Probability

Probability is nothing but chance (or) possibilities. Ex: India may be ~~win~~ win the toss (or) may not be. but we can't predict India will win the toss.

Experiment :- An operation which can produce some well defined outcomes is called experiment.

Ex: tossing a coin, playing a dies, playing cards etc.

Random experiment :- An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance is called a random experiment.

Ex: If we toss a coin the possibilities are Head (or) tail. we can't predict exact outcome

Sample space :- when we perform an experiment, then the set S of all possible outcomes is called sample space.

Ex: If toss a single coin, the possibilities are $S \to \{H, T\}$

If we toss a two coins the possibilities are $S \to \{HH, HT, TH, TT\}$

If we play a single dies, possibilities are $\{1, 2, 3, 4, 5, 6\}$

Event :- Any subset of a sample space is called an event.

Ex: 1) $S \to \{H, T\}$ $\Rightarrow E \subseteq S$.

$E = \{H\}$

2) If we through a dies. $S \to \{1, 2, 3, 4, 5, 6\}$

Event is prime number $\Rightarrow E = \{2, 3, 5\}$ which is subset of S.

Event is even number $\Rightarrow E\{2, 4, 6\} \subseteq S$.

(or)

Probability of getting even number

$P(E)$:- probability of event.

Ex: event is probability of getting head is $P(H)$

Formula: $P(E) = \dfrac{\text{No of favourable trial (or) case}}{\text{total no of possible trials (or) cases.}}$ (or) ~~P(E) = no favorable~~

Probability of getting + Probability of ~~getting~~ not getting = 1

& Probability of getting + Probability of not getting = 1 (or) 100% (or $p+q=1$)

ie $P(E) + P(\bar{E}) = 1$

$P(\text{tail}) = 100$

$P(\text{pass}) + P(\text{not pass}) = 100$

$70 + P(F) = 100$

$P(F) = 100 - 70 = 30\%$

2) Roll dies (single dice).

$P(2) = \frac{1}{6}$, then probability of not getting 2 is ?

$P(2) + P(\text{not getting } 2) = 1$

$\frac{1}{6} + P(\bar{2}) = 1$

$P(\bar{2}) = 1 - \frac{1}{6}$

$= \frac{5}{6}$

$\therefore P(2) + P(\bar{2}) = 1$

$\frac{1}{6} + \frac{5}{6} = 1$

$1 = 1$

And → multiplication

OR → addition.

Exhaustive event :- An event consisting of all the various possibilities is called exhaustive event.

Mutually exclusive event :- Two (or) more events are said to be mutually exclusive if the happening of one event prevent the simultaneous happening of the others.

Ex :- In tossing a coin, getting head & tail are mutually exclusive as getting head & tail is not possible inview of the fact that if head is the turn out getting tail is not possible.

Independent events :- Two (or) more events are said to be independent if the happening or nonhappening of one event does not prevent the happening or non happening of the others.

Ex : when two coins are tossed the event of getting head is an independent event as both the coins can turn out heads.

Note : 1) If A & B are two mutually exclusive events then

$P(A \text{ or } B) = P(A) + P(B)$

2) If A & B are independent events then $P(A \text{ and } B) = P(A) \cdot P(B)$

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Problems :-

1) The probability of getting head in tossing of coin

The possible outcomes are head & Tail.

No. of possible (exhaustive) cases $(n) = 2$.

No. of favourable cases $(m) = 1$

$\therefore$ Probability of getting head $(p) = \frac{m}{n} = \frac{1}{2}$.

2) The probability of getting (a) king (b) king (or) queen, when a card is drawn at random from a pack of 52 cards.

No. of possible (exhaustive) cases $(n) = 52$

(a) No of favourable cases $(m) = 4$

$\therefore$ Probability of getting king $(p) = \frac{m}{n} = \frac{4}{52} = \frac{1}{13}$

(b) No of favourable cases $(m) = 4+4 = 8$

$\therefore$ Probability of getting king (or) queen $(p) = \frac{m}{n} = \frac{8}{52} = \frac{2}{13}$ //

3) The probability of getting (a) number greater than 2 (b) an odd no. (c) an even number when a die is thrown.

Ans: No of possible cases/outcomes $(n) = 6$

(a) No of favourable outcomes $(m) = 4$ (Since 3, 4, 5, 6 are no's greater than 2)

$\therefore$ probability of getting a no greater than 2 $= \frac{4}{6} = \frac{2}{3}$.

(b) No of favourable outcomes $(m) = 3$ ($\because$ 1, 3, 5 are odd nos)

$\therefore$ probability of getting odd no $= \frac{3}{6} = \frac{1}{2}$

(c) no of favourable outcomes $(m) = 3$ (2, 4, 6 are even nos)

$\therefore$ probability of getting even no $= \frac{3}{6} = \frac{1}{2}$

Random variables :-

... is a real variable is associated

4) Probability of (a) getting a total more than 10 (b) getting a total less than 10 (c) getting a total equal to 10 when two dice are thrown ~~simult~~ simultaneously.

