

SISHYA SCHOOL, HOSUR
ANNUAL EXAMINATIONS (2024-25)
XI (JEE)

CLASS:	XI (SCIENCE)	TIME:	3 HOURS
SUBJECT:	MATHEMATICS (041)	MAXIMUM MARKS:	80

General Instructions:

1. This Question paper contains - **five sections A, B, C, D and E.** Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has **18 MCQs and 02 Assertion-Reason based** questions of 1 mark each.
3. **Section B** has **5 Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has **6 Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has **4 Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has **3 case-based** questions of 4 marks each, with sub parts.

Q. No.	Question	M	S	T
	SECTION A MULTIPLE CHOICE QUESTIONS <i>Each question carries 1 mark</i>			
1.	The solution to the inequality $2x + 8 < 13, x \in N$ is -----. (a) $[3, \infty)$ (b) $[1, 2]$ (c) $\{1, 2\}$ (d) $\{\dots, -1, 0, 1, 2\}$	1	U	2
	Answer: (c)			
2.	The number of six digit numbers, the digits of which are all odd is -----. (a) 6^5 (b) 5^6 © $5 \cdot 5!$ (d) Not possible	1	U	2
	Answer: (b)			
3.	If $x, 2y, 3z$ are in AP, where the distinct numbers x, y, z are in GP, then the common ratio of the GP is (a) 3 (b) $\frac{1}{3}$ (c) 2 (d) $\frac{1}{2}$	1	An	3
	Answer: (b)			

4.	If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then the value of $A + B = \text{-----}$. (a) 45° (b) 60° © 30° (d) 0°	1	U	2
	Answer: (a)			
5.	The degree measure of $\frac{7\pi}{18} =$ (a) 60° (b) 70° © 100° (d) 80°	1	U	3
	Answer: (b)			
6.	The fourth term from the end of the G.P. 3, 6, 12, 24, ..., 3072 is (a) 348 (b) 1536 © 438 (d) 384	1	U	3
	Answer: (d)			
7.	A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number", Then, E and F are (a) mutually exclusive (b) mutually exclusive and exhaustive © exhaustive (d) neither mutually exclusive nor exhaustive	1	U	3
	Answer: (d)			
8.	The XOZ-Plane divides the join of (2, 3, 1) and (6, 7, 1) in the ratio (a) 3: 7 (b) 2: 7 © -3: 7 (d) -2: 7	1	U	2
	Answer: ©			
9.	If $A = \{1, 2, 3\}$ and $B = \{4, 5\}$ then the number of relations from A to B is: -----. (a) 6 (b) 8 (c) 9 (d) 64	1	U	2
	Answer: (d)			
10.	The domain of the real valued function $f(x) = \frac{1}{3x-2}$ is -----. (a) $Q - \left\{\frac{2}{3}\right\}$ (b) $R - \left\{\frac{2}{3}\right\}$ (c) $R - \{2\}$ (d) N	1	An	3
	Answer: (b)			
11.	$\sin(-330^\circ)$ is equal to -----.	1	U	3

	(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) $-\frac{\sqrt{3}}{2}$			
	Answer: (b)			
12.	The value of $\frac{1-\tan^2 15^\circ}{1+\tan^2 15^\circ}$ is -----. (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{\sqrt{2}}{3}$ (d) $\frac{1}{\sqrt{2}}$	1	App	2
	Answer: (a)			
13.	x, y, z are in GP, where x, y and z are non-zero numbers if and only if (a) $x^2=yz$ (b) $y^2 = xz$ (c) $z^2 = xy$ (d) $xyz = 1$	1	An	3
	Answer: (b)			
14.	If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then $\tan(2A + B)$ is equal to -----. (a) 1 (b) 2 (c) 3 (d) 4	1	U	3
	Answer: (c)			
15.	The equation of the straight line passing through the point (3, 2) and perpendicular to the line $y = x$ is -----. (a) $x - y = 5$ (b) $x + y = 5$ (c) $x + y = 1$ (d) $x - y = 1$	1	App	3
	Answer: (c)			
16.	The number of terms in the expansion of $(x + y)^6 + (x - y)^6$ is -----. (a) 4 (b) 3 (c) 7 (d) 6	1	App	3
	Answer: (a)			
17.	The conjugate of $i^{-35} =$ (a) 1 (b) -1 (c) i (d) $-i$	1	U	3
	Answer: (d)			
18.	The common ratio of the G.P. of which the ratio of the 7 th and the third term is 16 is (a) 2 (b) ± 2 (c) 4 (d) ± 4	1	An	3
	Answer: (b)			

