Project Report group - 6

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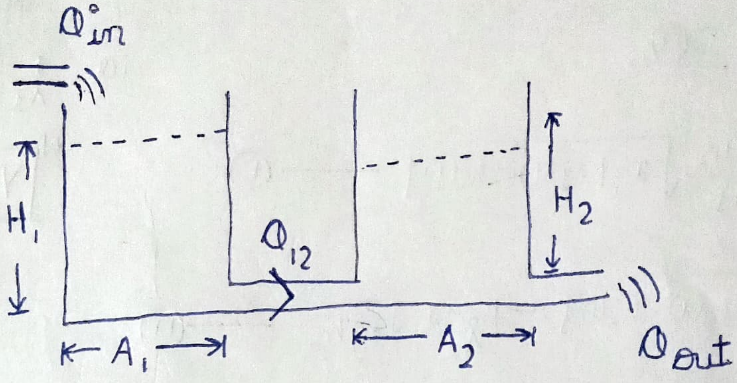

Rate of flow in a tank = $Q_{in} - Q_{out} = A\frac{dh}{dt}$

For tank 1

$$A_1 \frac{dh_1(t)}{dt} = Q_{in} - B_{12}\alpha_{12}\sqrt{2g[h_1(t)-h_2(t)]} \qquad \therefore Q_{in} = u(t)$$

$$\Rightarrow \frac{dh_1(t)}{dt} = \frac{u(t)}{A_1} - \frac{B_{12}\alpha_{12}}{A_1}\sqrt{2g[h_1(t)-h_2(t)]} \quad \text{—— (1)}$$

For tank 2

$$A_2 \frac{dh_2(t)}{dt} = B_{12}\alpha_{12}\sqrt{2g[h_1(t)-h_2(t)]} - B_2\alpha_2\sqrt{2gh_2(t)}$$

$$\frac{dh_2(t)}{dt} = \frac{B_{12}\alpha_{12}}{A_{12}}\sqrt{2g[h_1(t)-h_2(t)]} - \frac{B_2\alpha_2}{A_2}\sqrt{2gh_2(t)} \quad \text{—— (2)}$$

at eq^n for constant water level setpoint, the $\frac{dy}{dx}$ must be zero i.e $\dot{h}_1 = \dot{h}_2 = 0$. In addition for the case when $h_1 = h_2$ the system model is decoupled. so $h_1 > h_2$.

Let.
$z_1 = h_2 > 0$ and $z_2 = h_1 - h_2 > 0$.

$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$ $u = q(t)$ $b = \frac{\beta_{12}\alpha_{12}}{A_2}\sqrt{2g}$ $c = \frac{\beta_2 a_2}{A_2}\sqrt{2g}$

dynamic eq from ① & ⑪ $a = \frac{1}{A_1}$

$\boxed{Y = z_1}$

$$\frac{dh_1(t)}{dt} = \frac{u(t)}{A_1} - \frac{\beta_{12} a_{12}}{A_1}\sqrt{2g[h_1(t) - h_2(t)]} \quad \text{——} ①$$

$$\frac{dh_2(t)}{dt} = \frac{\beta_{12} a_{12}}{A_2}\sqrt{2g[h_1(t) - h_2(t)]} - \frac{\beta_2\alpha_2}{A_2}\sqrt{2gh_2} \quad \text{——} ⑪$$

Modifying ②

$$\cancel{\frac{dz(t)}{dt} = au - b\sqrt{z}}$$

$$\frac{dz_1(t)}{dt} = b\sqrt{z_2} - c\sqrt{z_1} \Rightarrow \boxed{\dot{z}_1 = b\sqrt{z_2} - c\sqrt{z_1}}$$

$$\frac{d(z_2 + z_1)(t)}{dt} = au - b\sqrt{z_2}$$

$$\Rightarrow \dot{z}_2 = au - b\sqrt{z_2} - \dot{z}_1$$

$$\Rightarrow \boxed{\dot{z}_2 = au - 2b\sqrt{z_2} + c\sqrt{z_1}}$$

Let $x = T(z)$ States are funs of z.

by inverse transformation.

