14.7 DETERMINISTIC INVENTORY MODEL

In this model, the demand is assumed to be fixed and completely predetermined i.e. *static demand*. Such models are referred to as economic lot size model.

There are 4 types under this category, namely,

(i) Purchasing model with no shortages

(ii) Manufacturing model with no shortages

(iii) Purchasing model with shortages

(iv) Manufacturing model with shortages.

14.7.1 EOQ Model without Shortages

Model I: Purchasing model with no shortages (a)

The Economic lot size system with uniform demand.

In this model we have to derive an economic lot size formula for the optimum production quantity q per cycle of a single product so as to minimise the total average variable cost per unit time.

The assumptions for this model are as follows.

(i) Demand rate is uniform

(ii) Lead time is zero

(iii) Production rate is infinite i.e. production is instantaneous

(iv) Shortages are not allowed

(v) Holding cost is rupees C_1 per quantity unit per unit time

(vi) Set-up cost is rupees C_3 per time setup.

Let each production cycle be made at fixed interval t and therefore the quantity q already present in the beginning is $q = Rt$ where R is the demand rate.

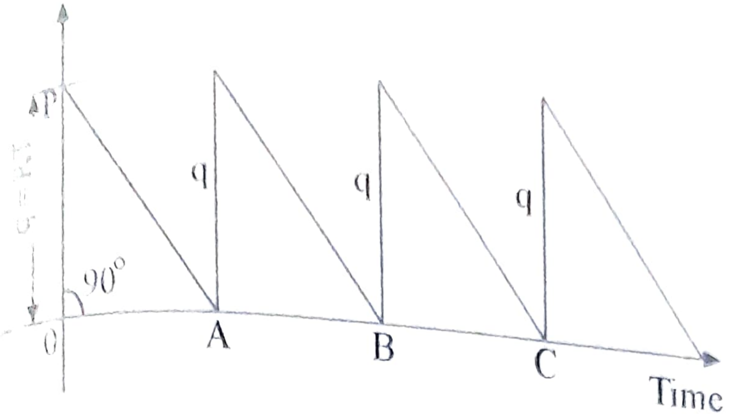
Since the stock is small time dt is Rdt, the stock in total time t will be

$$= \int_0^t Rtdt = \frac{1}{2} Rt^2 = \frac{1}{2} qt$$

$$[\because Rt = q]$$

= Area of the inventory ΔPOA.

The graphical solution of this inventory is shown below.



The rate of replenishment = slope of line OP

$$= \tan 90^\circ = \infty$$

Thus the cost of holding inventory per production run.

$$= C_1 \times \text{Area of } \Delta OPA = C_1\left(\frac{Rt^2}{2}\right)$$

The setup cost per production $= C_3$ per production per time interval.

The cost equation is given by the average total cost.

$$= C(t) = \frac{1}{t}\left[\frac{C_1Rt^2}{2} + C_3\right]$$

$$= \frac{C_1Rt}{2} + \frac{C_3}{t}$$

We want to find the minimum average total cost, by using the property of minimum, we have

$$\frac{dC(t)}{dt} = 0 \Rightarrow \frac{C_1R}{2} - \frac{C_3}{t^2} = 0$$

$$\text{which gives } t = \sqrt{\frac{2C_3}{C_1R}}$$

$$\frac{d^2C(t)}{dt^2} = 0 + \frac{2C_3}{t^3} > 0$$

Hence $C(t)$ is minimum for optimum time interval

$$\boxed{t^* = \sqrt{\frac{2C_3}{C_1R}}}$$

The optimum quantity to be produced (ordered) at each interval t^* is

$$q^* = Rt^* = R\sqrt{\frac{2C_3}{C_1 R}} = \sqrt{\frac{2C_3 R}{C_1}}$$

$$q^* = \sqrt{\frac{2C_3 R}{C_1}}$$

$$\boxed{C^* = \sqrt{\frac{2C_3 R}{C_2}}}$$

which is called the optimal lot size formula.

$$C^* = C_{\min} = \tfrac{1}{2} RC_1\sqrt{\frac{2C_3}{C_1 R}} + C_3\sqrt{\frac{C_1 R}{2C_3}}$$

$$= \sqrt{\frac{C_1 C_3 R}{2}} + \sqrt{\frac{C_1 C_3 R}{2}}$$

$$\boxed{C^* = \sqrt{2C_1 C_3 R}}$$

Characteristics of Model 1

(i) Optimum number of orders placed per year.

$$n^* = R/q^* = \sqrt{\frac{RC_1}{2C_3}}$$

(ii) Optimum length of time between orders.

$$t^* = \sqrt{\frac{2C_3}{RC_1}}$$

(iii) Minimum total annual inventory cost

$$C^* = \sqrt{2C_1 C_3 R}$$

(iv) Optimal lot size formula.

$$q^* = \sqrt{\frac{2C_3 R}{C_1}}$$

Model I (b) *EOQ problem with no shortages and several production runs of unequal length.*

In this problem all the assumptions are the same as in Model I(a) except that the demand is uniform and the production run differ in ... By replacing R by $R = D/T$ where D is the total demand to be satisfied during the period T in the above formula we get the following optimum quantities.

$$q^* = \sqrt{\frac{2C_3 D/T}{C_1}}$$

$$t^* = \sqrt{\frac{2C_3}{C_1 D/T}}$$

$$C^* = \sqrt{2C_1 C_3 D/T}$$

Model II: Manufacturing model with no shortages

Let

$C_1 =$ holding cost per item per unit time

$C_2 =$ i.e. shortages are not permitted

$C_3 =$ set-up cost per production cycle

$R =$ number of items required per unit time i.e. demand rate

$(K > R)$ is the production rate which is finite and greater than R

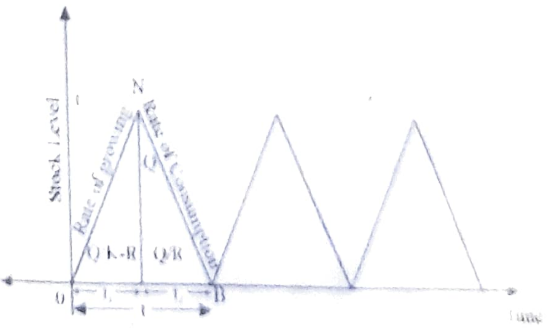
$t =$ interval between production cycles

$q =$ Rt the number of items produced per production run.