	<p style="text-align: center;">ASSERTION-REASON BASED QUESTIONS</p> <p>In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.</p> <p>(a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.</p>			
19.	<p>Assertion (A): $nC_7 = nC_5, n = 12$</p> <p>Reason (R): $nC_p = nC_q, p = q$ or $p + q = n$</p> <p>Answer: (a)</p>	1	AN	3
20.	<p>Assertion (A): The equation of a straight line parallel to x-axis and passing through the point $(-2, -2)$ is $y = -2$</p> <p>Reason (R): A line passing through the origin and having a slope of 45° will always be of the form $y = x$</p> <p>Answer: (b)</p>	1	AN	3
	<p style="text-align: center;">SECTION B</p> <p style="text-align: center;"><i>This section comprises of very short answer type questions (VSA) of 2 marks each</i></p>			
21.	<p>Let $A = \{1, 2\}$, $B = \{4, 5, 6\}$, $C = \{5, 6\}$. Verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.</p> <p>Solution:</p> <p>$B \cap C = \{5, 6\}$</p> <p>$\therefore A \times (B \cap C) = \{(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)\}$</p> <p>$A \times B = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\}$</p> <p>$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)\}$</p> <p>$\therefore (A \times B) \cap (A \times C) = \{(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)\}$</p> <p>$\therefore A \times (B \cap C) = (A \times B) \cap (A \times C)$</p> <p style="text-align: center;">OR</p> <p>If $f(x) = x + 1$ and $g(x) = [x] - 10x$ where $[\cdot]$ represents the greatest integer function, verify if $f(2x) + 2g(x) = 2f(x) + g(2x)$ when $x = -3.5$</p> <p>Solution:</p> <p>$LHS = 8 + 2[(-4 + 35)] = 70$</p> <p>$RHS = 9 + (-7 + 70) = 72$. They are not equal.</p>	2	AN	4

	$2({}^4C_1a^3b + {}^4C_3ab^3) = 2(4a^3b + 4ab^3)$ $= 8ab(a^2 + b^2)$ <p>In this, by substituting $a = \sqrt{3}, b = \sqrt{2}$</p> $(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4$ $= 8\sqrt{3} \cdot \sqrt{2} \left[(\sqrt{3})^2 + (\sqrt{2})^2 \right]$ $= 8\sqrt{6}(3 + 2) = 40\sqrt{6}$	1		
27.	<p>Find the equations of the lines that pass through (3, 4), and whose sum of the intercepts on the axes is 14.</p> <p>Solution:</p> <p>Equation of the line in intercept form:</p> $\frac{x}{a} + \frac{y}{b} = 1, \quad \text{where } a + b = 14 \implies b = 14 - a.$ <p>Substitute point (3, 4):</p> $\frac{3}{a} + \frac{4}{14 - a} = 1.$ <p>Simplify and eliminate fractions:</p> $3(14 - a) + 4a = a(14 - a).$ $a^2 - 13a + 42 = 0.$ <p>Solve the quadratic equation:</p> $(a - 6)(a - 7) = 0 \implies a = 6 \text{ or } a = 7.$ <p>Equations of the line:</p> <ul style="list-style-type: none"> For $a = 6, b = 8: 4x + 3y = 24.$ For $a = 7, b = 7: x + y = 7.$ <p style="text-align: center;">OR</p> <p>Find the equations of the medians of the triangle, the sides of which are given by the lines $x + y - 6 = 0$, $x - 3y - 2 = 0$ and $5x - 3y + 2 = 0$</p> <p>Solution:</p>	1	U	7

