t - Distribution

If $x_1, x_2, \ldots, x_n$ is a random sample size n from a normal population with mean μ and sample standard deviation S, the t- Distribution is defined as,

$$t = \frac{\bar{x} - \mu}{S} \sqrt{n}$$

where $\bar{x}$ is sample mean
μ is population mean.
n is sample size & S is SD.

& in t -distribution DOF is $(n-1)$. ie $(\nu = n-1)$.

Confidence limits for μ :-

For 95 % confidence limits (level of significance 5%)

are $\bar{x} \pm \frac{S}{\sqrt{n}} t_{(0.05)}$.

For 99 % confidence limits (level of significance 1%)

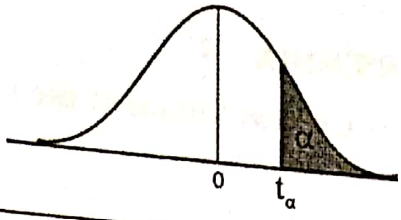
are $\bar{x} \pm \frac{S}{\sqrt{n}} t_{(0.01)}$

Note:

1) $\bar{x} = \frac{\sum x_i}{n}$

2) $S^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$

t_α – Critical Values of the t –Distribution



v	α						
	0.40	0.30	0.20	0.15	0.01	0.025	0.05
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Problems

1) Find the student's t. for the following variable values in a sample of eight -4, -2, -2, 0, 2, 2, 3, 3 taking the mean of the universe to be zero.

Sol^n. Given $\mu = 0$, $n = 8$, $(x_1, x_2, \dots, x_8) = (-4, -2, -2, 0, 2, 2, 3, 3)$

we have to find $t = \frac{\bar{x} - \mu}{s}\sqrt{n}$

$$\bar{x} = \frac{\Sigma f}{n} = \frac{-4-2-2+0+2+2+3+3}{8} = \frac{1}{4}$$

$$\bar{x} = 0.25$$

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

$$= \frac{1}{(8-1)}\left\{(-4-0.25)^2 + (-2-0.25)^2 + (-2-0.25)^2 + (0-0.25)^2 + (2-0.25)^2 + (2-0.25)^2 + (3-0.25)^2 + (3-0.25)^2\right\}$$

$$= \frac{1}{7}(49.5)$$

$$s^2 = 7.07$$

$$\therefore s = 2.66$$

$$\therefore t = \frac{0.25 - 0}{2.66}\sqrt{8}$$

$$t = 0.266$$

2) Consider the samples consisting of 9 no. 45, 47, 50, 52, 48, 47, 49, 53, 51. The sample is drawn from the population mean is 47.5 (μ). Find whether the sample mean differs significantly from the population mean at 5% level of significance?

Sol^n: $\mu = 47.5$, $\bar{x} = \frac{\Sigma x}{n} = \frac{442}{9} = 49.11$, $\nu = n-1 = 8$, $n = 9$.

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n} = \frac{1}{9}(54.88) = 6.098, \quad s = 2.4694.$$

$$t = \frac{\bar{x} - \mu}{s}\sqrt{n} = \frac{49.11 - 47.5}{2.4694}\sqrt{9} = 1.956$$

but $t_{0.05}(8) = 2.31$. $\therefore 1.956 < 2.31$ $\therefore$ Hypothesis is accepted.

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2) A certain stimulus administered to each of 12 patients resulted in the following change in blood pressure, 5, 2, 8, −1, 3, 0, 6, −2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure? ($t_{0.05}$ for 11 df = 2.201)

Sol:- Given $\{x_1, x_2, \ldots, x_{12}\} = \{5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4\}$.

$n = 12$.

$$\bar{x} = \frac{\sum x}{n} = \frac{5+2+8-1+3+0+6-2+1+5+0+4}{12}$$

$$= 2.5833$$

$$\bar{x} \approx 2.58$$

$$s^2 = \frac{1}{n-1}\sum(x-\bar{x})^2$$

$$= \frac{1}{11}\Big\{(5-2.58)^2+(2-2.58)^2+(8-2.58)^2+(-1-2.58)^2+(3-2.58)^2 + (0-2.58)^2+(6-2.58)^2+(-2-2.58)^2+(1-2.58)^2+(5-2.58)^2 + (0-2.58)^2+(4-2.58)^2\Big\}$$

$$s^2 = 9.538$$

$$\therefore \quad s = 3.088$$

Hence $\quad t = \dfrac{\bar{x}-\mu}{s}\sqrt{n}$

Let us suppose that the stimulus administration is not accompanied with increase in blood pressure, we can take $\mu = 0$.

$$\therefore \quad t = \frac{2.5833 - 0}{3.088}\sqrt{12}$$

$$t = 2.8979$$

& DOF $= n-1 = 12-1 = 11$

i.e $\nu = 11$.

at 5% level for Dof 11 $t = 2.201$.

