(3) The demand for a certain item is 16 units per period. Unsatisfied demand causes a shortage cost of R. 0.75 per unit per short period. The cost of inventory purchasing action is Rs. 15 per purchase and the holding cost is 15% of average inventory valuation per period. Item cost is Rs. 8000 per unit. Find the minimum cost purchase quantity.

(**Ans.** $q^* = 32$ units, $TC = $ Rs. 15 app.)

(4) A manufacturing company has to supply 3000 units per year to a customer who does not have enough space for storing the material. A penalty of Rs. 40 will be levied when the supplier fails to supply the material. The inventory carrying cost in Rs. 20 per unit per month and the ordering cost is Rs. 400 per run. Calculate the expected number of shortage at the end of each scheduling.

(**Ans.** 41 units per period)

(5) A particular item has a demand of 9000 units per year. The cost of one procurement is Rs. 100 and the holding cost is Rs. 240 per unit per year. The replacement is instantaneous and shortage is given as Rs. 5 per unit per year. Determine

(i) the economic lot size

(ii) the number of orders per year

(iii) the time between orders

(iv) the total cost per year if the cost of one unit is Re 1.

(Ans. (i) $q^* = 1053$ units per run; (ii) Number of orders per year; (iii) 0.117 year; (iv) Rs. 10712)

(6) A company has a demand of 12000 units per year for an item and it can produce 2000 such items per month. The cost of one set-up is Rs 400 and the holding cost per unit per month is 0.15. The shortage cost of one unit is Rs 20 per year. Determine the optimum lot size and the total cost per year assuming the cost of 1 unit as Rs. 4.

(**Ans.** $q^* = 3413$ units; $TC = $ Rs. 51, 336)

(7) The demand of an item is uniform at a rate of 20 units per month. The fixed cost is Rs 10 each time a production run is made. The production cost is Rs 1 per item and the inventory carrying cost is Re. 0.25 per item per month. If the shortage cost is Rs. 1.25 per item per month, determine how frequently the production run is to be made and of what size.

(**Ans.** $q^* = 44$ items; $t^* = 2.2$ month)

14.8 INVENTORY MODELS WITH PROBABILISTIC DEMAND

In this we consider the situations where demand is not known exactly but the probability distribution of demand is somehow known. The control variable in such cases is assumed to be either the scheduling period or the order level or both. The optimum order level can be derived by minimising the total expected cost rather than the actual cost involved. Expected cost are obtained by multiplying the actual costs for a particular situation with the probability of occurrence of that situation.

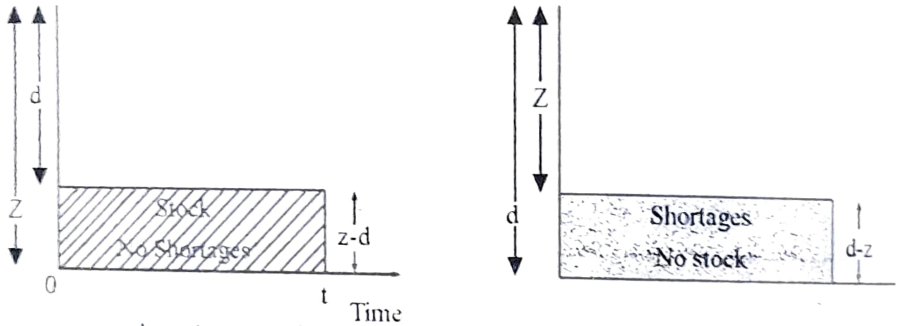
14.8.1 Model V (a)

Instantaneous demand, no set-up cost, stock, in discrete units.

The assumption in this model are

(i) It is the constant interval between orders. (It may also be considered as unity).
(ii) z is the stock (in discrete) units for time t
(iii) d is the estimated demand with probability $p(d)$
(iv) C_1 is the holding cost per item per t time unit
(v) C_2 is the shortage cost per item per t time unit
(vi) Lead time is zero to determine the optimum order level z.

Solution In this model, it is assumed that with instantaneous demand the total demand is filled at the beginning of each period. Thus, depending on the amount d demanded, the inventory position just before the demand occurs may either be surplus or shortage.