	<p>Find the vertices of the triangle: Solve the equations pairwise:</p> <ul style="list-style-type: none"> Intersection of $x + y - 6 = 0$ and $x - 3y - 2 = 0$: $A(5, 1)$. Intersection of $x - 3y - 2 = 0$ and $5x - 3y + 2 = 0$: $B(-1, -1)$. Intersection of $5x - 3y + 2 = 0$ and $x + y - 6 = 0$: $C(2, 4)$. <p>Find the midpoints of the sides:</p> <ul style="list-style-type: none"> Midpoint of BC: $M_1 = \left(\frac{-1+2}{2}, \frac{-1+4}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$. Midpoint of CA: $M_2 = \left(\frac{2+5}{2}, \frac{4+1}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right)$. Midpoint of AB: $M_3 = \left(\frac{5-1}{2}, \frac{1-1}{2}\right) = (2, 0)$. <p>Final Equations of the Medians:</p> <ol style="list-style-type: none"> $x + 9y = 14$, $7x - 9y = 2$, $x = 2$. 	1		
		1		
		1		
28.	<p>Show that $9^{n+1} - 8n - 9$ is divisible by 64, whenever 'n' is a positive integer.</p> <p>Solution:</p> <p>Consider,</p> $9^{n+1} = (1 + 8)^{n+1} = {}^{n+1}C_0 + {}^{n+1}C_1(8) + {}^{n+1}C_2(8)^2 + \dots + {}^{n+1}C_{n+1}(8)^{n+1}$ $= 1 + (n+1)(8) + 8^2[{}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + \dots + {}^{n+1}C_{n+1}(8)^{n-1}]$ $= 9 + 8n + 64[{}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + \dots + {}^{n+1}C_{n+1}(8)^{n-1}]$ $\Rightarrow 9^{n+1} - 8n - 9 = 64k,$ <p>Where $k = {}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + \dots + {}^{n+1}C_{n+1}(8)^{n-1}$ which is a natural number</p> <p>Thus, $9^{n+1} - 8n - 9$ is divisible by 64, whenever n is a positive integer.</p>	1	U	7
		1		
		1		
29.	<p>Find the equations of the line which pass through the points (4,5) and make equal angles with the lines $5x - 12y + 6 = 0$ and $3x = 4y + 7$.</p> <p>Solution:</p>		AP	7

	<p>Let the Slope of line be m</p> <p>Now, slope of other lines are $\frac{3}{4}, \frac{12}{5}$</p> <p>The Slope is Equally inclined so it must lie between these slopes such that</p> $\frac{3}{4} < m < \frac{12}{5}$ <p>As it is equally inclined so angle between the line and the two given line must be same</p> $\Rightarrow \theta_1 = \theta_2 \Rightarrow \tan \theta_1 = \tan \theta_2$ $\Rightarrow \frac{\frac{12}{5} - m}{1 + \frac{12}{5}m} = \frac{m - \frac{3}{4}}{1 + \frac{3}{4}m} \Rightarrow 63m^2 - 32m - 63 = 0$ $\Rightarrow m = \frac{32 \pm \sqrt{32^2 + 4 * 63 * 63}}{2 * 63} \Rightarrow m = \frac{-7}{9}, \frac{9}{7}$ <p>Now, Equation of line as it passes through (4, 5) will be</p> <p>Putting the obtained values of m in above Equation we get</p> $9x - 7y = 1$ $7x + 9y = 73$	1		
		1		
		1		
30.	<p>If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$, prove that $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$</p> <p>Solution:</p>	1	U	7
		1		

$$\begin{aligned}
 \text{L.H.S.} &= \frac{1 + \cos 2x}{2} + \frac{1 + \cos \left(2x + \frac{2\pi}{3}\right)}{2} + \frac{1 + \cos \left(2x - \frac{2\pi}{3}\right)}{2} \\
 &= \frac{1}{2} \left[3 + \cos 2x + \cos \left(2x + \frac{2\pi}{3}\right) + \cos \left(2x - \frac{2\pi}{3}\right) \right] \\
 &= \frac{1}{2} \left[3 + \cos 2x + 2 \cos 2x \cos \frac{2\pi}{3} \right] \\
 &= \frac{1}{2} \left[3 + \cos 2x + 2 \cos 2x \cos \left(\pi - \frac{\pi}{3}\right) \right] \\
 &= \frac{1}{2} \left[3 + \cos 2x - 2 \cos 2x \cos \frac{\pi}{3} \right] \\
 &= \frac{1}{2} [3 + \cos 2x - \cos 2x] = \frac{3}{2} = \text{R.H.S.}
 \end{aligned}$$

