

SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF MATHEMATICS

Subject Title: Resource Management Techniques

Subject Code: SPR 1307

Course: B.E.

Semester: VIII

Unit – 4: INVENTORY MODELS

Inventory Models- ABC Analysis, Cost involved in inventory management- EOQ Calculation, Deterministic Demand Inventory Models, Quantity Discount Models. Waiting Line Models- Feature of Waiting Line Models- Kendall Notations- M/M/1; M/M/C; FIFO/N/N Models only.

Prepared by Mrs. Meena Kumari. R

5.1. INVENTORY

Inventory may be defined a stock of goods, commodities or other economic resources that are stored or reserved for smooth and efficient running of business. The inventory may be kept in any one of the following forms:

1. Raw material
2. Work-in progress
3. Finished goods

If an order for a product is receive, we should have sufficient stock of materials required for manufacturing the item in order to avoid delay in production and supply. Also there should not be over stock of materials and goods as it involves storage cost and wastage in storing. Therefore inventory control is essential to promote business. Maintaining inventory helps to run the business smoothly and efficiently and also to provide adequate service to the customer. Inventory control is very useful to reduce the cost of transportation and storage.

A good inventory system, one has to address the following questions quantitatively and

qualitatively.

- What to order?
- When to order?
- How much to order?
- How much to carry in an inventory?

5.1.1 Objectives of inventory management/Significance of inventory management

To maintain continuity in production.

To provide satisfactory service to customers.

To bring administrative simplicity.

To reduce risk.

To eliminate wastage.

To act as a cushion against high rate of usage.

To avoid accumulation of inventory.

To continue production even if there is a break down in few machinery.

To ensure proper execution of policies.

To take advantages of price fluctuations and buy economically.

5.1.2 Costs involved in inventory

1. Holding Cost (Carrying or Storage Cost)

It is the cost associated with the carrying or holding the goods in stock. It includes storage cost, depreciation cost, rent for godown, interest on investment locked up, record keeping and administrative cost, taxes and insurance cost, deterioration cost, etc. It is denoted by 'C'.

2. Setup Cost/ Ordering Cost

Ordering cost is associated with cost of placing orders for procurement of material or finished goods from suppliers. It includes, cost of stationery, postage, telephones, travelling expenses, handling of materials, etc. (Purchase Model) Setup cost is associated with production. It includes, cost involved in setting up machines for production run. (Production Model). Both are denoted by 'S'.

3. Purchase Cost/Production Cost

When the organization purchases materials from other suppliers, the actual price paid for the material will be called the purchase cost.

When the organization produces material in the factory, the cost incurred for production of material is called as production cost. Both are denoted by 'P'.

4. Shortage Cost

If the inventory on hand is not sufficient to meet the demand of materials or finished goods, then it results in shortage of supply. The cost may include loss of reputation, loss of customer, etc.

Total incremental cost = Holding Cost + Setup Cost/ Ordering Cost

Total Cost = Purchase Cost/ Production Cost + Shortage Cost + Total Incremental cost.

5.2 Demand is one of the most important aspects of an inventory system.

Demand can be classified broadly into two categories:

5.2.1 Deterministic i.e., a situation when the demand is known with certainty. And, deterministic demand can either be *static* (where demand remains constant over time) or it could be *dynamic* (where the demand, though known with certainty, may change with time).

5.2.2 Probabilistic (Stochastic) refers to situations when the demand is *random* and is governed by a *probability density function* or *probability mass function*. Probabilistic demand can also be of two types - *stationary* (in which the demand probability density function remains unchanged over time), and *non-stationary*, where the probability densities vary over time.

Deterministic Inventory Models

- i. Model I: Purchasing model without shortages
- ii. Model II: Production model without shortages
- iii. Model III: Purchasing model with shortages

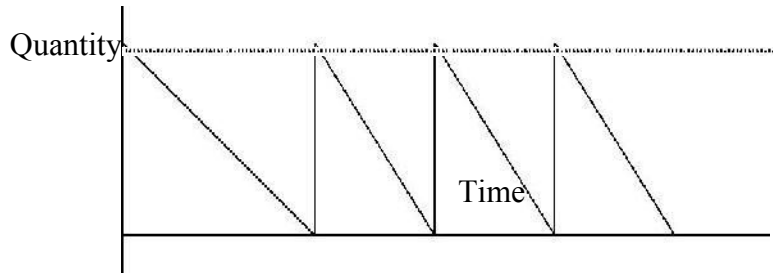
iv. Model IV: Production model with shortages

5.2.1.1 Model I: Purchasing model without shortages

Assumptions

- Demand(D) per year is known and is uniform
- Ordering cost(S) per order remains constant

- Carrying cost(C) per unit remains constant
- Purchase price(P) per unit remains constant
- No Shortages are allowed. As soon as the level of inventory reaches zero, the inventory is replenished back. Lead time is Zero.

