

1) Given $x_1 = (0, 2)$, $x_2 = (0, 0)$, $x_3 = (1, 5)$, $x_4 = (5, 0)$, $x_5 = (5, 2)$ Given clusters
 $C_1 = \{x_1, x_2, x_4\}$ $C_2 = \{x_3, x_5\}$

Sol Find the centroid.

$$M_k = \frac{1}{n_k} \sum_{i=1}^{n_k} x_{ik}$$

$$M_1 = \left(\frac{0+0+5}{3}, \frac{2+0+0}{3} \right)$$

$$= \left(\frac{5}{3}, \frac{2}{3} \right)$$

$$= (1.66, 0.66)$$

$$M_2 = \left(\frac{1+5}{2}, \frac{0+2}{2} \right)$$

$$= \left(\frac{6}{2}, \frac{2}{2} \right)$$

$$= (3, 1)$$

Step 2: Find the error

$$E_k = \sum_{i=1}^{n_k} (x_{ik} - M_{ik})^2$$

$$E_1 = [(0 - 1.66)^2 + (2 - 0.66)^2 + (0 - 1.66)^2 + (0 - 0.66)^2] \\ + [(5 - 1.66)^2 + (0 - 0.66)^2]$$

$$= 2.756 + 1.796 + 2.756 + 0.436 + 11.156 \\ + 0.436$$

$$= 19.336$$

$$E_2 = [(1 - 3)^2 + (0 - 1)^2 + (5 - 3)^2 + (2 - 1)^2]$$

$$= 3.063 + 1.00 + 3.063 + 1.00$$

$$= 8.126$$

$$E_{1c}^v = E_1^v + E_2^v$$

$$= 19.336 + 80.124$$

$$= 27.462$$

$$m_1 = (1.66, 0.66)$$

$$m_2 = (3.25, 2)$$

$$x_1 = (0, 2)$$

$$\begin{aligned} d(x_1, m_1) &= \sqrt{(0 - 1.66)^2 + (2 - 0.66)^2} \\ &= \sqrt{2.756 + 1.796} \\ &= \sqrt{4.552} = 2.134 \end{aligned}$$

$$d(x_2, m_2)$$

$$\begin{aligned} &= \sqrt{(0 - 3.25)^2 + (2 - 1)^2} \\ &= \sqrt{10.563 + 1.00} \\ &= \sqrt{11.563} \\ &= 3.400 \end{aligned}$$

$$\therefore m_2 > m_1$$

$$x_2 = (0, 0)$$

$$\begin{aligned} d(x_2, m_1) &= \sqrt{(0 - 1.66)^2 + (0 - 0.66)^2} \\ &= \sqrt{2.756 + 0.436} \\ &= \sqrt{3.192} \\ &= 1.787 \end{aligned}$$

$$d(x_2, m_2)$$

$$\begin{aligned} &= \sqrt{(0 - 3.25)^2 + (0 - 1)^2} \\ &= \sqrt{10.563 + 1.00} \\ &= \sqrt{11.563} \\ &= 3.400 \end{aligned}$$

$$\therefore m_2 > m_1$$

$$x_3 = (1.5, 0)$$

$$d(x_3, m_2)$$

$$\begin{aligned} d(x_3, m_1) &= \sqrt{(1.5 - 1.66)^2 + (0 - 0.66)^2} \\ &= \sqrt{0.026 + 0.436} \\ &= \sqrt{0.462} \\ &= 0.680 \end{aligned}$$

$$\begin{aligned} &= \sqrt{(1.5 - 3.25)^2 + (0 - 1)^2} \\ &= \sqrt{3.063 + 1.00} \\ &= \sqrt{4.063} \\ &= 2.016 \end{aligned}$$

$$\therefore m_2 > m_1$$

$$x_4 = (5, 0)$$

$$d(x_4, m_1) = \sqrt{(5-1.66)^2 + (0-0.66)^2} = \sqrt{11.156 + 0.436} = \sqrt{11.592} = 3.405$$

$$d(x_4, m_2)$$

$$= \sqrt{(5-3.25)^2 + (0-1)^2} = \sqrt{3.063 + 1.000} = \sqrt{4.063} = 2.016$$

$\therefore m_1 > m_2$
 $\therefore m_1$ is $< m_2$ so it belongs to the cluster 2

$$x_5 = (5, 2)$$

$$d(x_5, m_1) = \sqrt{(5-1.66)^2 + (2-0.66)^2} = \sqrt{11.156 + 1.796} = \sqrt{12.952} = 3.599$$

$$d(x_5, m_2) = \sqrt{(5-3.25)^2 + (2-1)^2} = \sqrt{3.063 + 1.00} = \sqrt{4.063} = 2.016$$

