

Original Article

Graphical Application of The Colebrook-White Equation to the Songloulou and Lagdo Dam Penstocks

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Abstract - This work is an implementation of research carried out over the last few years. It is about obtaining the coefficient of head loss (also known as the coefficient of friction) in a straight pipe using a graphical approach. The aim is to estimate it using an approach deemed to be direct and capable of producing more convincing results. Our approach consists of modeling two structures on which we have been working for some time in order to deduce the friction on the walls of their penstocks. It results from this work that, once the input data have been mastered, the graphical approach can be very useful for determining the friction on the walls of a pipe, whatever the fluid in transit.

Keywords - Modeling, Colebrook-White equation, Pressure drop coefficient, Penstock, CFD.

1. Introduction

Nowadays, there are several relationships available for determining the linear head loss coefficient. Although they have been adapted to each flow regime, they are more or less appreciated depending on their use. For laminar flow, the formula proposed by Poisseille [1] appears to be one of the simplest. When dealing with a turbulent regime, it becomes necessary to differentiate the type of turbulence: smooth or rough.

The work of Osborne Reynolds (1842-1912) helped define the boundaries between these two types. From this, two famous relationships emerged for each type: the Blasius relationship [2] for smooth turbulent flow and the Blenck relationship [3] for rough turbulent flow [4]. It should be added that many researchers have developed formulas and approaches which, despite their complexity for some and their limitations for others, are used for different regimes.

This is the case of [5-9], not to mention derivative studies such as that by [10] or that of [11] approximations or [11-23] relations. We also note that one of the direct methods currently in use is the use of the diagram proposed



by L. F. Moody [24], a diagram obtained under input conditions (input data) almost similar to our approach. The purpose of this paper is, therefore, to present another approach that can be used to determine this pressure loss coefficient.

2. Materials and Methods

To carry out this work, we based on two structures which we have been studying for several years now. These are the Lagdo Dam in northern Cameroon and the Songloulou Dam, the main energy supplier in the southern part of the country. These structures were modeled in Gambit 2.2, and the data collected in each structure at a given time were entered in the Fluent 6.2.2 solver. Figures 1 and 2 below show the models of our structures.

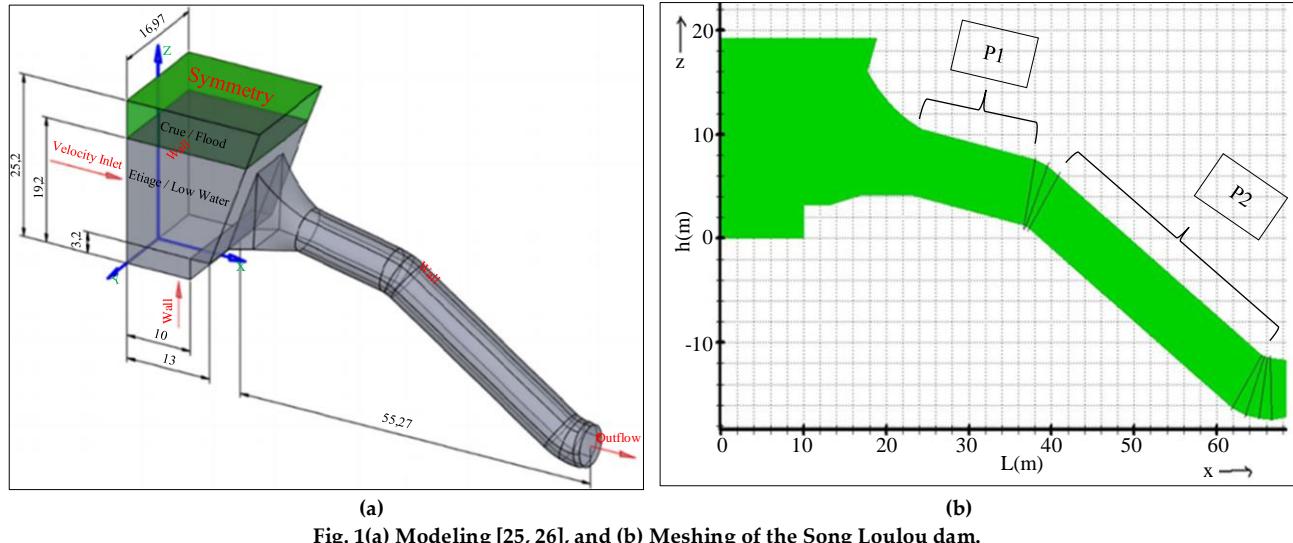


Fig. 1(a) Modeling [25, 26], and (b) Meshing of the Song Loulou dam.

