SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

Unit – 5: IQUEUING THEORY AND REPLACEMENT MODELS

Subject Title: Resource Management Techniques Subject Code: SPR 1307

Course: B.E. Semester: VIII

5.QUEUING THEORY

Queuing theory concerns the mathematical study of queues or waiting lines (seen in banks, post offices, hospitals, airports etc.). The formation of waiting lines usually occurs whenever the current demand for a service exceeds the current capacity to provide that service.

The objective of the waiting line model is to minimize the cost of idle time & the cost of waiting time.

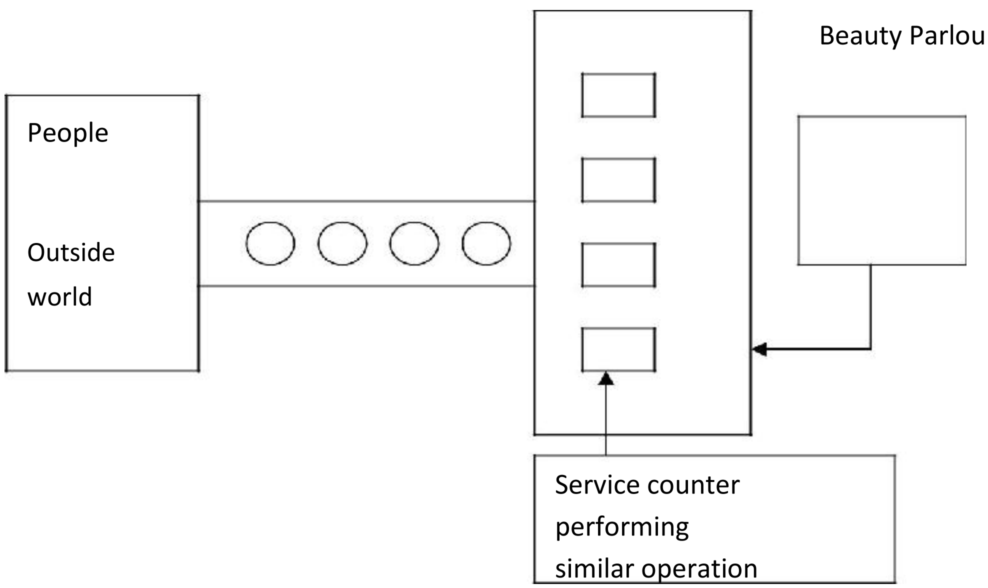
IDLE TIME COST: If an organization operates with many facilities and the demand from customers is very low, then the facilities are idle and the cost involved due to the idleness of the facilities is the *idle time cost*. The cost of idle service facilities is the payment to be made to the services for the period for which they remain idle.

*WAITING TIME COST:*If an organization operates with few facilities and the demand from customer is high and hence the customer will wait in queue. This may lead to dissatisfaction of customers, which leads to *waiting time cost*. The cost of waiting generally includes the indirect cost of lost business.

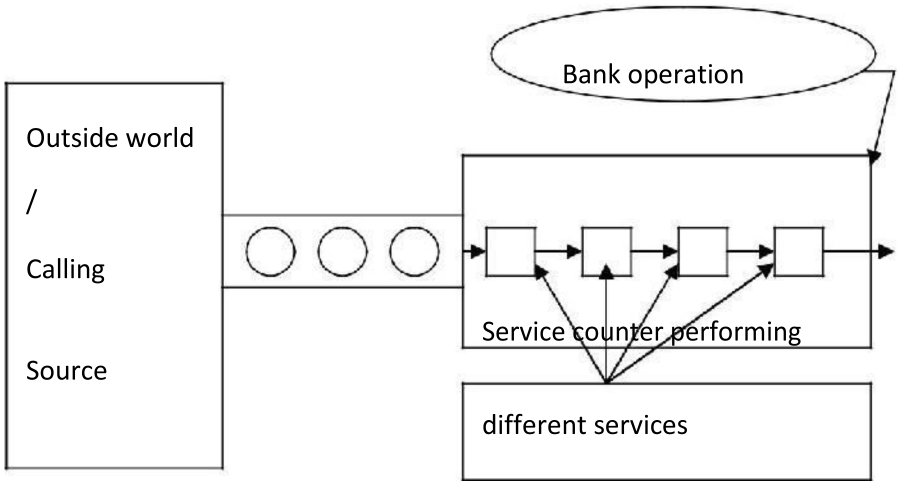
5.6.1TYPE OF QUEUE

a) Parallel queues. b) Sequential queues.

5.6.1.1PARALLEL QUEUES: If there is more than one server performing the same function, then queues are parallel.



5.6.1.2SEQUENTIAL QUEUES : If there is one server performing one particular function or many servers performing sequential operations then the queue will be sequential.



a. **Limited Queue:**

In some facilities, only a limited number of customers are allowed in the system and new arriving customers are not allowed to join the system unless the number below less the limiting value. (Number of appointments in hospitals)

b. **Unlimited Queue:**

In some facilities, there is no limit to the number of customer allowed in the system. (Entertainment centers).

a. **Infinite queue:** If the customer who arrives and forms the queue from a very large population the queue is referred to as infinite queue.

b. **Finite Queue:** if the customer who arrives and forms the queue from a small population then the queue is referred to as finite queue.

DEFINITIONS:

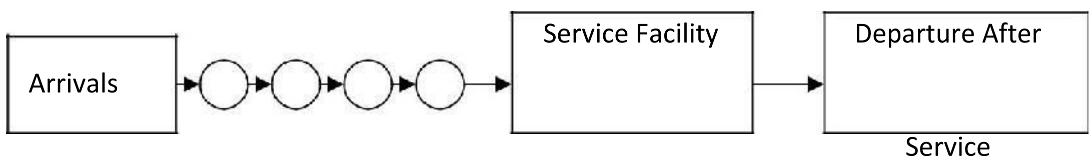
1. **The customer:** The arriving unit that requires some service to be provided.
2. **Server**: A server is one who provides the necessary service to the arrived customer.

3. **Queue (Waiting line):** The number of customers, waiting to be serviced. **The queue does not include the customer being serviced.**
4. **Service channel**: The process or system, which performs the service to the customer.

Based on the number of servers available.