$x_1 = z_1$ $z_1 = x_1$

$x_2 = b\sqrt{z_2} - c\sqrt{z_1}$ $z_2 = \left(\frac{c\sqrt{x_1} + x_2}{b}\right)^2$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{b\dot{z}_2}{2\sqrt{z_2}} - \frac{c\dot{z}_1}{2\sqrt{z_1}}$$

Putting the value back in eq ③

$$\dot{x}_2 = \frac{\alpha c}{2}\left[\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right] + \frac{c^2-b^2}{2} + \frac{ab}{2\sqrt{z_2}}u(t)$$

$$0 = \frac{\alpha c}{2}\left[\frac{\sqrt{3}}{\sqrt{3}c}\times b - \frac{\sqrt{3}c}{b\times\sqrt{3}}\right] + \frac{c^2}{2} - b^2 + \frac{ab}{2\sqrt{3}c}\times b \times u(t).$$

$$\Rightarrow \boxed{u(t) = \frac{\sqrt{3}\alpha c}{a}} \qquad u^* = \frac{\sqrt{3}c}{a}$$

$$\Delta\dot{\bar{x}} = A\,\Delta\bar{x} + B\Delta\bar{u}$$

$y = z_1$

The objective of the control scheme to regulate $Y(t) = z_1(t) = h_2(t)$

$$A = \left.\frac{\partial f}{\partial \bar{x}}\right|_{\substack{\bar{x}=\bar{x}^* \\ \bar{u}=u^*}} \qquad B = \left.\frac{\partial f}{\partial Y}\right|_{\substack{\bar{x}=\bar{x}^* \\ u=\bar{u}^*}}$$

$$A = \left.\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}\right|_{\substack{\bar{x}=\bar{x}^* \\ u=\bar{u}^*}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad B = \left.\begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{bmatrix}\right|_{\substack{\bar{x}=\bar{x}^* \\ u=\bar{u}^*}} = \begin{bmatrix} 0 \\ \phi \end{bmatrix}$$

$$Y = C\bar{x} + D\bar{u}$$

$$B = \begin{bmatrix} 0 \\ \frac{ab^2}{2\sqrt{3}c} \end{bmatrix}$$

$$C = [1, 0] \qquad D = [0]$$

∴ Final matrix are-

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \frac{ab^2}{2\sqrt{3}c} \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D\,[0]$$

$$\dot{x}_2 = \frac{bc}{2}\left[\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right] + \frac{c^2}{2} - b^2 + \frac{ab}{2\sqrt{z_2}} u(t). \quad \text{——} \quad (3).$$

let $f = \frac{bc}{2}\left[\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right] + \frac{c^2}{2} - b^2$ $\qquad \phi = \frac{ab}{2\sqrt{z_2}}.$

finally

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ f + \phi u \end{bmatrix} \begin{matrix} \rightarrow f_1 \\ \rightarrow f_2 \end{matrix}$$

this is f matrix.

$$\dot{x} = f(\bar{x}^*, \bar{u}^*) + \left.\frac{\partial f}{\partial \bar{x}}\right|_{\substack{x = \bar{x}^* \\ u = \bar{u}^*}} (x - \bar{x}^*) + \left.\frac{\partial f}{\partial \bar{u}}\right|_{\substack{x = \bar{x}^* \\ u = \bar{u}^*}} (u - u^*)$$

$\dot{x} = \bar{f}(\bar{x}, \bar{u}) \rightarrow$ original syste.

$$\dot{x} - \dot{x}^* = \left.\frac{\partial f}{\partial \bar{x}}\right|_{\substack{x = \bar{x}^* \\ u = \bar{u}^*}} (x - \bar{x}^*) + \left.\frac{\partial f}{\partial \bar{u}}\right|_{\substack{x = \bar{x}^* \\ u = \bar{u}^*}} (u - u^*)$$