In this model each production cycle consists of two parts t_1 and t_2 where

(i) t_1 is the period during which the stock is growing up at a rate $(K-R)$ items per unit time.

(ii) t_2 is the period during which there is no replenishment (or supply or production) but there is only a constant demand at the rate R. Further we assume that q is the stock available at the end of time t_1 which is expected to be consumed during the remaining period t_2 at the consumption rate R.



$$\frac{q^*}{R} = \sqrt{\frac{2C_3K}{C_1R(K-R)}} \quad \text{(optimal time interval)}$$

$$C^* = C_{\min} = \sqrt{2C_1\left(1-\frac{R}{K}\right)C_3R}$$

Characteristics of Model 2

1. Optimum number of production run per year.

$$n^* = R/q^* = \sqrt{\frac{C_1R}{2C_3}}\sqrt{\frac{K-R}{K}}$$

2. Optimum length of each lot size production run.

$$t^* = \sqrt{\frac{2C_3}{C_1R}}\sqrt{\frac{K}{K-R}}$$

3. Optimum lot size.

$$q^* = \sqrt{\frac{2C_3R}{C_1}}\sqrt{\frac{K}{K-R}}$$

4. Total minimum production inventory cost.

$$TC^* = \sqrt{2RC_3C_1}\sqrt{\frac{K-R}{K}}$$

Note: (i) If $K=R$, then $C^*=0$ i.e. there will be no holding cost and set-up cost.
(ii) If $K=\infty$ i.e. production rate is finite this model reduces to model I.

Example 14.1 The annual demand of an item is 3200 units. The unit cost is Rs. 6/- and inventory carrying charges 25% per annum. If the cost of one procurement is Rs. 150. Determine.

(i) EOQ; (ii) No. of orders per year; (iii) time between two consecutive order; (iv) the optimal cost.

Solution Given $R=3200$ units

$$C_1 = C \times I \quad C = 6, \quad I = \frac{25}{100} = \frac{1}{4}$$

$$C_3 = \text{Rs. } 150 \qquad C_1 = 6 \times \frac{1}{4} = \frac{3}{2}$$

(i) The optimum lot size $q^* = \sqrt{\dfrac{2C_3R}{C_1}}$

$$= \sqrt{\frac{2 \times 3200 \times 150}{3/2}} = 800 \text{ units.}$$

(ii) Number of orders $= N = \dfrac{R}{q^*} = \dfrac{3200}{800} = 4$

(iii) Time between two consecutive orders $= t^* = 1/N^* = 1/4$ year or 3 months.

(iv) The optimal cost $= 6 \times 3200 + \sqrt{2C_1C_3R}$

$$= 6 \times 3200 + \sqrt{2 \times \frac{3}{2} \times 150 \times 3200}$$

$$= \text{Rs. } 20{,}400.$$

Example 14.2 A company purchases 9000 parts of a machine for its annual requirements, ordering one month's usage at a time. Each part cost Rs. 20. The ordering cost per order is Rs. 15 and the carrying charges are 15% of the average inventory per year. You have been asked to suggest a more economical purchasing policy for the company. What advice would you offer and how much would it save the company per year?

(M.U., B.E, Apr.99)

Solution Given $R = 9000$ parts per year

$C_1 = 15\%$ of the average inventory per year

$= 20 \times 15/100 = $ Rs. 3 each part per year

$C_3 = $ Rs 15 per order

$$q^* = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 15 \times 9000}{3}} = 300 \text{ units}$$

$$t^* = \frac{q^*}{R} = \frac{300}{9000} = \frac{1}{30} \text{ year} = \frac{365}{30} = 12 \text{ days}$$

$$C^* = C_{min} \sqrt{2C_1C_3R} = \sqrt{2 \times 15 \times 9000} = 300 \text{ units}$$

If the company follows the policy of ordering every month then the annual ordering cost becomes $= 12 \times 15 = $ Rs. 180.

Lot size of inventory each month $q = \dfrac{9000}{12} = 750$ parts

Average inventory at any time $\dfrac{q}{2} = \dfrac{750}{2} = 375$ parts

Storage cost at any time $= 375 \times C_1 = 375 \times 3$

$= $ Rs. 1125

Total cost $= 1125 + 180 = $ Rs. 1305.

The company purchases 300 parts at time intervals of 12 days instead of ordering 750 parts each month. So there will be a net saving of Rs. 1305 − Rs. 900 = Rs. 405 per year.

Example 14.3 The demand rate of a particular item is 12,000 units per year. The set-up cost per run is Rs. 350 and the holding cost is Rs. 0.20 per unit per month. If no shortages are allowed and the replacement is instantaneous, determine (i) The optimum run size. (ii) The optimum scheduling period (iii) Minimum total expected annual cost.

Solution Demand rate $= R = 12{,}000$ per year

Holding cost $C_1 =$ Rs. 0.2 per unit per month

$=$ Rs. 2.4 per unit per year

Set-up cost $C_3 =$ Rs. 350 per run

(i) Optimum lot size $= q^* = \sqrt{\dfrac{2RC_3}{C_1}} = \sqrt{\dfrac{2 \times 12000 \times 350}{2.4}}$

$= 1870$ units

(ii) Optimum scheduling period

$$t^* = \frac{Q^*}{r} = \frac{1870}{12000} \text{ year}$$

$$= \frac{1870 \times 12}{12000} \text{ month} = 18.87 \text{ months}$$

(iii) Minimum total expected annual cost

$$= \sqrt{2RC_1C_3}$$

$$= \sqrt{2 \times 2.4 \times 350 \times 120000} = \text{Rs. } 4490 \text{ per year.}$$

Example 14.4 The annual requirement for a product is 3000 units. The ordering cost is Rs. 100 per order. The cost per unit is Rs. 10. The carrying cost per unit per year is 30% of the unit cost. (a) Find the EOQ (b) by using better organisational methods the ordering cost per order is brought down to Rs. 80 per order, but the same quantity as determined above were ordered. (c) If a new EOQ is found by using the ordered cost as Rs. 80, what would be further savings in cost?