$\therefore \quad 2.8979 \nless 2.201$.

Hence the Hypothesis is rejected at 5% level of significance. We conclude ~~that~~ with 95% confidence that the stimulus in general is accompanied with increase in blood pressure.

3) Ten individuals are chosen at random from a population & their hights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71, Test the hypothesis that the mean height of the universe is 66 inches. ($t_{0.05} = 2.262$ for 9 d.f).

Soln: Given $\mu = 66$, $n = 10$.

$$\bar{x} = \frac{\Sigma x}{n} = \frac{678}{10} = 67.8$$

$$s^2 = \frac{1}{n-1} \Sigma (x - \bar{x})^2$$

$$= \frac{1}{9} [(63 - 67.8)^2 + \dots + (71 - 67.8)^2]$$

$$s^2 = 9.067$$

$$\Rightarrow s = 3.011$$

$$\therefore t = \frac{\bar{x} - \mu}{s} \sqrt{n} = \frac{67.8 - 66}{3.011} \sqrt{10} = 1.89$$

$$\therefore t = 1.89 < 2.262$$

Thus the hypothesis is accepted at 5% level of significance.

4) A machine is expected to produce nails of length 3 inches. A random sample of 25 nails gave an average length of 3.1 inch with standard deviation 0.3. Can it be said that the machine is producing nails as per specification? ($t_{0.05}$ for 24 d.f is 2.064).

Soln: $\mu = 3$, $\bar{x} = 3.1$, $n = 25$, $s = 0.3$

$$t = \frac{\bar{x} - \mu}{s} \sqrt{n}$$

$$= \frac{0.1}{0.3} \sqrt{25}$$

$$t = 1.67 < 2.064.$$

Thus the hypothesis that the machine is producing nails as per specification is accepted at 5% level of significance

5) A sample of 10 measurement of the diameter of a sphere gave a mean of 12 cm & a standard deviation 0.15 cm. Find the 95% confidence limits for the actual diameter.

Sol^n: Given $n = 10$, $\bar{x} = 12$, $S = 0.15$ & $\nu = DOF = n-1 = 9$

Also $t_{0.05}$ for 9 df $= 2.262$.

∴ 95% confidence limits for the actual diameter is given by

$$\bar{x} \pm \left[\frac{S}{\sqrt{n}}\right] t_{0.05} = 12 \pm \frac{0.15}{\sqrt{10}}(2.262)$$

$$= 12 \pm 0.1079$$

ie $12 + 0.1079 = 12.1079$, $12 - 0.1079 = 11.893$

Thus 11.893 cm to 12.1079 cm is the confidence limits for the actual diameter.

6) A sample of 12 measurements of diameter of a metal ball given $\bar{x} = 7.38$ mm, S.D $= 1.24$ mm. Find
ⓐ 95% ⓑ 99%. Confidence limits. $\left[\begin{array}{l} t_{0.05}(11) = 2.20 \\ t_{0.01}(11) = 3.11 \end{array}\right]$

Sol^n: Given $\bar{x} = 7.38$, $S = 1.24$, $n = 12$, $\nu = 12 - 1 = 11$. (DOF)

ⓐ Confidence limit at 95%,

$$\bar{x} \pm \frac{S}{\sqrt{n}} t_{0.05}^{(11)} = 7.38 \pm \frac{1.24}{\sqrt{12}}(2.20)$$

$$\Rightarrow 7.38 + \frac{1.24}{\sqrt{12}}(2.20) = \boxed{8.167}$$

$$\& \; 7.38 - \frac{1.24}{\sqrt{12}}(2.20) = \boxed{6.592}$$

} 95% confidence limits.

ⓑ Confidence limits at 90%

$$\bar{x} \pm \frac{S}{\sqrt{n}} t_{0.01}^{(11)} = 7.38 \pm \frac{1.24}{\sqrt{12}}(3.11)$$

$$\Rightarrow 7.38 + \frac{124}{\sqrt{12}}(3.11) = \boxed{8.493}$$

$$7.38 - 1.24/\sqrt{12}\,(3.11) = \boxed{6.267}$$

} 90% confidence limits.