Case 1: When demand d exceeds the stock z i.e. $d \leqslant z$
Now holding cost becomes

$$= \begin{cases} (Z-d)\,C_1 & d \le Z \\ C_1 \times 0 & \text{for } d > Z \text{ (no stock)} \end{cases}$$

Case 2: When $d > z$ then shortage cost

$$= C_2 \times 0 \; d \; z$$
$$= (d-z)\,C_2 \text{ for } d > z$$

The total expected cost

$$C(Z) = \sum_{d=0}^{Z} (Z-d) C_1 p(d) + \sum_{d=Z+1}^{\infty} C_1 \cdot 0 p(d)$$

$$+\sum_{d=0}^{Z} C_2 \cdot 0\, p(d) + \sum_{d=Z+1}^{\infty} (d-Z)C_2\, p(d)$$

$$C(Z) = \sum_{d=0}^{Z} (z-d)C_1 p(d) + \sum_{d=Z+1}^{\infty} (d-Z)C_2 p(d)$$

For $C(Z)$ to be minimum

$$\Delta C(Z-1) < 0 < \Delta C(Z)$$

$$\Delta C(Z) = C_1 \sum_{d=0}^{Z} [(Z+1)-d-(Z-d)]\, p(d)$$

$$+ C_2 \sum_{d=Z+1}^{\infty} [(d-Z+1)-(d-Z)]\, p\,(d)$$

$$= C_1 \sum_{d=0}^{Z} p(d) - C_2 \sum_{d=Z+1}^{\infty} p(d)$$

$$= C_1 \sum_{d=0}^{Z} p(d) - C_2 \sum_{d=0}^{\infty} p(d) - \sum_{d=0}^{Z} p(d)$$

$$= (C_1 + C_2) \sum_{d=0}^{Z} p(d)\, C_2 \quad \because \sum_{d=0}^{Z} p(d) = 1$$

For minimum $\Delta C(z) > 0$

$$\therefore (C_1 + C_2) \sum_{d=0}^{Z} p(d) - C_2 > 0$$

$$\boxed{\sum_{d=0}^{\infty} p(d) > \frac{C_2}{C_1 + C_2}}$$

$\therefore$ The required relationship is

$$\sum_{d=0}^{Z-1} p(d) < \frac{C_2}{C_1 + C_2} < \sum_{d=0}^{Z} p(d)$$

14.8.2 Model V (b)- Stock levels in Continuous Units

In this model is the stock levels are in continuous units then we replace p(d) by f(x) dx where f(x) is p.d.f.

Proof Let $\int_{x_1}^{x_2} f(x)\, dx$ be the probability of an order within range x_1 to x_2

The cost equation for this model is same as the previous model by replacing $p(d)$ by $f(x)\,dx$

$$C(Z) = C_1\int_0^Z (Z-x)f(x)dx + C_2\int_Z^\infty (x-Z)f(x)dx$$

The optimal value of z is obtained by equating to the first derivative of

$$\frac{dC(Z)}{dZ} = C_1\int_0^Z [(1-0)f(x)\,dx] + C_1\left[(Z-x)\,f(x)\,\frac{dx}{dZ}\right]_{x=0}^{Z}$$

$$+\, C_2\int_0^\infty (0-1)f(x)\,dx + C_2\left((x-Z)\,f(x)\frac{dx}{dZ}\right)_0^\infty$$

$$= C_1\int_0^Z f(x)dx - C_2\int_Z^\infty f(x)dx$$

$$= C_1\int_0^Z f(x)dx - C_2\left[\int_Z^\infty f(x)dx - \int_0^Z f(x)dx\right]$$

$$\left[\because \int_Z^\infty f(x)dx = 1\right]$$

$$= (C_1 + C_2)\int_0^Z f(x)dx - C_2$$

$$\frac{dC(Z)}{dZ} = 0 \Rightarrow (C_1 + C_2)\int_0^Z f(x)dx = C_2$$

$$\int_0^Z f(x)dx = \frac{C_2}{C_1 + C_2}$$

Also $$\frac{d^2C(Z)}{dZ^2} = (C_1 + C_2)\left(f(x)\frac{dx}{dZ}\right)_0^Z$$

$$= (C_1 + C_2)f(z) > 0$$

$(\because f(Z) > 0,\ C_1$ and C_2 are not zero

Thus, we can get the optimum value of z satisfying

$$\int_0^Z f(x)dx = \frac{C_2}{C_1 + C_2}$$

Example 14.16 A newspaper boy buys papers for 30 paise each and sells them for 70 paise. He cannot return unsold newspapers. Daily demand has the following distribution

