Operation Research:

1. OR is an aid for the executive in making decisions by providing him with the quantitative information, based on the scientific method analysis.

2. OR is the art of giving bad answers to problems, to which, otherwise worse answers are given.

3.OR is the art of wining wars without actually fighting them.

4.OR is the application of scientific methods by interdisciplinary teams to problems involving the control of organised (man-machine) systems so as to provide solutions which best serve the purpose of the organisation as a whole.

The characteristics of OR Model

A model is defined as idealised representation of the real life situation. It represents one or few aspects of reality.

Characteristics of OR—

1. The number of simplifying assumption should be as few as possible.

2. Model should be simple but close to reality.

3. It should be adaptable to parametric type of treatment.

4. It should be easy and economical to construct.

Main advantage and limitation of OR model.

Advantage

1 It provides a logical and systematic approach to the problem.

2. It indicate the scope .as well as limitation of a problem.

3. It makes the overall structure of the problem more comprehensible and helps in dealing with problem in its entirety.

Limitations—

Models are only idealised representation Of reality and should not be regarded as absolute in any case.

Distinguish between:

(i) Iconic or Physical Model and Analogue or schematic model.

(ii) Deterministic and Probabilistic model.

(i) Iconic Model—It represent the system by enlarging or reducing the size on some scale. In other words it is an image.

Example-toy aeroplane, photographs, drawings, maps etc.

Schematic Model—The models, in which one set of properties is used to

represent, another set of properties are called schematic or analogue models.

For example-graphs used to show different quantities.

(ii) Deterministic model—Such models assume conditions of complete certainty and perfect knowledge.

Example-LPP, transportation, assignment etc.

Probabilistic (or stochastic) Model—These type of models' usually handle such

situation in which the consequences or payoff of managerial actions cannot be predicted with certainty. However it is possible to forecast a pattern of events, based on which managerial decision can be made.

For example insurance companies are willing to insure against risk of fire, accident, sickness and so on. Here the pattern of events have been compiled in the form of probability distribution.

The objective of operation Research

The objective of OR is to provide a scientific basis to the managers of an organization for solving problems involving interaction of the components of the system, by employing a system approach by a team of scientists drawn from different disciplines, for finding a solution which is in the best interest of the organization as a whole.

The characteristics of operation research.

1. System Orientation

2. Use of interdisciplinary teams

3. Application of Scientific methOds

4. Uncovering of new problems

5. Improvement in the quality of decisions

6. Use of computer

7. Quantitative solutions

8. Human factors

The various phases of operation research

Or

The steps involved in the solution of OR Problem.

Operation research is based on scientific methodology which proceeds as:

1. Formulating the problem.

2 Constructing a model to represent the system under study

3. Deriving a solution from the model.

4. Testing the model and the solution derivq4 from it.

5. Establishing controls over the solution.

6. Putting the solution to work i.e. implementation.

(i) Assignment of job to machine

(ii) Product mix

(iii) Advertising media selection

(iv) Transportation.

2. Dynamic programming

(i) Capital budgeting

(ii) Employment smoothening

(iii) Cargo loading.

3. Inventory control

(i) Economic order quantity

MATHEMATICAL FORMULATION OF LPP

Egg contains 6 units of vitamin A per gram and 7 units of vitamin B per gram and cost 12 paise per gram. Milk contains 8 units of vitamin A per gram and 12 units of vitamin B per gram and costs 20 paise per gram. The daily minimum requirement of vitamin A and vitamin B are 100 units and 120 units respectively. Find the optimal product mix. Formulate this problem as a LPP.

Let x_1 = number of grams of eggs to be consumed
x_2 = number of grams of milk to be consumed
The LPP is: Min Z = $12x_1 + 20x_2$
Subject to

$$6x_1 + 8x_2 \geq 100$$
$$7x_1 + 12x_2 \geq 120$$
$$x_1, x_2 \geq 0$$

GRAPHICAL METHOD

Working procedure

Step 1: Consider each inequality constraint as equation.

Step 2: Plot each equation on the graph, as each will geometrically represent a straight line.

Step 3: Mark the region. If the inequality constraint corresponding to a line is $\leq$ type then the region below the line lying in the first quadrant is shaded. For the inequality constraint $\geq$ type, the region above the line in the first quadrant is shaded. The points lying in the common region will satisfy all the constraints simultaneously. The common region is the feasible region.

Step 4: Plot the objective function by assuming Z = 0. This will be a straight line passing through the origin. As the value of Z is increased from zero, the line starts moving, parallel to itself. Move the line till it is farthest away from the origin for maximization of the objective function. For a minimization problem the line will be nearest to the origin. A point of the feasible region through which this line passes will be the optimal point.

Step 5: Alternatively find the co-ordinates of the extreme points of the feasible region and find the value of the objective function at each of these extreme points. The point at which the value is maximum (or minimum) is the optimal point and its coordinates give the optimal solution.

Solve the following LPP by graphical method.

Minimize Z = $20x_1 + 10x_2$

Subject to

$x_1 + 2x_2 \le 40$

$3x_1 + x_2 \ge 30$

$4x_1 + 3x_2 \ge 60$

& $x_1, x_2 \ge 0$

Solution:

Replace all the inequalities of the constraints by equation

$x_1 + 2x_2 = 40$ if $x_1 = 0 \Rightarrow x_2 = 20$

$x_2 = 0 \Rightarrow x_1 = 40$

$\therefore$ $x_1 + 2x_2 = 40$ passes through (0,20)(40,0)

$3x_1 + x_2 = 30$ passes through (0,30)(10,0)

$4x_1 + 3x_2 = 60$ passes through (0,20)(15,0)

Plot each equation on the graph. The feasible region is ABCD.

