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SCHOOL OF COMPUTING

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

UNIT-III Data Mining and Warehousing – SIT1301

ASSOCIATION RULE MINING

3.1 Frequent patterns are patterns (e.g., itemsets, subsequences, or substructures) that appear frequently in a data set. For example, a set of items, such as milk and bread, that appear frequently together in a transaction data set is a ***frequent itemset***.

A subsequence, such as buying first a PC, then a digital camera, and then a memory card, if it occurs fre- quently in a shopping history database, is a (*frequent*) ***sequential pattern***.

A *substructure* can refer to different structural forms, such as subgraphs, subtrees, or sublattices, which may be combined with itemsets or subsequences. If a substructure occurs frequently, it is called a **(*frequent*) *structured pattern***.

Association Mining

- Association rule mining:
  - Finding frequent patterns, associations, correlations, or causal structures among sets of items or objects in transaction databases, relational databases, and other information repositories.

- Applications:
  – Basket data analysis, cross-marketing, catalog design, loss-leader analysis, clustering, classification, etc.
- Examples.
  - Rule form: ―Body ® Head [support, confidence]‖.
  - buys(x, ―diapers‖) ® buys(x, ―beers‖) [0.5%, 60%]
  - major(x, ―CS‖) ^ takes(x, ―DB‖) ® grade(x, ―A‖) [1%, 75%]

3.2 Association and Correlations

Association Rule: Basic Concepts

- Given: (1) database of transactions, (2) each transaction is a list of items (purchased by a customer in a visit)
- Find: <u>all</u> rules that correlate the presence of one set of items with that of another set of items
  – E.g., 98% of people who purchase tires and auto accessories also get automotive services done
  - Applications
    – * ⇒*Maintenance Agreement* (What the store should do to boost Maintenance Agreement sales)
    – *Home Electronics* ⇒* (What other products should the store stocks up?)
    – Attached mailing in direct marketing
    – Detecting - ping-pong‖ing of patients, faulty - collisions‖

Rule Measures: Support and Confidence

- Find all the rules $X \;\&\; Y \Rightarrow Z$ with minimum confidence and support
  – support, *s*, probability that a transaction contains {X 4 Y 4 Z}
  – confidence, *c*, conditional probability that a transaction having {X 4 Y} also contains *Z*

Let minimum support 50%, and minimum confidence 50%, we have

– $A \Rightarrow C$ (50%, 66.6%)

– $C \Rightarrow A$ (50%, 100%)