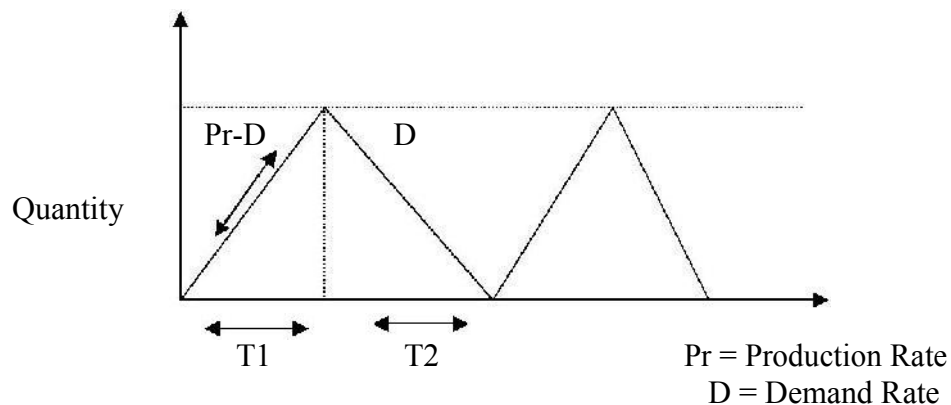


Inventory decreases at the rate of ' D ' As soon as the level of inventory reaches zero, the inventory is replenished back

5.2.1.2 Model II: Production model without shortages

Assumptions

- Demand(D) per year is known and is uniform
- Setup cost (S) per production run remains constant
- Carrying cost(C) per unit remains constant
- Production cost per unit(P) per unit remains constant
- No Shortages are allowed. As soon as the level of inventory reaches zero, the inventory is replenished back.



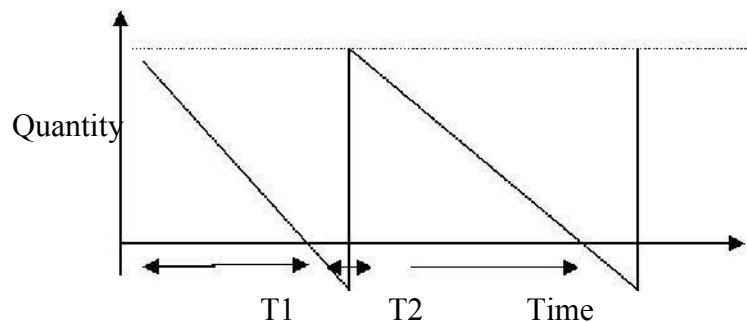
T_1 is the time taken when manufacturing takes place at the rate of Pr and demand at the rate of

D. So the stock is built up at the rate of $(Pr - D)$. During t_2 there is no production only usage of stock. Hence, stock is decreased at the rate of 'D'. At the end of t_2 , stock will be nil.

5.2.1.3 Model III: Purchasing model with shortages

Assumptions

- Demand(D) per year is known and is uniform
- Ordering cost(S) per order remains constant
- Carrying cost(C) per unit remains constant
- Purchase price(P) per unit remains constant
- Shortages are allowed. As soon as the level of inventory reaches zero, the inventory is replenished back with lead time.
- Shortage cost (sh) per unit remains constant



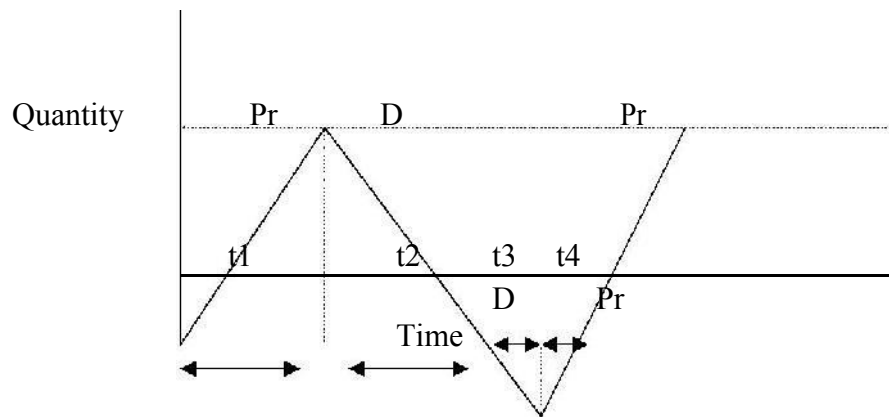
T_1 is the time during which stock is nil. During T_2 shortage occur and at the end of T_2 stock is replenished back.