C and D are points of intersection of lines

$\therefore$ $x_1 + 2x_2 = 40$, $3x_1 + x_2 = 30$,

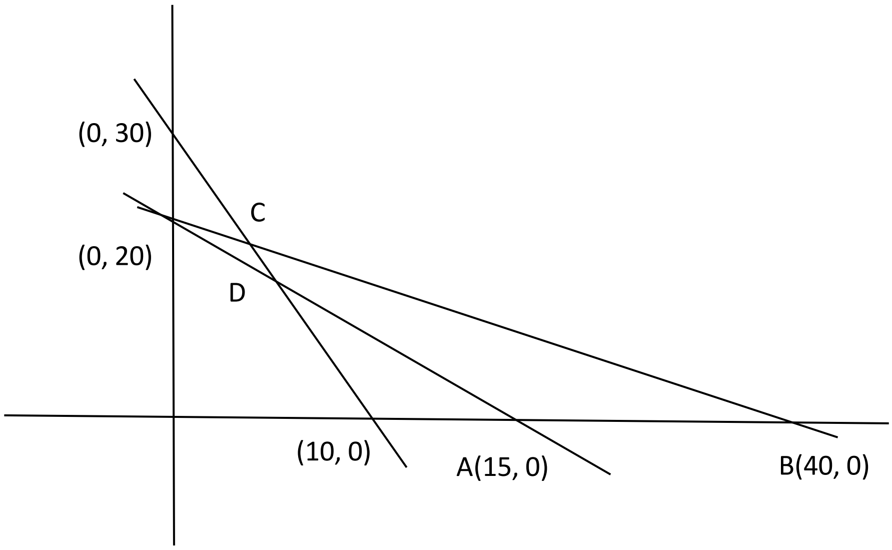
and $4x_1 + 3x_2 = 60$, $x_1 + x_2 = 30$

On solving we get C = (4, 18) D = (6, 12)

Corner points	Value of Z = 20 $x_1 + 10x_2$
A (15,0)	300
B (40,0)	800
C (4,18)	260
D (6,12)	240 minimum value

$\therefore$ The minimum value of Z occurs at D (6, 12).

Hence the optimal solution is $x_1 = 6, x_2 = 12$.



Problem. An advertising company wishes to plan its advertising strategy in three different media-television, radio and magazines. The purpose of advertising is to reach as large a number of potential customers as possible. Following data have been obtained from market survey-

	Television	Radio	Magazine I	Magazine II
Cost of an advertising unit	Rs. 30000	20000	15000	10000
No. of potential customer reached per unit	20000	600000	150000	100000
No. of female customer reached per unit	1, 50000	400000	70000	50000

The company wants to spend not more than Rs. 450000 on advertising. Following are the further requirements.

1. at least 1 million exposures take place among female customers.

2. advertising on magazines be limited to Rs 1, 50000

3. at least 3 advertising units to be bought on magazine 1 and 2 units on magazine II.

4. The number of advertising units on television and radio should each be between 5 and 10.

Formulate an LPP model for the problem.

Solution : Let x1- no. of advertising unit of television

no. of advertising unit of radio

x3- no. of advertising unit of Magazine I

x4— no. of advertising unit of Magazine II

Objective function

Maximize Z = i05 (2x1+ 6x2 + 1.5x3 + x4)

Constraints are

$$30000x_1 + 20000x_2 + 15000x_3 + 10000x_4 \leq 450000$$

$$150000x_1 + 400000x_2 + 70000x_3 + 50000x_4 \geq 1000000$$

$$15000x_3 + 10000x_4 \leq 150000$$

$$x_3 \geq 3$$

$$x_4 \geq 2$$

$$5 \leq x_1 \leq 10 \text{ or } x_1 \geq 5,\ x_1 \leq 10$$

$$5 \leq x_2 \leq 10 \text{ or } x_2 \geq 5,\ x_2 \leq 10$$

where $x_1, x_2, x_3\, x_4$ each ≥ 0

Problem. A person requires 10, 12 and 12 units of chemicals A, B and C respectively for herbal garden. A liquid product contains 5, 2 and 1 units of A,B and C respectively per Jar. A dry product contains 1, 2 and 4 units of A, B and C per cartoon. If the liquid product sells for Rs. 3 per Jar and dry product sells for Rs. 2 per cartoon, how many of each should be purchased to minimise the cost and meet the requirements.

Solution : Requirement

	A	B	C	
	10	12	12 units	
Liquid product	5	2	1	units Rs. 3/-per jar
Dry product	1	2	4	Rs. 2/-per Cartons

1. Select decision variable

   x_1 – no. of jars of liquid product

   x_2 – no. of cartoons of dry product

2. Objective function

   Minimize cost $(z) = 3x_1 + 2x_2$

3. Constraints :

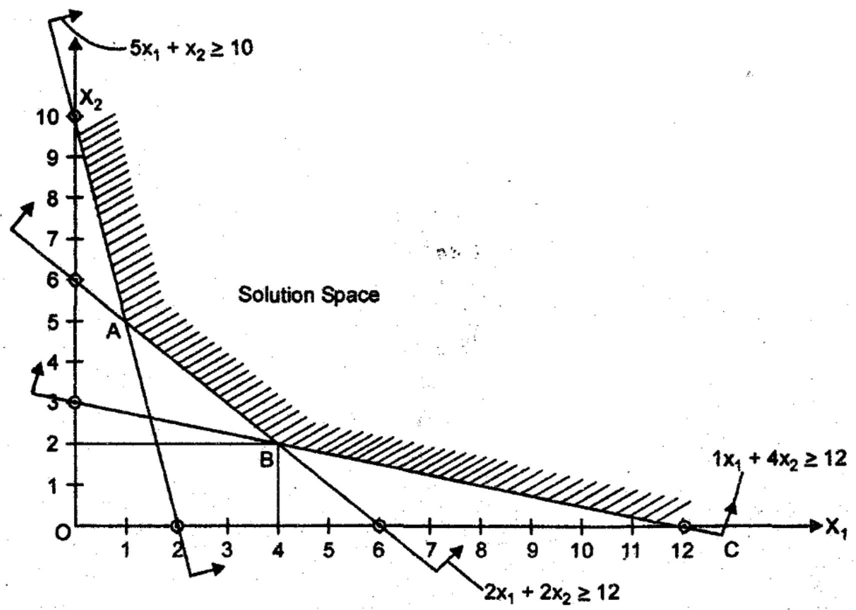
$$5x_1 + x_2 \geq 10$$
$$2x_1 + 2x_2 \geq 12$$
$$1x_1 + 4x_2 \geq 12$$