No. of customers	23	24	25	26	27	28	29	30	31	32
Probability	.01	.03	0.06	0.10	0.20	0.25	0.15	0.10	0.05	0.05

If each boy's demand is independent of the previous day's how many papers should he order each day.

Solution

$C_1 = 0.30$ $\quad C_2 = 0.70 - 30 = 0.40$

	23	24	25	26	27	28	29	30	31	32
$P(d)$	0.01	0.03	0.06	0.10	0.20	0.25	0.15	0.10	0.05	0.05
$\Sigma P(d)$	0.01	0.04	0.10	0.20	0.40	0.65	0.80	0.90	0.95	1.00

Now $\dfrac{C_2}{C_1 + C_2} = \dfrac{40}{30+40} = 0.5714.$

Now $0.40 < 0.5714 < 0.65$

$\therefore$ No of papers ordered = 28.

Example 14.17 The cost of holding an item in stock is Rs. 2 per unit and the shortage cost is Rs. 8. If Rs. 2 is the purchasing cost per unit, determine the optimal order level of inventory. Following probability distribution.

D	0	1	2	3	4	5
P_D	0.05	0.25	0.20	0.15	0.20	0.15

Solution

Given, $C_1 = \text{Rs. } 2$

$C_2 = \text{Rs. } 8$

Purchasing cost is Rs. 2

D	0	1	2	3	4	5
P_D	0.05	0.25	0.20	0.15	0.20	0.15
ΣP_D	0.05	0.30	0.50	0.65	0.85	1.00

$$\frac{C_2}{C_1+C_2} = \frac{8}{10} = 0.8$$

$$.65 < \frac{C_2}{C_1-C_2} < 0.85$$

Example 14.18 The probability distribution of a monthly sales of a certain item is as follows

Monthly sales	0	1	2	3	4	5	6
Probability	0.02	0.05	0.30	0.27	0.20	0.10	0.06

The cost of carrying inventory is Rs. 10 per unit per month. The current policy is to maintain a stock of four items at the beginning of each month. Assuming that the cost of shortage is proportional to both time and quantity, obtain the imputed cost of shortage of one item for one unit.

(MU. B.E. Apr. 93).

Solution

Monthly sales	0	1	2	3	4	5	6
Probability	0.02	0.05	0.30	0.27	0.20	0.10	0.06
Cumulative	0.02	0.07	0.37	0.64	0.84	0.94	1.00

We know $\sum_{d=0}^{Z-1} p(d) < \frac{C_2}{C_1+C_2} < \sum_{d=0}^{Z} p(d)$

here $Z = 4$ $\sum_{d=0}^{3} p(d) < \frac{C_2}{C_1+C_2} < \sum_{d=0}^{4} p(d)$

$$\Rightarrow 0.64 < \frac{C_2}{C_1+C_2} < 0.84$$

$$0.64 < \frac{C_2}{10+C_2} < 0.84$$

$$0.64 < \frac{C_2}{10+C_2} \Rightarrow C_2 = 17.7$$

From $\frac{C_2}{C_1+C_2} < 0.84$

$C_2 = 52.5$

Shortage Cost $17.7 < C_2 <$ Rs. 52.5

Example 14.19 The demand for a certain product has a rectangular distribution between 4000 and 5000. Find the optimal order quantity if the storage cost is Re. 1 per unit and shortage cost is Rs. 7 per unit.