Transaction ID	Items Bought
2000	A,B,C
1000	A,C
4000	A,D
5000	B,E,F

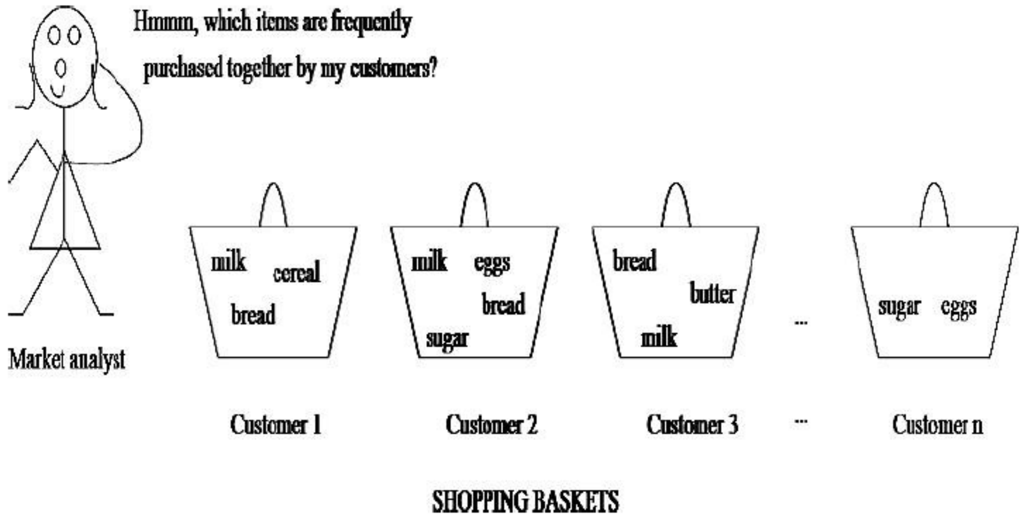
Association Rule Mining: A Road Map

- Boolean vs. quantitative associations (Based on the types of values handled)
  - buys(x, ―SQLServer‖) ^ buys(x, ―DMBook‖) ® buys(x, ―DBMiner‖) [0.2%, 60%]
  - age(x, ―30..39‖) ^ income(x, ―42..48K‖) ® buys(x, ―PC‖) [1%, 75%]
- Single dimension vs. multiple dimensional associations (see ex. Above)
- Single level vs. multiple-level analysis
  - What brands of beers are associated with what brands of diapers?
- Various extensions
  - Correlation, causality analysis
- Association does not necessarily imply correlation or causality
  - Maxpatterns and closed itemsets
  - Constraints enforced
- E.g., small sales (sum < 100) trigger big buys (sum > 1,000)?

Market – Basket analysis

A market basket is a collection of items purchased by a customer in a single transaction, which is a well-defined business activity. For example, a customer's visits to a grocery store or an online purchase from a virtual store on the Web are typical customer transactions. Retailers accumulate huge collections of transactions by recording business activities over time. One common analysis run against a transactions database is to find sets of items, or *itemsets*, that appear together in many transactions. A business can use knowledge of these patterns to improve the Placement of these items in the store or the layout of mail- order catalog page and Web pages. An itemset containing *i* items is called an *i- itemset*. The percentage of transactions that contain an itemset is called the itemset's *support*. For an itemset to be interesting, its support must be higher than a user-specified minimum. Such itemsets are said to be frequent.

Figure : Market basket analysis



Computer ⇒ financial_management_ software [support = 2%, confidence = 60%]

Rule support and confidence are two measures of rule interestingness. They respectively reflect the usefulness and certainty of discovered rules. A support of 2% for association Rule means that 2% of all the transactions under analysis show that computer and financial management software are purchased together. A confidence of 60% means that 60% of the customers who purchased a computer also bought the software. Typically, association rules are considered interesting if they satisfy both a minimum support threshold and a minimum confidence threshold.

3.3 Mining Methods

- Mining Frequent Pattern without candidate generation
- Mining Frequent Pattern without candidate generation

3.4 Mining Frequent Patterns with candidate Generation

The method that mines the complete set of frequent itemsets with candidate generation.

Apriori property & The Apriori Algorithm. Apriori property

- All nonempty subsets of a frequent item set most also be frequent.
  - An item set I does not satisfy the minimum support threshold, min-sup, then I is not frequent, i.e., support(I) < min-sup
  - If an item A is added to the item set I then the resulting item set (I U A) can not occur more frequently than I.
- Monotonic functions are functions that move in only one direction.
- This property is called anti-monotonic.
- If a set can not pass a test, all its supersets will fail the same test as well.
- This property is monotonic in failing the test.

The Apriori Algorithm

- Join Step: Ck is generated by joining Lk-1with itself
- Prune Step: Any (k-1)-itemset that is not frequent cannot be a subset of a frequent k-itemset

Method

```
L_1 = find_frequent_1 itemsets(D);
for (k = 2; L_{k-1}≠ϕ; k++) {
        C_k = apriori_gen (L_{k-1}, min_sup);
        For each transaction t ∈ D { // scan D for counts
                C_t = subset (C_k, t); // get the subsets of t that are candidates
                for each candidate c ∈ C_t
                    c.count++;
        }
        L_k = {c ∈ C_k|c.count ≥ min_sup}
}
return L = U_k L_L;
```

Procedure a priori_gen (L_{k-1}: frequent (k=t) itemsets; min_sup; minimum support)

1) for each itemset $l_1 \in L_{k-1}$
2) for each itemset $l_2 \in L_{k-1}$
3) If $(l_1[1] = l_2[1]) \wedge (l_1[2] = l_2[2]) \wedge \ldots \wedge (l_1[k-2] = l_2[k-2]) \wedge (l_1[k-1] < l_2[k-1])$ that {
4) c = l_1 x l_2; // join step: generate candidates
5) if has_infrequent_subset (c, L_{k-1}) then
6) Delete c, // prune step: remove unfruitful candidate
7) else add c to C_k;
8) }
9) Return C_k;