4. Add non negativity constraints :

$$x_1 \geq 0 \quad ; \quad x_2 \geq 0$$

Graphical Method :

$$5x_1 + x_2 = 10 \Rightarrow x_1 = 0; x_2 = 10 \text{ and } x_2 = 0; x_1 = 2$$
$$2x_1 + 2x_2 = 12 \Rightarrow x_1 = 0; x_2 = 6 \text{ and } x_2 = 0; x_1 = 6$$
$$1x_1 + 4x_2 = 12 \Rightarrow x_1 = 0; x_2 = 3 \text{ and } x_2 = 0; x_1 = 12$$



Point A (1,5) $Z(A) = 3 \times 1 + 2 \times 5 = 13$

Point B (4, 2) $Z(B) = 3 \times 4 + 2 \times 2 = 16$

Point C (12,0) $Z(C) = 3 \times 12 + 2 \times 0 = 36$

Minimum cost at point A *i.e.* Rs. 13

x_1 (no. of Jar of Liquid product) = 1

x_2 (no. of carton of dry product) = 5

Minimum cost (Z) = Rs. 13.

Problem. A firm manufactures pain relieving pills in two sizes A and B, size A contains 4 grains of element a, 7 grains of element b and 2 grains of element c, size B contains 2 grains of element a, 10 grains of element b and 8 grains of c. It is found by users that it requires at least 12 grains of element a, 74 grains of element b and 24 grains of element c to provide immediate relief It is required to determine that least no. of pills a patient should take to get immediate relief. Formulate the problem as standard LPP.

Solution : Pain relieving pills

	a	b	c
Size A	4	7	2
Size B	2	10	8
Min. requirement	12	74	24

Step 1. Select decision variable

x_1 – no. of pills of size A

x_2 – no. of pills of size B

Step 2. Objective function

Minimum (no. of pills) $z = x_1 + x_2$

Step 3. Constraints

$$4x_1 + 2x_2 \geq 12$$
$$7x_1 + 10x_2 \geq 74$$
$$2x_1 + 8x_2 \geq 24$$

Step. 4. Add non negativity constraints

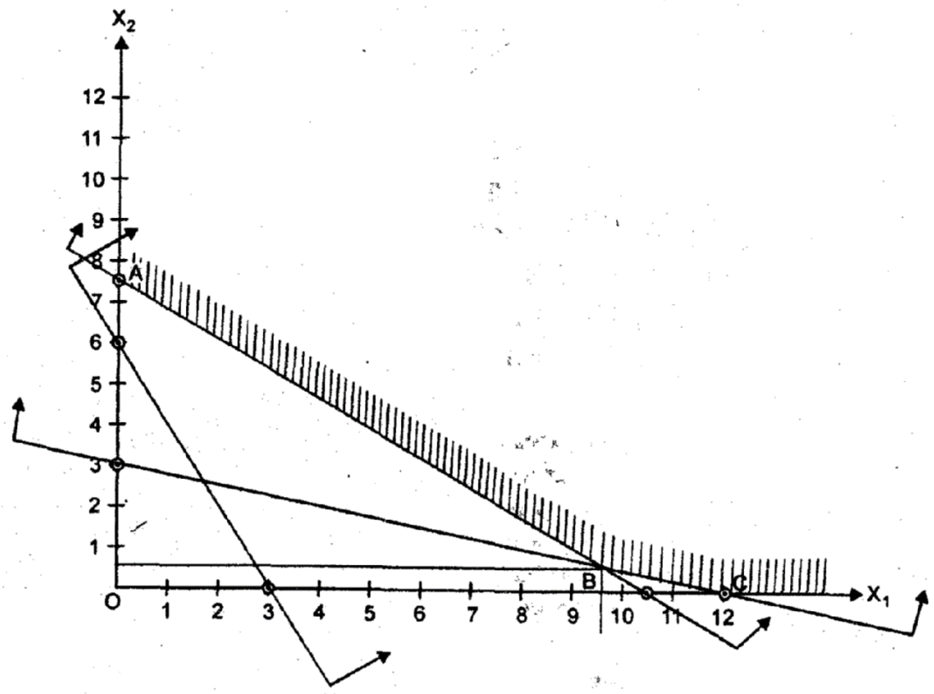
$$x_1 \geq 0 \quad ; \quad x_2 \geq 0$$

Determining the value of x_1 and x_2 by graphical method

$4x_1 + 2x_2 = 12$ $x_1 = 0; x_2 = 6$ and $x_2 = 0; x_1 = 3$

$7x_1 + 10x_2 = 74$ $x_1 = 0; x_2 = 7.4$ and $x_2 = 0; x_1 = 10.57$

$2x_1 + 8x_2 = 24$ $x_1 = 0; x_2 = 3$ and $x_2 = 0; x_1 = 12$



Point A (0, 7.4) Z(A) = 0 + 7.4 = 7.4 (Minimum)

Point C (12, 0) Z (C) = 12 + 0 = 12

Point B (9.6, 0.6) Z(B) = 9.6 + 0.6 = 10.2

No. of pills of size A = 0

No. of pills of size B = 7.4 ≃ 8 pills

Minimum no. of pills = 8 pills.

Problem. An automobile manufacturer makes automobiles and trucks in a factory that is divided into two shops. Shop A which perform the basic assy operation must work 5 man days on each truck but only 2 man days on each automobile. Shop B which perform finishing operations must work 3 man days for each automobile or truck that it produces. Because of men and machine limitations shop A has 180 man days per week available while shop B has 135.man days per week. If the manufacturer makes a profit of Rs. 300 on each truck and Rs. 200 on each automobile; how many of each should be produced to maximize his profit?