Solution

Given: $C_1 = \text{Re. } 1;\ C_2 = \text{Rs. } 7$

$$\int_0^Q f(x)\,dx = \frac{C_2}{C_1 + C_2}$$

Since the distribution is rectangular the p.d.f is given by $\frac{1}{b-a}$

$$\text{Hence } \int_{4000}^{Q} \frac{1}{5000-4000}\,dx = 7/8$$

$$\Rightarrow \int_{4000}^{Q} \frac{1}{1000}\,dx = 7/8$$

$$\frac{1}{1000}(x)_{4000}^{Q} = 7/8$$

$$\frac{1}{1000}(Q-4000) = 7/8$$

$$Q - 4000 = \frac{7000}{8}$$

$$Q = \frac{7000}{8} + 4000 = \frac{39000}{8}$$

$$= 4875$$

$$Q = 4875 \text{ units}$$

Example 14.20 An ice cream company sells one of its type of ice creams by weight. If the product is not sold on the day it is prepared, it can be sold for a loss of 50 paise per pound. But there is an unlimited market for one day old ice creams. On the other hand the company makes a profit of Rs. 3.20 on every pound of ice creams sold on the day it is prepared. If daily orders form a distribution with $f(x) = 0.02 - 0.0002x \quad 0 \leq x \leq 100$, how many pounds of ice creams should the company prepare every day.

Solution

Given, $C_1 = \text{Re. } 0.50;\ C_2 = \text{Rs. } 3.20$

Let Q be the amount of ice cream prepared every day

$$\text{Now} \int_0^Q f(x)dx = \frac{C_1}{C_1+C_2}$$

$$\int_0^Q (0.02 - 0.0002x)dx = \frac{3.2}{3.2+0.5}$$

$$\left(0.02x - \frac{0.0002x^2}{2}\right)_0^Q = 0.865$$

$$0.0002Q^2 + 0.04Q + 1.730 = 0$$

On solving, $Q = 136.7$ and 63.5
But $Q = 136.7$ is not possible ($\because x \leq 100$)
Hence, $Q = 63.5$ pounds is the optimum quantity.

Example 14.21 Let the probability density of a demand of a certain item during a day be

$$f(x) = \begin{cases} 0\cdot 1 & 0 \leq x \leq 10 \\ 0 & x > 10 \end{cases}$$

The demand is assumed to occur at a uniform pattern during the whole day. Assuming that the unit carrying cost of the item in inventory is Rs. 0.5 per day, and unit shortage cost is Rs. 4.5 per day. If the purchasing cost per unit is Rs. 0.5. Determine the optimum level of inventory.

Solution Given $C_1 =$ Rs. 0.50, $C_2 =$ Rs. 4.5, $C_3 =$ Rs. 0.5
Let Q be the amount of ice cream prepared every day

$$\int_0^Q f(x)dx = \frac{C_2 - C}{C_1 + C_2}$$

$$\int_0^Q 0.1\, dx = \frac{4.5 - 0.5}{4.5 + 0.5}$$

$$((0.1)x)_0^Q = 0.8$$

$$0.1Q = 0.8$$

$$\Rightarrow Q = 8 \text{ units}$$

EXERCISES

1. A newspaper boy buys paper for Rs. 1.40 and sells them for Rs. 2.45 each. He cannot refund unsold newspapers. Daily demand has the following distribution

Customers	25	26	27	28	29	30	31	32	33	34	35	36
Probability	0.03	0.05	0.05	0.10	0.15	0.15	0.12	0.10	.10	0.07	0.06	0.02

If each day's demand is independent of the previous day's demand. How many papers he should order each day.

(**Ans:** 30 news papers)

2. A contractor of second hand motor trucks maintains a stock of trucks every month. Demand of the trucks occurs at a relatively constant rate but not in a constant size. The demand is shown in the following probability distribution

Demand	0	1	2	3	4	5	6 or more
Probability	0.40	0.24	0.20	0.10	0.05	0.01	0.00

The holding cost of an old truck in stock for one month is Rs. 100.00 and the penalty for a truck if not supplied on demand is Rs 1000.00. Determine the optimal size of the stock of the contractor?

(**Ans:** 3 trucks).