Procedure has_infrequent_subset (c: candidate k itemsetm; L_{k-1}: frequent (k-1) itemsets); // use prior knowledge

1) for each (k-1) subset s of c
2) if s $\notin L_{k-1}$ then
3) return TRUE;
4) return FALSE;

Example

Scan D for count of each candidate →

C_1

Itemset	Sup.
{I1}	2
{I2}	3
{I3}	3
{I4}	3
{I5}	1

Compare candidate support with minimum support count →

L_1

Itemset	Sup.
{I1}	2
{I2}	3
{I3}	3
{I4}	3

Generate C_2 candidates from L_1 →

C_2

Itemset
{I1,I2}
{I1,I3}
{I1,I4}
{I2,I3}
{I2,I4}
{I3,I4}

Scan D for count of each candidate →

C_2

Itemset	Sup.
{I1,I2}	2
{I1,I3}	1
{I1,I4}	1
{I2,I3}	2
{I2,I4}	2
{I3,I4}	3

Compare candidate support with minimum support count →

L_2

Itemset	Sup.
{I1,I2}	2
{I2,I3}	2
{I2,I4}	2
{I3,I4}	3

Generate C_3 candidates from L_2 →

C_3

Itemset
{I2,I3,I4}

Scan D for count of each candidate →

C_3

Itemset	Sup.
{I2,I3,I4}	2

Compare candidate support with minimum support count →

L_3

Itemset	Sup.
{I2,I3,I4}	2

3.6 Mining Frequent Item set without Candidate Generation

Frequent Pattern Growth Tree Algorithm

It grows long patterns from short ones using local frequent items

- "abc" is a frequent pattern
- Get all transactions having "abc": DB|abc
- "d" is a local frequent item in DB | abc € abcd is a frequent pattern

Construct FP-tree from a Transaction Database

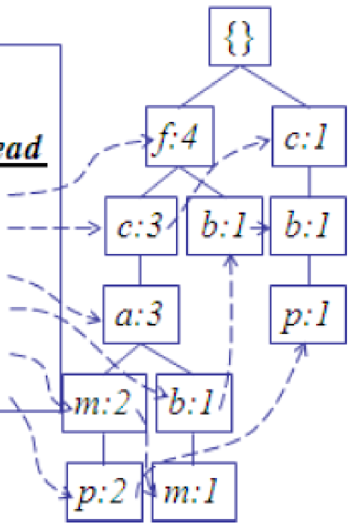
TID	*Items bought*	*(ordered) frequent items*
100	{f, a, c, d, g, i, m, p}	{f, c, a, m, p}
200	{a, b, c, f, l, m, o}	{f, c, a, b, m}
300	{b, f, h, j, o, w}	{f, b}
400	{b, c, k, s, p}	{c, b, p}
500	{a, f, c, e, l, p, m, n}	{f, c, a, m, p}

min_support = 3

1. Scan DB once, find frequent 1-itemset (single item pattern)
2. Sort frequent items in frequency descending order, f-list
3. Scan DB again, construct FP-tree

Header Table

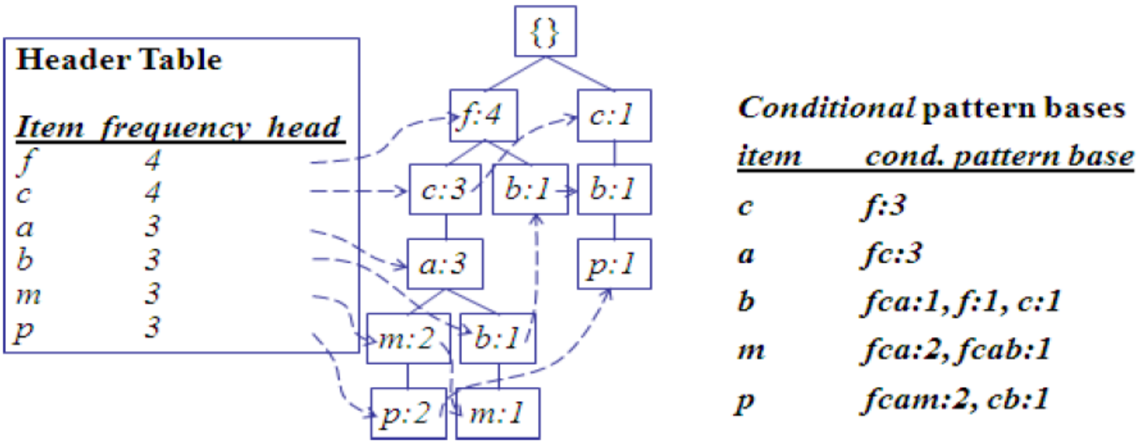
Item	*frequency*	*head*
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