Solution :

	Shop A	Shop B	Profit
Automobile	2 man days	3 man days	Rs. 200
Trucks	5 man days	3 man days	Rs. 300
Availability	180 man days/week	135 man days/week	

1. Select decision variable
   x_1 – no. of automobile to be produced/week
   x_2 – no. of trucks to be produced/week
2. Objective function
   Maximize $Z = 200x_1 + 300x_2$
3. Constraints

$$2x_1 + 5x_2 \le 180$$
$$3x_1 + 3x_2 \le 135$$

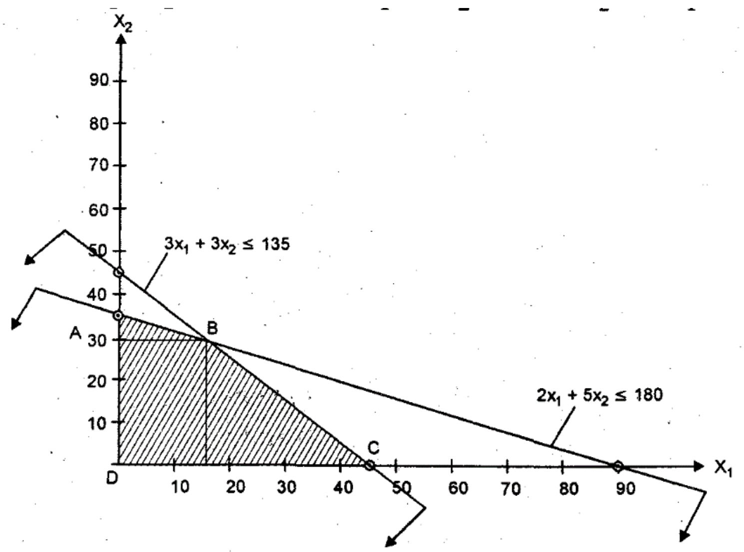
4. Add non negativity constraints

$$x_1 \ge 0; x_2 \ge 0$$

Determine the value of x_1 and x_2 by graphical method

$2x_1 + 5x_2 = 180$ $\quad x_1 = 0; x_2 = 36$ and $x_2 = 0; x_1 = 90$

$3x_1 + 3x_2 = 135$ $\quad x_1 = 0; x_2 = 45$ and $x_2 = 0; x_1 = 45$



Point D (0, 0) Z (D) $= 200 \times 0 + 300 \times 0 = 0$

Point A (0, 36) Z (A) $= 200 \times 0 + 300 \times 36 = 10800$

Point C (45, 0) Z (C) $= 200 \times 45 + 300 \times 0 = 9000$

Point B (15 , 30) Z (B) $= 200 \times 15 + 300 \times 30 = 3000 + 9000 = 12000$

Maximum Profit at Point B (15 , 30) *i.e.* Rs.12000/-

x_1 = no. of automobile/week = 15

x_2 = no. of trucks/week = 30

Maximum profit = 12000/-

Problem. On completing the construction of house a person discovers that J square feet of plywood scrap and 80 square feet of white pine scrap are in useable m for the construction of tables and book cases. It takes 16 square feet of plywood 8 square feet of white pine to make a table, 12 square feet of plywood and 16 Llare feet of white pine are required to contruct a book case. By selling the finishing duct to a local furniture store the person can realize a profit of Rs. 25 on each table d Rs. 290 on each book case. How may the man most profitably use the left over ood ? Use graphical method to solve problem.

Solution :

	Plywood	White pine	Profit	
Table	16	8	Rs. 25	each table
Book case	12	16	Rs. 290	each book case
Availability	100	80		

1. Select decision variable

x_1 – no. of table

x_2 – no. of book case

2. Objective function

Maximize profit (Z) = $25x_1 + 290x_2$

3. Constraints

$$16x_1 + 12x_2 \leq 100$$

$$8x_1 + 16x_2 \leq 80$$

4. Add non negativity constraints

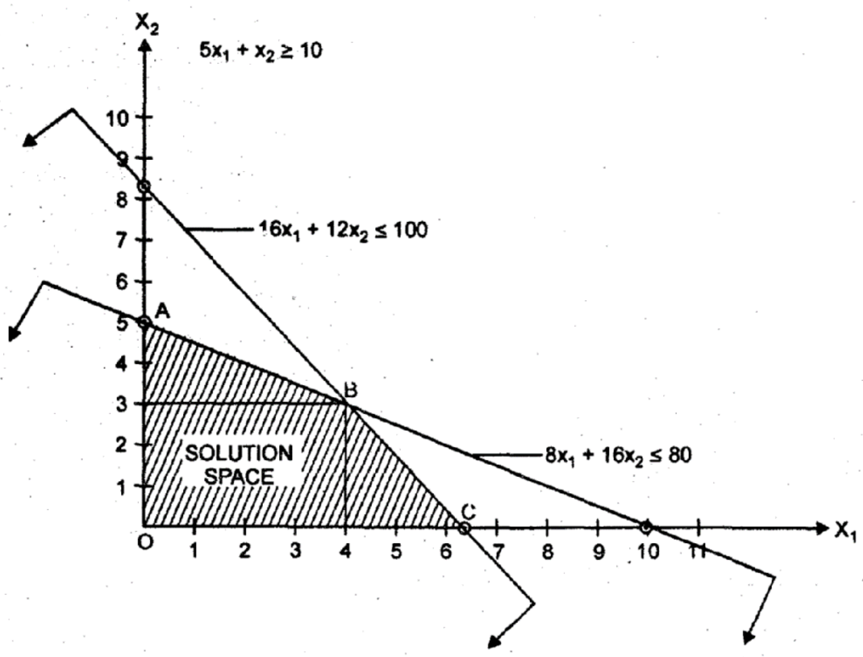
$$x_1 \geq 0$$

$$x_2 \geq 0$$

Determine the value of x_1 and x_2 using graphical method

$16x_1 + 12x_2 = 100$ $\quad x_1 = 0; x_2 = 8.3$ and $x_2 = 0; x_1 = 6.25$

$8x_1 + 16x_2 = 80$ $\quad x_1 = 0; x_2 = 5$ and $x_2 = 0; x_1 = 10$



Point O (0, 0) Z(O) = 25 × 0 + 290 × 0 = 0

Point A (0, 5) Z (A) = 25 × 0 + 290 × 5 = 1450

Point C (6.25, 0) Z (C) = 25 × 6.25 + 290 × 0 = 156.25

Point B (4, 3) Z (B) = 25 × 4 + 290 × 3 = 100 + 870 = 970

Maximum profit (Z) at point A *i.e.* Rs. 1450.

x_1 – no. of table = 0 Max. Profit (Z) = Rs. 1450/-

x_2 – no.of bookcase = 5.

Basic definition:

1) Define a feasible region.
Solution:

A region in which all the constraints are satisfied simultaneously is called a feasible region.