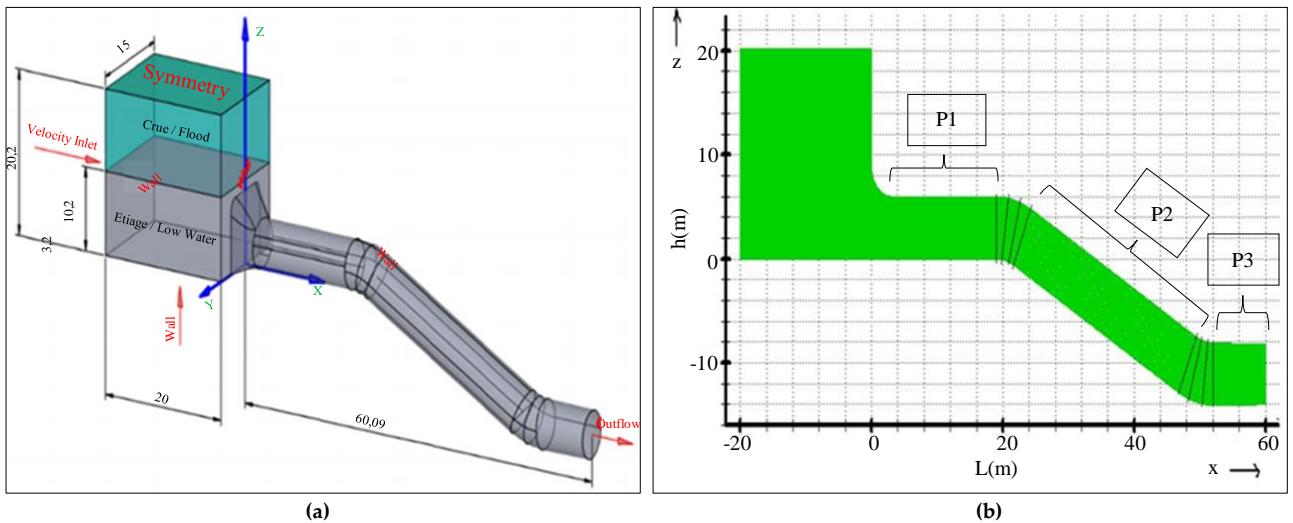


Fig. 2(a) Modeling [25, 26], and (b) Meshing of the Lagdo dam.

The first step was to clamp the mesh to the walls. Next, the mesh was cut close to the walls in steps of 0.5mm up to a height of 20mm. We then introduced the software formula 1 below, developed by F.C. Scobey [8], which, for each step considered, gave a very distinct profile.

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left[\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}} \right] \quad (1)$$

This is how we obtained the maximum height above which friction begins to fall. It should be noted that the integration of this formula required a prior step in a solver, which implied a very high computation time. It should also be pointed out that the fluid considered in this work was just clear water [26].

3. Results and Discussion

The height for the maximum head loss coefficient found is 5mm from the walls (i.e., $e = \varepsilon_{max} = 5\text{mm}$). For each Section, we extracted the evolution of friction in the pipe for 5 velocities, from the minimum velocity observable in each structure to the maximum velocity

3.1. Application to the Lagdo Dam

Velocities in this dam range from 3.1 m/s to 3.87 m/s.