F-list=f-c-a-b-m-p

Find Patterns Having P from P-conditional Database

- Starting at the frequent item header table in the FP-tree
- Traverse the FP-tree by following the link of each frequent item P
- Accumulate all of transformed prefix paths of items *p* to form *P*'s conditional pattern base



Benefits of the FP-tree Structure

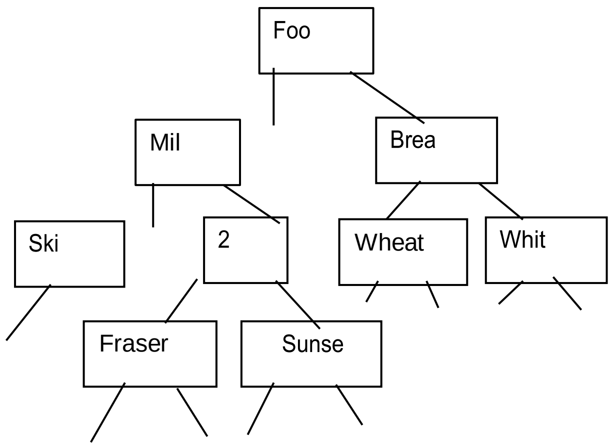
- Completeness:
  - never breaks a long pattern of any transaction
  - preserves complete information for frequent pattern mining
- Compactness
  - reduce irrelevant information—infrequent items are gone
  - frequency descending ordering: more frequent items are more likely to be shared
  - never be larger than the original database (if not count node-links and counts)
  - Example: For Connect-4 DB, compression ratio could be over 100

3.7 Mining various kinds of Association Rule

- Mining Multi-level association rule
- **Mining Multi dimensional Association Rule**

Mining multilevel association rules from transactional databases.

- Items often form hierarchy.
- Items at the lower level are expected to have lower support.
- Rules regarding itemsets atappropriate levels could be quite useful.
- Transaction database can be encoded based on dimensions and levels
- We can explore shared multi-level mining



TID	Items
T1	{111, 121, 211, 221}
T2	{111, 211, 222, 323}
T3	{112, 122, 221, 411}
T4	{111, 121}
T5	{111, 122, 211, 221, 413}

3.8 Mining Multi-Level Associations

- A top_down, progressive deepening approach:
  - First find high-level strong rules:

    milk ® bread [20%, 60%].
  - Then find their lower-level ―weaker‖ rules:

    2% milk ® wheat bread [6%, 50%].
- Variations at mining multiple-level association rules.
  - Level-crossed association rules:

    2% *milk* ® *Wonder wheat bread*
  - Association rules with multiple, alternative hierarchies**:**

    2% *milk* ® *Wonder bread*

Multi-level Association: Uniform Support vs. Reduced Support

- Uniform Support: the same minimum support for all levels
  - + One minimum support threshold. No need to examine itemsets containing any item whose ancestors do not have minimum support.
  - – Lower level items do not occur as frequently. If support threshold
- too high ⇒ miss low level associations
- too low ⇒ generate too many high level associations
- Reduced Support: reduced minimum support at lower levels
  - There are 4 search strategies:
    - Level-by-level independent
    - Level-cross filtering by k-itemset
    - Level-cross filtering by single item
    - Controlled level-cross filtering by single item

Multi-level Association: Redundancy Filtering

- Some rules may be redundant due to ―ancestor‖ relationships between items.
- Example
  - milk ⇒ wheat bread [support = 8%, confidence = 70%]
  - 2% milk ⇒ wheat bread [support = 2%, confidence = 72%]
- We say the first rule is an ancestor of the second rule.
- A rule is redundant if its support is close to the ―expected‖ value, based on the rule's ancestor

Multi-Level Mining: Progressive Deepening

- A top-down, progressive deepening approach:
  - First mine high-level frequent items: milk (15%), bread (10%)
  - Then mine their lower-level ―weaker‖ frequent itemsets: 2% milk (5%), wheat bread (4%)
- Different min_support threshold across multi-levels lead to different algorithms:
  - If adopting the same *min_support* across multi-levels then toss *t* if any of *t*'s ancestors is infrequent.