$= \dot{\Delta x}$

Now
let us try to linearize about point $x_1^* = 3\text{ cm}$ (h_2)

$x_1^* = 3 \qquad x_2^* = \dot{x}_1^* = 0. \qquad x^* = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

For finding u^* we need z_1 & z_2

$\boxed{z_1 = x_1 = 3.}$

$$z_2 = \left(\frac{c\sqrt{x_1} + x_2}{b}\right)^2$$

$$= \frac{c^2 (3)^{2/2}}{b^2} = \boxed{\frac{3c^2}{b^2} = z_2^*}$$

$$\frac{Y(S)}{U(S)} = C(SI-A)^{-1}B + D \qquad \therefore D = [0]$$

$$\frac{Y(S)}{U(S)} = C(SI-A)^{-1}B$$

$$= \begin{bmatrix}1 & 0\end{bmatrix}\left[S\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\right]^{-1}\begin{bmatrix}0\\ \frac{ab^2}{2\sqrt{3}c}\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}S & -1\\ 0 & S\end{bmatrix}^{-1}\begin{bmatrix}0\\ \frac{ab^2}{2\sqrt{3}c}\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\end{bmatrix}\frac{\begin{bmatrix}S & \cancel{0}1\\ 0 & S\end{bmatrix}}{S^2}\begin{bmatrix}0\\ \frac{ab^2}{2\sqrt{3}c}\end{bmatrix}$$

$$= \begin{bmatrix}\frac{S \quad 1}{S^2}\end{bmatrix}\begin{bmatrix}0\\ \frac{ab^2}{2\sqrt{3}c}\end{bmatrix}$$

$$\frac{Y(S)}{U(S)} = \frac{ab^2}{2\sqrt{3}cS^2}$$

Let ~~abc~~ $\frac{ab^2}{2\sqrt{3}c} = K.$

$\frac{K}{S^2}$ so we can use PD controller.

$$\Rightarrow G_{cL}(S) = \frac{\frac{(K_p + K_d S)K}{S^2}}{1 + \frac{(K_p + K_d S)K}{S^2}}$$

using Routh Hurwitz criteria

$K_d K > 0$ & $K_p K > 0$

for stability

$$\boxed{G_{cL}(S) = \frac{(K_p + K_d S)K}{S^2 + K_d K S + K_p K}}$$

h_1 and h_2 → height of liquid in tank 1 and 2

A_1 and A_2 → cross sectional area of tank 1 and 2

α_2 → cross sectional area of outlet pipe in tank 2

α_{12} → cross sectional area of interaction pipe between tank 1 and 2

β_{12} → valve ratio of interaction pipe between tank 1 and 2

β_2 → valve ratio of ~~interaction~~ outlet pipe of tank 2

$A_1, A_2 = 154\ cm^2$

$\alpha_2, \alpha_{12} = 0.5\ cm^2$

$\beta_{12} = 1.53$

$\beta_2 = 0.68$

$g = 981\ cm/s^2$

[7]

Trying using PID controller.

$$\frac{K}{s^2}$$

$$G_{CL}(s) = \frac{\dfrac{K(K_p + K_d s + K_i/s)}{s^2}}{1 + \dfrac{K(K_p + K_d s + K_i/s)}{s^2}}$$

$$\Rightarrow \frac{\dfrac{K(K_p s + K_d s^2 + K_i)}{\cancel{s \times s^2}}}{s^3 + \dfrac{K(K_p s + K_d s^2 + K_i)}{\cancel{s \times s^2}}} \Rightarrow \frac{K(K_p s + K_d s^2 + K_i)}{s^3 + K_d s^2 + \underset{K_p \times}{\cancel{K_p}} K s + K_i}$$

$$\boxed{G_{CL}(s) = \frac{K(K_p s + K_d{}^2 + K_i)}{s^3 + K_d K_p s^2 + (K_p K_d) s + K_i}}$$