$\therefore m_1 > m_2$ so it belongs to the cluster 2

$$C_1 = \{x_1, x_2, x_3\}, C_2 = \{x_4, x_5\}$$

Step ①:

$$m_1 = \left\{ \frac{0+0+1.5}{3}, \frac{2+0+0}{2} \right\}$$

$$= \left\{ \frac{1.5}{3}, \frac{2}{2} \right\}$$

$$= \{0.500, 0.667\}$$

$$m_2 = \left(\frac{5+5}{2}, \frac{0+2}{2} \right)$$

$$= \left(\frac{10}{2}, \frac{2}{2} \right)$$

$$= (5, 1)$$

Step 2: Find the error

$$E_1 = \{ (0-0.500)^2 + (2-0.667)^2 + (0-0.500)^2 + (0-0.667)^2 + (1.5-0.500)^2 + (0-0.667)^2 \}$$

$$= 0.250 + 1.777 + 0.250 + 0.445 + 1.00 + 0.445$$

$$= 4.167$$

$$E_2 = \{ (5-5)^2 + (0-5)^2 + (5-5)^2 + (2-5)^2 \}$$

$$= \{ 0 + 1 + 0 + 1 \}$$

$$= 2$$

$$E_{1c} = E_1 + E_2$$

$$= 4.167 + 2$$

$$= 6.167$$

Reassign all the data objects to the cluster with the max distance.

$$X_1 = (0, 2)$$

$$M_1 = (0.500, 0.667)$$

$$M_2 = (5, 2)$$

$$d(X_1, M_1) = \sqrt{(0-0.500)^2 + (2-0.667)^2}$$

$$= \sqrt{0.250 + 1.777}$$

$$= \sqrt{2.027}$$

$$= 1.424$$

$$d(X_1, M_2) = \sqrt{(0-5)^2 + (2-2)^2}$$

$$= \sqrt{25.00 + 1.00}$$

$$= \sqrt{26.00}$$

$$= 5.099$$

$$\therefore M_2 > M_1$$

$$x_2 = (0, 0)$$

$$\begin{aligned} d(x_2, m) &= \sqrt{(0-0.500)^2 + (0-0.667)^2} \\ &= \sqrt{0.250 + 0.445} \\ &= \sqrt{0.695} \\ &= 0.834 \end{aligned}$$

$$\begin{aligned} d(x_2, m_2) &= \sqrt{(0-5)^2 + (0-1)^2} \\ &= \sqrt{25.00 + 1.00} \\ &= \sqrt{26.00} \\ &= 5.099 \end{aligned}$$

$\therefore m_2 > m_1$

$$x_3 = (1.5, 0)$$

$$\begin{aligned} d(x_3, m) &= \sqrt{(1.5-0.500)^2 + (0-0.667)^2} \\ &= \sqrt{1.00 + 0.445} \\ &= \sqrt{1.445} \\ &= 1.202 \end{aligned}$$

$$\begin{aligned} d(x_3, m_2) &= \sqrt{(1.5-5)^2 + (0-1)^2} \\ &= \sqrt{12.250 + 1.00} \\ &= \sqrt{13.250} \\ &= 3.640 \end{aligned}$$

$\therefore m_2 > m_1$

$$x_4 = (5, 0)$$

$$\begin{aligned} d(x_4, m) &= \sqrt{(5-0.500)^2 + (0-0.667)^2} \\ &= \sqrt{20.250 + 0.445} \\ &= \sqrt{20.695} \\ &= 4.549 \end{aligned}$$

$$\begin{aligned} d(x_4, m_2) &= \sqrt{(5-5)^2 + (0-1)^2} \\ &= \sqrt{0 + 1.00} \\ &= \sqrt{1.00} \\ &= 1.00 \end{aligned}$$

$$x_5 = (5, 2)$$

$$\begin{aligned} \text{def. } d(x_5, m) &= \sqrt{(5-0.5)^2 + (2-0.667)^2} \\ &= 4.548 \end{aligned}$$

$m_1 \neq m_2$

$$d(x_5 | m_2) = \sqrt{(5-5)^2 + (2-1)^2} \\ = 1.00$$

$$C_1 = (x_1, x_2, x_3)$$

$$C_2 = (x_4, x_5)$$

The alteration stops here as
we got the same set of clusters
as the previous step.