5.2.1.4 Model IV: Production model with shortages

Assumptions

Demand(D) per year is known and is uniform

- ❖ Setup cost (S) per production run remains constant
- ❖ Carrying cost(C) per unit remains constant
- ❖ Production cost per unit(P) per unit remains constant
- ❖ Shortages are allowed. As soon as the level of inventory reaches zero, the inventory is replenished back with lead time.
- ❖ Shortage cost (Sh) per unit remains constant



T1 is the time taken when manufacturing takes place at the rate of Pr and demand at the rate of D . So the stock is built-up at the rate of $(Pr - D)$. During $t2$ there is no production only usage of stock. Hence, stock is decreased at the rate of ' D '. At the end of $t2$, stock will be nil. During $T3$ shortage exists at the rate of ' D '. During $T4$ production begins stock builds and shortage decreases at the rate of ' $Pr-D$ '

5.2.2.1 Inventory basic terminologies

- EOQ- Economic order quantity – The optimum order per order quantity for which total inventory cost is minimum.
- EBQ- Economic batch quantity – The optimum manufacturing quantity in one batch for which total inventory cost is minimum.
- Demand Rate – rate at which items are consumed
- Production rate- rate at which items are produced
- Stock replenishment rate
 - Finite rate – the inventory builds up slowly /step by step(production model)
 - Instantaneous rate – rate at which inventory builds up from minimum to maximum instantaneously (purchasing model)
- Lead time- Time taken by supplier to supply goods
- Lead time demand it is the demand for goods in the organization during lead time.
- Reorder level- the level between maximum and minimum inventory at which purchasing or manufacturing activities must start from replenishment.
Reorder level = Buffer stock+ Lead time demand
- Buffer stock- to face the uncertainties in consumption rate and lead time , an extra stock is

maintained. This is termed as buffer stock:

Buffer stock = (Maximum Lead time – Average Lead time) x Demand per month

- Maximum Inventory Level: Maximum quantity that can be allowed in the stock: Maximum Inventory = EOQ + Buffer stock
- Minimum Inventory Level is the level that is expected to be available when the supply is due: Minimum Inventory level = Buffer stock
- Average Inventory = (Minimum Inventory + Maximum Inventory)/2
- Order cycle is the period of time between two consecutive placements of orders.

5.3 Inventory system followed in a organization:

- Q – System (fixed order quantity system)
- P - System (fixed period system)

5.3.1 Q – System

In a fixed order quantity system means every time an order is placed the quantity order is EOQ.

In Q – System, the period between the orders is not constant:

Ex. 1st – 1 month –

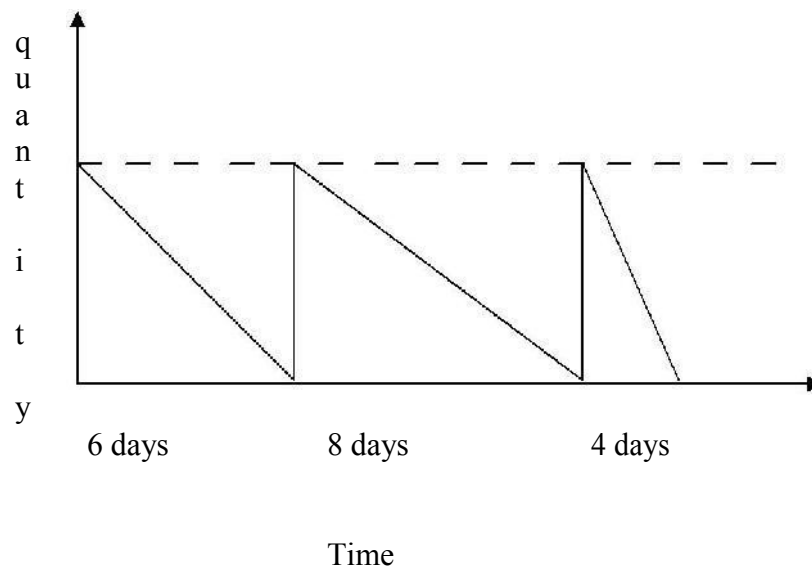
EOQ 2nd – 1 ½ month –

EOQ

3rd – 2 month – EOQ

4th – 15 days - EOQ

Whenever the stock reaches reorder level, next order is placed.





- Reorder level- the level between maximum and minimum inventory at which purchasing or manufacturing activities must start from replenishment.

Reorder level = Buffer stock + Lead time demand

- Lead time is the time taken by supplier to supply goods
- Lead time demand it is the demand for goods in the organization during lead time.
- Buffer stock: To face the uncertainties in consumption rate and lead time, an extra stock is maintained. This is termed as buffer stock:

Buffer stock = (Maximum Lead time – Average Lead time) x Demand per month

- Maximum Inventory Level: Maximum quantity that can be allowed in the stock: Maximum Inventory = EOQ + Buffer stock
- Minimum Inventory Level is the level that is expected to be available when the supply is due:

Minimum Inventory level = Buffer stock

Average Inventory = (Minimum Inventory + Maximum Inventory)/2

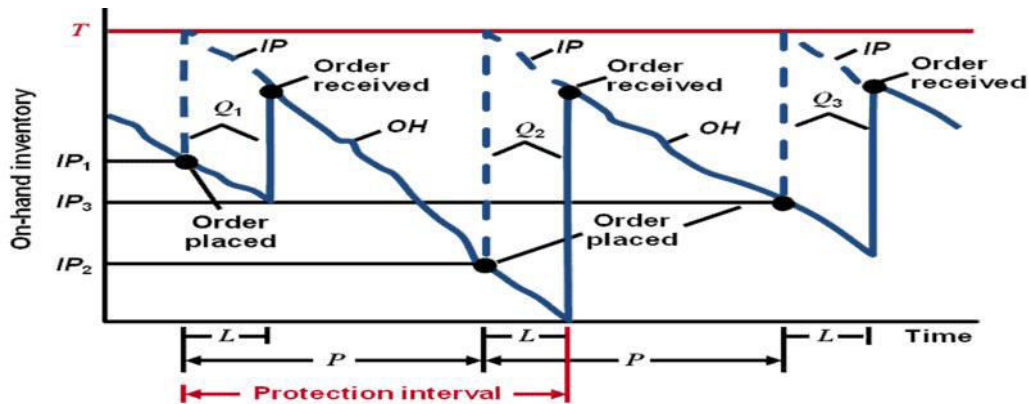
5.3.2P – System

Time period between the orders is fixed; hence it is called as Fixed Period System. Period of order is fixed but the quantity will vary. Ex:

1st – 1 month – 1000 units
2nd – 1 month – 1200 units
3rd – 1 month – 950 units

A predetermined level of inventory is fixed and the order quantity is determined by deducting the level of stock at the time review from P determine level of inventory.

Order quantity = Predetermined level of inventory – level of stock at the time of review



5.4 Inventory Selective Control Techniques

Every organization consumes several items of store. Since all the items are not of equal importance, a high degree control on inventories of each item is neither applicable nor useful. So it becomes necessary to classify items in group depending upon their utility importance. Such type of classification is name as the principle of selective control.

5.4.1ABC Analysis (Always Better Control)

- A - High value items
- B - Moderate value items
- C - Low value items

ABC analysis is one of the methods for classification of materials. It is based on Pareto's law that a few high usage value items constitute a major part of the inventory while a large bulk of items constitute to very low usage value.

5.4.1.1PROCEDURE FOR ABC ANALYSIS:

1. Note down the material code.
2. Note down the annual usage in terms of units.
3. Note down the price per unit.
4. Calculate the Annual usage value.

5. Arrange the materials according to the value in descending order.
6. Find out the percentage contribution of each material to the total value.
7. Find out the percentage contribution of each material towards the total quantity.

8. Cumulate the % contribution towards value.
9. The classification is as follows.

A = 80% contribution B = 15% contribution C = 5% contribution.

5.4.1.2SIGNIFICANCE OF ABC ANALYSIS

ABC analysis is a very useful technique to classify the materials.

- The control procedure is based on which category the item belongs to.
A = Tight control
B = Moderate control
C = Very little control.
- The inventory to be maintained is again based on the category
A = Low Inventory
B = Moderate Inventory
C = High Inventory.
- The number of suppliers is also based on the category to which it belongs.
A = Many suppliers
B = Moderate No. of suppliers
C = Few suppliers.

5.4.2VEDAnalysis

- V Vital items
- E Essential items
- D Desirable or Durable items

5.4.3HML Analysis

- High price items
- Moderate price items
- Low price items

5.4.4FNSD Analysis

- F Fast Moving items
- N Normal Moving items
- S Slow Moving items

- D Dead items

5.5 Probabilistic Inventory Model

One such model is fixed order quantity model (FOQ).

In this model,

1. The demand (D) is uncertain, you can estimate the demand through any one of the forecasting techniques and the probability of demand distribution is known.
2. Lead time (L) is uncertain, probability of lead time distribution is known.
3. Cost(C) all the costs are known.
 - a. –Inventory holding costs C1
 - b. –shortage cost C2
4. The optimum order level Z is determined by the following relationship

$$\sum_{d=0}^{z-1} p(d) < \frac{C_2}{C_1 + C_2} < \sum_{d=z}^{\infty} p(d)$$

5.5.1 Stock out Cost/Shortage cost

It is difficult to calculate stock out cost because it consists of components difficult to quantify so indirect way of handling stock out cost is through service levels. Service levels means ability of organization to meet the requirements of the customer as on when he demands for the product. It is measured in terms of percentage.

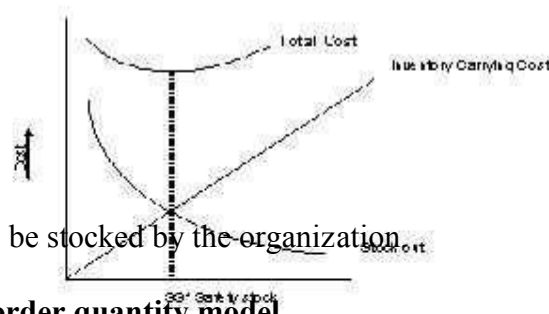
For example: if an organization maintains 90% service level, this means that 10% is “stock out” level. This way the stock out level is addressed.

