McAsey, Libin Mou, and Weimin Han, "Convergence of the Forward-Backward Sweep Method in Optimal Control," *Computational Optimization and Applications*, vol. 53, pp. 207-226, 2012. [CrossRef] [Google Scholar] [Publisher Link]

[12] K.O. Okosun, Rachid Ouifki, and Nizar Marcus, "Optimal Control Analysis of Malaria Disease Transmission Model that Includes Treatment and Vaccination with Waning Immunity," *Biosystems*, vol. 106, no. 2-3, pp. 136-145, 2011. [CrossRef] [Google Scholar] [Publisher Link]

[13] Suzanne Lenhart, and John T. Workman, *Optimal Control Applied to Biological Model*, 1st ed., Chapman and Hall/CRC, New York, 2007. [CrossRef] [Google Scholar] [Publisher Link]

[14] L.S. Pontryaggin et al., *Mathematical Theory of Optimal Process*, 1st ed., Routledge, London, 1987. [CrossRef] [Google Scholar] [Publisher Link]

[15] Abdulfatai Atte Momoh et al., "Curtailing the Spread of Drug-Abuse and Violence Co-Menace: An Optimal Control Approach," *Alexandria Engineering Journal*, vol. 61, no. 6, pp. 4399-4422, 2022. [CrossRef] [Google Scholar] [Publisher Link]