(*MU, Mech,. Oct. 1997*)

Solution Given $R = 3000$ units per year

$C_1 = C \times I = 10 \times 30/100 =$ Rs. 3 per unit per year

$C_3 =$ Rs. 100 per order

(a) Optimal lot size $q^* = \sqrt{\dfrac{2C_3R}{C_1}} = \sqrt{\dfrac{2 \times 100 \times 3000}{3}}$

$= 447$ units

$$\text{Total inventory cost} = \sqrt{2RC_1C_3}$$
$$= \sqrt{2 \times 3000 \times 3 \times 100}$$
$$= \text{Rs. } 1342 \text{ per year.}$$

(b) $R = 3000$ units per year

$C_1 =$ Rs. 3 per unit year

$$\text{Optimal size } q^* = \sqrt{\frac{2RC_3}{C_1}} = \sqrt{\frac{2 \times 3000 \times 80}{3}}$$
$$= 400 \text{ units per order}$$

$C_3 =$ Rs. 80 per order

$$\text{Total inventory cost} = \frac{R}{q^{**}} \cdot C_3 + \frac{q^{**}}{2} \times C_1$$
$$= \frac{3000}{400} \times 100 + \frac{400}{2} \times 3$$
$$= \text{Rs. } 1350$$

Net change in the total cost or saving in cost $= 1350–1348$
$= 8 = 0.0.8\%$

Example 14.5 A company has to supply 1000 times per month at uniform rate and each time a production run is started it costs Rs. 200. Cost of storing is Rs. 20 per item per month. The number of items to be produced per run has to be ascertained. Determine the total set-up cost and average inventory cost if the run size is 500, 600, 700, 800. Find the optimal production run size using EOQ formula.

Solution Given

The demand $R = 1000$ per month

Setup cost $C_3 =$ Rs. 200 per order

Carrying cost $C_1 =$ Rs. 20 per item per month

Run size	*Set-up Cost*	*Average inventory cost*	*Total cost*
500	$\frac{1000}{500} \times 200 = 400$	$\frac{500}{2} \times 20 = 5000$	5400
600	$\frac{1000}{600} \times 200 = 333.3$	$\frac{600}{2} \times 20 = 6000$	6333.3
700	$\frac{1000}{700} \times 200 = 285.7$	$\frac{700}{2} \times 20 = 7000$	7285.7
800	$\frac{1000}{800} \times 200 = 250$	$\frac{800}{2} \times 20 = 8000$	8250

From the above table we conclude that the total cost increases when the batch size increases.

$$\text{EOQ } q^* = \sqrt{\frac{2RC_3}{C_1}} = \sqrt{\frac{2 \times 1000 \times 200}{20}}$$

$$= 141 \text{ units.}$$

Example 14.6 The following table gives the annual demand and unit price of 4 items.

Item	*A*	*B*	*C*	*D*
Annual demand (units)	800	400	392	13 800
Unit price (Rs)	0.02	1.00	8.00	0.20

Order cost is Rs. 5 per order and holding cost is 10% of unit price.

(i) Determine the EOQ in units
(ii) Total variable cost
(iii) Compute EOQ in Rs
(iv) Compute EOQ in years of supply
(v) Number of orders per year.

Solution

Item A:

Given $R = 800$ units per year
$C_3 =$ Rs. 5 per order
$C_1 = 10/100 \times 0.02 = 0.002$

$$\text{EOQ} = \sqrt{\frac{2RC_3}{C_1}} = \sqrt{\frac{2 \times 800 \times 5}{0.002}} = 2000 \text{ units}$$

$$\text{Total variable cost} = \sqrt{2C_1C_3R} = \sqrt{2 \times 0.002 \times 5 \times 800}$$

EOQ in Rs. $= 2000 \times 0.0 =$ Rs. 40 = Rs. 4

$$\text{EOQ in years supply} = \frac{2000}{800} = 2.5 \text{ years}$$

$$\text{No. of orders per year} = \frac{R}{q^*} = \frac{800}{2000} = \frac{1}{2.5} = 0.4$$

Item B:

$R = 400$ units per year
$C_3 =$ Rs. 5 per order
$C_1 = 10/100 \times 1 = 1/10 =$ Re. 0.1

$$\text{EOQ} = \sqrt{\frac{2RC_3}{C_1}} = \sqrt{\frac{2\times 392\times 5}{0.1}} = 200 \text{ units}$$

$$\text{Total variable cost} = \sqrt{2\times C_3 \times C_1 \times R}$$

$$= \sqrt{2\times 400\times 5\times 0.1} = \text{Rs. } 20$$

$$\text{EOQ in Rs.} = 200\times 1 = \text{Rs. } 200$$

$$\text{EOQ in years supply} = \frac{200}{400} = 0.5 \text{ year}$$

$$\text{No. of orders per year} = \frac{400}{200} = 2$$

Item C:

$R = 392$ units per year
C_3 = Rs. 5 per order
$C_1 = 10/100 \times 8 = 0.8$

$$\text{EOQ} = \sqrt{\frac{2RC_3}{C_1}} = \sqrt{\frac{2\times 392\times 5}{0.8}} = 70 \text{ units}$$

$$\text{Total variable cost} = \sqrt{2RC_1C_3}$$

$$= \sqrt{2\times 392\times 0.8\times 5} = \text{Rs. } 56$$

$$\text{EOQ in Rs.} = 70 \times 8 = \text{Rs. } 560$$

$$\text{EOQ in years of supply} = \frac{70}{392} = 0.18 \text{ year}$$

$$\text{Number of orders per year} = \frac{392}{70} = 5.6$$

Similarly we can calculate for item D. Thus we arrive at following. (i) 2627 units (ii) Rs. 52.54 (iii) Rs. 525.40 (iv) 0.19 year (v) 5.26.