5.5.2 Safety stock

It is the extra stock or buffer stock or minimum stock. This is kept to take care of fluctuations in

demand and lead time.

If you maintain more safety stock, this helps in reducing the chances of being “stock out”. But at the same time it increases the inventory carrying cost. Suppose the organization maintains less service level that results in more stock out cost but less inventory carrying cost. It requires a tradeoff between inventory carrying cost and stock out cost. This is explained through following Fig.



Safety stock ($S.S^*$) is to be stocked by the organization

5.5.3 Working of fixed order quantity model

Fixed order quantity system is also known as continuous review system or perpetual inventory system or Q system.

In this system, the ordering quantity is constant. Time interval between the orders is the variable. The system is said to be defined only when if the ordering quantity and time interval between the orders are specified. EOQ provides answer for ordering quantity. Reorder level provides answers for time between orders.

The working and the fixed order quantity model is shown in the below Fig

5.5.4 Application of Fixed Order Quantity System

1. It requires continuous monitoring of stock to know when the reorder point is reached.
2. This system could be recommended to "A" class because they are high consumption items. So we need to have fewer inventories. This system helps in keeping less inventory comparing to other inventory systems.

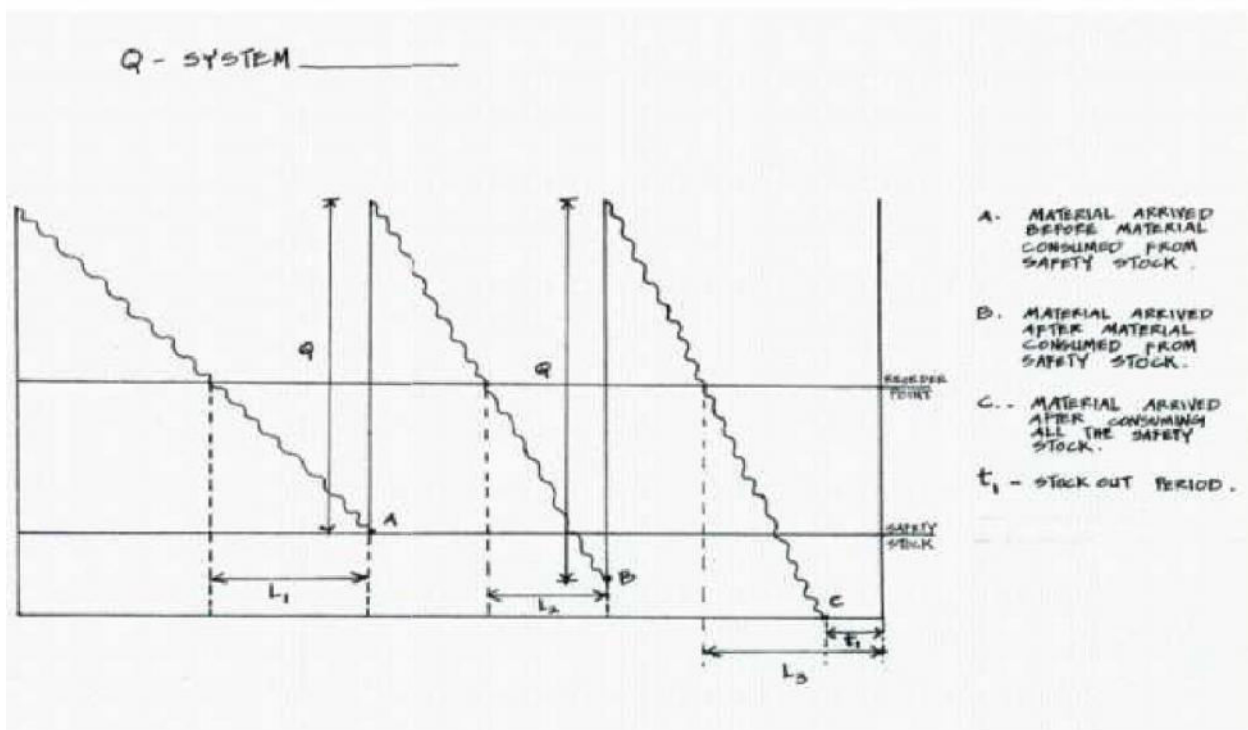
5.5.4.1 Advantages:

1. Since the ordering quantity is EOQ, comparatively it is meaningful. You need to have less safety stock. This model relatively insensitive to the forecast and the parameter changes.
2. Fast moving items get more attention because of more usage.