3.1.1. Bottom Wall

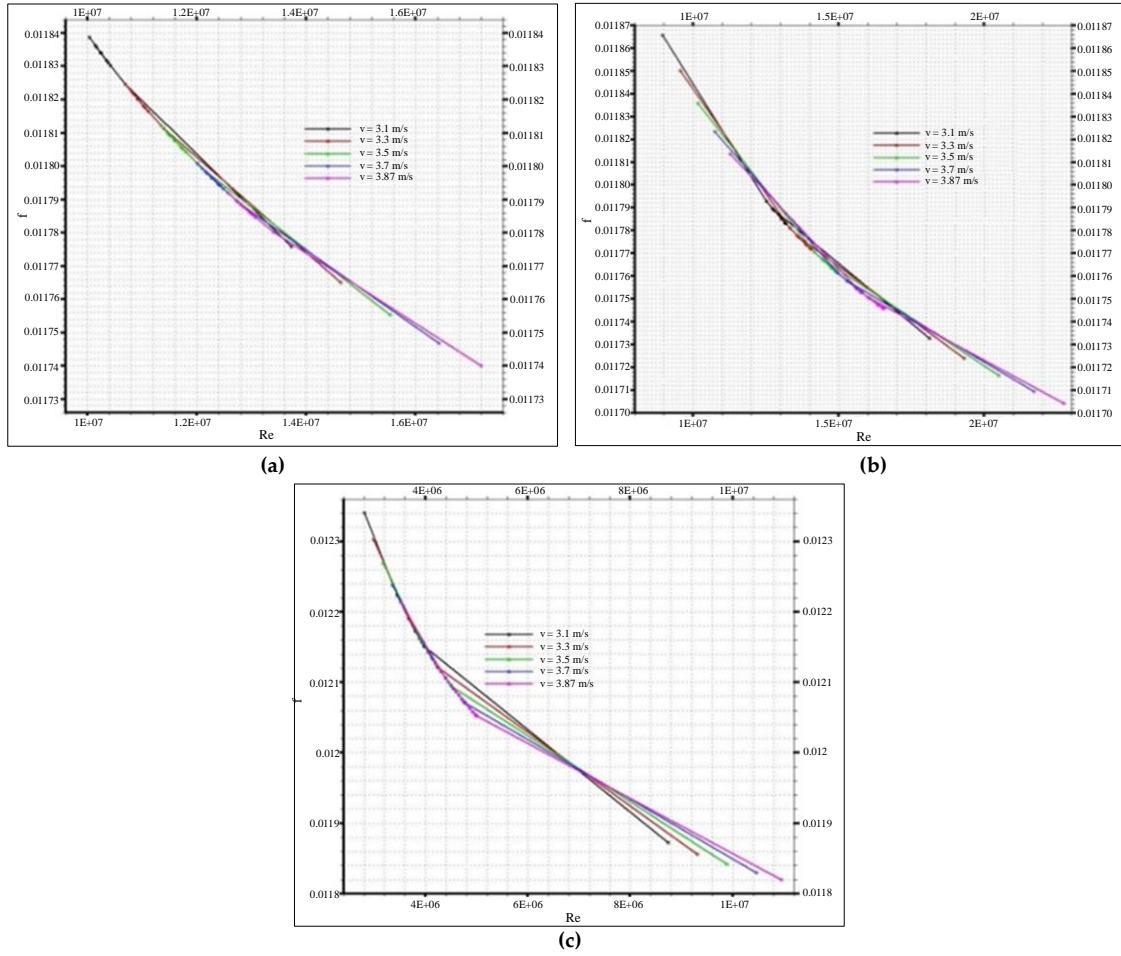


Fig. 3 Evolution of turbulent friction in (a) Section 1, (b) Section 2, and (c) Section 3 for different Re.