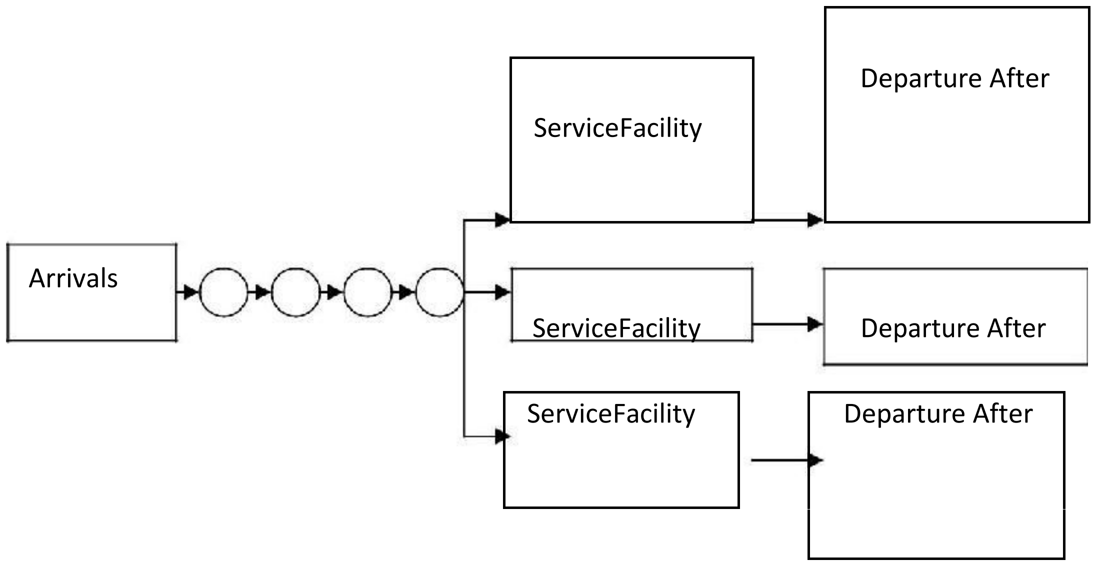
4A. **Single Channel:** If there is a single service station, customer arrivals from a single line to be serviced then the channel is said to Single Channel Model or Single Server Model.

Eg. Doctor's clinic



4B. **Multiple Channel Waiting Line Model:** If there are more than one service station to handle customer who arrive then it is called Multiple Channel Model. Symbol “c” is used.

E.g., Barber shop



5. **Arrival rate:** The rate at which the customers arrive to be serviced. It is denoted by λ. λ indicates take average number of customer arrivals per time period.

6. **Service rate:** The rate at which the customers are actually serviced. It in indicated by μ . μ indicates the mean value of customer serviced per time period.

7. **Infinite queue:** If the customers who arrive and form the queue from a very large population the queue is referred to as infinite queue.

8. **Priority:** This refers to method of deciding as to which customer will be serviced. Priority is said to occur when an arriving customer is chosen for service ahead of some other customer

already in the queue.

9. **Expected number in the queue“Lq”**: This is average or mean number of customer waiting to be serviced. This is indicated by “Lq”.

10. **Expected number in system Ls.:** This is average or mean number of customer either waiting to be serviced or being serviced. This is denoted by Ls.

11. **Expected time in queue Wq".:** This is the expected or mean time a customer spends waiting in the queue. This is denoted by "Wq".
12. **The Expected time in the system "Ws':** This is the expected time or mean time customers spends for waiting in the queue and for being serviced. This is denoted by "Ws'.
13. **Expected number in a non-empty queue:** Expected number of customer waiting in the line excluding those times when the line is empty.
14. **System utilization or traffic intensity**: This is ratio between arrival and service rate.
15. **Customer Behaviour:** The customer generally behaves in 4 ways:

a) **Balking:** A customer may leave the queue, if there is no waiting space or he has no time to wait.

b) **Reneging:** A customer may leave the queue due to impatience
c) **Priorities:** Customers are served before others regardless of their arrival
d) **Jockeying**: Customers may jump from one waiting line to another.

16. **Transient and Steady State**:

A system is said to be in Transient state when its operating characteristics are dependent on time.A system is said to be in Steady state when its operating characteristics are not dependent on time.

5.6.2CHARACTERISTICS OF QUEUING MODELS:

a) Input or arrival (inter –arrival) distribution.
b) Output or Departure (Service) distribution.
c) Service channel
d) Service discipline.
e) Maximum number of customers allowed in the system.
f) Calling source or Population.

a)ARRIVAL DISTRIBUTION:

It represents the rate in which the customer arrives at the system. Arrival rate/interval rate:

►Arrival rate is the rate at which the customers arrive to be serviced per unit of time.

►Inter-arrival time is the time gap between two arrivals.

►Arrival may be separated

1) By **equal** interval of time
2) By **unequal** interval of time which is **definitely known.**

3) Arrival may be **unequal** interval of time whose **probability is known.**

►Arrival rate may be

1. Deterministic (D)
2. Probabilistic
    a. Normal (N)
    b. Binomial (B)
    c. Poisson (M/N)
    d. Beta (β)
    e. Gama (g)
    f. Erlongian (Eh)

The typical assumption is that arrival rate is randomly distributed according to Poisson distribution it is denoted by λ. λ indicates average number of customer arrival per time period.

b) SERVICE OR DEPARTURE DISTRIBUTON:

It represents the pattern in which the customer leaves the system. Service rate at which the customer are actually serviced. It indicated by μ. μ indicates the mean value of service per time period. Interdeparture is the rate time between two departures.

Service time may be

- ► **Constant.**
- ► **Variable** with **definitely known** probability.
- ► **Variable** with **known probability**.

Service Rate Or Departure Rate may be:

1. Deterministic
2. Probabilistic.

   a. Normal (N)

   b. Binomial (B)

   c. Poisson (M/N)

   d. Beat (β)

   e. Gama (g)

   f. Erlongian (Ek)

   g. Exponential (M/N)

   The typical assumption used is that service rate is randomly distributed according to exponential distribution. Service rate at which the customer are actually serviced. It indicated by μ . μ indicates the mean value of serv ice per time period.

c) SERVICE CHANNELS:

The process or system, which is performing the service to the customer.

Based on the number of channels**:**

Single channel

If there is a single service station and customer arrive and from a single line to be serviced, the channel is said to single channel. **Single Channel – 1.**

Multiple channel

If there is more than one service station to handle customer who arrive, then it is called multiple channel model. **Multiple Channel - C.**

d) SERVICE DISCIPLINE: Service discipline or order of service is the rule by which customer are selected from the queue for service.

FIFO: First In First Out – Customer are served in the order of their arrival. Eg. Ticket counter, railway station, banks.
LIFO: Last In First Out – Items arriving last come out first.

Priority: is said to occur when a arriving customer is chosen ahead of some other customer for service in the queue.
SIRO: Service in random order

Here the common service discipline "First Come, First Served".

e) **MAXIMUM NUMBER OF CUSTOMER ALLOWED IN THE SYSTEM:**

Maximum number of customer in the system can be either finite or finite.

a. **Limited Queue:**

In some facilities, only a limited number of customers are allowed in the system and new arriving customers are not allowed to join the system unless the number below less the limiting value. (Number of appointments in hospitals)

b. **Unlimited Queue:**

In some facilities, there is no limit to the number of customer allowed in the system. (Entertainment centers).

f) POPULATION:

The arrival pattern of the customer depends upon the source, which generates them. a. **Finite population (<40):**

If there are a few numbers of potential customers the calling source is finite. b. **Infinite calling source or population:**

If there are large numbers of potential customer, it is usually said to be infinite.