Sol: No of possible out comes when two dice are thrown simultaneously is given by $(n) = 6^2 = 36$

(a) Favourable outcomes are (5,6), (6,5) & (6,6).
That is $m = 3$

$\therefore$ required probability $(p) = \frac{3}{36} = \frac{1}{12}$

(b) Favourable outcomes are all the outcomes except those in the first case.
That is $(m) = 36 - 3 = 33$

$\therefore$ required probability $= \frac{33}{36} = \frac{11}{12}$

(c) Favourable outcomes are (4,6), (6,4) & (5,5).
That is $m = 3$

$\therefore$ required probability $= \frac{3}{36} = \frac{1}{12}$

Probability that a leap year will have 53 sundays.

5) Probability that a leap year there are 366 days when it comprises of

Sol: In a leap year there are 366 days when it comprises of 52 weeks & 2 days. These two days can be
(i) Sunday & Monday (ii) Monday & Tuesday (iii) Tue & wed (iv) wed & Thur (v) Thur & friday (vi) friday & sat (vii) Sat & Sunday

$\therefore$ No of possible outcomes $(n) = 7$

No of favourable cases $(m) = 2$ (i & vii)

$\therefore$ required probability $= \frac{2}{7}$

Random variables :-

In a random experiment, if a real variable is associated with every outcome then it is called a random variable (or) stochastic variable. In other words a random variable is a function that assigns a real number to every sample point in the sample space of a random experiment. Random variables are usually denoted by X, Y, Z, ... and it may be noted that different random variables may be associated with the same sample space S. The set of all real numbers of a random variable X is called the range of X.

(OR)

A random variable is a function which can take on any value from the sample space & having range of some set of real numbers is known as the random variable.

Ex:- Tossing a coin,

$S = \{H, T\}$ & the random variable X is ~~head then~~ number of head, then

$X = \{1, 0\}$

$\therefore$ Range of $X = \{0, 1\}$

Ex:- Two coin is tossed, then

$S = \{HH, HT, TH, TT\}$ & the random variable is X is Number of heads in the outcomes then

Outcome	HH	HT	TH	TT
Random variable X	2	1	1	0

$\therefore$ Range of $X = \{0, 1, 2\}$.

Suppose Y = Number of tails in the outcome, then

Random variable Y	0	1	1	2

Range of $Y = \{0, 1, 2\}$

Classification of Random Variables :

1) Discrete Random Variable
2) continuous Random Variable

1. Discrete Random Variable :-

If a random variable takes finite (or countably infinite no) number of values then it is called a discrete random variable. So that we can say that discrete random variables has distinct values that can be counted.

Ex:- 1) Tossing a coin & observing the outcome.
2) Tossing coins & observing the numbers.
3) Throwing a die and observing the numbers on the face.

2. Continuous Random variable :-

If a random variable takes non countable infinite number of values then it is called a continuous (or non discrete) Random variable. Many physical experiments can produce infinite no of outputs in a finite time of observation. In such cases we use continuous variables to define outputs of such systems.

Ex:- 1) Weight of articles.
2) Conducting a survey on the life of electric bulbs.
3) height of students, age etc.

Types of probability Distribution

① Discrete probability Distribution
② Continuous probability distribution.

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1. **Discrete probability Distribution** (or) **Probability mass function**

Definitions:

Let $x_1, x_2, \ldots x_n$ be the values of X and their probabilities are $P(x_i)$ then $P(X)$ is said to be probability mass function if

(i) $p(x_i)$ (or) $P(X) \geq 0$ (ii) $\sum_i p(x_i) = 1$ (or) $\sum P(X) = 1$. $\quad | \because p(x_i) = P(X)$.

The set of values $[x_i, p(x_i)]$ is called discrete probability distribution of the discrete random variable X. [The function $P(X)$ is called Probability ~~distribution~~ density function (pdf) (or) probability mass function (PMf)]

The distribution function $f(x)$ defined by

$f(x) = P(X \leq x) = \sum_{i=1}^{x} p(x_i)$, x is an integer is called the cumulative distribution function (cdf).

Mean and Variance of the discrete probability distribution:-

mean $(\mu) = \sum_i x_i \cdot p(x_i)$ (or) $\mu = \sum x P(x)$.

Variance $(V) = \sum_i x_i^2 p(x_i) - [\sum_i x_i p(x_i)]^2$ (or) $V = \sum x^2 P(x) - [\sum x P(x)]^2$

(or)

$V = \sum (x_i - \mu)^2 \cdot p(x_i)$.

Standard deviation $(\sigma) = \sqrt{V}$

Problems:-

1) A coin is tossed twice. A random variable X represents the number of heads turning up. Find the discrete probabity distribution for X. Also find its mean and variance

Soln: Random experiment: tossing a coin twice

sample space: $S = \{HH, HT, TH, TT\}$, $n = 4$.

Random variable X: no. of Heads turning up.

$X = 0, 1, 2$

Probability Distribution:

$X = x_i$	0	1	2
$P(X = x_i)$	$\frac{1}{4}$	$\frac{2}{4} = \frac{1}{2}$	$\frac{1}{4}$

0 means ~~no~~ head turn up is equal to 1. (TT).