– If adopting reduced *min_support* at lower levels then examine only those descendents whose ancestor's support is frequent/non-negligible.

3.9 Mining Multidimensional Association mining

Mining our *AllElectronics* database, we may discover the Boolean association rule
buys(X, "*digital camera*"*)* ⇒*buys(X*, "*HP printer*"*).*   (7.6)

Following the terminology used in multidimensional databases,

single- dimensional or **intradimensional association rule** because it contains a single distinct predicate (e.g., *buys*) with multiple occurrences (i.e., the predicate occurs more than once within the rule). Such rules are commonly mined from transactional data.

Considering each database attribute or warehouse dimension as a predicate, we can therefore mine association rules containing *multiple* predicates such as

age(X, "20 . . . 29"*)* A*occupation(X*, "*student*"*)* ⇒*buys(X*, "*laptop*"*).*

Association rules that involve two or more dimensions or predicates can be referred to as **multidimensional association rules**. Rule contains three predicates (*age*, *occupation*, and *buys*), each of which occurs *only once* in the rule. Hence, we say that it has **no repeated predicates**.

Multidimensional association rules with no repeated predicates are called **interdimensional association rules**. We can also mine multidimensional association rules with repeated predicates, which contain multiple occurrences of some predicates. These rules are called **hybrid-dimensional association rules**.

An example of such a rule is the following, where the predicate *buys* is repeated:

age(X, "20 . . . 29"*)* A*buys(X*, "*laptop*"*)* ⇒*buys(X*, "*HP printer*"*).*

Database attributes can be nominal or quantitative. The values of **nominal** (or categorical) attributes are "names of things." Nominal attributes have a finite number of possible values, with no ordering among the values (e.g., *occupation*, *brand*, *color*).

Quantitative attributes are numeric and have an implicit ordering among values (e.g., *age, income, price*). Techniques for mining multidimensional association rules can be categorized into two basic approaches regarding the treatment of quantitative attributes.

In the first approach, *quantitative attributes are discretized using predefined concept hierarchies*. This discretization occurs before mining. For instance, a concept hierarchy for *income* may be used to replace the original numeric values of this attribute by interval labels such as "0..20K," "21K..30K," "31K..40K," and so on.

Here, discretization is *static* and predetermined. Chapter 3 on data preprocessing gave several techniques for discretizing numeric attributes. The discretized numeric attributes, with their interval labels, can then be treated as nominal attributes (where each interval is considered a category).

Mining Quantitative Association Rules

- Determine the number of partitions for each quantitative attribute
- Map values/ranges to consecutive integer values such that the order is preserved
- Find the support of each value of the attributes, and combine when support is less than MaxSup. Find frequent itemsets, whose support is larger than MinSup
- Use frequent set to generate association rules
- Pruning out uninteresting rules

Partial Completeness

- R : rules obtained before partition
- R': rules obtained after partition
- Partial Completeness measures the maximum distance between a rule in R and its closest generalization in R'
- $\hat{X}$ is a generalization of itemset X: if

$$\forall\ x \in \text{attributes (X)} \ [< x, l, u > \in X \wedge x, l', u' > \in \hat{X} \Rightarrow l' \leq l \leq u \leq u']$$

- The distance is defined by the ratio of support

K-Complete

- *C* : the set of frequent itemsets
- For any K ≥ 1, *P* is K-complete w.r.t *C* if:
  1. P C
  2. For any itemset *X* (or its subset) in *C*, there exists a generalization whose support is no more than *K* times that of *X* (or its subset)
- The smaller K is, the less the information lost