5.6.3KENDALL'S NOTATION: a/b/c;

d/e/f. Where, a – Arrival rate.

b – Service rate.

c – Number of service s 1 or c.

d – Service discipline (FIFO)

e - Number of persons allowed in the queue (N or∞)

f - Number of people in the calling source (∞ or N)

1. **M/M/1, FIFO/**∞/∞**:**

Means Poisson arrival rate, Exponential service rate/one server /FIFO service discipline/Unlimited queues & Unlimited queue in the calling source.

2. **M/M/C, FIFO/**∞/∞**:**

Poisson arrival rate, Exponential service rate, more than one server, FIFO service discipline Unlimited queues and unlimited persons in the calling source.

3. **M/M/I, FIFO/N/**∞**:**

Means Poisson arrival rate, Exponential service rate, One server, FIFO, Limited queue & Unlimited population.

5.6.4SINGLE CHANNEL /MULTIPLE CHANNELPOPULATION MODEL:

1. Find an expression for probability of n customer in the system at time (Pn) in terms of λ and μ
2. Find an expression for probability of zero customers in the system at time t.(Po)
3. Having known Pn, find out the expected number of units in the Queue (Lq)
4. Find out the expected number of units in the system (Ls)
5. Expected waiting time in system (Ws)

6. Expected waiting time queue (Wq)

5.6.5 SOLUTION PROCESS

1. Determine what quantities you need to know.
2. Identify the server
3. Identify the queued items
4. Identify the queuing model
5. Determine the service time
6. Determine the arrival rate
7. Calculate ρ
8. Calculate the desired values

$$= \frac{1}{20} \times 60 = 3 \text{ min.}$$

(*c*) Avg. length of queue

$$L_q = \frac{\lambda}{\mu}\left(\frac{\lambda}{\mu - \lambda}\right)$$

$$L_q = \frac{10}{20}\left(\frac{10}{20-10}\right) = \frac{1}{2} = 0.5 \text{ vehicles}$$

(*d*) Probability that a customer arriving at the pump will have to wait

$$= \frac{\lambda}{\mu} = \frac{10}{20} = 0.5$$

(*e*) The utilisation factor for the pump unit

$$= \frac{\lambda}{\mu} = \frac{10}{20} = 0.5$$

(*f*) Probability that the number of customer in the system is 2

$$P(n) = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n$$

$$P(2) = \left(1 - \frac{10}{20}\right)\left(\frac{10}{20}\right)^2$$

$$= \left(1 - \frac{1}{2}\right)\left(\frac{1}{2}\right)^2 = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8} = 0.125.$$

Problem. 8.14. Arrival at a telephone booth are considered are to be poission, with an average time of 10 minutes between one arrival and next. The length of phone call assumed to be distributed exponentially with mean 3 minutes then
(a) What is. the probability that a person arriving at the booth will have to wait?
(b) What is the average length of the queues that form from time to time.

Ans. Arrival rate $\lambda = \frac{1}{10}$ per minute

Service rate $\mu = \frac{1}{3}$ per minute

(*a*) Probability that a person arriving at the booth will have to wait

$$= \frac{\lambda}{\mu} = \frac{\frac{1}{10}}{\frac{1}{3}} = \frac{3}{10} = 0.3.$$

(b) Average queue length that is formed from time to time

$$= \frac{\mu}{\mu-\lambda} = \frac{\frac{1}{3}}{\frac{1}{3}-\frac{1}{10}} = \frac{\frac{1}{3}}{\frac{7}{30}}$$

$$= \frac{30}{21} = 1.42 \text{ customer.}$$

Problem. 8.15. Customers arrive at one-window drive according to a poission distribution with mean of 10 mm and service time per customer is exponential with mean of 6 minutes. The space in front of the window can accommodate only three vehicles including the serviced one. Other vehicles have wait outside the space.
Calculate.
(a) Probability that an arriving customer can drive directly to the space in front of the window.
(b) Probability that an arriving customer will have to wait outside the directed space. -
(c) How long is an arriving customer expected to wait before starting service?

Ans. Arrival rate $\lambda = \frac{1}{10}$ customers/minute

$= 6$ customers/hour

Service rate $\mu = \frac{1}{6}$ customers/minute

$= 10$ customers/hour

(a) The probability that an arriving customer can drive to the space in front of the window can be obtained by summing up the probabilities of the events in which this can happen. A customer can drive directly to the space if
(1) three is no. customer car already.
(2) there is already 1 customer car.
(3) there are 2 cars in the space.

Thus the required probability $= P_0 + P_1 + P_2$

$$= \left(1-\frac{\lambda}{\mu}\right)+\frac{\lambda}{\mu}\left(1-\frac{\lambda}{\mu}\right)+\left(\frac{\lambda}{\mu}\right)^2\left(1-\frac{\lambda}{\mu}\right)$$

$$= \left(1-\frac{\lambda}{\mu}\right)\left[1+\frac{\lambda}{\mu}+\frac{\lambda^2}{\mu^2}\right]$$

$$= \left(1-\frac{6}{10}\right)\left[1+\frac{6}{10}+\frac{36}{100}\right]$$

$$= \left(\frac{2}{5}\right)\left[\frac{196}{100}\right] = \frac{392}{500} = 0.78$$

(*b*) The probability that an arriving customer has to wait outside the directed space

$$= 1 - 0.78 = 0.22$$

(*c*) Avg. waiting time of a customer in the queue

$$= \frac{1}{\mu}\frac{\lambda}{\mu-\lambda} = \frac{1}{10}\left(\frac{6}{10-6}\right) = \frac{1}{10}\left(\frac{6}{4}\right) = \frac{6}{40} = \frac{3}{20}$$

$= 0.15$ hours $= 9$ minutes.

Problem 8.16. Arrival of machinists at a tool crib are considered to be poission distribution at an avg. rate of 6 per hour. The length of time the machinists must remain at the tool crib is exponentially distributed with an average time being 0.05 hours.
(a) What is the probability that the machinists arriving at tool crib will have to wait.
(b) What is the average number of machinists at the tool crib.
(c) The company will install a second tool crib when convinced that a machinist would expect to have spent at least 6 mins waiting and being serviced at the tool crib. By how much must the flow of machinists to toolcrib increase to justify the addition of second tool crib?

Ans. Arrival rate of machinist 2 = 6 per hour time spent by machinist at the tool crib = 0.05 hours.