$\therefore \sum P(x) = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1.$

we observe that $p(x_i) > 0$, $\sum P(x) = 1.$

$\therefore$ mean $= \mu = \sum x \cdot p(x) = 0 + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4}$

$= 0 + \frac{1}{2} + \frac{1}{2}$

$\mu = 1$

Variance $= V = \sum (x_i - \mu)^2 p(x_i)$

$= (0-1)^2 \cdot \frac{1}{4} + (1-1)^2 \cdot \frac{1}{2} + (2-1)^2 \cdot \frac{1}{4}$

$= \frac{1}{4} + 0 + \frac{1}{4}$

$V = \frac{1}{2}$

$\therefore$ mean $= 1$, variance $= \frac{1}{2}$.

2. Show that the following distribution represents a discrete probability distribution. Find mean & variance.

x	10	20	30	40
$p(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Soln: we observe that $p(x) > 0$ & $\sum P(x) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{8}{8}$

$\sum P(x) = 1.$

mean $= \mu = \sum p(x) \cdot x = \frac{10}{8} + \frac{3 \times 20}{8} + \frac{3}{8} \times 30 + \frac{1}{8} \times 40$

$= \frac{10 + 60 + 90 + 40}{8}$

$\boxed{\mu = 25}$

Variance $= V = \sum (x - \mu)^2 p(x)$

$= (10-25)^2 \frac{1}{8} + (20-25)^2 \frac{3}{8} + (30-25)^2 \cdot \frac{3}{8} + (40-25)^2 \frac{1}{8}$

$= \frac{225 + 25 + 25 + 225}{8} = \frac{600}{8}$

$\boxed{V = 75}$

3) Find the value of k such that the following distribution represents a finite probability distribution. Hence find its mean and standard deviation. Also find $p(x \le 1)$, $P(x>1)$ & $P(-1<x\le 2)$

x	-3	-2	-1	0	1	2	3
$p(x)$	k	2k	3k	4k	3k	2k	k

Sol^n: Given distribution represent probability distribution hence it must satisfies the conditions

$$p(x) \ge 0 \quad \& \quad \Sigma P(x) = 1.$$

& by the condition $\Sigma P(x) = 1$

$$\Rightarrow k + 2k + 3k + 4k + 3k + 2k + k = 1$$
$$16k = 1$$
$$k = \frac{1}{16}$$

∴ The discrete / finite distribution probability is

x	-3	-2	-1	0	1	2	3
$p(x)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

Mean $= \mu = \Sigma x p(x) = \frac{-3}{16} + -2 \cdot \frac{2}{16} + (-1)\frac{3}{16} + 0 \cdot \frac{4}{16} + 1 \cdot \frac{3}{16} + 2 \cdot \frac{2}{16} + 3 \cdot \frac{1}{16}$

$$= \frac{1}{16}\left[\cancel{-3} - \cancel{4} - \cancel{3} + 0 + \cancel{3} + \cancel{4} + \cancel{3}\right]$$

$$\mu = 0$$

Variance $(V) = \Sigma (x-\mu)^2 \cdot p(x)$

$$= (-3-0)^2 \cdot \frac{1}{16} + (-2-0)^2 \frac{2}{16} + (-1-0)^2 \frac{3}{16} + (0-0)^2 \frac{4}{16} + \frac{(1-0)^2 3}{16} + \frac{(2-0)^2 2}{16} + (3-0)^2 \frac{1}{16}$$

$$= \frac{1}{16}\left[9 + 8 + 3 + 0 + 3 + 8 + 9\right] = \frac{40}{16} = \frac{5}{2}.$$

$\boxed{V = \frac{5}{2}}$ & standard deviation $(\sigma) = \sqrt{5/2}$

Now, $P(x \le 1) = p(-3) + p(-2) + P(-1) + p(0) + p(1) = \frac{1}{16} + \frac{2}{16} + \frac{3}{16} + \frac{4}{16} + \frac{3}{16} = \frac{13}{16}$

$P(x>1) = P(2) + P(3) = \frac{2}{16} + \frac{1}{16} = \frac{3}{16}$

$P(-1 < x \le 2) = p(0) + p(1) + p(2) = \frac{4}{16} + \frac{3}{16} + \frac{2}{16} = \frac{9}{16}$ //

4). The pdf of a variable X is given by the following table

x	0	1	2	3	4	5	6
$P(x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

For what value of k, this represents a valid probability distribution? Also find $P(x \geq 5)$, and $P(3 < x \leq 6)$.

Soln:- The probability distribution is valid if $P(x) \geq 0$ & $\sum P(x) = 1$.

we must have $k \geq 0$ & $k + 3k + 5k + 7k + 9k + 11k + 13k = 1$

$$\Rightarrow k = \frac{1}{49}.$$

Also $P(x \geq 5) = P(5) + P(6) = 11k + 13k$

$$= 24k$$
$$= \frac{24}{49}.$$

$$P(3 < x \leq 6) = P(4) + P(5) + P(6)$$
$$= 33k$$
$$= \frac{33}{49}.$$

5). The probability distribution of a finite random variable X is given by the following table.

x_i	-2	-1	0	1	2	3
$P(x_i)$	0.1	k	0.2	$2k$	0.3	k

find the value of k, mean and variance.