Example 14.7 The demand rate for an item in a company is 18000 units per year. The company can produce at the rate of 3000 per month. The set-up cost is Rs. 500 per order and the holding cost is 0.15 per units per month. Calculate

(i) Optimum manufacturing quantity
(ii) The maximum inventory
(iii) Time between orders
(iv) The number of orders per year
(v) The time of manufacture
(vi) The optimum annual cost if the cost of an item is Rs. 2 per unit.

Solution Given,

Item cost C = Rs. 2 per unit
Set-up cost C_3 = Rs. 500 per order

Carrying cost = Rs. 0.15 per unit per month
Demand rate R = 18000 units per year
= 1500 units per month
Production rate $K = 3000$ units per month

(i) Optimum manufacturing quantity

$$q^* = \sqrt{\frac{2RC_3}{C_1}}\sqrt{\frac{K}{K-R}}$$

$$= \sqrt{\frac{2\times 1500\times 500}{0.15}}\sqrt{\frac{3000}{3000-1500}} = 4470 \text{ units}$$

(ii) Maximum inventory $= \dfrac{q}{K}(K-R)$

$$= \frac{4470}{3000}(3000-1500) = 2235 \text{ units}$$

(iii) Times between orders $= \dfrac{q^*}{R} = \dfrac{4470}{1500} = 3$ month(approximately)

(iv) Number of orders per year $\dfrac{12}{3} = 4$

(v) Times between orders $= \dfrac{q^*}{K} = \dfrac{4470}{3000} = 1.5$ months

(vi) The optimum annual cost

= item cost + ordering cost + holding cost

$= 1800 \times 2 + \dfrac{18000}{4470} \times 500 + \dfrac{2235}{36000}(36000 - 18000) \times 1.8$

= Rs. 40025.

Example 14.8 A contractor has to supply 10,000 bearings per day to an automobile manufacturer. He finds that when he starts production run he can produce 25000 bearings per day. The holding cost of a bearing in stock is Rs. 0.02 per year. Set-up cost of a production is Rs. 18. How frequently should production run be made?

Solution Given

Demand rate $R = 10000$ units per day
Production rate $K = 25000$ units per day
Set-up cost C_3 = Rs. 18 per order
Carrying or holding cost $C_1 = 0.02$ per unit per year
= Rs. 0.000055 per unit per day

Optimum order quantity $q^* = \sqrt{\dfrac{2C_3R}{C_1}}\sqrt{\dfrac{K}{K-R}}$

$$= \sqrt{\frac{2\times 18\times 10000}{0.000055}}\sqrt{\frac{25000}{25000-10000}}$$

$= 1{,}04{,}447$ bearings

Times between orders $= \dfrac{q^*}{R} = \dfrac{104447}{10000}$

$= 10.4$ days

Times of manufacture $= \dfrac{q^*}{R} = \dfrac{104447}{25000} = 4$ days (approx).

∴ The manufacture cycle or production cycle starts at an interval of 10.4 days and production continues for 4 days.
In each cycle a batch of 1,04,447 bearings is produced.

Example 14.9 An item is produced at the rate of 50 per day. The demand occurs at the rate of 25 items per day. If the set-up cost is Rs. 100 per run and the holdings cost is Rs. 0.01 per unit of item per day. Find the economic lot size for one run assuming the shortages are not permitted. Also find the time of the cycle and minimum cost for one run.

Solution Given, Demand rate $R = 25$ items per day

Production rate $K = 50$ items per day
Setup cost $C_3 =$ Rs. 100 per run
Holding cost $C_1 =$ Rs. 0.01 per unit per day.

(i) Economic lot size $q^* = \sqrt{\dfrac{2C_3R}{C_1}}\sqrt{\dfrac{K}{K-R}}$

$$= \sqrt{\frac{2\times 100\times 25}{0.01}}\sqrt{\frac{50}{50-25}} = 1000 \text{ units}$$

(ii) $t^* = \dfrac{q^*}{R} = \dfrac{1000}{25} = 40$ days

Minimum daily cost is given by

$$C_1^* = \sqrt{2C_1C_3R}\sqrt{\frac{K-R}{K}}$$

$$= \sqrt{2\times 0.01\times 100\times 25}\sqrt{\frac{50-25}{50}} = \text{Rs. } 5$$

∴ Minimum cost per run $= 5\times 40 =$ Rs. 200.

...mple 14.10 A company has a demand of 12,000 units per year ...item and it can produce 2000 such items per month. The cost of one ...up is Rs. 400 and the holding cost per unit per month is Rs. 0.015. ...d the optimum lot size, max inventory manufacturing time and the ...al time.

Solution Given, $R = 12000$ units per year

$K = 2000$ units per month
$= 2000 \times 12 = 24000$ units per year

Set-up cost per run $C_3 = $ Rs. 400 per run

Holding cost $C_1 = $ Rs. 0.15 per unit per month
$= 0.15 \times 12 = $ Rs. 1.8 per unit per year

Optimum lot size $$q^* = \sqrt{\frac{2C_3R}{C_1}}\sqrt{\frac{K}{K-R}}$$

$$= \sqrt{\frac{2\times 400\times 12000}{1.8}}\sqrt{\frac{24000}{24000-12000}}$$

$= 3266$ units (app)

Maximum inventory $$= \frac{K-R}{K}q^* = \frac{24000-12000}{24000}(3266)$$

$= 1632$ units

Manufacturing times $$= \frac{q^*}{K} = \frac{3266}{24000} = 0.136 \text{ years}$$