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2) Define a feasible solution.
Solution:

A solution to the LPP which satisfies the non-negativity restrictions of the LPP is called a feasible solution.

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3) Define optimal solution.
Solution:

Any feasible solution which optimizes the objective function is called its optimal solution.

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4) What is the difference between basic solution and basic feasible solution?
Solution:

Given a system of m linear equations with n variables (m<n), the solution obtained by setting n-m variables equal to zero and solving for the remaining m variables is called a basic solution. A basic solution in which all the basic variables are non-negative is called a basic feasible solution.

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5) Define unbounded solution.
Solution:

If the values of the objective function Z can be increased or decreased indefinitely, such solutions are called unbounded solutions.

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6) What are slack and surplus variables?
Solution:

The non-negative variable which is added to LHS of the constraint to convert the inequality $\leq$ into an equation is called slack variable.

$\sum_{j=1}^{n} a_{ij} x_i + s_i = b_i (i = 1, 2, ..., m)$ where s_i are called slack variables.

The non-negative variable which is subtracted from the LHS of the constraint to convert the inequality $\geq$ into an equation is called surplus variable.

$\sum_{j=1}^{n} a_{ij} x_i - s_i = b_i (i = 1, 2, ..., m)$ where s_i are called surplus variables.

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7) What is meant by optimality test in a LPP?
Solution:

By performing optimality test we can find whether the current feasible solution can be improved or not. This is possible by finding the $Z_j - C_j$ row. In the case of a maximization problem if all $Z_j - C_j$ are nonnegative, then the current solution is optimal.

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8) What are the methods used to solve an LPP involving artificial variables?

Solution:

i) Big M method or penalty cost method
ii) Two-phase simplex method

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9) Define artificial variable

Solution:

Any non negative variable which is introduced in the constraint in order to get the initial basic feasible solution is called artificial variable.

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10) When does an LPP posses a pseudo-optimal solution?

Solution:

An LPP possesses a pseudo-optimal solution if at least one artificial variable is in the basis at positive level even though the optimality conditions are satisfied.

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11) What is degeneracy?

Solution:

The concept of obtaining a degenerate basic feasible solution in a LPP is known as degeneracy. In the case of a BFS, all the non basic variables have zero value. If some basic variables also have zero value, then the BFS is said to be a degenerate BFS.

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12) How to resolve degeneracy in a LPP?

Solution:

a) Divide each element of the rows (with tie) by the positive coefficients of the key column in that row.
b) Compare the resulting ratios, column by column, first in the identity and then in the body from left to right.
c) The row which first contains the smallest ratio contains the leaving variable.

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13) State the characteristics of canonical form.

Solution:

The characteristics of canonical form are

i) The objective function is of maximization type
ii) All constraints are "$\leq$" type
iii) All variables X_i are non negative.

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14) State the characteristics of standard form.

Solution:

The characteristics of standard form are

i) The objective function is of maximization type
ii) All constraints are expressed as equations
iii) RHS of each constraint is non- negative
iv) All variables X_i are non-negative.

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15) Define basic feasible solution

Solution:

Given a system of m linear equations with n variables (m<n), the solution obtained by setting n-m variables equal to zero and solving for the remaining m variables is called a basic solution. A basic solution in which all the basic variables are non-negative is called a basic feasible solution.

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16) Define non-degenerate solution

Solution:

A non-degenerate basic feasible solution is the basic feasible solution which has exactly m positive X_i (i = 1, 2,------m) ie, none of the basic variables are zero.

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17) Define degenerate solution

Solution:

A basic feasible solution is said to be degenerate if one or more basic variables are zero.

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18) Write the general mathematical model of LPP in matrix form.

Solution:

Max or Min Z = CX

Subject to AX ($\leq = \geq$) b

$X \geq 0$

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19) Define basic solution:

Solution:

Given a system of m linear equations with n variables (m<n), the solution obtained by setting n-m variables equal to zero and solving for the remaining m variables is called a basic solution.

Simplex method(algorithm)

Step 1: Check whether the objective function of the given LPP is to be maximized or minimized. If it is to be minimized then we convert it into a problem of maximization by Min Z = - Max (-Z)

Step 2: Check whether all b_i are positive. If any one of b_i is negative then multiply the inequation of the constraint by -1 so as to get all $b_i$to be positive.

Step 3: Express the problem in standard form by introducing slack/surplus variables, to convert the inequality constraints into equations.

Step 4: Obtain an initial basic feasible solution to the problem in the form X_B = $\mathbf{B}^{-1}$b and put it in the first column of the simplex table.

Step 5: Compute the net evaluations $Z_J - C_J$ by using the relation

$Z_J - C_J = C_B X_J - C_J$.

Examine the sign of $Z_J - C_J$.

(i) If all $Z_J - C_J \geq 0$, then the current BFS is the optimal solution.

(ii) If at least one $Z_J - C_J < 0$, then proceed to the next step.

Step 6: If there are more than one negative $Z_J - C_J$ choose the most negative

them. Let it be $Z_r - C_r$.

(i) If all $X_{ir} \leq 0 (i = 1, 2, ...m)$ then there is an unbounded solution to the given problem.

(ii) If at least one $X_{ir} > 0 (i = 1, 2, ...m)$ then the variable X_r (key column) enters the basis.

Step 7: Compute the ratio $\left\{ \frac{X_{Bi}}{X_{ir}} / X_{ir} > 0 \right\}$. Let the minimum of these ratios be

$\frac{X_{Bk}}{X_{kr}}$. Then choose the variable X_k (key row) to leave the basic. The element

at the intersection of the key column and the key row is called the key element.

Step 8: Form a new basis by dropping the leaving variable and introducing the

entering variable along with the associated value under C_B column. Convert the leading element to unity by dividing the key row by the key element and convert all other elements in the simplex table by using the formula

New element = Old element –

{(product of elements in key row and key column) / key element}

Go to Step 5 and repeat the procedure until either an optimal solution is obtained or there is an indication of unbounded solution.

The procedure of the big M method

Step 1: Express the problem in the standard form.

Step 2: Add non-negative artificial variables to the left side of each of the equations corresponding to constraints of the type ≥ or = . Assign a very large penalty cost (-M for Maximization and M for Minimization) with artificial variables in the objective function.