5.5.4.2 Weakness:

1. We can't club the order for items which are to be procured from one supplier to reduce the ordering cost.
2. There is more chance for high ordering cost and high transaction cost for the items, which follow different reorder level.
3. You can not avail supplier discount. While the reorder level fall in different time periods.

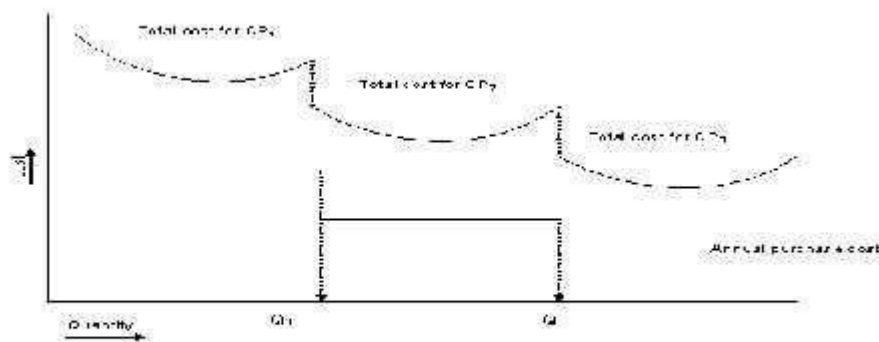
Figure –Fixed Order Quantity Model



5.5.5 QUANTITY DISCOUNT MODEL

As it is mentioned already, the purchase cost becomes relevant with respect to the quantity of order only when the supplier offers discounts. Discounts means if the ordering quantity exceeds particular limit supplier offers the quantity at lesser price per unit. This is possible because the supplier produces more quantity. He could achieve the economy of scale the benefit achieved through economy of scale that he wants to pass it onto customer. This results in lesser price per unit if customer orders more quantity.

If you look at in terms of the customer's perspective customer has also to see that whether it is advisable to avail the discount offered, this is done through a trade off between his carrying inventory by the result of acquiring more quantity and the benefit achieved through purchase price. Suppose if the supplier offers discount schedule as follows,



If the ordering quantity is less than or equal to Q_1 then purchase price is C_{p1} .

If the ordering quantity is more than Q_1 and less than Q_2 then purchase price is C_{p2} .

If the ordering quantity is greater than or equal to Q_2 then purchase price is C_{p3} .

Then the curve you get cannot be a continuous total cost curve, because the annual purchase

cost

breaks at two places namely at Q1 and Q2.

5.5.5.1 STEPS TO FIND THE QUANTITY TO BE ORDERED

1. Find out EOQ for the all price break events. Start with lowest price
2. Find the feasible EOQ from the EOQ's we listed in step 1.
3. Find the total annual inventory cost using the formulae for feasible EOQ $= \sqrt{2DSC} + D \cdot P$
4. Find the total annual inventory cost for the quantity at which price break took place using the following formula.

$$\text{Total annual inventory cost} = TC = (D/Q) \cdot S + (Q/2) \cdot C + D \cdot P$$

5. Compare the calculated cost in steps 3 and 4. Choose the particular quantity as ordered Quantity at which the total annual inventory cost is minimum.

5.6 QUEUING THEORY

Queuing theory concerns the mathematical study of queues or waiting lines (seen in banks, post offices, hospitals, airports etc.). The formation of waiting lines usually occurs whenever the current demand for a service exceeds the current capacity to provide that service.

The objective of the waiting line model is to minimize the cost of idle time & the cost of waiting time.

IDLE TIME COST: If an organization operates with many facilities and the demand from customers is very low, then the facilities are idle and the cost involved due to the idleness of the facilities is the *idle time cost*. The cost of idle service facilities is the payment to be made to the services for the period for which they remain idle.

WAITING TIME COST: If an organization operates with few facilities and the demand from customer is high and hence the customer will wait in queue. This may lead to dissatisfaction of

Problems

Problem 1. A particular item has a demand of 9000 units/year. The cost of one procurement is Rs. 100 and the holding cost per unit is Rs. 2.40 per year. The replacement is instantaneous and no shortance are allowed.

Determine

1. Economic lot size.
2. The no. of order per year.
3. The time between orders.
4. Total cost per year if the cost of one unit is Rs. 1.

Solution.

$$R = 9000 \text{ unit/year}$$

$$C_3 = \text{Rs. 100/procurement}$$

$$C_1 = \text{Rs. 2.40/unit/year}$$

1. $q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 100 \times 9000}{2.40}} = 866 \text{ units/Procurement.}$

2. $n_0 = \frac{1}{t_0} = \sqrt{\frac{C_1R}{2C_3}} = \sqrt{\frac{2.40 \times 9000}{2 \times 100}} = \sqrt{108} = 10.4 \text{ order/year.}$

3. $t_0 = \frac{1}{n_0} = \frac{1}{10.4} = 0.0962 \text{ years} = 1.15 \text{ months between procurement.}$

4.
$$\begin{aligned} C_0 &= 9000 \times 1 + \sqrt{2C_1C_3R} \\ &= 9000 + \sqrt{2 \times 2.40 \times 100 \times 9000} \\ &= \text{Rs. 11080 per year.} \end{aligned}$$

Problem 2. A manufacturing company purchases 9000 parts of a machine for its annual requirements, ordering one month usage at a time. Each part cost Rs. 20. The order cost per ordering is Rs. 15 and the carrying charges are 15% of the average inventory per year. You have been asked to suggest a more economical purchasing policy for the company. What advice would you offer and how much would it save the company per year.