3.1.2. Top Wall

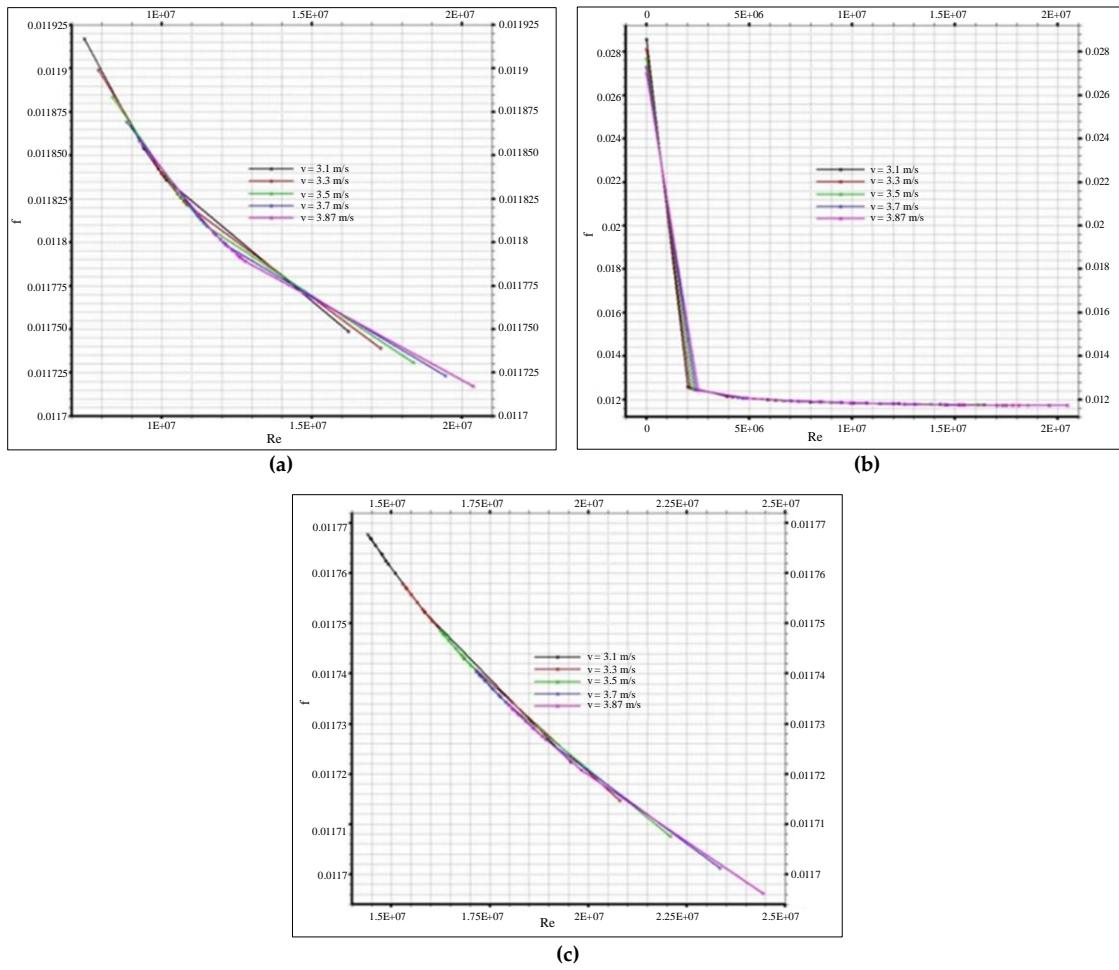


Fig. 4 Evolution of turbulent friction in (a) Section 1, (b) Section 2, and (c) Section 3 for different Re.

These results are more interesting in that they highlight the limitations of current approaches. We can clearly see that the head loss coefficient is not identical for the same section of a given pipe, as shown by current approaches. Even at a given instant t , we find different values for the same section of pipe. This can be seen most clearly in Figure 5 below.

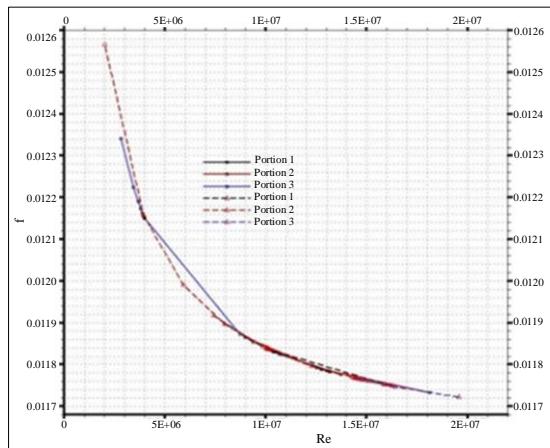


Fig. 5 Turbulent friction in the two pipe sections on the bottom wall (solid line) and the top wall (mixed line) for $Re=26.9 \times 10^6$

3.2. Application to the Songloulou Dam

Velocities in this dam range from 4.045m/s to 4.73m/s.