Service rate to machinist $\mu = \frac{1}{0.05} = 20$ per hour

Probability that the machinists arriving at tool crib will have to wait

$$= \frac{\lambda}{\mu} = \frac{6}{20} = \frac{3}{10} = 0.3$$

Avg. no. of machinists at the tool crib

$$(L_S) = \frac{\lambda}{\mu-\lambda} = \frac{6}{20-6} = \frac{6}{14} = \frac{3}{7} \text{ machinists}$$

(*c*) Waiting time + Service time:

Time spent in the system $W_s = 6$ minutes $= \frac{1}{10}$ hour

λ_1 – new arrival rate of machinist

$$W_s = \frac{1}{\mu-\lambda_1} = \frac{1}{20-\lambda_1}$$

$$\frac{1}{10} = \frac{1}{20-\lambda_1} \Rightarrow 20 - \lambda_1 = 10 \qquad \Rightarrow \lambda_1 = 10 \text{ machinist/hour}$$

Increase in the flow of machinists to toolcrib increase to justify the addition of a second tool crib =10 — 6 = 4/hour.

Problem 8.17 On an average 96 patients per 24 hours day require the service of an emergency clinic. Also an average a patient requires 10 miii. of active attention. Assume that the facility can handle one emergency at a time. Suppose that it cost the clinic Rs. 100 per patient treated to obtain an average servicing time of 10 minutes, and that each minute of decrease in his average time would cost Rs. 10/-per patient treated. How much would have to be budgeted by the clinic to decrease the average size of

the queue from $1\frac{1}{3}$ patients to $\frac{1}{2}$ patient.

Ans. $\lambda = \frac{96}{24} = 4$ patients/hour

$$\mu = \frac{1}{10} \times 60 = 6 \text{ patients/hour}$$

Avg. no. of patients in the queue.

$$L_q = \frac{\lambda}{\mu}\left(\frac{\lambda}{\mu - \lambda}\right) = \frac{4}{6}\left(\frac{4}{6-4}\right)$$

$$L_q = \frac{4}{2}(2) = \frac{8}{6} = 1\frac{1}{3}$$

This number is to be reduced from $1\frac{1}{3}$ to $\frac{1}{2}$. This can be achieved by increasing the service rate to say μ'

$$L_q' = \frac{\lambda}{\mu'}\left(\frac{\lambda}{\mu' - \lambda}\right)$$

$$\frac{1}{2} = \frac{4}{\mu'}\left(\frac{4}{\mu' - 4}\right)$$

$$\mu'^2 - 4\mu' - 32 = 0 \text{ or } (\mu' - 8)(\mu' + 4) = 0$$

$\mu' = 8$ patients/hour ($\mu' = -4$ is illogical and hence neglected)

Avg. time required by each patient $= \frac{1}{8} hr$

$$= \frac{15}{2} \text{ minutes}$$

Therefore the budget required for each patient

$$= \text{Rs. } (100 + \frac{5}{2} \times 10) = \text{Rs. } 125/-$$

Thus to decrease the size of the queue, the budget per patient should be increased from Rs. 100 to Rs. 125/—

Problem 8.18. In a large maintenance department, fitters draw parts from the

parts stores which is at present staffed by one storeman. The maintenance foreman is concerned about the time spent by fitters getting parts and wants to know if the employment of a stores labourer to assist the storeman would be worth while. On investigation it is found that
(a) a simple queue situation exists.
(b) fitters cost Rs. 2.50 per hour.
(c) the storeman costs Rs. 2 per hour and can deal, on the avg. with 10 fitters per hour.
(d) a labourer could be employed at Rs. 1.75 per hour and would, increase the service capacity of the stores to 12 per hour.
(e) on the average 8 fitters visit the stores each hour.

Ans. We calculate the avg. number of customers in the system before and after the labouer is employed and compare the reduction in the resulting queuing cost with the increase in service cost.

Without labourer:

Number of customers in the system

$$L_s = \frac{\lambda}{\mu-\lambda} = \frac{8}{10-8} = 4$$

Cost/hr = 4 × Rs. 2.50 = Rs. 10/-

With labourer :

$\lambda = 8/\text{hr},\ \mu = 12/\text{hr}.$

Number of customer in the system

$$L_s = \frac{\lambda}{\mu-\lambda} = \frac{8}{12-8} = 2$$

Cost/hr = Cost of fitters per hour + cost of labourer per hour
= 2 * Rs. 2.50 + Rs. 1.75 = Rs. 6.75.

Since there is net saving of Rs. 3.25/- It is recommended to employs the labourer.

Problem 8.19. Customers arrive at the first class ticket counter of a theatre at the rate of 12 per hour. There is one clerk serving the customers at the rate of 30 per hour.
(a) What is the probability that there is no customer in the counter (i.e. that the system is idle) ?
(b) What is the probability that there are more than 2 customers in the counter?
(c) What is the probability that there is no customer waiting to be served?
(d) What is the probability that a customer is being served and no body is waiting.

Ans. Here $\lambda = 12/\text{hour},\ \mu = 30/\text{hour}$

(*a*) Probability that there is no customer in the system $P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{12}{30} = 0.6$

Probability that there are more than two customers in the counter

$$= P_3 + P_4 + P_5 + \dots\dots\dots$$

$$= 1 - (P_0 + P_1 + P_2)$$

$$= 1 - \left[\left(1-\frac{\lambda}{\mu}\right)+\frac{\lambda}{\mu}\left(1-\frac{\lambda}{\mu}\right)+\left(\frac{\lambda}{\mu}\right)^2\left(1-\frac{\lambda}{\mu}\right)\right]$$

$$= 1 - \left[\left(1-\frac{\lambda}{\mu}\right)+\left[1+\frac{\lambda}{\mu}+\frac{\lambda^2}{\mu^2}\right]\right]$$

$$= 1 - \left[0.6\left(1+\frac{12}{30}+\frac{144}{900}\right)\right]$$

$$= 0.064$$

Probability that there is no customer waiting to be served = Probability that there is at most one customer in the counter.

$$= P_0 + P_1 = 0.6 + 0.6\left(\frac{12}{30}\right) = 0.84$$

Probability that a customer is being served and no body is waiting.

$$= P_1 = \left(1-\frac{\lambda}{\mu}\right)\frac{\lambda}{\mu}$$

$$= 0.6\left(\frac{12}{30}\right) = 0.24.$$

Problem 8.20. In a bank there,is only one window, a solitary employee performs all the service required and the window remains continuously open from 7 am to 1 pm. It has been discovered that average number of clients is 54 during the day and the average service-time is of 5 mins per person.

Calculate

(a)Average number of clients in the system (including the one bring served)

(b)The average number of clients in the waiting line. (including the one being served)

(c) Average waiting time.

(d) Average time spends in the system. Ans. Working hours per day = 6 hrs.

Ans.