Solution.

$$R = 9000 \text{ parts/year.}$$

$$q = \frac{9000}{12} = 750 \text{ parts}$$

$$C = \text{Rs. } 20/\text{ parts}, C_3 = \text{Rs. } 15/\text{order.}$$

$$C_1 = \text{Rs. } 20 \times \frac{15}{100} = \text{Rs. } 3/\text{part/year}$$

$$\text{Total annual variable cost} = \frac{q}{2} \cdot C_1 + \frac{R}{q} \cdot C_3$$

$$= \text{Rs. } \left[\frac{750}{2} \times 3 + \frac{9000}{750} \times 15 \right]$$

$$= \text{Rs. } 1305.$$

$$\text{E.O.Q. } (q) = \sqrt{\frac{2RC_3}{C_1}} = \sqrt{\frac{2 \times 9000 \times 15}{3}} = 300 \text{ units.}$$

$$\text{Total annual variable cost} = \sqrt{2RC_1C_3} = \sqrt{2 \times 9000 \times 3 \times 15}$$

$$(\text{with E.O.Q.}) = \text{Rs. } 900.$$

Hence if the company purchases 300 units each time and places 30 orders in the year, the net saving to the company will be Rs. $(1305 - 900) = \text{Rs. } 405$ a year.

Problem 3. You have to supply your customers 100 units of a certain product every monday- You obtain the product from a local supplier at Rs. 60 per unit. The cost of ordering and transportation from the supplier are Rs. 150 per order. The cost of carrying inventory is estimated at 15% per year of the cost of the product carried.

1. Find the lot size which will minimize the cost of the system.
2. Determine the optimal cost.

Solution.

$R = 100$ units/week ; $C = \text{Rs. } 60$.

$C_3 = \text{Rs. } 150$ per order

$C_1 = 15\%$ per year of the cost of the product.

$= (15 \times 60)/(100 \times 52)$ per unit per week.

$= \text{Rs. } 9/52$ per unit per week.

1.
$$q \text{ (EOQ)} = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 150 \times 100 \times 52}{9}} = 416 \text{ units.}$$

2.
$$C_{\min} = CR + \sqrt{(2C_1C_3R)}$$
$$= 60 \times 100 + \sqrt{2 \times \left(\frac{9}{52}\right) \times 150 \times 100}$$
$$= \text{Rs. } 6072.$$

Optimal cost = Rs. 6072.

E.O.Q. = 416 units.

Problem 4. Daily demand for a product is normally distributed with mean, 60 units and a standard deviation of 6 units. The lead time is constant at 9 days. The cost of placing an order is Rs. 200, and the annual holding costs are 20% of the unit price of Rs. 50. A 95% service level is desired for the customers, who place orders during the reorder period. Determine the order quantity and the reorder level for the item in question, assuming that there are 300 working days during a year.

Solution.

(R) Demand/day = 60 units

(C_3) order cost = Rs. 200/order

(C_1) holding cost = $0.20 \times 50 = \text{Rs. } 10/-$ per unit per year
working day/year = 300

Demand/year = $60 \times 300 = 18000$ units

$$\text{EOQ } q = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 200 \times 18000}{10}} = 848.52 \text{ units.}$$

Lead time = 9 days

Standard deviation of daily demand = 6 units

Now variance of demand during the lead time is equal to the sum of variance of daily demand during the lead time period

$$\begin{aligned}\text{variance} &= 6^2 + 6^2 + 6^2 + \dots + 6^2 \text{ (9 times)} \\ &= 9 \times 6^2 = 324\end{aligned}$$

Standard deviation of demand during the lead time

$$\text{Period} = \sqrt{324} = 18.$$

With E.O.Q. of 848.52 the no. of order during the year

$$(n_0) = \frac{18000}{848.52} = 21.21$$

Service level = 0.95

the normal deviate Z as found from probability table is 1.65.

$$\begin{aligned}\text{Safety stock} &= Z \times \text{standard deviation} \\ &= 1.65 \times 18 = 29.7 \text{ units.}\end{aligned}$$

$$\begin{aligned}\text{Reorder level} &= \text{Expected demand during lead time} + \text{safety stock} \\ &= 60 \times 9 + 29.7 = 569.7 \text{ units.}\end{aligned}$$

Ans. Economic order quantity EOQ = 848.52 units.

Reorder level = 569.7 units.