3.10 Correlation Analysis

- Interest (correlation, lift)
  - taking both P(A) and P(B) in consideration
  - P(A^B)=P(B)*P(A), if A and B are independent events
  - A and B negatively correlated, if the value is less than 1; otherwise A and B positively correlated

X2 Correlation

- X2 measures correlation between categorical attributes

X	1	1	1	1	0	0	0	0
Y	1	1	0	0	0	0	0	0
Z	0	1	1	1	1	1	1	1

Itemset	Support	Interest
X,Y	25%	2
X,Z	37.50%	0.9
Y,Z	12.50%	0.57

$$X =^2 = \sum \frac{(observe\_expe\ cted)^2}{exp\ ected}$$

	game	not game	sum(row)
video	4000(4500)	3500(3000)	7500
not video	2000(1500)	500 (1000)	2500
sum(col.)	6000	4000	10000

- expected(i,j) = count(row i) * count(column j) / N
- X2 = (4000 - 4500)2 / 4500 - (3500 - 3000)2 / 3000 - (2000 - 1500)2 / 1500 - (500 -1000)2 / 1000 = 555.6
- X2 > 1 and observed value of (game, video) < expected value, there is a negative correlation

Numeric correlation

- Correlation concept in statistics
  - Used to study the relationship existing between 2 or more numeric variables
  - A correlation is a measure of the linear relationship between variables Ex: number of hours spent studying in a class with grade received
  - Outcomes:
    - → positively related
    - → Not related
    - → negatively related
  - Statistical relationships
    - Covariance
    - Correlation coefficient

3.11 Constraint-Based Association Mining

- Interactive, exploratory mining giga-bytes of data?
  - Could it be real? — Making good use of constraints!
- What kinds of constraints can be used in mining?
  - Knowledge type constraint: classification, association, etc.
  - Data constraint: SQL-like queries
- Find product pairs sold together in Vancouver in Dec.'98.
  - Dimension/level constraints:
- in relevance to region, price, brand, customer category.
  - <u>Rule constraints</u>
- small sales (price < $10) triggers big sales (sum > $200).
  - Interestingness constraints:
- strong rules (min_support ≥ 3%, min_confidence ≥ 60%).

Rule Constraints in Association Mining

- Two kind of rule constraints:
  - Rule form constraints: meta-rule guided mining.
- P(x, y) ^ Q(x, w) ® takes(x, ―database systems‖).
  - Rule (content) constraint: constraint-based query optimization (Ng, et al., SIGMOD'98).
- sum(LHS) < 100 ^ min(LHS) > 20 ^ count(LHS) > 3 ^ sum(RHS) > 1000
- 1-variable vs. 2-variable constraints (Lakshmanan, et al. SIGMOD'99):
  - 1-var: A constraint confining only one side (L/R) of the rule, e.g., as shown above.
  - 2-var: A constraint confining both sides (L and R).
- sum(LHS) < min(RHS) ^ max(RHS) < 5* sum(LHS)

Constrain-Based Association Query

- Database: (1) trans (TID, Itemset), (2) itemInfo (Item, Type, Price)
- A constrained asso. query (CAQ) is in the form of {(*S1, S2*)|*C* },
  - where C is a set of constraints on S1, S2 including frequency constraint
- A classification of (single-variable) constraints:
  - Class constraint: : $S \subset A$. *e.g.* $S \subset Item$
  - Domain constraint:
    - $S\ \theta\ v,\ \theta \in \{ =, \neq, <, \leq, >, \geq \}$. *e.g. S.Price < 100*
    - $v\ \theta\ S$, θ *is* $\in$ *or* $\notin$ *e.g. snacks* $\notin$ *S. Typev*
    - $V\ \theta\ S$, *or* $S\ \theta\ V$, $\theta \in \{\subseteq, \subset, \not\subset, =, \neq)$
    - *- e.g. {snacks, sodas}* $\subseteq$ *S.Type*
  - Aggregation constraint: *agg(S)* $\theta\ v$, where *agg* is in {*min, max, sum, count, avg*}, and $\theta \in \{ =, \neq, <, \leq, >, \geq \}$.
    - *e.g. count(S1.Type) = 1 , avg(S2.Price) < 100*

Constrained Association Query Optimization Problem

- Given a CAQ = { (*S1, S2*) | *C* }, the algorithm should be :
  - sound: It only finds frequent sets that satisfy the given constraints C
  - complete: All frequent sets satisfy the given constraints C are found

- A naïve solution:
  - Apply Apriori for finding all frequent sets, and then to test them for constraint satisfaction one by one.
- Our approach:
  - Comprehensive analysis of the properties of constraints and try to push them as deeply as possible inside the frequent set computation.