Arrival rate $\lambda = 54$ clients/day

$$= \frac{54}{6} = 9 \text{ clients/hr}$$

Service rate $\mu = 5$ min. per person

$$= \frac{1}{5} \text{ person/min}$$

$$= \frac{1}{5} \times 60 = 12 \text{ clients/hr.}$$

(*a*) Avg. no. of client in the system

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{9}{12-9} = \frac{9}{3} = 3 \text{ clients}$$

(*b*) Avg. no. of clients in the Queue

$$L_q = \frac{\lambda}{\mu}\left(\frac{\lambda}{\mu - \lambda}\right) = \frac{9}{12}(3) = \frac{9}{4} = 2.25 \text{ clients}$$

(*c*) Avg. waiting time $w_q = \dfrac{\lambda}{\mu(\mu - \lambda)} = \dfrac{1}{12}(3) = \dfrac{1}{4}$ hr.

$$= 15 \text{ min per client.}$$

(*d*) Avg. time spends in the system

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{12-9} = \frac{1}{3} \text{ hr.}$$

$$= \frac{1}{3} \times 60 = 20 \text{ min per client.}$$

QUESTION BANK

INVENTORY MANAGEMENT

INVENTORY MANAGEMENT

Deterministic cost Inventory Models

Model I: Purchasing model without shortages

(Demand rate Uniform, Production rate Infinite)

1. Find the economic order quantity and the number of orders if demand for the year is 2000 units. Ordering cost is Rs500 per order and the carrying cost for one unit per year is Rs2.50. calculate the Total Incremental Cost and Total cost if the purchase price of 1 unit is Rs25/-.

2. A manufacturing company uses an item at a constant rate of 4000 per year. Each unit costs Rs2. The company estimates that it will cost Rs50 to place an order and the carrying cost is 20% of stock value per year. Find economic order quantity and the Total Cost.

Model II: Production model without shortages

(Demand rate Uniform, Production rate finite)

3. A company needs 12000 units per year. The set up cost is Rs 400 per production run. Holding cost per unit per month is Rs15. The production cost is Rs4. The company can produce 2000 units per month. Find out the economic batch quantity, total incremental cost, total cost.

4. Demand = 2000 units/yr. The organization can produce @ 250 units per month. The set up cost is Rs1500/set up, running cost is 10% of average cost of the inventory pr year. If the organization incurs the cost of Rs100, determine how frequently the organization has to go for producing the required material.

Model III: Purchasing model with shortages

(Demand rate Uniform, Production rate Infinite, Shortages allowed)

5. The demand for an item is 20 units per month. The inventory carrying cost is Rs25 per item/month. The fixed cost (ordering cost) is Rs10 for each item a order is made. The purchase cost is Re.1 per item. The shortage cost is Rs15 per year. Determine how often a order should be made and what is the economic order quantity. Find the No. of orders, Total Incremental Cost and Total cost.

6. Demand = 9000 units. Cost of 1 procurement Rs100, holding cost – Rs2.40 per unit, shortage cost = Rs5 per unit. Find economic order quantity and how often should it be ordered. If price is Rs10 find Total Incremental Cost and Total Cost.

Model IV: Production model with shortages

(Demand rate Uniform, Production rate finite, Shortages allowed)

7. A company demands 12000 units per year. The set up cost is Rs 400 per production run. Holding cost per unit per month is Rs0.15. The shortage cost is Rs20 per year. The company can produce 2000 units per month. Find out the economic batch quantity, total incremental cost, total cost per year assuming cost of one unit is Rs 4.

8. The demand for an item in a company is 18,000 units per year, and the company can produce the item at a rate of 3000 per month. The cost of one set up is Rs.500 and the holding cost of one unit per month is 15 paise. The shortage cost of one unit is Rs.20 per month. Determine the optimum manufacturing quantity and the number of shortages. Also determine the manufacturing time and time between set-ups.

Buffer Stock - Deterministic Model

9. A Company uses annually 50,000 units, Each order costs Rs.45 and inventory carrying costs are .18 per unit. i) Find economic order quantity ii) If the company operates 250 days a year and the procurement lead time is 10 days and safety stock is 500 units, find reorder level, maximum, minimum and average inventory.

10. Annual Demand = 12000, Ordering cost = Rs 12, Carrying cost = 10% of inventory per unit cost per unit is Rs 10. The company operates for 250 days per year .The procurement lead time in the past is 10 days, 8 days, 12 days, 13 days and 7 days. find EOQ, Buffer stock reorder level, maximum, minimum and average inventory.

PROBABILISTIC INVENTORY MODEL

1. The probability distribution of the demand for certain items is as follows

Monthly sales	0	1	2	3	4	5	6
Probability	.01	.06	.25	.35	.20	.03	.10

The cost of carrying inventory is Rs 30 per unit per month and cost of unit short is Rs 70 per month. Determine the optimum stock level that would minimize the total expected cost.

2. A news paper boy buys paper for Rs 1.40 and sells them for Rs 2.45 .He cannot return unsold news papers .Daily Demand for the following distribution is as follows

Customers	25	26	27	28	29	30	31	32	33	34	35	36
Probability	.03	.05	.05	.10	.15	.15	.12	.10	.10	.07	.06	.02

If the days demand is independent of the previous day, how many papers he should order each day?

3. The probability distribution of the demand for certain items is as follows

Monthly sales	0	1	2	3	4	5	6
Probability	.02	.05	.30	.27	.20	.10	.06

The cost of carrying inventory is Rs 10 per unit per month .The current policy is to maintain a stock of 4 items at the beginning of each month. Determine the shortage cost per one unit for one time unit.

4. A company orders a new machine after certain fixed time. It is observed that one of the parts of the parts of the machine is very expensive if it is ordered without the machine. The cost of spare part when ordered with the machine is Rs 500 and the cost of down time of the machine and cost of arranging the new part is Rs10, 000. From the past records it is observed that spare parts required with probabilities mentioned below

Demand	0	1	2	3	4	5	6
Probability	.90	.05	.02	.01	.01	.01	0.00

Find the optimal no of spare parts which should be ordered along with the machine.

QUANTITY DISCOUNT MODEL

5. Find the optimal order quantity for a product for which price break up is as follows :

Quantity	**Unit Cost(Rs)**
$0 \leq Q1 < 50$	10
$50 \leq Q2 < 100$	9
$100 \leq Q3$	8

The monthly demand for the product is200 units, the cost of storage is 25%of the unit cost and ordering cost is Rs 20 per order.

6. Find the optimal order quantity for a product for which price break up is as follows :

Quantity	Unit Cost(Rs)
$0 \le Q1 < 500$	10
$500 \le Q2$	9.25

The monthly demand for the product is200 units, the cost of storage is 2%of the unit cost and ordering cost is Rs 350 per order.