...time $$t^* = \frac{q^*}{R} = \frac{3266}{12000} = 0.272 \text{ years}$$

EXERCISES

1. ...annual requirements for a particular raw material are 2000 units, costing Re. 1 each to manufacture. The ordering cost is Rs. 10.00 per order and the ...ing cost 16% per annum of the average inventory value. Find the EOQ and the total inventory cost per annum.
(Ans. $q^* = 500$ units
C_{min} Rs. 80)

2. ...an item the production is instantaneous. The storage cost of one item is Re. 1 per month and the set-up cost is Rs. 25 per run. If the demand is 200 units per month find the optimum quantity to be produced per set-up and ...find the total cost of storage and set up per month.
(Ans. $q^* = 100$ units $t^* = 15$ days C^* Rs. 100
Total costs of storage and set up
$25 + 1 \times 100 =$ Rs. 125)

3. XYZ company buys in lots of 2000 units which is only 3months supply. The cost per unit is Rs. 125 and the order cost is Rs. 250. Then inventory carrying charge is Rs. 20% of unit value. How much money can be saved by using economic order quantity? **(Ans:** = Rs. 1600)

4. The annual demand for a product is 1,00,000 units. The rate of production is 2,00,000 units per year. The set-up cost per production run is Rs. 5,000 and the variable product cost of each items is Rs. 10. The annual holdings cost per unit is 20% of its value. Find the optimum production lot size and the length of the production run.
(Ans: $q^* = 3162$ units $t^* = 115$ days)

5. A contractor has to supply 20,000 units per day. He can produce 30,000 units per day. The cost of holding a unit in stock is Rs. 3 per year and the set-up cost per run is Rs. 50. How frequently and of what size should the production run be made? **(Ans:** $q^* = 1414$ units $t = 1.68$ hours)

6. An item is produced at the rate of 128 units per day. The annual demand is 6440 units. The set up cost for each production run is Rs. 24 and inventory carrying charges cost is Rs. 3 per unit per year. There are 250 working days for production each year. Develop an inventory policy for this item.
(Ans. $q^* = 358$ units, $t^* = 14$ days.
Manufacturing time = 2.8 days
C^* = Rs. 858.65)

7. A stockist has to supply 400 units of a product every monday to his customer. He gets the product at Rs 50 per unit from the manufactures. The cost of ordering and transportation from the manufactures is Rs 75 per order. The cost of carrying inventory is 7.5 % per year of the cost of the product. Find (i) the economic lot size (ii) the total optimal cost (including the capital cost)
(Ans. $q^* = 912$ units per order
$C^* = 20{,}065.80$ per week)

8. A certain item costs Rs. 250 per ton. The monthly requirements are 10 tons and each time the stock is replenished there is a set-up cost of Rs 1000. The cost of carrying inventory has been estimated as 12 % of the value of the stock per year. What is the optimal order quantity and how frequently should orders be placed?
(*MU MBA Apr. 96*)
(Ans. $q^* = 89.44$ units $t^* = 9$ months)

14.7.2 EOQ Model with Shortages

Model III:Purchasing model with shortages

Case I: This is the extension of model I allowing shortages. The assumptions are

(i) C_1 is the holding cost per quantity unit per unit time
(ii) C_2 is the shortage cost per quantity per unit time
(iii) R is the demand rate
(iv) t_p is the scheduling time period which is constant

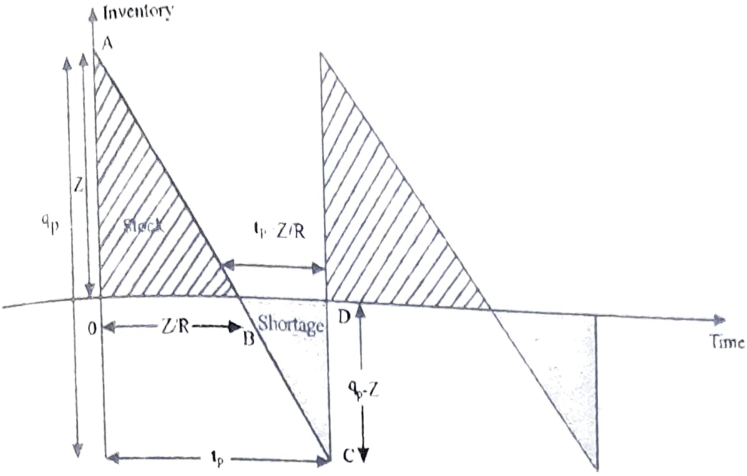
(v) q_p is the fixed lot size $q_p = Rt_p$

(vi) Z is the order level to which the inventory is raised in the beginning of each scheduling period. Shortages if any, have to be made up. Here, Z is a variable.

(vii) Production rate is infinite

(viii) Lead time is zero.

In this model we can easily observe that the inventory carrying cost C_1 as well as shortage cost C_2 will be involved only when $0 \leq z \leq qp$.



In the above figure the dotted area (ΔDBC) represents the failure to meet the demand and the shaded area (ΔAOB) shows the inventory.

Since q_p is the lot size sufficient to meet the demand for time t_p but $(<q_p)$ amount of stock is planned in order to meet the demand for time Z/R, where R is the demand rate. Shortage of amount $(q_p - z)$ will arise for the entire remaining period $t_p - Z/R$

Holding cost per unit time becomes

$$= C_1\,(\Delta OAB)/t_p$$

$$= \frac{C_1}{t_p}\left[\frac{1}{2} z \cdot z/R\right]$$

$$= \frac{1}{2}\frac{z^2 C_1}{Rt_p} = \frac{1}{2}\frac{z^2 C_1}{q_p}$$

Shortage cost per unit time is

$$= C_2\,(\Delta BDC)/t_p = C_2\left(\tfrac{1}{2}\,BD \cdot DC\right)/t_p$$

$$= \frac{C_2}{2t_p}\left[\left(t_p - z/R\right)\left(q_p - z\right)\right] = \frac{C_2}{2Rt_p}\left(Rt_p - z\right)\left(q_p - z\right)$$

$$= \frac{1}{2}\frac{C_2}{q_p}\left(q_p - z\right)^2$$

The cost equation for this model is

$$\boxed{C(z) = \frac{1}{2}\frac{z^2 C_1}{q_p} + \frac{1}{2}\frac{C_2}{q_p}(q_p - z)^2}$$

the set-up cost C_3 and period t_p are constant. The average set-up cost C_3/t_p being constant are not to being considered in the cost equation.