3. A ship building company has launched a program for the construction of a new class of ships. Certain spare parts like prime mover each costing Rs. 2 lakhs have to be purchased. If these units are not available when needed a very serious loss is incurred which is in the order of 1 crore in each instance. Requirements of spares with the correspounding probabilities are given below

No of spares	0	1	2	3	4	5
Probability of requirement	0.876	0.062	0.041	0.015	0.005	0.001

How many spare parts should the company buy in order to optimise the inventory decision?

(**Ans:** 3 spare parts)

4. A banking company sells cakes by the kg weight. It makes a profit of Rs. 5.00 per kg, on each kg sold on the day it is baked. It disposes of all cakes not sold on the day it is baked at a loss of Rs. 1.20 a kg. If the demand is known to be rectangular between 2000 and 3000 kg. Determine the optimal order quantity baked.

(**Ans:** 2807 kg.)

5. A T.V dealer finds that the cost of holding a television in stock for a week is Rs. 20, customers who cannot obtain new television immediately tend to go to other dealers and he estimates that for every customer who does not get immediate delivery he loses on the average Rs. 200. For one particular model of television the probabilities for a demand of 0, 1, 2, 3, 4 and 5 television in

14

chapter

INVENTORY CONTROL

14.1 INTRODUCTION

Inventory is defined as any idle resources of an enterprise. It is a physical stock of goods kept for future use. In a factory the inventory may be in the form of raw materials, parts, semi-finished goods. Inventory also include furniture, machinery etc.

14.2 REASONS FOR MAINTAINING INVENTORIES

The need of the management to make decisions regarding the inventory arises because of the various alternative course of action available with the enterprise. It is essential for an enterprises to have inventory due to the following reasons.

(i) It helps in smooth and efficient running of the business.
(ii) It provides adequate service to the customers.
(iii) It reduces the possibility of duplication of orders.
(iv) It helps in maintaining economy by absorbing some of the fluctuations when the demand of an item fluctuates or is seasonal.
(v) It helps in minimising the loss due to the deterioration, obsolescence, damage etc.
(vi) It acts as a buffer stock when raw materials are received late and shop rejections are too many.
(vii) Takes advantages of price discounts by bulk purchasing.

Though the inventories are essential and provide an alternative to production/purchase in the future, it also locks up the capital of the enterprise. It includes the expenses of stores, equipment, personnel, insurance etc., therefore, excess inventories are undesirable. Larger inventories do not necessarily lead to a high volume of output instead it might hamper the production.

Our problem is to balance between the advantages of having inventories and cost of carrying them to arrive at an optimal level of inventories to minimise the total inventory cost. This calls for controlling the inventories in the most profitable way. The basic objective of inventory control is to release capital for more productive use.

14.3 TYPES OF INVENTORY

There are five types of inventory, namely:

(i) Transportation inventories
(ii) Buffer inventories
(iii) Anticipation inventories
(iv) De coupling inventories
(v) Lot-size inventories.

Transportation inventories This arises due to the transportation of inventory items to the various distribution centers and customers from the various production centers. The amount of transportation inventory depends on the time consumed in transportation and the nature of the demand.

Buffer inventories These are maintained to meet the uncertainty of demand and supply.

Seasonal

Anticipation inventories These are built in advance by anticipating or foreseeing the future demand. (e.g) Production of crackers before the Diwali festival, electric fans, or coolers before the on-set of summer season.

De-coupling inventories The inventories used to reduce the interdependence of various stages of production system are known as de–coupling inventories.

Lot-size inventories Generally the rate of consumption is different from the rate of production or purchasing. Therefore, items are produced in larger quantities which result in lot-size, also called as cycle inventories.

14.4 INVENTORY COSTS

There are four categories of inventory cost associated with keeping inventories of items. They are:

(i) Item (or production or purchase) cost
(ii) Ordering or set-up cost

(ii) Carrying or holding cost
(iv) Shortage or stock out cost

Item cost It refers to the cost associated with an item whether it is manufactured or purchased. The purchase price will be considered when discounts are allowed for any purchase above a certain quantity.

Set-up cost (C_3) These costs include the fixed cost associated with obtaining the goods through placing of an order or purchasing or manufacturing or setting-up a machinery before starting the production. They include the costs of —purchase, requisition, follow up, receiving the goods, quality control etc. These are also called as order costs or replenishment costs usually denoted by C_3 per production run (cycle). They are assumed to be independent of the quantity ordered or produced.