3.2.1. Bottom Wall

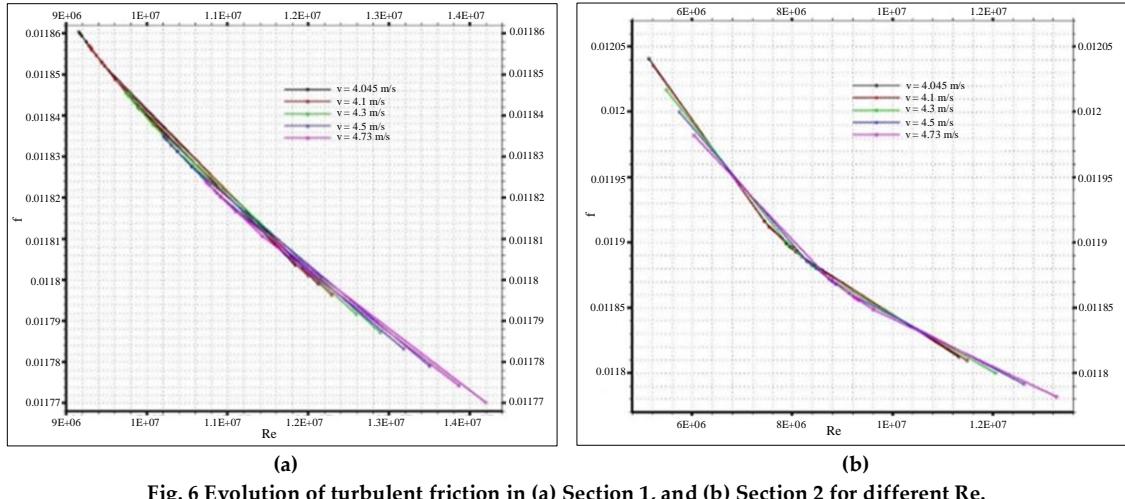


Fig. 6 Evolution of turbulent friction in (a) Section 1, and (b) Section 2 for different Re.

3.2.2. Top Wall

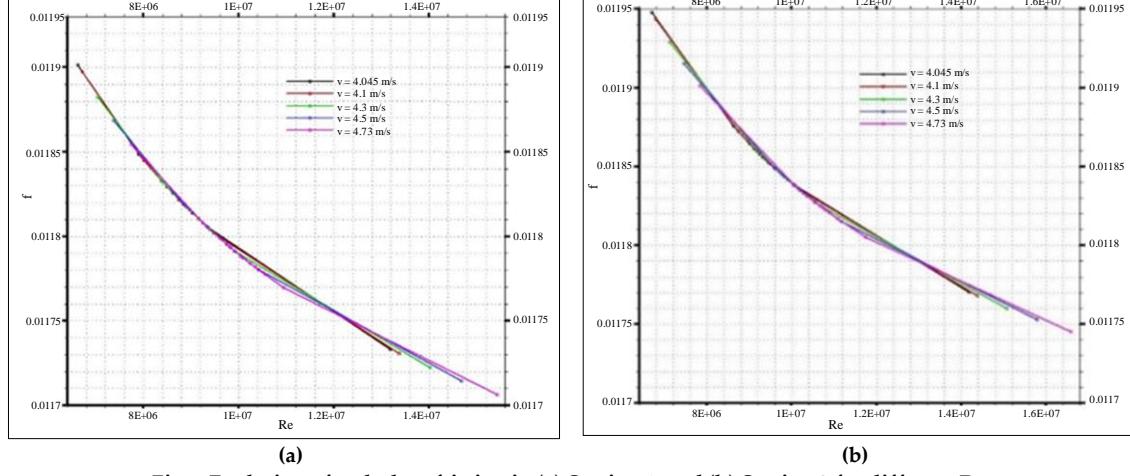


Fig. 7 Evolution of turbulent friction in (a) Section 1, and (b) Section 2 for different Re.

As in the previous case, Figure 8 below shows the difference between the two walls.

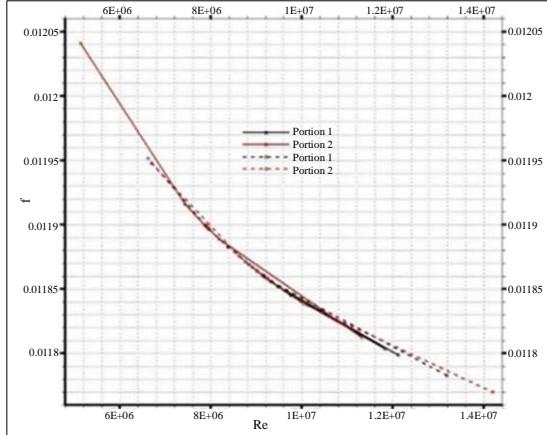


Fig. 8 Turbulent friction in both sections of the pipe on the bottom wall (solid line) and the top wall (mixed line) for $Re=26.9 \times 10^6$

4. Comparative Study

We took the Moody diagram completed by [27, 28] after his work on various penstock linings. We then inserted the profiles obtained and assessed our approach. Figures 9(a) and 9(b) below show the results obtained.