To obtain the optimum order level z we differentiate $C(z)$ w.r to z and equate it to zero. we get

$$\frac{dc}{dz} = \frac{1}{2}\frac{C_1}{q_p}(2z) + \frac{1}{2}\frac{C_2}{q_p}2(q_p - z)(-1) = 0$$

$$\Rightarrow z = \frac{C_2}{C_1 + C_2} q_p$$

or

$$\boxed{z = \frac{C_2}{C_1 + C_2} R t_p}$$

Condition for minimum cost is also satisfied because

$$\frac{d^2 C}{dz^2} = \frac{C_1}{q_p} + \frac{C_2}{q_p} > 0$$

Substituting this value of z in cost equation and on simplification we get

$$C_{min} = \frac{1}{2}\frac{C_1 + C_2}{C_1 + C_2} R t_p$$

Case II All the assumptions in this case are same as in case I except that the scheduling period is not constant here. Hence, we have to consider the average set up cost C_3/t in the cost equation, so that comparisons can be made between different values of t.

The cost equation becomes

$$C(t, z) = \frac{1}{t}\left[\frac{C_1 z^2}{2R} + \frac{C_2}{2R}(Rt - z)^2 + C_3\right]$$

To minimise the cost $C(t, z)$ which is the function of 2 independent variables t, z

...or this we have

$$\frac{\partial c}{\partial z} = 0, \quad \frac{\partial c}{\partial t} = 0$$

$$\frac{\partial c}{\partial z} = 0 \text{ gives } \boxed{z = \frac{C_2 Rt}{C_1 + C_2}}$$

$\frac{\partial c}{\partial t} = 0$ gives

$$\boxed{t^* = \sqrt{\frac{2C_3}{RC_1}\left(\frac{C_1 + C_2}{C_2}\right)}} \text{ (optimum period)}$$

optimal order quantity q is given by

$$q^* = Rt^* = R\sqrt{\frac{2C_3}{RC_1}\left(\frac{C_1 + C_2}{C_2}\right)}$$

$$\boxed{q^* = \sqrt{\frac{2C_3}{C_1}\frac{C_1 + C_2}{C_2}}}$$

$$\boxed{C_{\min} = C^* = \sqrt{2C_1C_3R}\sqrt{\frac{C_2}{C_1 + C_2}}}$$

Model IV: Manufacturing model with shortages

In this model the assumptions are

(i) R is the uniform demand rate

(ii) Lead time is zero.

(iii) Production rate is finite (k units per unit time)

(iv) Inventory carrying cost is R_s $C_1 = K > R$ (IP) per quantity unit per unit time

(v) Shortages are not allowed and back logged and this cost is Rs. 12 per quantity per unit time

(vi) Set up cost is Rs C_3 per set-up.

To obtain the optimum order level z we differentiate $C(z)$ w.r t z and equate it to zero, we get

$$\frac{dc}{dz} = \frac{1}{2}\frac{C_1}{q_p}(2z) + \frac{1}{2}\frac{C_2}{q_p}2(q_p - z)(-1) = 0$$

$$\Rightarrow z = \frac{C_2}{C_1 + C_2} q_p$$

or
$$\boxed{z = \frac{C_2}{C_1 + C_2} Rt_p}$$

(Condition for minimum cost is also satisfied because)

$$\frac{d^2C}{dz^2} = \frac{C_1}{q_p} + \frac{C_2}{q_p} > 0$$

(Substituting this value of z in cost equation and on simplification)

we get

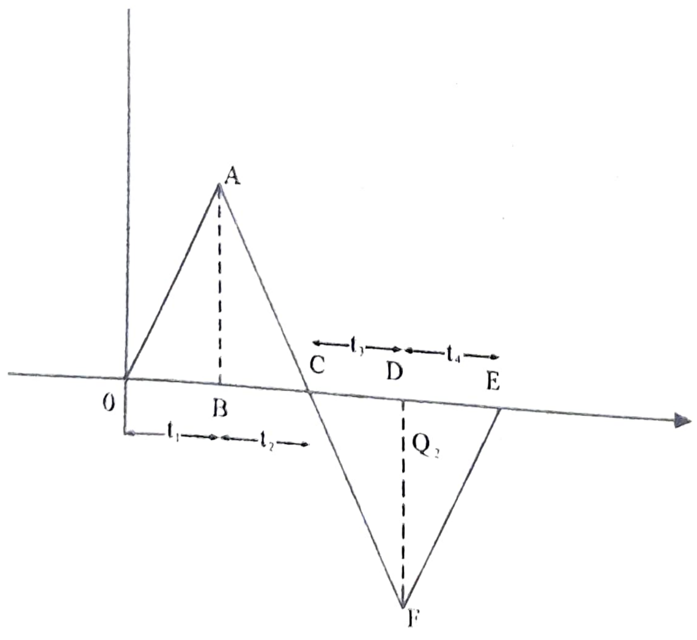
$$C_{\min} = \frac{1}{2} \frac{C_1 + C_2}{C_1 + C_2} Rt_p$$

Case II All the assumptions in this case are same as in case I except that the scheduling period it is not constant here. Hence we have to consider the average set-up cost C_3/t in the cost equation, so that comparision can be made between different values of t.

The cost equation becomes

$$C(t, z) = \frac{1}{t}\left[\frac{C_1 z^2}{2R} + \frac{C_2}{2R}(Rt - z)^2 + C_3\right]$$

To minimise the cost $C(t, z)$ which is the function of 2 independent variables t, z.