Categories of Constraints.

1. Anti-monotone and Monotone Constraints

- constraint Ca is anti-monotone iff. for any pattern S not satisfying Ca, none of the super-patterns of S can satisfy Ca
- A constraint Cm is monotone iff. for any pattern S satisfying Cm, every super-pattern of S also satisfies it

2. Succinct Constraint

- A subset of item Is is a succinct set, if it can be expressed as σp(I) for some selection predicate p, where σ is a selection operator
- SP$\subseteq$2I is a succinct power set, if there is a fixed number of succinct set I1, …, Ik $\subseteq$I,s.t. SP can be expressed in terms of the strict power sets of I1, …, Ik using union and minus
- A constraint Cs is succinct provided SATCs(I) is a succinct power set

3. Convertible Constraint

- Suppose all items in patterns are listed in a total order R
- A constraint C is convertible anti-monotone iff a pattern S satisfying the constraint implies that each suffix of S w.r.t. R also satisfies C
- A constraint C is convertible monotone iff a pattern S satisfying the constraint implies that each pattern of which S is a suffix w.r.t. R also satisfies C

Property of Constraints: Anti-Monotone

- Anti-monotonicity: *If a set S violates the constraint, any superset of S violates the constraint.*
- Examples:
  - *sum(S.Price)* ≤*v* is anti-monotone
  - *sum(S.Price)* ≥*v* is not anti-monotone
  - *sum(S.Price)* = *v* is partly anti-monotone
- Application:
  - Push ―*sum(S.price)* ≤ 1000‖ deeply into iterative frequent set computation.

Example of Convertible Constraints: Avg(S) θ V

- Let R be the value descending order over the set of items
  - E.g. I={9, 8, 6, 4, 3, 1}
- Avg(S) ≥ v is convertible monotone w.r.t. R
  - If S is a suffix of S1, avg(S1) □ avg(S)
    - {8, 4, 3} is a suffix of {9, 8, 4, 3}
    - avg({9, 8, 4, 3})=6 ≥ avg({8, 4, 3})=5
  - If S satisfies avg(S) ≥v, so does S1
    - {8, 4, 3} satisfies constraint avg(S) ≥ 4, so does {9, 8, 4, 3}

Property of Constraints: Succinctness

- Succinctness:
  - For any set S1 and S2 satisfying C, S1 □ S2 satisfies C
  - Given A1 is the sets of size 1 satisfying C, then any set S satisfying C are based on A1 , i.e., it contains a subset belongs to A1 ,
- Example :
  - *sum(S.Price)* ≥*v* is not succinct
  - *min(S.Price)* ≤*v* is succinct

- Optimization:
  - If C is succinct, then C is pre-counting prunable. The satisfaction of the constraint alone is not affected by the iterative support counting.
    - ed based on the training set
    - Unsupervised learning (clustering)
    - The class labels of training data is unknown
    - Given a set of measurements, observations, etc. with the aim of establishing the existence of classes or clusters in the data.

QUESTIONS

PART A

1. What is association rule mining? Explain with example?
2. How association rules are mined in large databases?
3. Design a method that mines the complete set of frequent item sets without candidate generation?
4. Explain iceberg queries with example?
5. Differentiate mining quantitative association rules and distance based association rules?
6. Explain how to improve the efficiency of apriori algorithm?
7. List out different kinds of constraint based association mining?
8. How to transform from association analysis to correlation analysis?

PART B

1. Explain market basket analysis with an motivating example for association rule mining?
2. Define association rule mining and explain how the apriori algorithm works with suitable examples
3. Explain mining multi level association rules from transactional database?
4. Explain FP-Growth Method: Mining Frequent Itemsets without Candidate Generation?
5. What is the principle of multi-level association? How do you mine data from relational databases and data warehouses for multidimensional association rules?

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