M/M/1, FIFO/∞/∞:

SINGLE CHANNEL/INFINITE POPULATION

Arrival Rate: Poisson

Service Rate: Exponential

No of Channels: Single

Service Discipline: FIFO

Queue Discipline: Infinite

Population: Infinite

1. Consider a self-service store with one cashier. Assume Poisson arrival and exponential service times. Suppose 9 customers arrive on an average for every 5 minutes and the cashier can service 10 in 5 minutes. Find the average number of customer in the system and average time a customer spends in the store.

2. In a public telephone booth, the arrivals are on an average 15 per hour. A call on the average takes 3 minutes. If there are just one phone (Poisson arrivals and exponential service), find the expected number of customer in the booth and the idle time of the booth.

M/M/1, FIFO/N/∞:

SINGLE CHANNEL/FINITE POPULATION

Arrival Rate: Poisson

Service Rate: Exponential

No of Channels: Single

Service Discipline: FIFO

Queue Discipline: finite

Population: Infinite

4. At a one-man barbershop, the customer arrives according to Poisson process at an average rate of 5 per hour and they are served according to exponential distribution with an average service rate of 10 minutes. There are only 5 seats available for waiting of the customer and customer do not wait if they find no seat available. Find the average number of customer in the system, average queue length and the average time a customer spends in the barbershop. Also find the idle time of the barber.

5. Consider a single server queuing system with poisson input and exponential service times. Suppose mean arrival rate is 3 units per hour and expected service time is 0.25 hours and the maximum calling units in the system is two. Calculate expected number in the system .

5. In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. the line capacity is 9 trains Calculate the following:

    a) The probability that the yard is empty
    b) Average queue length

MODEL QUESTION PAPER

PART – A

1. Write short notes on i) Re-order level ii) Safety stock
2. Explain the different models of inventory.
3. What do you mean by Buffer stock and write the formula to find buffer stock?
4. Discuss the various types of deterministic inventory models.
5. Define a) EOQ b) EBQ c) Lead time d) Shortage cost.
6. List out the inventory selective control techniques.
7. What do you mean by a) Parallel queues b) Sequential queues.
8. Briefly explain the characteristics of queuing model.
9. Explain the objectives of waiting line model.
10. Describe the queuing models M/M/1 and M/M/C.

PART – B

11. From the following information calculate EOQ, frequency of orders, Number of orders, Total cost, and Total incremental cost:

    Annual Demand - 20000 units/yr
    Ordering cost – Rs.30 per order
    Carrying cost – 12.5% on inventory cost
    Purchase price – Rs.1.50 per unit per year

12. A company orders a new machine after certain fixed time. It is observed that one of the parts of the parts of the machine is very expensive if it is ordered without the machine. The cost of spare part when ordered with the machine is Rs 500 and the cost of down time of the machine and cost of arranging the new part is Rs10, 000. From the past records it is observed that spare parts required with probabilities mentioned below

Demand	0	1	2	3	4	5	6
Probability	.90	.05	.02	.01	.01	.01	0.00

Find the optimal no of spare parts which should be ordered along with the machine.

13. Find the optimal order quantity for a product for which price break up is as follows :

Quantity	Unit Cost(Rs)
$0 \le Q1 < 100$	20
$100 \le Q2$	19.25

The monthly demand for the product is100 units, the cost of storage is 2%of the unit cost and ordering cost is Rs 250 per order.

14. A news paper boy buys paper for 0.30p and sells them for 0.50p .He cannot return unsold news papers .Daily Demand for the following distribution is as follows

No. of copies sold	10	11	12	13	14	
Probability		0.1	0.15	0.20	0.25	0.30

If the days demand is independent of the previous day, how many papers he should order each day?

15. A Company uses annually 15,000 units, Each order costs Rs.25 and inventory carrying costs are .9 per unit. i) Find economic order quantity ii) If the company operates 200 days a year and the procurement lead time is 15days and safety stock is 250 units, find reorder level, maximum, minimum and average inventory.

18. Find the optimal order quantity for a product for which price break up is as follows :

Quantity	Unit Cost(Rs)
$0 \le Q1 < 25$	5
$25 \le Q2 < 50$	4
$50 \le Q3$	3

The monthly demand for the product is200 units, the cost of storage is 25%of the unit cost and ordering cost is Rs 20 per order.

19. The demand for an item in a company is 20,000 units per year, and the company can produce the item at a rate of 5000 per month. The cost of one set up is Rs.500 and the holding cost of one unit per month is 15 paise. The shortage cost of one unit is Rs.15 per month. Determine the optimum manufacturing quantity and the number of shortages. Also determine the manufacturing time and time between set-ups.

20. At a one-man barbershop, the customer arrives according to Poisson process at an average rate of 2 per hour and they are served according to exponential distribution with an average service rate of 5 minutes. There are only 4 seats available for waiting of the customer and customer do not wait if they find no seat available. Find the average number of customer in the system, average queue length and the average time a customer spends in the barbershop. Also find the idle time of the barber.

21. Consider a bank with one cashier. Assume Poisson arrival and exponential service times. Suppose 9 customers arrive on an average for every 5 minutes and the cashier can service 10 in 5 minutes. Find the average number of customer in the system and average time a customer spends in the bank.

4.1REPLACEMENT MODEL

If any equipment or machine is used for a long period of time, due to wear and tear, the item tends to worsen. A remedial action to bring the item or equipment to the original level is desired. Then the need for replacement becomes necessary. This may be due physical impairment, due to normal wear and tear, obsolescence etc. The resale value of the item goes on diminishing with the passage of time.

The depreciation of the original equipment is a factor, which is responsible not to favor replacement because the capital is being spread over a long time leading to a lower average cost. Thus there exists an economic trade-off between increasing and decreasing cost functions. We strike a balance between the two opposing costs with the aim of obtaining a minimum cost.

Replacement model aims at identifying the **time** at which the assets must be replaced in order to minimize the cost.

4.2REASONS FOR REPLACEMENT OF EQUIPMENT:

1. Physical impairment or malfunctioning of various parts refers to

- The physical condition of the equipment itself
- Leads to a decline in the value of service rendered by the equipment
- Increasing operating cost of the equipment
- Increased maintenance cost of the equipment
- Or a combination of the above.

2. Obsolescence of the equipment, caused due to improvement in the existing tools and machinery mainly when the technology becomes advanced.

3. When there is sudden failure or breakdown.

4.3REPLACEMENT MODELS:

- **Assets that fails Gradually:**

  Certain assets wear and tear as they are used. The efficiency of the assets decline with time. The maintenance cost keeps increasing as the years pass by eg. Machinery, automobiles, etc.

  1. Gradual failure without taking time value of money into consideration
  2. Gradual failure taking time value of money into consideration

- **Assets which fail suddenly**

  Certain assets fail suddenly and have to be replaced from time to time eg. bulbs.