OR

Prove that $\frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A} = \tan 6A$

Solution:

$$\frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A}$$

When we rearrange we get,

$$= \frac{(\sin 9A + \sin 3A) + (\sin 7A + \sin 5A)}{(\cos 9A + \cos 3A) + (\cos 7A + \cos 5A)}$$

$$= \frac{\left(2 \sin \frac{9A + 3A}{2} \cos \frac{9A - 3A}{2}\right) + \left(2 \sin \frac{7A + 5A}{2} \cos \frac{7A - 5A}{2}\right)}{\left(2 \cos \frac{9A + 3A}{2} \cos \frac{9A - 3A}{2}\right) + \left(2 \cos \frac{7A + 5A}{2} \cos \frac{7A - 5A}{2}\right)}$$

$$= \frac{\left(2 \sin \frac{12A}{2} \cos \frac{6A}{2}\right) + \left(2 \sin \frac{12A}{2} \cos \frac{2A}{2}\right)}{\left(2 \cos \frac{12A}{2} \cos \frac{6A}{2}\right) + \left(2 \cos \frac{12A}{2} \cos \frac{2A}{2}\right)}$$

$$= \frac{(2 \sin 6A \cos 3A) + (2 \sin 6A \cos A)}{(2 \cos 6A \cos 3A) + (2 \cos 6A \cos A)}$$

$$= \frac{2 \sin 6A (\cos 3A + \cos A)}{2 \cos 6A (\cos 3A + \cos A)}$$

$$= \frac{\sin 6A}{\cos 6A}$$

$$= \tan 6A$$

$$= \text{RHS}$$

2

1

1

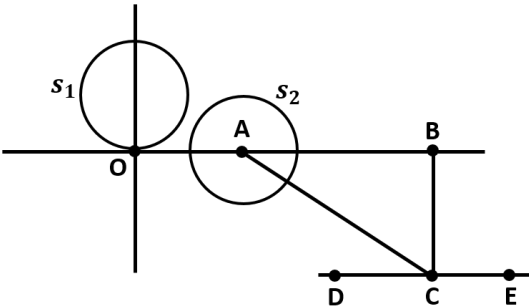
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2

1

1

	<p>Length of AB</p> $AB = \sqrt{(-5 - 1)^2 + (5 - 3)^2 + (2 - 0)^2}$ $= \sqrt{(-6)^2 + 2^2 + 2^2} = \sqrt{36 + 4 + 4} = \sqrt{44}$ <p>Length of CD</p> $CD = \sqrt{(-9 + 3)^2 + (-1 + 3)^2 + (2 - 0)^2}$ $= \sqrt{(-6)^2 + 2^2 + 2^2} = \sqrt{36 + 4 + 4} = \sqrt{44}$ <p>✓ Since $AB = CD$, one pair of opposite sides is equal.</p> <p>Length of BC</p> $BC = \sqrt{(-9 + 5)^2 + (-1 - 5)^2 + (2 - 2)^2}$ $= \sqrt{(-4)^2 + (-6)^2 + 0^2} = \sqrt{16 + 36} = \sqrt{52}$ <p>Length of DA</p> $DA = \sqrt{(-3 - 1)^2 + (-3 - 3)^2 + (0 - 0)^2}$ $= \sqrt{(-4)^2 + (-6)^2 + 0^2} = \sqrt{16 + 36} = \sqrt{52}$ <p>Length of AC</p> $AC = \sqrt{(-9 - 1)^2 + (-1 - 3)^2 + (2 - 0)^2}$ $= \sqrt{(-10)^2 + (-4)^2 + 2^2} = \sqrt{100 + 16 + 4} = \sqrt{120}$ <p>Length of BD</p> $BD = \sqrt{(-3 + 5)^2 + (-3 - 5)^2 + (0 - 2)^2}$ $= \sqrt{(2)^2 + (-8)^2 + (-2)^2} = \sqrt{4 + 64 + 4} = \sqrt{72}$ <p>$AC \neq BD$. Therefore the quadrilateral is not a rectangle. Therefore, it is a parallelogram and Bella wins the round.</p>	3		
35.	Find x and y, for which the complex numbers $-3 + ix^2y$ and $x^2 + y + 4i$ are conjugate for each other, Given $x, y \in R$.	2	U	10
	<p style="text-align: center;">SECTION E</p> <p><i>This section comprises of 3 case study/passage-based questions of 4 marks each.</i></p>			