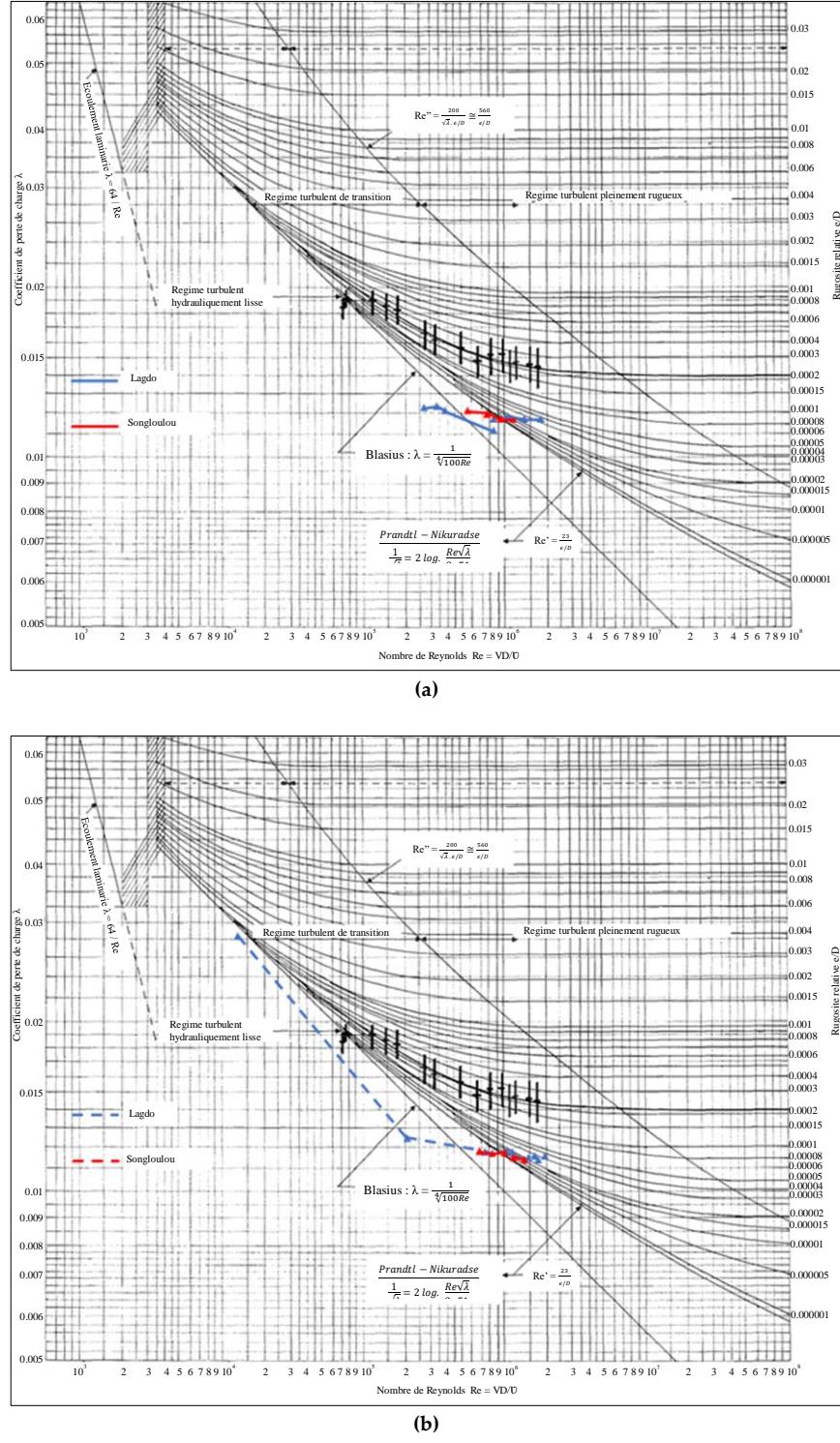


Fig. 9 Evolution of turbulent friction in the Lagdo penstock (blue) in the Songloulou penstock (red) for (a) The bottom wall, and (b) The top wall.

The profiles left by the low walls (Figure 9(a)) and high walls (Figure 9(b)) on both structures follow a pattern that is similar to that of the Moody diagram, and also to that deduced by Levin. However, since we did not consider the same diameter and used clear water, the difference is justified. This discrepancy is also justified by the size of the structure and the approach used.

5. Conclusion

At the end of this work, in which we proposed to carry out a graphical study of the coefficient of head loss in a pipe, it emerged that this approach could be an interesting solution for hydraulic engineers. After identifying the structures to be studied and modeling them using software proven by researchers to be effective in the study of complex environments, we integrated the parameters recorded at various sites. This enabled us to obtain results that can be directly exploited. These results also show that the coefficient of head loss is not identical over a section of the same pipe. This means that the coefficient of friction in a section of pipe at the same speed, is not identical as determined in current practices.