...above figure shows that there is an inventory cycle. Stocks start at ...and increases for a period t_1. They decline for a period t_2 until they ...reach zero at the point where a backlog piles up for the time t_3. At ...end of t_3 production starts and backlog is diminished for the time t_4 ...the backlog reaches zero. The cycle then repeats itself after total ...$(t_1 + t_2 + t_3 + t_4)$.

...holding cost $= C_1 \times \Delta OAC = C_1 \cdot \frac{1}{2} Q_1 (t_1 + t_2)$.

...shortage cost $= C_2 \times \Delta EFC = C_2 \cdot \frac{1}{2} Q_2 (t_3 + t_4)$

The set - up cost per set - up is equal to C_3

$$C = \frac{1}{2}\left\{\frac{C_1 Q_1 (t_1 + t_2) + C_2 Q_2 (t_3 + t_4) + C_3}{t_1 + t_2 + t_3 + t_4}\right\}$$

...C is a function of variables $(Q_1, Q_2, t_1, t_2, t_3, t_4)$. There are four ...relationships which permits us to eliminate two variables.

The inventory is zero at O and during the period t_1 an amount kt_1 is produced but because orders are being filled up at a rate the net increase ...in inventory during t_1 is given by

$$Q_1 = kt_1 - Rt_1 = t_1 (K - R)$$

...after time t_1 the production is stopped and the stock Q_1 is used up ...during t_2 and because the rate of use is R we have.

$$Q_1 = Rt_2$$

$$Q_1 = t_1 (K - R) \Rightarrow Rt_2 = t_1 (K - R)$$

$$t_1 = \frac{Rt_2}{K - R}$$

...period t_4 shortages accumulate at a rate R

$$Q_2 = Rt_3$$

...period t_4 production rate is K and demand rate is R so that the net ...reduction of shortage becomes $K - R$, and thus we have ...$t_4 (K - R)$

$$Q_2 = t_4 (K - R)$$

$$t_4 = \frac{Q_2}{K - R} = \frac{Rt_3}{K - R}$$

...because the total cycle $(t_1 + t_2 + t_3 + t_4)$ and production q is just ...to meet the demand at the rate R, we have

$$Q = R (t_1 + t_2 + t_3 + t_4)$$

Substituting the values of t_1 and t_4 in q. We get,

$$q = R\left(\frac{Rt_2}{K-R} + t_2 + t_3 + \frac{Rt_3}{K-R}\right).$$

$$q = \frac{(t_2 + t_3)K}{K-R}.$$

Now eliminating t_1, t_4, Q_1, and Q_4 from the cost equation. We get,

$$C(t_2, t_3) = \frac{\frac{1}{2}\left(C_1 t_2^2 + C_2 t_3^2\right) RK + C_3(K-R)}{K(t_2 + t_3)}$$

To find the best values t_2* and t_3* of t_2 and t_3. Differentiate the above equation partially with respect to t_2 and t_3 and set the results equal to zero. We get,

$$t_2^* = \sqrt{\frac{2\, C_3\, C_2 (1 - R/K)}{R\,(C_1 + C_2)\, C_1}}$$

$$t_3^* = \sqrt{\frac{2\, C_3\, C_1 (1 - R/K)}{R\,(C_1 + C_2)\, C_2}}$$

Using this result we obtain the optimum order quantity

$$\boxed{q^* \quad \sqrt{\frac{2\, R\, C_3}{C_1 C_2}}\,(C_1 + C_2)\sqrt{\frac{K}{K-R}}}$$

$$Q_2^* \quad \sqrt{\frac{2\, R\, C_1\, C_3}{(C_1 + C_2)\, C_2}}\sqrt{\frac{K-R}{K}}$$

Finally the minimum cost is given by

$$\boxed{C^* \quad \sqrt{\frac{2\, R\, C_1\, C_3}{C_1 + C_2}}\sqrt{\frac{K-R}{K}}}$$

Example 14.11 The demand of an item is uniform at a rate of 25 units per month. The fixed cost is Rs. 15 each time a production run is made. The production cost is Re 1 per item and the inventory carrying cost is Re. 0.30 per item per month. If the shortage cost is Rs. 1.50 per item per month, determine how often to make a production run and of what size it would be?

Solution Given:

$C_1 = 0.30$ per item per month
$C_2 =$ Rs. 1.50 per item per month
$C_3 =$ Rs. 15.00 per item per month
$R = 25$ units per month.

The optimum value of q is given by

$$q^* = \sqrt{\frac{2RC_3}{C_1}}\sqrt{\frac{C_1 + C_2}{C_2}}$$

$$= \sqrt{\frac{2 \times 15 \times 25}{0.3}}\sqrt{\frac{0.3 + 1.5}{1.5}} = 54 \text{ items}$$

$$\boxed{t^* = \frac{q^*}{R}} = \frac{54}{25} \text{ months} = 2.16 \text{ month}$$

Example 14.12 The demand for an item is 18000 units per year. The holding cost is Rs. 1.20 per unit time and the cost of shortage is Rs. 5.00. The production cost is Rs. 400.00. Assuming that replacement rate is instantaneous determine the optimum order quantity.

(MU, B.E ,OCT. 1998)

Solution Given:

$R = 18000$ units per year
Holding cost $C_1 =$ Rs. 1.20 per unit
Shortage cost $C_2 =$ Rs. 5.00
Set-up cost $C_3 =$ Rs. 400 per run

The optimum order quantity

$$q^* = \sqrt{\frac{2RC_3}{C_1}}\sqrt{\frac{C_1 + C_2}{C_2}}$$

$$= \sqrt{\frac{2 \times 18000 \times 400}{1.2}}\sqrt{\frac{1.2 + 5}{5}}$$

$= 3857$ units.

$$\boxed{t^* = \frac{q^*}{R}} = \frac{3857}{18000} = 0.214 \text{ year.}$$

$$\boxed{N^* = \frac{R}{q^*}} = 4.67 \text{ order per year}$$

Example 14.13 The demand for an item is deterministic and constant over time and it is equal to 600 units per year. The per unit cost of the item is Rs. 50 while the cost of placing an order is Rs. 5. The inventory carrying cost is 20% of the cost of inventory per annum and the cost of shortage is Rs. 1 per unit per month. Find the optimal ordering quantity and when stock outs are permitted. If the stock outs are not permitted what would be the loss to the company.