Soln: we must have $p(x_i) \geq 0$ & $\sum P(x_i) = 1$. for a probability distribution

$$\Rightarrow \sum P(x_i) = 1$$

$$0.1 + k + 0.2 + 2k + 0.3 + k = 1$$
$$4k + 0.6 = 1$$
$$4k = 1 - 0.6 = 0.4$$
$$k = 0.1$$

mean$(\mu) = \sum x_i P(x_i) = 0.1(-2) + (-1)(0.1) + 0 + 1.2(0.1) + 2(0.3) + 3(0.1)$

$$\boxed{\mu = 0.8}$$

Variance $(V) = \sum (x - \mu)^2 P(x)$

$$= (-2 - 0.8)^2 \cdot 0.1 + (-1 - 0.8)^2 (0.1) + (0 - 0.8)^2 (0.2) + (1 - 0.8)^2 2(0.1) + (2 - 0.8)^2 (0.3) + (3 - 0.8)^2 (0.1)$$

$$\boxed{V = 2.16}$$

6) A random variable X has the following probability function for various values of x

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Find k (ii) Evaluate $P(x<6)$, $P(x\geq 6)$ & $P(3<x\leq 6)$. Also find the probability distribution and the distribution function of X.

Soln: We must have $P(x)\geq 0$, $\Sigma P(x)=1$.

& $P(x)=1$

$0+k+2k+2k+3k+3k^2+2k^2+(7k^2+k)=1$

$10k^2+9k-1=0$

$(10k-1)(k+1)=0$

$k=1/10$ & $k=-1$

If $k=-1$, the first condition fails & hence $k\neq -1$

$\therefore k=1/10$

∴ Probability distribution is as follows.

x	0	1	2	3	4	5	6	7
$P(x)$	0	$\frac{1}{10}$	$1/5$	$1/5$	$3/10$	$1/100$	$1/50$	$17/100$
		0.1	0.2	0.2	0.3	0.01	0.02	0.17

$P(x<6)=P(0)+P(1)+P(2)+P(3)+P(4)+P(5)$

$=0+1/10+1/5+1/5+3/10+1/100$

$=81/100=0.81$.

$P(x\geq 6)=P(6)+P(7)$

$=1/50+17/100$

$=19/100=0.19$.

$P(3<x\leq 6)=P(4)+P(5)+P(6)$

$=3/10+1/100+1/50=\frac{33}{100}$

$=0.33$

The distribution function of X is $F(x)=P(X\leq x)=\sum_{i=1}^{x}P(x_i)$ is also called cumulative distribution function & the same is as follows

x	0	1	2	3	4	5	6	7
$F(x)$	0	$0+0.1=0.1$	$0.1+0.2=0.3$	$0.3+0.2=0.5$	$0.5+0.3=0.8$	$0.8+0.01=0.81$	$0.81+0.02=0.83$	$0.83+0.17=1$

7) A random variable X take the values $-3, -2, -1, 0, 1, 2, 3$ such that $P(X=0) = P(X<0)$ and $P(X=-3) = P(X=-2) = P(X=-1) = P(X=1) = P(X=2) = P(X=3)$. Find the probability distribution.

Sol:- Let the distribution $[X, P(X)]$ be as follows.

X	-3	-2	-1	0	1	2	3
$P(X)$	P_1	P_2	P_3	P_4	P_5	P_6	P_7

By date $P(X=0) = P(X<0)$

ie $P(X=0) = P(X=-1) + P(X=-2) + P(X=-3)$.

$$P_4 = P_3 + P_2 + P_1 \longrightarrow (1)$$

Also by date, (2nd condition), $P(X=-3) = P(X=-2) = P(X=-1) = P(X=1) = P(X=2) = P(X=3)$

ie ~~$P_1+P_2+P_3+P$~~ $P_1 = P_2 = P_3 = P_5 = P_6 = P_7 \longrightarrow (2)$

Further we must have,

$$P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 = 1 \longrightarrow (3)$$

use (2) in (1) we get

$$P_4 = 3P_1$$

use (2) in (3) we get

$6P_1 + P_4 = 1$, but $P_4 = 3P_1$

$9P_1 = 1$

$$\boxed{P_1 = 1/9}$$

& $P_4 = 3P_1$

$= 3 \cdot 1/9$

$$\boxed{P_4 = 1/3}$$

Thus probability distribution is as follows

X	-3	-2	-1	0	1	2	3
$P(X)$	$1/9$	$1/9$	$1/9$	$1/3$	$1/9$	$1/9$	$1/9$

Binomial Distribution :-

If p is the probability of success and q is the probability of failure, the probability of x success out of n trials is given by

$$p(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

$n \to$ no. of repeated trial
$p \to$ probability of success
$q \to$ probability of failure

For binomial distribution, mean (μ) = np

Variance (V) = npq

Standard deviation (σ) = $\sqrt{V} = \sqrt{npq}$

Poisson Distribution :-

Poisson distribution is regarded as the limiting form ~~of~~ the binomial distribution when n is very large ($n \to \infty$) and p is the Probability of success is very small ($p \to 0$) so that np tends to a fixed finite constant say m. and it is given by :

$$P(x) = \frac{m^{x} e^{-m}}{x!}$$ is called Poisson distribution of the random variable

where $m = np$.