Data Availability

Data was shown in the document. If more, the author has data that supports the conclusions.

Authors' Contributions

Conceptualization, Tchawé Tchawé Moukam and Tcheukam-Toko Dénis; Methodology, Tchawé Tchawé Moukam; Software, Tchawé Tchawé Moukam and Tientcheu Nsiewe Maxwell; Validation, Tchawé Tchawé Moukam, Tcheukam-Toko Dénis and Kenmeugne Bienvenu; Formal Analysis, Tchawé Tchawé Moukam; Investigation, Tchawé Tchawé Moukam; Resources, Tchawé Tchawé Moukam and Djiaoko Thomas; Data Curation, Tchawé Tchawé Moukam and Nkontchou Ngongang François Legrand; Writing – Original Draft Preparation, Tchawé Tchawé Moukam; Writing – Review & Editing, Tchawé Tchawé Moukam, Djiaoko Thomas and Tcheukam-Toko Dénis; Visualization, Tchawé Tchawé Moukam and Djiaoko Thomas; Supervision, Tcheukam-Toko Dénis and Kenmeugne Bienvenu; Project Administration, Kenmeugne Bienvenu; Funding Acquisition, None.

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References

- [1] Jean Leonard Poiseuille, *Recherches Expérimentales Sur Le Mouvement Des Liquides Dans Les Tubes De Très Petits Diamètres, Mémoire lu (3^e partie, suite)*, Imprimerie Royale, 1844. [[Google Scholar](#)] [[Publisher Link](#)]
- [2] H. Blasius, "Das Aehnlichkeitgesetz bei Reibungsvorgangen in Flüssigkeiten," *Forschungsheft*, vol. 131, pp. 1-41, 1913. [[CrossRef](#)] [[Google Scholar](#)] [[Publisher Link](#)]
- [3] T. Blench, *Regime Behaviour of Canals and Rivers*, Butterworths, London, 1957. [[Google Scholar](#)] [[Publisher Link](#)]
- [4] Hug, Michel, *Mécanique des Fluides Appliquée Aux Problèmes D'aménagement et D'énergétique*, Eyrolles, 1975. [[Google Scholar](#)] [[Publisher Link](#)]
- [5] Julius Weisbach, *Lehrbuch der Ingenieur- und Maschinen-Mechanik*, F. Vieweg und Sohn, 1850. [[Google Scholar](#)] [[Publisher Link](#)]
- [6] Darcy, Henry, "Recherches Experimentales Relatives au Mouvement de L'Eau dans les tuyaux," *Mallet-Bachelier*, Paris, vol. 2, pp. 268, 1857. [[Google Scholar](#)] [[Publisher Link](#)]
- [7] Manning, Robert, "On the Flow of Water in Open Channels and Pipes," *Transactions of the Institution of Civil Engineers of Ireland*, vol. 20, pp. 161-207, 1891. [[Google Scholar](#)]
- [8] Frederick Charles Scobey, "The Flow of Water in Riveted and Analogous Pipes," *USDA*, Washington, vol. 150, 1930. [[Google Scholar](#)] [[Publisher Link](#)]