Problem 5. The demand per month for a product is distributed normally with a mean of 100 and standard deviation 25. The lead time distribution is given below. What service level will be afforded by a reorder level of 500 units?

Lead time (months) :	1	2	3	4	5
Probability :	0.10	0.20	0.40	0.20	0.10

Solution. It is given that the demand is distributed normally with

$$\text{Mean } (\bar{D}) = 100 \text{ units} \quad \text{SD } (\sigma d) = 25 \text{ units}$$

lead time (L) = 1, 2, 3, 4 and 5

Reorder level (M) = 500 units

We shall use iterative method of computing service level for the reorder level policy when the demand per unit time is distributed normally and distribution of lead time is known

$$Z = \frac{M - L\bar{D}}{\sigma d \sqrt{L}}$$

By iterative method

Lead time	Value of Z when M = 500	Probability of not running out of stock corresponding to the value of Z (from table)	Probability of this particular lead time occurring	Conditional Probability of not running out of stock
1.	$\frac{500 - 100 \times 1}{25\sqrt{1}} = 16.0$	100	0.10	10
2.	$\frac{500 - 100 \times 2}{25\sqrt{2}} = 8.49$	100	0.20	20
3.	$\frac{500 - 100 \times 3}{25\sqrt{3}} = 4.49$	100	0.40	40
4.	$\frac{500 - 100 \times 4}{25\sqrt{4}} = 2.00$	97.7	0.20	19.5
5.	$\frac{500 - 100 \times 5}{25\sqrt{5}} = 0.00$	50.0	0.10	5.0

Total conditional probability of not running out of stock
 $= 10 + 20 + 40 + 19.5 + 5 = 94.5$.

Hence a reorder level of 500 units will give 94.5% service level.

Problem 6. The annual demand for a product is 500 units. The cost of storage per unit per year is 10% of the unit cost, The ordering cost is Rs. 180 for each order. The unit cost depends upon the amount ordered. The range of amount ordered and the unit cost price are as follows

Range of amount ordered	$0 \leq Q_1 \leq 500$	$0 \leq Q_2 \leq 1500$	$1500 \leq Q_3 \leq 3000$	$3000 < Q_4$
Unit cost (Rs.)	25.00	24.80	24.60	24.4

Solution. Here $R = 500$ units

$I = 0.10$

$C_3 = \text{Rs. } 180$

$$\text{EOQ for unit price of Rs. } 24.40 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 180 \times 500}{24.40 \times 0.10}}$$

$$= 271.6 \text{ units.}$$

But this is not feasible because the unit price of Rs. 24.40 is not available for an order size of 271.6 units.

$$\text{EOQ for unit price of Rs. 24.60} = \sqrt{\frac{2 \times 180 \times 500}{24.60 \times 0.10}} = 270.5 \text{ units (infeasible)}$$

$$\text{EOQ for unit price of Rs. 24.80} = \sqrt{\frac{2 \times 180 \times 500}{24.80 \times 0.10}} = 269.4 \text{ units (infeasible)}$$

$$\text{EOQ for unit price of Rs. 25.00} = \sqrt{\frac{2 \times 180 \times 500}{25 \times 0.10}} = 268.3 \text{ units (feasible)}$$

Total annual cost for order quantity of 268.3 units (optimal size)

$$\begin{aligned} &= \sqrt{2C_1C_3R} + CR \\ &= \sqrt{2 \times 25 \times 0.10 \times 180 \times 500} + 25 \times 500 \\ &= \text{Rs. 13170.82.} \end{aligned}$$

Total annual cost for order quantity corresponding to cut off point of 500 units.

$$\begin{aligned} &= \frac{q}{2}C_1 + C_3\frac{R}{q} + CR \\ &= \frac{500}{2} \times 24.80 \times 0.10 + 180 \times \frac{500}{500} + 24.80 \times 500 \\ &= \text{Rs. 13200.} \end{aligned}$$

Total annual cost for order quantity corresponding to cut off point of 3000 units.

$$\begin{aligned} &= \text{Rs.} \left(\frac{1500}{2} \times 24.60 \times 0.10 + 180 \times \frac{500}{1500} + 24.60 \times 500 \right) \\ &= \text{Rs. 14745.} \end{aligned}$$

Total annual cost for order quantity corresponding to cut off point of 3000 units.

$$\begin{aligned} &= \text{Rs.} \left(\frac{3000}{2} \times 24.40 \times 0.10 + 180 \times \frac{500}{3000} + 24.40 \times 500 \right) \\ &= \text{Rs. (3660 + 30 + 12200)} = \text{Rs. 15890.} \end{aligned}$$

Since the total cost is minimum at $q_0 = 271.6$ units. It represents the optimal order quantity.

List of Formulas

1. Expected number of units in the system (waiting + being served) (or)
Length of the system

$$L_s = \frac{\lambda}{\mu - \lambda}$$