Step 3: Solved the modified LPP by simplex method, until any one of the three cases that may arise.

1. If no artificial variable appears in the basis and the optimality conditions

are satisfied, then the current solution is an optimal basic feasible solution.

2. If at least one artificial variable in the basis is at zero level and the optimality condition is satisfied then the current solution is an optimal basic feasible solution (though degenerate).

3. If at least one artificial variable appears in the basis at positive level and the optimality condition is satisfied then the original problem has no feasible solution .The solution satisfies the constraints but does not optimize the objective function, since it contains very large penalty M and it is called pseudo optimal solution.

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1. Solve the following LPP using simplex method

   Max Z = $15x_1 + 6x_2 + 9x_3 + 2x_4$

   Subject to

$$2x_1 + x_2 + 5x_3 + 6x_4 \leq 20$$
$$3x_1 + x_2 + 3x_3 + 25x_4 \leq 24$$
$$7x_1 + x_4 \leq 70$$
$$\& x_1, x_2, x_3, x_4 \geq 0$$

Solution:

Rewrite the inequality of the constraint into an equation by adding slack variables S_1, S_2 and S_3 the LPP becomes,

Max Z = $15x_1 + 6x_2 + 9x_3 + 2x_4 + 0.S_1 + 0.S_2 + 0.S_3$

Subject to

$$2x_1 + x_2 + 5x_3 + 6x_4 + S_1 = 20$$
$$3x_1 + x_2 + 3x_3 + 25x_4 + S_2 = 24$$
$$7x_1 + x_4 + S_3 = 70$$
$$\& \; x_1, x_2, x_3, x_4, S_1, S_2, S_3 \geq 0$$

Initial basic feasible solution is

S_1=20
S_2=24
S_3=70

Initial simplex table

C_j	15	6	9	2	0	0	0

C_B	B	X_B	X_1	X_2	X_3	X_4	S_1	S_2	S_3	Min $\frac{X_B}{X_1}$
0	S_1	20	2	1	5	6	1	0	0	10
0	S_2	24	3	1	3	25	0	1	0	8
0	S_3	70	7	0	0	1	0	0	1	10
	Z_j	0	0	0	0	0	0	0	0	
	Z_J-C_J		-15	-6	-9	-2	0	0	0	

Since all $Z_J - C_J \le 0$ and the current basic feasible solution is not optimum.

First iteration: X_1 enters the basis and S_2 leaves the basis.

	B	X_B	X_1	X_2	X_3	X_4	S_1	S_2	S_3	Min $\frac{X_B}{X_2}$
$\leftarrow 0$	S_1	4	0	1/3	3	$-32/3$	1	2/3	0	12
15	X_1	8	1	1/3	1	25/3	0	1/3	0	24
0	S_3	14	0	$-7/3$	-7	$-172/3$	0	$-7/3$	1	-
	Z_J	120	15	5	15	125	0	5	0	
	Z_J-C_J		0	-1 $\uparrow$	6	123	0	5		

some ≤ 0 the basic solution optimum

Since $Z_J - C_J$ current feasible is not

Second iteration:

X_2 enters the basis and S_1 leaves the basis.

The new table is

C_B	B	X_B	X_1	X_2	X_3	X_4	S_1	S_2	S_3
6	X_2	12	0	1	9	-32	3	-2	0
15	X_1	4	1	0	-2	57/3	-1	1	0
0	S_3	42	0	0	14	-132	7	-7	1
	Z_j	132	15	6	24	93	3	3	0
	Z_J-C_J		0	0	15	91	3	3	0

Since all $Z_J - C_J \ge 0$ and the current basic feasible solution is optimum

and is given by Max Z = 132, $X_1 = 4$, $X_2 = 12$, $X_3 = 0$, $X_4 = 0$.

----

2. Solve by the big M method

Minimize Z = $4x_1 + 3x_2$

Subject to

$$2x_1 + x_2 \ge 10$$
$$-3x_1 + 2x_2 \le 6$$
$$x_1 + x_2 \ge 6$$
$$\& \ x_1, x_2 \ge 0$$

Solution:

Given Minimize Z = $4x_1 + 3x_2$

Subject to

$$2x_1 + x_2 \ge 10$$
$$-3x_1 + 2x_2 \le 6$$
$$x_1 + x_2 \ge 6$$
$$\& \ x_1, x_2 \ge 0$$

That is Max Z = $-4x_1 - 3x_2$

Subject to

$$2x_1 + x_2 \ge 10$$
$$-3x_1 + 2x_2 \le 6$$
$$x_1 + x_2 \ge 6$$
$$\& \ x_1, x_2 \ge 0$$

By introducing the non negative slack, surplus and artificial variables, the standard form of the LPP is

$$\text{Max } Z = -4x_1 - 3x_2 + 0.s_1 + 0.s_2 - MR_1 - MR_2$$

Subject to

$$2x_1 + x_2 - s_1 + R_1 = 10$$
$$-3x_1 + 2x_2 + s_2 = 6$$
$$x_1 + x_2 - s_3 + R_2 = 6$$
$$\& \ x_1, x_2, s_1, s_2, s_3, R_1, R_2 \ge 0$$

Initial basic feasible solution is

R_1=10
R_2=6
S_2=6

Initial iteration:

C_j -4 -3 0 0 0 -M -M

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3	R_1	R_2	Min
-M	R_1	10	2	1	-1	0	0	1	0	5
0	S_2	6	-3	2	0	1	0	0	0	-
-M	R_2	6	1	1	0	0	-1	0	1	6
	$Z_j - C_j$	-16M	-3M+4	-2M+3	M	0	M	0	0	

Since some $Z_J - C_J \le 0$ and the current basic feasible solution is not optimum.

First iteration:

C_j -4 -3 0 0 0 -M -M

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3	R_2	Min
-4	x_1	5	1	1/2	-1/2	0	0	0	10
0	S_2	21	0	7/2	-3/2	1	0	0	42/7
-M	R_2	1	0	½	1/2	0	-1	1	2
	$Z_j - C_j$	-M-20	0	$\frac{-M+2}{2}$	$\frac{-M+4}{2}$	0	M	0	

Since some $Z_J - C_J \le 0$ and the current basic feasible solution is not optimum.