- [9] C.F. Colebrook, and C.M. White, "Experiments with Fluid Friction in Roughened Pipes," *Proceedings of the Royal Society A*, vol. 161, no. 906, pp. 367-381, 1937. [CrossRef] [Google Scholar] [Publisher Link]
- [10] Prabhata K. Swamee, and Akalank K. Jain, "Explicit Equations for Pipe-Flow Problems," *Journal of the Hydraulics Division*, vol. 102, no. 5, pp. 657-664, 1976. [CrossRef] [Google Scholar] [Publisher Link]
- [11] Dejan Brkić, "New Explicit Correlations For Turbulent Flow Friction Factor," *Nuclear Engineering and Design*, vol. 241, no. 9, pp. 4055-4059, 2011. [CrossRef] [Google Scholar] [Publisher Link]
- [12] Maiquel López-Silva et al., "Explicit Pipe Friction Factor Equations: Evaluation, Classification, and Proposal," *Faculty of Engineering Magazine*, 2024. [CrossRef] [Google Scholar] [Publisher Link]
- [13] S. Lukman, and I. A. Oke, "Accurate Solutions of Colebrook-White's Friction Factor Formulae," *Nigerian Journal of Technology*, vol. 36, no. 4, pp. 1039-1048, 2017. [CrossRef] [Google Scholar] [Publisher Link]
- [14] G. Manadilli, "Replace Implicit Equations with Signomial Functions," *Chemical Engineering Journal*, vol. 104, no. 8, pp. 129-130, 1997. [Google Scholar]
- [15] Marco Alfaro-Guerra, Rodrigo Guerra-Rojas, and Alan Olivares-Gallardo, "Experimental Evaluation of Exact Analytical Solution of the Colebrook-White Equation," *Engineering, Research and Technology*, vol. 20, pp. 1-11, 2019. [CrossRef] [Google Scholar] [Publisher Link]
- [16] Jim McGovern, "Friction Factor Diagrams for Pipe Flow," Technical Note, Technological University Dublin, 2011. [Google Scholar] [Publisher Link]
- [17] B.J. McKeon et al., "Friction Factors for Smooth Pipe Flow," *Journal of Fluid Mechanics*, vol. 511, pp. 41-44, 2004. [CrossRef] [Publisher Link]
- [18] B.J. McKeon, C.J. Swanson, and A.J. Smits, "A New Friction Factor Relationship for Fully Developed Pipe Flow," *Journal of Fluid Mechanics*, vol. 538, pp. 429-443, 2005. [CrossRef] [Google Scholar] [Publisher Link]
- [19] Yozo Mikata, and Walter S. Walczak, "Exact Analytical Solutions of the Colebrook-White Equation," *Journal of Hydraulic Engineering*, vol. 143, no. 2, 2015. [CrossRef] [Google Scholar] [Publisher Link]
- [20] Viktor Mileikovskyi, and Tetiana Tkachenko, "Precise Explicit Approximations of the Colebrook-White Equation for Engineering Systems," *Lecture Notes in Civil Engineering*, vol. 100, pp. 303-310, 2020. [CrossRef] [Google Scholar] [Publisher Link]
- [21] Marko Milošević et al., "Hydraulic Losses in Systems of Conduits with Flow from Laminar to Fully Turbulent: A New Symbolic Regression Formulation," *Axiomes*, vol. 11, no. 5, pp. 1-11, 2022. [CrossRef] [Google Scholar] [Publisher Link]
- [22] Renata T. de A. Minhoni et al., "The Performance of Explicit Formulas for Determining the Darcy-Weisbach Friction Factor," *Engenharia Agrícola*, vol. 40, no. 2, pp. 258-265, 2020. [CrossRef] [Google Scholar] [Publisher Link]
- [23] Saeed Kazemi Mohsenabadi, Mohammad Reza Biglari, and Mahdi Moharrampour, "Comparison of Explicit Relations of Darcy Friction Measurement with Colebrook- White Equation," *Applied Mathematics in Engineering, Management and Technology*, vol. 2, no. 4, pp. 570-578, 2014. [Google Scholar]
- [24] Lewis F. Moody, "Friction Factors for Pipe Flow," *Transactions of the ASME*, vol. 66, no. 8, pp. 671-684, 1944. [CrossRef] [Google Scholar] [Publisher Link]
- [25] Energy of Cameroon (ENE) S.A. [Online]. Available: <https://eneocameroon.cm/index.php/en/l-entreprise-a-propos-d-eneo-l-entreprise-en/l-entreprise-a-propos-d-eneo-en#:~:text=In%20generation%2C%20Eneo%20has%20an,Eneo%20generation%20is%20from%20hydro>
- [26] Tchawé Tchawé Moukam et al., "Numerical Study of the Flow Upstream of a Water Intake Hydroelectric Dam in Stationary Regime," *American Journal of Energy Research*, vol. 6, no. 2, pp. 35-41, 2018. [CrossRef] [Google Scholar] [Publisher Link]
- [27] L. Levin, "Etude Hydraulique de Huit Revêtements Intérieurs de Conduites Forcées," *La Houille Blanche*, vol. 58, no. 4, pp. 263-278, 1972. [CrossRef] [Google Scholar] [Publisher Link]
- [28] L. Levin, "Difficultés du Calcul des pertes de Charge Linéaires dans les Conduites Forcées," *La Houille Blanche*, no. 1, pp. 41-54, 1966. [CrossRef] [Google Scholar] [Publisher Link]
- [29] Fluent 6.3, User Guide, 2006. [Online]. Available: <https://romeo.univ-reims.fr/documents/fluent/fluentUserGuide.pdf>