$p(x)$ = poisson probability function

x = poisson variate.

For Poisson distribution, mean (μ) = m

Variance (V) = m

Standard deviation (σ) = $\sqrt{V} = \sqrt{m}$

Note :-

$${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

$${}^{n}P_{r} = \frac{n!}{(n-r)!} \quad (\text{or}) \quad {}^{n}P_{r} = {}^{n}C_{n-r}$$

$${}^{n}C_{0} = 1$$

$${}^{n}C_{n} = 1$$

$${}^{n}C_{1} = n$$

Problems

1) Find the binomial probability distribution which has mean 2 and Variance $4/3$.

Sol^n: WKT binomial distribution mean $= np$ & variance $= npq$

Given $np = 2$, $npq = 4/3$

$\Rightarrow 2q = 4/3$

$\boxed{q = 2/3}$

$\Rightarrow p + q = 1$

$p = 1 - q$

$= 1 - 2/3$

$\boxed{p = 1/3}$

Since $np = 2 \Rightarrow n = \frac{2}{p} = \frac{2}{1/3}$

$\boxed{n = 6}$

$\therefore$ The binomial distribution function $P(x) = {}^nC_x\, p^x\, q^{n-x}$ becomes

$P(x) = {}^6C_x \left(\frac{1}{3}\right)^x (2/3)^{6-x}$

$\therefore {}^6C_0 = 1$, ${}^6C_6 = 1$.

$\therefore$ The distribution of probability is as follows:

x	0	1	2	3	4	5	6
$P(x)$	${}^6C_0 (2/3)^6 = (2/3)^6$	${}^6C_1 (1/3)(2/3)^5$	${}^6C_2 (1/3)^2 (2/3)^4$	${}^6C_3 (1/3)^3 (2/3)^3$	${}^6C_4 (1/3)^4 (2/3)^2$	${}^6C_5 (1/3)^5 (2/3)$	${}^6C_6 (1/3)^6 \cdot 1 = (1/3)^6$

2) When a coin is tossed 4 times, find the probability of getting (i) Exactly one head (ii) atmost 3 heads (iii) atleast 2 heads.

Sol^n: $p = P(H) = 1/2 = 0.5$, $q = 0.5$, $n = 4$ $\quad | \; p + q = 1$.

WKT the probability of x successes out of n trials is given by

$P(x) = {}^nC_x\, p^x q^{n-x}$

(i) $P(1 \text{ head}) = P(x=1) = {}^4C_1 (0.5)^1 (0.5)^{4-1} = {}^4C_1 (0.5)(0.5)^3 = 0.25$

(ii) $P[\text{at most 3 heads (ie} \le 3)] = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= {}^4C_0 (0.5)^0 (0.5)^4 + {}^4C_1 (0.5)^1 (0.5)^3 + {}^4C_2 (0.5)^2 (0.5)^2 + {}^4C_3 (0.5)(0.5)^3$

$= (0.5)^4 (1 + 4 + 6 + 4) = 0.9375$.

(iii) $P(\text{atleast 2 heads}) = 1 - [P(x=0) + P(x=1)]$

$= 1 [(0.5)^4 + {}^4C_1 (0.5)(0.5)^3] = 0.6875$

6. 3) The probability that a pen manufactured by a factory be defective is $\frac{1}{10}$. If 12 such pens are manufactured, what is the Probability that (i) exactly 2 are defective (ii) atleast 2 are defective (iii) none of them are defective.

Soln: Total no. of pens $= n = 12$.

Probability of a defective pen, $p = \frac{1}{10} = 0.1$

Probability of non defective pen, $q = 1-p = 1-0.1 = 0.9$

& WKT $P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

(i) Probability of exactly two defectives,

$$P(x=2) = P(2) = {}^{12}C_2 (0.1)^2 (0.9)^{12-2}$$

$$= \frac{12 \times 11}{1 \times 2} (0.1)^2 (0.9)^{10}$$

$$P(2) = 0.2301.$$

(ii) Probability of atleast two defectives

$$P(2)+P(3)+\cdots+P(12) = 1-[P(0)+P(1)]$$

$$= 1-\left[{}^{12}C_0 (0.1)^0 (0.9)^{12} + {}^{12}C_1 (0.1)^1 (0.9)^{11}\right]$$

$$= 1-[0.2824 + 12 (0.1)^1 (0.9)^{11}]$$

$$= 1-[0.2824 + 0.3766]$$

$$= 0.341.$$

(iii) Probability of no defective

$$P(0) = {}^{12}C_0 (0.1)^0 (0.9)^{12}$$

$$= 1 \times 1 \times (0.9)^{12}$$

$$= 0.2824$$

4) The probability that a person aged 60 years will live upto 70 is 0.65. What is the probability that out of 10 persons aged 60 atleast 7 of them will live upto 70.