Carrying or holding cost (C_1) The cost associated with carrying or holding the goods in stock is known as holding or carrying cost, which is denoted by C_1 per unit of goods for a unit of time. Holding cost is assumed to vary directly with the size of inventory as well as the time the item is held in stock. The following components constitute the holding cost.

(i) Invested capital cost: This is the interest charge over the capital invested.
(ii) Record keeping and administrative cost.
(iii) Handling cost: These include costs associated with movement of stock such as cost of labour etc.
(iv) Storage costs.
(v) Depreciation costs.
(vi) Taxes and insurance etc.

If P is the purchase price of an item, I is the stock holding cost per unit time as a fraction of stock value then the holding cost is $C_1 = IP$.

Shortage cost or stock out cost (C_2) The penalty costs that are incurred as a result of running out of stock (ie. shortage) are known as shortage or stock out costs. These are denoted by C_2 per unit of goods for a specified period.

If the unfilled demand for the goods can be satisfied at a latter date (backlog case) these costs are assumed to vary directly with the shortage quantity and the delaying time both. If the unfilled demand is lost (no backlog case) shortage cost becomes proportional to shortage quantity only.

14.5 VARIABLES IN THE INVENTORY PROBLEM

The variables involved in the inventory model are of two types:
(i) Controlled variable (ii) Uncontrolled variables

Controlled variables These variables include three basic questions namely—

1) How much quantity of an item that should be ordered?
2) When should the order be placed? i.e. the frequency or timing of acquisition.
3) The completion stage of stocked items.

Uncontrolled variables These include holding costs, shortage cost and set-up cost.

Note: Total inventory cost = Purchase cost of inventory items + Ordering cost + Carrying cost + Shortage costs.

14.6 OTHER FACTORS INVOLVED IN INVENTORY ANALYSIS

14.6.1 Demand

Demand refers to the number of items required per period. It may be known exactly or known in terms of probabilities or may be completely unknown.

The demand pattern of items may be either deterministic or probabilistic. Problems in which demand is known and fixed are called deterministic problem. Whereas those problems in which the demand is assumed to be a random variable are called stochastic or probabilistic problems.

In case of deterministic demand it is assumed that the quantities needed over subsequent periods of time are known exactly. Further the known demand may be fixed or variable with time. Such demands are called *static or dynamic demands* respectively.

The probabilistic demand occurs when the demand over a certain period of time is not known with certainty; but it is described by a known probability distribution. A probabilistic demand may be either stationary or non-stationary overtime.

14.6.2 Lead Time

The time gap between the placing of an order and the actual arrival of the inventory is known as leadtime. If the leadtime is known and is not

...zero, and if the demand is deterministic, all that one requires to order in advance by the time equal to the lead-time. If the lead-time is zero, there is no need to order in advance.

...case, the leadtime is a variable which is known only probabilistically, then the question of when to order is more difficult. The amount and timing of replenishment is found by considering the *expected* total holding and shortage over the leadtime required.

14.6.3 Amount Delivered (Supply of Goods)

...supply of goods may be instantaneous or spread over a period of time. If a quantity q is ordered or purchased or produced, the amount delivered may vary around q with a known probability density function.

14.6.4 Order Cycle

The time period between placement of two successive orders is referred to as an order cycle. The order cycle may be placed on the basis of the following two types of inventory review systems.

Continuous review The record of the inventory level is checked continuously until a certain lower limit (known as recorder level) is reached when a new order is placed. This is often known as two-bin systems.

Periodic review In this the inventory levels are reviewed at equal time intervals and orders are placed at such intervals. The quantity ordered each time depends on the available inventory level at the time of review.

14.6.5 Time Horizon

The time period over which the inventory level will be controlled is known as time horizon.

14.6.6 Recorder Level

The level between the maximum and the minimum stock at which the purchasing (manufacturing) activities must start for the replenishment is known as recorder level.

The inventory model can be classified into two categories.

(i) Deterministic inventory model
(ii) Probabilistic inventory model