36.	<p>Case Study 1: Two circular shaped celestial bodies s_1 and s_2, with same radius of 2 units, were spotted by an astronomer. s_1 had its center on y-axis, touching the x-axis at the origin, while s_2 was located with its centre on x-axis at point A, which is 4 units away from the origin. She observed that both were moving at the same uniform speed. While s_1 took 1 hour to move horizontally from O to A, it took twice the time to move from A to B. s_2 moved from A to C in 5 hours. AB is parallel to DE and perpendicular to BC. Answer the following questions, based on the given information.</p> 			
	(i) Find the equation of the circle s_2 when its centre is C.	2	AN	3
	<p>Answer: From the given information, $AB = 8$ and $AC = 10$. Therefore, $BC = 6$. The point C is $(12, -6)$. The required equation is $(x - 12)^2 + (y + 6)^2 = 4$</p>			
	(ii) If s_1 moved from B vertically to C with DE as its tangent at C, find its equation at its new position.	2	AN	3
	<p>Answer: At its new position, From the given information, The point C is $(12, 6)$. The y-coordinate of the centre is $-(6 - 2) = -4$. \therefore Its centre is at the point $(12, -4)$. The required equation is $(x - 12)^2 + (y + 4)^2 = 4$.</p>			
37.	<p>Case-Study 2: The probability that a girl, preparing for competitive examination will get a State Government service is 0.12; the probability that she will get a Central Government job is 0.25 and the probability that she will get both is 0.07. Based on the information, answer the following questions:</p>			
	<p>(i) What is the probability that she won't get the central government job?</p> <p>Answer: Let S be the event of getting State Government service and C be the event of getting Central Government job. $P(S') = 1 - 0.25 = 0.75$</p>	1	AP	3
	<p>(ii) Find the probability that she will get at least one of the two jobs.</p> <p>Answer: $P(\text{at least one of the two jobs}) = P(S \text{ or } C) = P(S \cup C)$</p>	1	AP	3

	$= P(S) + P(C) - P(S \cup C)$ $= 0.12 + 0.25 - 0.07 = 0.30$			
	<p>(iii) Find the probability that she will get only one of the two jobs.</p> <p>Solution:</p> $P(\text{Only one of the two jobs}) = P(\text{only } S \text{ or only } C)$ $= P(S \cap \bar{C}) + P(\bar{S} \cap C)$ $= \{P(S) - P(S \cap C)\} + \{P(C) - P(S \cap C)\}$ $= \{0.12 - 0.07\} + \{0.25 - 0.07\}$ $= 0.23.$	2	AN	4
38.	<p>Case Study 3: Riya and her five friends went on a tour to Simla. They stayed in a hotel. There were 4 vacant rooms – A, B, C and D. Out of these, A and B were double rooms which could accommodate two persons each, while C and D could accommodate only one person each. Based on the information, answer the following questions:</p>			
	<p>(i) Find the number of ways in which room A can be filled.</p> <p>Solution:</p> <p>Since A is a double room, the number of ways = ${}^6C_2 = 15$.</p>	1	AP	3
	<p>(ii) If both the rooms A and B are filled, in how many ways can room C be filled?</p> <p>Solution:</p> <p>Since C is a single room and there are two people remaining, the number of ways = ${}^2C_1 = 2$.</p>	1	AP	3
	<p>(iii) If Riya wanted to stay with a particular friend, find the total number of ways of accommodating the 6 of them.</p> <p>Solution:</p> <p>The remaining four can be accommodated in the following ways: ${}^4C_2 \times 2!$</p> <p>But Riya and her friend can be accommodated in A or B.</p> <p>\therefore the total number of ways = $2 \times {}^4C_2 \times 2! = 24$</p>	2	AP	4

Declaration: Certified that the question paper does not have questions which have been given in unit tests or other assessments.

Prepared by: A.Pradhap

Reliability by : Guru IIT

Grammar Checked by : Ms. Latha

Grammar Oversight

Signature of the Principal:

A handwritten signature in green ink, consisting of a stylized 'U' followed by a horizontal line.