  1. Individual Replacement policy (IRP)
  2. Group Replacement policy (GRP)

4.3.1Assests that fails Gradually

4.3.1.1Gradual failure without taking time value of money into consideration

As mentioned earlier the equipments, machineries and vehicles undergo wear and tear with the passage of time. The cost of operation and the maintenance are bound to increase year by year. A stage may be reached that the maintenance cost amounts prohibitively large that it is better and economical to replace the equipment with a new one. We also take into account the salvage value of the items in assessing the appropriate or opportune time to replace the item. We assume that the details regarding the costs of operation, maintenance and the salvage value of the item are already known

- **Procedure for replacement of an asset that fails gradually (without considering Time value of money):**

a) Note down the years
b) Note down the running cost 'R' (Running cost or operating cost or Maintenance cost or other expenses)
c) Calculate Cumulative the running cost '$\sum R$'
d) Note down the capital cost 'C'
e) Note down the scrap or resale value 'S'
f) Calculate Depreciation = Capital Cost – Resale value
g) Find the Total Cost

   Total Cost = Cumulative Running cost + Depreciation
h) Find the average cost

   Average cost = Total cost/No. of corresponding year

i) Replacement decision: Average cost is minimum (Average cost will decrease and reach minimum, later it will increase)

Year	Running Cost	Cumulative Running Cost	Capital cost	Salvage value Or Resale value	Depn. = Capital cost – salvage value	Total cost= Cumulative running cost + Depreciation	Average annual cost P_n = Total cost / no. of corresponding year
N	R_n	$\sum R_n$	C	S_n	C - S_n	$\sum R_n + C - S_n$	$(\sum R_n + C - S_n)$ /n
1	2	3	4	5	6 (4-5)	7 (3+6)	8 (7/1)

4.3.1.2Gradual failure taking time value of money into consideration

In the previous section we did not take the interest for the money invested, the running costs and resale value. If the effect of time value of money is to be taken into account, the analysis must be based on an equivalent cost. This is done with the present value or present worth analysis.

For example, suppose the interest rate is given as 10% and Rs. 100 today would amount to Rs. 110 after a year's time. In other words the expenditure of Rs. 110 in year's time is equivalent to Rs. 100 today. Likewise one rupee a year from now is equivalent to (1.1)-1 rupees today and one-rupee in '*n*' years from now is equivalent to (1.1)-n rupees today. This quantity (1.1)-n is called the present value or present worth of one rupee spent 'n' years from now.

- **Procedure for replacement of an asset that fails gradually (with considering Time value of money):**

Assumption:

i. Maintenance cost will be calculated at the beginning of the year
ii. Resale value at the end of the year

Procedure:

a) Note down the years
b) Note down the running cost ‘R’ (Running cost or operating cost or Maintenance cost or other expenses)
c) Write the present value factor at the beginning for running cost
d) Calculate present value for Running cost
e) Calculate Cumulative the running cost ‘$\sum$ R’
f) Note down the capital cost ‘C’
g) Note down the scrap or resale value ‘S’
h) Write the present value factor at the end of the year and also calculate present value for salvage or scrap or resale value.
i) Calculate Depreciation = Capital Cost – Resale value

j) Find the Total Cost = Cumulative Running cost + Depreciation
k) Calculate annuity factor (Cumulative present value factor at the beginning)

l) Find the Average cost = Total cost / Annuity

m) Replacement decision: Average cost is minimum (Average cost will decrease and reach minimum, later it will increase)

Year n	R_n	Pv^{n-1}	R_nPv^{n-1}	$\sum R_nPv^{n-1}$	C	S_n	Pv^n	S_nPv^n	C - S_nPv^n	$\sum R_nv^{n-1}$ + C - S_nPv^n	$\sum PV^{n-1}$	W_n
1	2	3	4(2*3)	5	6	7	8	9(7*8)	10	11(5+10)	12	13

4.3.2ITEMS THAT FAIL COMPLETELY AND SUDDENLY

There is another type of problem where we consider the items that fail completely. The item fails such that the loss is sudden and complete. Common examples are the electric bulbs, transistors and replacement of items, which follow sudden failure mechanism.

4.3.2.1INDIVIDUAL REPLACEMENT POLICY (IRP):

Under this strategy equipments or facilities break down at various times. Each breakdown can be remedied as it occurs by replacement or repair of the faulty unit.

Examples: Vacuum tubes, transistors

Calculation of Individual Replacement Policy (IRP): n

Average life of an item = $\sum_{i-1}^{n} i * Pi$

Pi denotes Probability of failure during that week **i** denotes no. of weeks

No. of failures = $\frac{\text{Total no. of items}}{\text{Average life of an item}}$

Total IRP Cost = No. of failures * IRP cost

4.3.2.2GROUP REPLACEMENT

As per this strategy, an optimal group replacement period '*P*' is determined and common preventive replacement is carried out as follows.

(a) Replacement an item if it fails before the optimum period '*P*'.

(b) Replace all the items every optimum period of '*P*' irrespective of the life of individual item. Examples: Bulbs, Tubes, and Switches.

Among the three strategies that may be adopted, the third one namely the group replacement policy turns out to be economical if items are supplied cheap when purchased in bulk quantities. With this policy, all items are replaced at certain fixed intervals.

4.3.4.1Procedure for Group Replacement Policy (GRP):

1. Write down the weeks
2. Write down the individual probability of failure during that week
3. Calculate No. of failures:

   N_0 - No. of items at the beginning

   N_1 - No. of failure during 1^{st} week (N_0P_1)

   N_2 - No. of failure during 2^{nd} week ($N_0P_2 + N_1P_1$)

   N_3 - No. of failure during 3^{rd} week ($N_0P_3 + N_1P_2 + N_2P_1$)
4. Calculate cumulative failures
5. Calculate IRP Cost = Cumulative no. of failures * IRP cost
6. Calculate and write down GRP Cost = Total items * GRP Cost
7. Calculate Total Cost = IRP Cost + GRP Cost
8. Calculate Average cost = Total cost / no. of corresponding year

Problems

Problem 1. The cost of a machine is Rs. 6100/- and its scrap The maintenance costs found from experience are as follows:

Year	1	2	3	4	5	6	7	8
Maintenance cost	100	250	400	600	900	1200	1600	2000

When should the machine be replaced ?

Ans. Let it is profitable to replace the machine after n years. The n is determined by the minimum value of T_{avg}.

Years service	Purchase price-scrap value	Annual maintenance cost	Summation of maintenance cost	Total cost	Avg. annual cost (T_{avg})
1.	6000	100	100	6100	6100
2.	6000	250	350	6350	3175
3.	6000	400	750	6750	2250
4.	6000	600	1350	7350	1837.50
5.	6000	900	2250	8250	1650
6.	6000	1200	3450	9450	$\boxed{1575}$ Min
7.	6000	1600	5050	11050	1578
8.	6000	2000	7050	13050	1631

The avg. annual cost is minimum Rs. should be replaced after 6 years of use.