Soln: Let x be the number of persons aged 60 years living upto 70 years. For this variate we have by data

$p = 0.65$, $q = 1 - 0.65$
$q = 0.35$

Consider $P(x) = {}^nC_x p^x q^{n-x}$, $n = 10$

We have to find $P(x \geq 7)$.

ie $P(x \geq 7) = P(7) + P(8) + P(9) + P(10)$ (or) $P(x \geq 7) = 1 - [P(0)+P(1)+P(2)+P(3)+P(4)+P(5)+P(6)]$

$= {}^{10}C_7 (0.65)^7 (0.35)^3 + {}^{10}C_8 (0.65)^8 (0.35)^2 + {}^{10}C_9 (0.65)^9 (0.35) + {}^{10}C_{10} (0.65)^{10} (0.35)^0$

${}^{10}C_7 = {}^{10}C_3 = \frac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3} = 120$, ${}^{10}C_8 = {}^{10}C_2 = \frac{10 \times 9}{1 \times 2} = 45$, ${}^{10}C_9 = {}^{10}C_1 = 10$

$P(x \geq 7) = 0.5138$

5) The number of telephone lines busy at an instant of time is a binomial variate with probability 0.1 that a line is busy. If 10 lines are chosen at random, what is the probability that (i) no line is busy (ii) all lines are busy (iii) atleast one line is busy (iv) atmost 2 lines are busy.

Soln: Let x denote the number of telephone lines busy. ∴ By data

$p = 0.1$, $q = 1 - 0.1 = 0.9$, $n = 10$

$P(x) = {}^nC_x p^x q^{n-x} = {}^{10}C_x (0.1)^x (0.9)^{10-x}$

(i) Probability that no line is busy $= P(0) = {}^{10}C_0 (0.1)^0 (0.9)^{10} = 0.3487$.

(ii) Probability that all lines are busy $= P(10) = {}^{10}C_{10} (0.1)^{10} (0.9)^0 = (0.1)^{10}$

(iii) Probability that atleast one line is busy

ie $P(x \geq 1) = 1 -$ Probability of no line is busy

$= 1 - P(0) = 1 - 0.3487$

$= 0.6513$.

(iv) Probability that atmost two lines are busy

ie $P(x \leq 2) = P(0) + P(1) + P(2)$

$= {}^{10}C_0 (0.1)^0 (0.9)^{10} + {}^{10}C_1 (0.1)^1 (0.9)^9 + {}^{10}C_2 (0.1)^2 (0.9)^8$

$= 0.9298$ //

Thus the number of ...

6) In a quiz contest of answering 'Yes' or 'No' what is the probability of guessing atleast 6 answers correctly out of 10 questions asked? Also find the probability of the same if there are 4 options for a correct answer?

Soln: Let x denote the correct answer and we have in the first case

$p = \frac{1}{2}, \; q = \frac{1}{2}, \; n = 10$

$P(x) = {}^{n}C_x p^x q^{n-x} = {}^{10}C_x (1/2)^x (1/2)^{10-x} = {}^{10}C_x (1/2)^{10}$

We have to find $P(x \geq 6)$

$$P(x \geq 6) = \frac{1}{2^{10}}\left[{}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right]$$

$$= \frac{1}{2^{10}}\left[210 + 120 + 45 + 10 + 1\right] = \frac{386}{1024}$$

$P(x \geq 6) = 0.377$

In second case when there are 4 options

$p = 1/4, \; q = 3/4, \; n = 10$

$$P(x) = {}^{10}C_x (1/4)^x (3/4)^{10-x} = \frac{1}{4^{10}}\left[3^{10-x} \cdot {}^{10}C_x\right]$$

$$\therefore P(x \geq 6) = P(6) + P(7) + P(8) + P(9) + P(10)$$

$$= \frac{1}{4^{10}}\left[3^4 \cdot {}^{10}C_6 + 3^3 \, {}^{10}C_7 + 3^2 \cdot {}^{10}C_8 + 3^1 \, {}^{10}C_9 + 3^0 \, {}^{10}C_{10}\right]$$

$$= \frac{1}{4^{10}}\left[81 \times 210 + 27 \times 120 + 9 \times 45 + 3 \times 10 + 1\right]$$

$P(x \geq 6) = 0.019$

7) In sampling a large number of parts manufactured by a company the mean number of defectives in samples of 20 is 2. Out of 1000 such samples how many would be expected to contain atleast 3 defective parts.

Soln: mean $(\mu) = np = 2$, $\quad n = 20$.

$\Rightarrow 20p = 2 \Rightarrow p = 0.1, \quad \therefore q = 1 - p = 0.9$

Let x denote the defective part.