Second iteration:

C_j -4 -3 0 0 0

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3
-4	X_1	4	1	0	-1	0	1
0	S_2	14	0	0	-5	1	7
-3	x_2	2	0	1	1	0	-2
	$Z_j - C_j$	-22	0	0	1	0	2

Since all $Z_J - C_J \geq 0$ the current basic feasible solution is optimum.

and is given by Min Z = 22, X_1 = 4, X_2 = 2.

----

3. Use two phase simplex method to

Max Z = $5x_1 + 3x_2$

Subject to

$$2x_1 + x_2 \leq 1$$
$$x_1 + 4x_2 \geq 6$$
$$\& \ x_1, x_2 \geq 0$$

Solution:

By introducing the non negative slack, surplus and artificial variables, the standard form of the LPP is

Max Z = $5x_1 + 3x_2 + 0.s_1 + 0.s_2 - R_1$

Subject to

$$2x_1 + x_2 + s_1 = 1$$
$$x_1 + x_2 - s_2 + R_1 = 6$$
$$\& x_1, x_2, s_1, s_2, R_1 \geq 0$$

Initial basic feasible solution is

S_1=1

R1=6

Initial iteration:

C_j 0 0 0 0 -1

C_B	B	X_B	X_1	X_2	S_1	S_2	R_1	Min
0	S_1	1	2	1	1	0	0	1
-1	R_1	6	1	4	0	-1	1	6/4
	$Z_j - C_j$	-6	-1	-4	0	1	0	

Since some $Z_J - C_J \leq 0$ and the current basic feasible solution is not optimum.

First iteration:

C_j 0 0 0 -1 -1

C_B	B	X_B	X_1	X_2	S_1	S_2	R_1
0	x_2	1	2	1	1	0	0
-1	R_1	2	-7	0	-4	-1	1
	$Z_j - C_j$	-2	7	0	4	1	0

Since all $Z_J - C_J \geq 0$ and the current basic feasible solution is optimal to the auxiliary LPP. Since an artificial variable is in the current basis at positive level, the given LPP has no feasible solution.

Explain the meaning of duality in LPP

For every LP problem there is related unique L P problem mvolvmg the same data which also describes the original problem.

The given original problem is known as primal programme. The programme can be rewritten by transposing the rows and columns of the statement of the problem. Inverting the programme in this way results in dual programme. The two programmes have very closely related properties so that optimal solution of the dual problem gives complete information about the optimal solution of primal problem. Solving the problem by writing dual programme is known as duality in LP

If the dual of an LPP is solved, where will we get the value of decision variables of the primal LPP.

The value of decision variables of primal are given by the base row of the dual solution under the slack variable, neglecting the -ye sign if any, and under the artificial variables neglecting the —ye sign if any, after deleting the constant M.

What is the importance of duality?

1.If. the primal problem contains a large number of rows and a smaller number of columns, the computational procedure can be considerably reduced by converting it into dual and then solving it.

2. This can help managers in answer questions about alternative course of actions and their relative values.

3. Economic interpretation of the dual helps the management in making future decisions.

4. Calculation of the dual checks the accuracy of the primal solution.

Define dual of LPP.

For every LPP there is a unique LPP associated with it involving the same data and closely related optimal solution. The original problem is then called the primal problem while the other is called its dual problem. If the primal problem is

Maximize Z = CX

subject to $AX \le b$

$X \ge 0$

Then the dual is

Minimize $Z^* = b^T Y$

subject to $A^T Y \ge C^T$

$Y \ge 0$

Problems

Problem 4.6. Write the dual form for the following :

$$\text{Min } z = x_1 + x_2$$

Subjected to

$$2x_1 + x_2 \geq 4$$
$$x_1 + 7x_2 \geq 7$$
$$x_1, x_2 \geq 0$$

Solution :

The dual of the given problem will be

$$\text{Max W} = 4y_1 + 7y_2$$

Subject to

$$2y_1 + 1y_2 \leq 1$$
$$y_1 + 7y_2 \leq 1$$
$$y_1 , y_2 \geq 0$$

y_1 and y_2 are non negative dual variables.

Problem 4.7. Write the dual form for the following :

$$\text{Max } z = 4x_1 + 2x_2$$

Subject to

$$x_1 + x_2 \geq 3$$
$$x_1 - x_2 \geq 2$$
$$x_1 , x_2 \geq 0$$

Solution : As the given problem is of maximization type, all constraints should by $\leq$ type; Multiply all constraints with –ve.

$$-x_1 - x_2 \leq -3$$
$$-x_1 + x_2 \leq -2$$
$$x_1, x_2 \geq 0$$

The dual of the given problem will be

$$\text{Min W} = -3y_1 - 2y_2$$

Subject to

$$-y_1 - y_2 \geq 4$$
$$-y_1 + y_2 \geq 2$$
$$y_1 ; y_2 \geq 0$$

y_1 and y_2 are non negative dual variables.

Problem 4.8. Construct the dual of the problem.

Minimize $z = 3x_1 - 2x_2 + 4x_3$

Subject to constraints

$$3x_1 + 5x_2 + 4x_3 \geq 7$$
$$6x_1 + x_2 + 3x_3 \geq 4$$
$$7x_1 - 2x_2 - x_3 \leq 10$$
$$x_1 - 2x_2 + 5x_3 \geq 3$$
$$4x_1 + 7x_2 - 2x_3 \geq 2$$
$$x_1, x_2, x_3 \geq 0$$

Solution: As the given problem is of minimization all constraints should be of type

$$-7x_1 + 2x_2 + x_3 \geq -10$$

The dual of the given problem will be

Maximize $W = 7y_1 + 4y_2 - 10y_3 + 3y_4 + 2y_5$

Subject to

$$3y_1 + 6y_2 - 7y_3 + y_4 + 4y_5 \leq 3$$
$$5y_1 + y_2 + 2y_3 - 2y_4 + 7y_5 \leq -2$$
$$5y_1 + 3y_2 + y_3 + 5y_4 - 2y_5 \leq 4$$
$$y_1, y_2, y_3, y_4, y_5 \geq 0.$$