(1575/-) during the sixth year. Hence the m/c

Problem 2. A machine owner finds from his past records that the costs per year of maintaining a machine whose purchase price is Ks. 6000 are as given below

Year	1	2	3	4	5	6	7	8
Maintenance cost	1000	1200	1400	1800	2300	2800	3400	4000
Cost Resale price	3000	1500	750	375	200	200	200	200

Determine at what age is a replacement due?

Ans. Capital cost C = 6000/-. Let it be profitable to replace the. machine after n

years. Then n should be determined by the minimum value of Tav•

Year of service	Resale value	Purchase Price Resale value	Annual Maintenance cost	Summation of maintenance cost	Total Cost	Average annual cost
1.	3000	3000	1000	1000	4000	4000
2.	1500	4500	1200	2200	6700	3350
3.	750	5250	1400	3600	8850	2950
4.	375	5625	1800	5400	11025	2756.25
5.	200	5800	2300	7700	13500	$\boxed{2700}$
6.	200	5800	2800	10500	16300	2716.66
7.	200	5800	3400	13900	19700	2814.28
8.	200	5800	3400	17300	23100	2887.5

We observe from the table that avg. annual cost is minimum (Rs. 2700/-). Hence the m/c should replace at the end of 5th year.
Type B. Replacement of items whose maintenance costs increase with time and value of money also changes with time.
The machine should be replaced if the next period's cost is greater than weighted average of previous cost.
Discount rate [Present worth factor (PWF)

$$V = \frac{1}{1+i}$$

$$V_n = (V)^{n-1}$$

n – no. of year

i – annual interest rate

V_n – PWF of n^{th} year.

Problem 3. A machine costs Rs. 500/— Operation and Maintenance cost are zero for the first year and increase by Rs. 100/— every year. If money. is worth 5% every year, determine the best age at which the machine should be replaced. The resale value of the machine is negligible small. What is the weighted average cost of owning and operating the machine?

Ans. Discount rate $V = \frac{1}{1+i} = \frac{1}{1+0.05} = 0.9524$

Discount rate for Ist year $V_n = \left(\frac{1}{1+i}\right)^{n-1}$

$$V_1 = (0.9524)^0 = 1$$
$$2^{nd} \text{ year } V_2 = (0.9524)^1 = 0.9524$$
$$3^{rd} \text{ year } V_3 = (0.9524)^2 = 0.9070$$
$$4^{th} \text{ year } V_4 = (0.9524)^3 = 0.8638$$
$$5^{th} \text{ year } V_5 = (0.9524)^4 = 0.8227$$

Years of service (n)	Maintenance cost (Rs)	Discount factor $(V)^{n-1}$	Discounted cost	Summation of cost of m/c and maint. Cost	Summation of discount factor	Weighted average cost
1	0	1.0000	0.00	500.00	1.0000	500
2	100	0.9524	95.24	595.24	1.9524	304.88
3	200	0.9070	181.40	776.64	2.8594	$\boxed{217.61}$ min
4	300	0.8638	259.14	1035.78	3.7232	278.20
5	400	0.8227	329.08	1364.86	4.5459	300.25

M/c su1d be replaced at the end of 3C1 year.

Problem 3. Purchase price of a machine is Rs. 3000/— and its running cost is given in the table below. If should be replaced. the discount rate is 0.90. Find at what age the machine

Year	1	2	3	4	5	6	7
Running cost (Rs.)	500	600	800	1000	1300	1600	2000

Ans. V (Discount rate) = 0.90

Year of service (n)	Running cost (Rs.)	Discount factor $(V)^{n-1}$	Discounted cost	Summation of cost of m/c and maint. cost	Summation of discount factor	Weighted average cost
1	500	1	500	3500	1	3500
2	600	0.90	540	4040	1.9	2126.31
3	800	0.81	648	4688	2.71	1729.88
4	1000	0.729	729	5417	3.439	1575.16
5	1300	0.6561	852.93	6269.93	4.0951	$\boxed{1531.08}$ min.
6	1600	0.59049	944.78	7214.71	4.6855	1539.79
7	2000	0.5314	1062.8	8277.51	5.2169	1586.6

M/c should be replaced at the end of 5th year.

Problem 4. The following mortality ratio have been observed for a certain type of light bulbs in an installation with 1000 bulbs

End of week	1	2	3	4	5	6
Probability of failure to date	0.09	0.25	0.49	0.85	0.97	1.00

There are a large no. of such bulbs which are to be kept in working order. If a bulb fails in service, it cost Rs. 3 to replace but if all the bulbs all replaced in the same operation it can be done for only Rs. 0.70/— a bulb. It is proposed to replace all bulbs at fixed intervals, whether or not they have burnt out and continue replacing burnt out bulb as they fail.

(a) What is the best interval between group replacement?

(b) Also establish if the policy, as determined by you is superior to the policy of replacing bulbs as and when they, fail, there being nothing like group replacement.

(c) At what group replacement price per bulb, would a policy of strictly individual replacement become preferable to the adopted policy?

Solution : Let p. be the probability that a new light bulbs fail during the 1th wek of the life.

$P_1 = 0.09$

$P_2 = 0.25 - 0.09 = 0.16$

$P_3 = 0.49 - 0.25 = 0.24$

$P_4 = 0.85 - 0.49 = 0.36$

$P_5 = 0.97 - 0.85 = 0.12$

$P_6 = 1.00 - 0.97 = 0.03$

Week	Expected no. of failure (N)	
0	$N_0 = N_0$	
1	$N_1 = 1000 \times 0.09$	$= 90$
2	$N_2 = 1000 \times 0.16 + 90 \times 0.09$	$= 168$
3	$N_3 = 1000 \times 0.24 + 90 \times 0.16 + 168 \times 0.09$	$= 269$
4	$N_4 = 1000 \times 0.36 + 90 \times 0.24 + 168 \times 0.16 + 269 \times 0.09$	$= 432$
5	$N_5 = 1000 \times 0.12 + 90 \times 0.36 + 168 \times 0.24 + 269 \times 0.16 + 432 \times 0.09$	$= 274$
6.	$N_6 = 1000 \times 0.03 + 90 \times 0.12 + 168 \times 0.36 + 269 \times 0.24 + 432$	
	$\times 0.16 + 274 \times 0.09$	$= 260$
and so on		

(*a*) Determination of optimum group replacement interval

Week	Total cost of group replacement	Avg cost/week
1.	$1000 \times 0.70 + 90 \times 3 = 970$	970.00
2.	$1000 \times 0.70 + 3\ (90+168) = 1474$	$\boxed{737.00}$
3.	$1000 \times 0.70 + 3\ (90 + 168 + 269) = 2281$	760.33