Solution Given:

$R = 600$ units
$C_1 = 0.20 \times 50 = \text{Rs. } 10$
$C_3 = \text{Rs. } 5$ per order
$C_2 = \text{Re. } 1$ per unit per month or Rs. 12 per year per unit

(i) When stock outs are permitted, the optimal ordering quantity is given by

$$q^* = \sqrt{\frac{2\,R\,C_3}{C_1}}\sqrt{\frac{C_1 + C_2}{C_2}}$$

$$= \sqrt{\frac{2 \times 600 \times 5}{10}}\sqrt{\frac{10+1}{1}} = 33 \text{ units}$$

(ii) Maximum number of back orders

$$Z^* = q^*\left(\frac{C_1}{C_1 + C_2}\right) = 33\left(\frac{10}{10+12}\right) = 15 \text{ units}$$

(iii) Total expected yearly cost (with shortage allowed) is

$$C(q^*) = \sqrt{2\,R\,C_1\,C_3}\sqrt{\frac{C_2}{C_1 + C_2}}$$

$$= \sqrt{2 \times 600 \times 10 \times 5}\sqrt{\frac{1}{10+1}} = 180{\cdot}91$$

$$= 181$$

If stock outs or back ordering are not permitted the optimal order quantity is

$$q^* = \sqrt{\frac{2\,R\,C_3}{C_1}} = 24.5 \text{ units}$$

The total relevant cost is given by

$$C(q^*) = \sqrt{2\,C_1 C_3\,R} = \text{Rs. } 245$$

Thus the additional cost when back ordering is not allowed is Rs. (245–181) = Rs. 64.

Example 14.14 The demand for an item in a company is 18,000 units per year. The company can produce the items at a rate of 3,000 per nth. The cost of one set-up is Rs. 500 and the holding cost of 1 unit month is Rs. 0.15. The shortage cost of one unit is Rs. 20 per year. rmine the optimum manufacturing quantity and the number of tages. Also determine the manufacturing time and the time between t ups.

(MU, BE, Apr 99)

Solution

C_1=0.15 per month
C_2=Rs. 20.00
C_3=Rs. 500.00 $\quad K$=3000 units per month
R=18000 units per year or 1500 units per month

$$q^* = \sqrt{\frac{2RC_3}{C_1}\,\frac{C_1+C_2}{C_2}}\sqrt{\frac{K}{K-R}}$$

$$= \sqrt{\frac{2\times1500\times500}{0.15}\left(\frac{0.15+20}{20}\right)}\sqrt{\frac{3000}{3000-1500}}$$

$= 4489$ units (app).

Number of shortages

$$Q_2^* = q^*\left(\frac{C_1}{C_1+C_2}\right)\left(\frac{K-R}{K}\right)$$

$$= 4489\left(\frac{0.15}{0.15+20}\right)\left(\frac{3000-1500}{3000}\right) = 17 \text{ units (app)}$$

$$\text{Manufacturing time} = \frac{q^*}{K} = \frac{4489}{3000} = 1.5 \text{ months}$$

$$\text{Time between set - ups.} = \frac{q^*}{K} = \frac{4489}{1500} = 3 \text{ months}$$

R

Example 14.15 The demand for an item is 12,000 per year and rtage are allowed. If the unit cost is Rs. 15 and the holding cost is Rs per year per unit. Determine the optimum total yearly cost. The cost placing one order is Rs. 6,000 and the cost of one shortage is Rs. 100 year.

(MU, BE, Oct. 98)

Solution Given

R = 12000 units per year
Holding cost C_1 = Rs. 20 per unit per year
Set-up cost = C_3 = Rs. 6000 per order
Shortage cost C_2 = Rs. 100

Total annual cost = (Number of orders per year × total cost per period)

$$q^* = \sqrt{\frac{2\,R\,C_3}{C_1}}\sqrt{\frac{C_1+C_2}{C_2}}$$

$$= \sqrt{\frac{2\times 12000\times 6000}{20}}\sqrt{\frac{20+100}{100}} = 2939 \text{ units.}$$

$$\text{Number of orders per year } = \frac{12000}{2939} = \frac{R}{q^*}$$

$$= 4.08$$

Number of shortages:

$$Z^* = q^*\left(\frac{C_1}{C_1+C_2}\right)$$

$$= 2939\left(\frac{20}{20+100}\right) = 489 \text{ units.}$$

Total yearly cost is given by

$$= C\times R^* + \sqrt{2RC_1C_3}\sqrt{\frac{C_2}{C_1+C_2}}$$

$$= 15\times 12000 + \sqrt{2\times 12000\times 20\times 6000}\sqrt{\frac{100}{120}}$$

$$= \text{Rs } 108989.79.$$

EXERCISES

(1) A manufacturer has to supply his customer 24,000 units of his product per year. The demand is fixed and known. The customer has no storage space and so the manufacturer has to ship a day's supply each day. On the failure to supply, the manufacturer is liable to pay a penalty of Rs 0.20 per unit per month. The inventory holding cost amount to Re. 0.01 per unit per month and the set-up cost is Rs. 350 per production run. Find the optimum lot size for the manufactures.

[**Ans:** 4744 units per run]

(2) The demand for a product is 25 units per month and the items are withdrawn uniformly. The set-up cost each time a product is run is Rs. 15. The inventory holding cost is Re. 0.30 per item per month.

(i) Determine how often to make production run, if shortages are not allowed.

(ii) Determine how often to make production run, if shortages cost Rs 1.50 per item per month.

(**Ans:** (i) $q^* = 50$ units; (ii) $q^* = 54.7$ units)