Problem 4.9. Construct the dual of the problem

maximize $z = 3x_1 + 10x_2 + 2x_3$

Subject to

$$2x_1 + 3x_2 + 2x_3 \leq 7$$
$$3x_1 - 2x_2 + 4x_3 = 3$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

Solution : The equation $3x_1 - 2x_2 + 4x_3 = 3$ can be expressed as pair of inequality

$$3x_1 - 2x_2 + 4x_3 \leq 3$$
$$3x_1 - 2x_2 + 4x_3 \geq 3$$
$$-3x_1 + 2x_2 - 4x_3 \leq -3$$

Minimize

$$W = 7y_1 + 3(y_2' - y_2'')$$

Subject to

$$2y_1 + 3(y_2' - y_2'') \geq 3$$
$$3y_1 - 2(y_2' - y_2'') \geq 10$$
$$2y_1 + 4(y_2' - y_2'') \geq 2$$
$$y_1, y_2', y_2'' \geq 0$$

Subsituting $y_2' - y_2'' = y_2$

$$W = 7y_1 + 3y_2$$

Subject to

$$2y_1 + 3y_2 \geq 3$$
$$3y_1 - 2y_2 \geq 10$$
$$2y_1 + 4y_2 \geq 2$$
$$y_1 \geq 0$$

y_2 unrestricted variable.

Solve by the dual simplex method the following LPP

Min Z = $5x_1 + 6x_2$.

Subject to $x_1 + x_2 \geq 2$

$$4x_1 + x_2 \geq 4$$
$$x_1, x_2 \geq 0$$

Solution:

By introducing slack variables S_1, S_2 we get the standard form of LPP as given below.

Max Z = $-5x_1 - 6x_2 + 0s_1 + 0s_2$

Subject to

$$-x_1 - x_2 + s_1 = -2$$
$$-4x_1 - x_2 + s_2 = -4$$
$$x_1, x_2, s_1, s_2 \geq 0$$

Initial table

C_J -5 -6 0 0

C_B	B	X_B	X_1	X_2	S_1	S_2
0	S_1	-2	-1	-1	1	0
$\leftarrow 0$	S_2	-4	-4	-1	0	1
	Z_J	0	0	0	0	0
	$Z_J - C_J$		$5\uparrow$	6	0	0

Since all $Z_J - C_J \geq 0$ optimality conditions are satisfied. Since all $X_{Bi}< 0$, the current solution is not a basic feasible solution.

Since X_{B2} = -4 is most negative, the basic variable S_2 leaves the basis.

Since Max {5/-4, 6/-1} = -5/4, X_1 enters the basis.

First iteration: Drop S_2 and introduce X_1

C_J -5 -6 0 0

C_B	B	X_B	X_1	X_2	S_1	S_2
$\leftarrow 0$	S_1	-1	0	$-3/4$	1	$-1/4$
-5	x_1	1	1	$1/4$	0	$-1/4$
	Z_J	-5	-5	$-5/4$	0	$5/4$
	$Z_J - C_J$		0	19/4	0	$5/4$ $\uparrow$

Since all $Z_J - C_J \geq 0$ optimality conditions are satisfied. Since some $X_{Bi}< 0$ the current solution is not a basic feasible solution.

S_1 leaves the current basis. Since Max {19/-3, 5/-1} =5/-1, S_2 enters the basis.

Second iteration:

C_J -5 -6 0 0

C_B	B	X_B	X_1	X_2	S_1	S_2
0	S_2	4	0	3	-4	1

-5	x_1	2	1	1	-1	0
	Z_J	-10	-5	-5	5	0
	$Z_J - C_J$		0	1	5	0

Since all $Z_J - C_J \geq 0$ and all $X_{Bi} \geq 0$, the current basic feasible solution is optimum. The optimal solution is Z = 10, $x_1 = 2$.

Use dual simplex method to solve the LPP

Max Z = $-3x_1 - 2x_2$

Subject to

$x_1 + x_2 \geq 1$
$x_1 + x_2 \leq 7$
$x_1 + 2x_2 \geq 10$
$x_2 \leq 3$
$\& \ x_1, x_2 \geq 0$

Solution:

The given LPP is

Max Z = $-3x_1 - 2x_2$

Subject to

$-x_1 - x_2 \leq 1$
$x_1 + x_2 \leq 7$
$-x_1 - 2x_2 \leq 10$
$x_2 \leq 3$
$\& \ x_1, x_2 \geq 0$

By introducing the non negative slack variables s_1, s_2, s_3 and s_4 the LPP becomes

Max Z = $-3x_1 - 2x_2 + 0.s_1 + 0.s_2 + 0.s_3 + 0.s_4$

Subject to

$$-x_1 - x_2 + s_1 = -1$$
$$x_1 + x_2 + s_2 = 7$$
$$-x_1 - 2x_2 + s_3 = -10$$
$$x_2 + s_4 = 3$$
$$\& \; x_1, x_2, s_1, s_2, s_3, s_4 \geq 0$$

Initial iteration BFS is

S_1 = -1, S_2=7, S_3 =-10, S_4=3

Initial Iteration:

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3	S_4
0	S_1	-1	-1	-1	1	0	0	0
0	S_2	7	1	1	0	1	0	0
0	S_3	-10	-1	-2	0	0	1	0
0	S_4	3	0	1	0	0	0	1
	$Z_j - C_j$	0	3	2	0	0	0	0

First iteration

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3	S_4
0	S_1	4	-1/2	0	1	0	-1/2	0
0	S_2	2	1/2	0	0	1	1/2	0
-2	x_2	5	1/2	1	0	0	-1/2	0
0	S_4	-2	-1/2	0	0	0	1/2	1
	$Z_j - C_j$	-10	2	0	0	0	1	0

Second iteration:

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3	S_4
0	S_1	2	0	0	1	0	-1	-1
0	S_2	0	0	0	0	1	1	1
-2	x_2	3	0	1	0	0	0	1
-3	x_1	4	1	0	0	0	-1	-2
	$Z_j - C_j$	-18	0	0	0	0	3	4

Since all $Z_J - C_J \geq 0$ the current basic feasible solution is optimum and is given by Min Z = -18, X_1 = 4, X_2 = 3.