$P(x) = {}^{n}C_x p^x q^{n-x} = {}^{20}C_x (0.1)^x (0.9)^{20-x}$

Probability of atleast 3 defective parts $P(x \geq 3) = P(3) + P(4) + \dots + P(20)$

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 - [(0.9)^{20} + {}^{20}C_1 (0.1)(0.9)^{19} + {}^{20}C_2 (0.1)^2 (0.9)^{18}]$

$= 0.323$

Thus the number of defectives in 1000 samples is $1000 \times 0.323 = 323$ //

8) In 800 families with 5 children each how many families would be expected to have (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys (iv) atmost 2 girls by assuming probabilities for boys and girls to be equal.

Soln :- p = probability of having a boy $= 1/2$

q = probability of having a girl $= 1/2$

let x denote the number of boys in the family

$$P(x) = {}^nC_x\, p^x q^{n-x} \quad \text{where } n=5$$

$$P(x) = {}^5C_x (1/2)^x (1/2)^{5-x} = \frac{1}{2^5}\, {}^5C_x = \frac{{}^5C_x}{32}$$

Since we have to find the expected number in respect of 800 families we have

$$800\, P(x) = 800\, \frac{{}^5C_x}{32} = 25\, {}^5C_x = f(x),\ (\text{say})$$

(i) we have to find $f(3)$.

$$f(3) = 25 \cdot {}^5C_3 = 25 \times 10 = 250.$$

Expected no. of families with 3 boys is 250.

(ii) we have to find $f(0)$.

$$f(0) = 25 \cdot {}^5C_0 = 25 \times 1 = 25.$$

Expected no. of families with 5 girls is 25.

(iii) we have to find $f(2) + f(3)$.

$$f(2) + f(3) = 25 \cdot {}^5C_2 + 25 \cdot {}^5C_3 =$$
$$= 25\, {}^5C_2 + 25 \cdot {}^5C_2 = 50\, {}^5C_2 = 50 \times 10 = 500.$$

Expected no. of families with 2 or 3 boys is 500.

(iv) At most 2 girls means that, families can have 5 boys and 0 girls or 4 boys and 1 girl or 3 boys and 2 girls.

we have to find $f(5) + f(4) + f(3) = 25\, {}^5C_5 + 25 \cdot {}^5C_4 + 25 \cdot {}^5C_3$

$$= 25(1+5+10) = 25 \times 16$$
$$= 400.$$

Expected no. of families with atmost 2 girls is 400.

9) A lot contains 1% of defective items. What should be the number (n) of items in a random sample so that the probability of finding atleast one defective in it is atleast 0.75 ?

Solⁿ: let p be the probability of a defective item.

By data, $p = 1\% = 0.01$, $\therefore q = 0.99$.

If x denotes a defective item

$P(x) = {}^nC_x p^x q^{n-x} = {}^nC_x (0.01)^x (0.99)^{n-x}$

We need to find n such that the probability of finding atleast one defective is ≥ 0.75. That is to find n such that

$P(x \geq 1) \geq 0.75$

$1 - P(x < 1) \geq 0.75$

$1 - P(0) \geq 0.75$

$1 - (0.99)^n \geq 0.75$

(or) $0.25 \geq (0.99)^n$

Equivalently we have $\log(0.25) \geq n \log(0.99)$

$$n \leq \frac{\log(0.25)}{\log(0.99)} = 137.935$$

Hence required n is 138.

10) The probability of a shooter hitting a target is 1/3. How many times he should shoot so that the probability of hitting the target atleast once is more than 3/4.

Solⁿ. Let p = probability of hitting a target = 1/3, $\therefore q = 2/3$

$P(x) = {}^nC_x p^x q^{n-x} = {}^nC_x (1/3)^x (2/3)^{n-x}$

We have to find n such that

$P(x \geq 1) > 3/4$

$1 - P(x < 1) > 3/4$ or $1 - P(0) > 3/4$

$1 - (2/3)^n > 3/4$ or $(2/3)^n < 1/4$ ie $(2/3)^n < 0.25$

We can find n by inspection as we have

$2/3 \approx 0.67$, $(2/3)^2 \approx 0.44$, $(2/3)^3 \approx 0.3$, $(2/3)^4 \approx 0.2$

Hence required n is 4.

Problems on Poisson distribution:

1) Fit a poisson distribution for the following data and calculate the theoritical frequencies:

x	0	1	2	3	4
y	122	60	15	2	1

Sol: First we shall compute the mean (μ) for the given distribution

$$\mu = \frac{\Sigma fx}{\Sigma f} = \frac{0 + 60 + 30 + 6 + 4}{200} = 0.5$$

ie mean (μ) = m for the Poisson distribution

$\therefore$ The Poisson distribution is

$$P(x) = \frac{m^x e^{-m}}{x!},$$

& $f(x) = 200 \cdot P(x)$.

$$f(x) = 200 \cdot \frac{(0.5)^x e^{-0.5}}{x!}.$$

$$= 121.3 \frac{(0.5)^x}{x!}$$

Putting $x = 0, 1, 2, 3, 4$ in $f(x)$ we obtain the theoritical frequencies. They are as follows

$121.3 \frac{(0.5)^0}{0!} = 121$, $121.3 \frac{(0.5)^1}{1!} = 61$, $121.3 \frac{(0.5)^2}{2!} = 15$,

$121.3 \frac{(0.5)^3}{3!} = 3$, $121.3 \frac{(0.5)^4}{4!} = 0$.

$\therefore$ required theoritical frequencies are 121, 61, 15, 3, 0.

2) In a certain factory turning out razor blades there is a small probability of 1/500 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective (ii) one defective (iii) two defective blades in a consignment of 10,000 packets.

Sol: p = probability of a defective blade $= \frac{1}{500} = 0.002$.

In a packet of 10, the mean number of defective blade is

$$m = np = 10 \times 0.002 = 0.02$$

Poisson distribution is $P(x) = \frac{m^x e^{-m}}{x!} = \frac{e^{-0.02}(0.02)^x}{x!}$

$$f(x) = \frac{9802\,(0.02)^x}{x!}$$

(i) Probability of no defective $= f(0) = 9802$

(ii) Probability of one defective $= f(1) = 9802\,(0.02) = 196$

(iii) Probability of two defective $= f(2) = \frac{9802\,(0.02)^2}{2!} = 2.$

3) The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean 3. Out of 1000 taxi drivers find approximately the number of the drivers with
(i) no accident in a year
(ii) more than 3 accident in a year

soln: By data, mean $(\mu) = 3$, For Poisson distribution $\mu = m = 3$.

Poisson distribution is given by $P(x) = \frac{m^x e^{-m}}{x!} = \frac{3^x e^{-3}}{x!}$

let $f(x) = 1000\,p(x)$

$f(x) = 1000 \cdot \frac{3^x \cdot e^{-3}}{x!} = 50 \cdot \frac{3^x}{x!}$, $\left|\because e^{-3} = 0.05\right.$

(i) No. of drivers with no accident in a year $= f(0) = 50 \cdot \frac{3^0}{0!} = 50.$

(ii) Probability of more than 3 accident in a year $= 1 - P(X \leq 3)$

$$= 1 - [P(0)+P(1)+P(2)+P(3)]$$

$$= 1 - \left[e^{-3}\left(\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!}\right)\right]$$

$$= 1 - \left(0.05\,(1+3+4.5+4.5)\right)$$

$$= 0.35$$

$\therefore$ no of drivers out of 1000 with more than 3 accidents in a year is $f(x) = 1000\,(0.35)$

$= 350$

4) 2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains (i) no defective fuses (ii) 3 or more defective fuses.

Soln: Probability of a defective fuse $= p = 2/100 = 0.02$

$\therefore$ mean number of defectives $\mu = m = np = 200 \times 0.02 = 4$

$\therefore$ Poisson distribution is given by $P(x) = \dfrac{m^x e^{-m}}{x!}$

$$P(x) = \frac{4^x e^{-4}}{x!}$$

$$= 0.0813 \cdot \frac{4^x}{x!}$$

(i) Probability of no defective fuse $= P(0) = 0.0183$.

(ii) Probability of 3 or more defective fuses $= 1 - [P(0) + P(1) + P(2)]$

$$= 1 - 0.0183\left[1 + \frac{4^1}{1!} + \frac{4^2}{2!}\right]$$

$$= 1 - 0.0183(1+4+8)$$

$$= 0.7621.$$

5) If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get a bad reaction.

Soln:- As the probability of occurance (bad reaction) is very small this follows Poisson distribution and we have

$$P(x) = \frac{m^x e^{-m}}{x!}$$

mean $= m = np = 2000 \times 0.001 = 2$.

we have to find $P(x > 2)$.

$$P(x>2) = 1 - P(x \le 2)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - e^{-m}\left[1 + \frac{m}{1!} + \frac{m^2}{2!}\right]$$

$$P(x>2) = 1 - e^{-2}[1+2+2] = 1 - 5e^{-2}$$

$$= 0.3222.$$

6) A communication channel receives independent pulses at the rate of 12 pulses per micro second. The probability of transmission error is 0.001 for each micro second. Compute the probabilities of (i) no error during a micro second (ii) one error per micro second (iii) atleast one error per micro second (iv) two errors (v) atmost two errors.

Sol:- Given, $n = 12$, $p = 0.001$.

$\therefore \mu = np = m = 12 \times 0.001 = 0.012$

The Poisson distribution is $P(x) = \frac{m^x e^{-m}}{x!}$

$$P(x) = e^{-0.012} \frac{(0.012)^x}{x!}$$

(i) no error $= P(0) = 0.988072$

(ii) one error $= P(1) = e^{-0.012} \frac{(0.012)^1}{1} = 0.01193.$

(iii) Probability of atleast one error $= 1 - P(0)$

$$= 1 - e^{-0.012} \frac{(0.012)^0}{0!} = 0.01193.$$

(iv) two errors $= P(2) = e^{-0.012} \frac{(0.012)^2}{2!} = 0.000071.$

(v) atmost two errors $= P(0) + P(1) + P(2)$

$$= e^{-0.012} \left[1 + 0.012 + \frac{(0.012)^2}{2!} \right]$$

$$= 0.99999714$$

$$\approx 1.$$

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