7) For a random sample of 16 values with mean 41.5 and the sum of square of deviations from the mean = 135. Find at 95% and 99% confidence limits for the mean of the population. [$t_{0.05}(15) = 2.13$, $t_{0.01}(15) = 2.95$]

Sol^n: Given $\bar{x} = 41.5$, $n = 16$, $\nu = n-1 = 16-1$, $\sum(x-\bar{x})^2 = 135$,

$\nu = 15$

$$s^2 = \frac{135}{16} = 8.4375$$

$$s = 2.904$$

(i) 95% confidence limit,

$$\bar{x} \pm \frac{s}{\sqrt{n}} t_{0.05}(15) = 41.5 \pm \frac{2.904}{\sqrt{16}}(2.13)$$

$$\Rightarrow 41.5 + \frac{2.904}{\sqrt{16}}(2.13) = \boxed{43.05}$$

$$\& \; 41.5 - \frac{2.904}{\sqrt{16}}(2.95) = \boxed{39.95}$$

(ii) 90% confidence limits,

$$\bar{x} \pm \frac{s}{\sqrt{n}} t_{0.01}(11) = 41.5 \pm \frac{2.904}{\sqrt{16}}(2.95)$$

$$\Rightarrow 41.5 + \frac{2.904}{\sqrt{16}}(2.95) = \boxed{43.64}$$

$$\& \; 41.5 - \frac{2.904}{\sqrt{16}}(2.95) = \boxed{39.36}$$

8) 11 school boys were given a test in maths, carrying a maximum marks of 25. They were given a months extra coaching and a second test of equal difficulties was held. The following table gives the marks in two tests. Do the marks give evidence that the student have benifited by extra coaching? Use 5% level of significance.

Boys	1	2	3	4	5	6	7	8	9	10	11
I test (x)	23	20	19	21	18	20	18	17	23	16	19
II test (y)	24	19	22	18	20	22	20	20	23	20	17

Soln: First find $X = (x_i - y_i)$

$$X = \{1, -1, 3, -3, 2, 2, 2, 3, 0, 4, -2\}$$

$$\therefore \bar{X} = \frac{\Sigma x}{n} = \frac{11}{11} = 1$$

$n = 11$, $\nu = n-1 = 10$, $\mu = 0$.

$$s^2 = \frac{(x_i - \bar{x})^2}{n} = \frac{50}{11} = 4.545$$

$$\therefore S = 2.1320$$

$$\therefore t = \frac{\bar{x} - \mu}{S}\sqrt{n}$$

$$= \frac{1-0}{2.1320}\sqrt{11}$$

$$t = 1.556$$

& $t_{0.05}^{(10)} = 2.23$.

$\therefore 1.556 < 2.23$. $\therefore$ Hypothesis is accepted.

Note :-

Test significance for difference b/w sample means,

Consider two independent samples $x_i\,(i=1,2,3,\dots,n_1)$, & $y_j\,(j=1,2,\dots,n_2)$ drawn from a normal population. and $(\bar{x}, \sigma_x)$ & $(\bar{y}, \sigma_y)$ respectively mean & ~~variance~~ S.D of the Then ~~to~~ test hypothesis of difference b/w the samples means is given by

$$t = \frac{\bar{x}-\bar{y}}{S\sqrt{1/n_1 + 1/n_2}}$$

where $S^2 = \frac{1}{n_1+n_2-2}\left\{\sum_{i=1}^{n_1}(x_i-\bar{x})^2 + \sum_{j=1}^{n_2}(y_j-\bar{y})^2\right\}$ (or) $S^2 = \frac{n_1 S_1^2 + n_2 S_2^2}{n_1+n_2-2}$.

& dof $= \nu = n_1+n_2-2$.

Problems:

9) A group of boys & girls were given an intelligence test. The mean score, S.D score and numbers in in each group are as follows,

	Boys	Girls
mean	74	70
S.D	8	10
n	12	10

Is the difference b/w the means of the two groups significant at 5% level of significance ($t_{0.05} = 2.086$ for 20 d.f).

Sol^n: By data, $\bar{x} = 74$, $S_1 = 8$, $n_1 = 12$ [Boys]

$\bar{y} = 70$, $S_2 = 10$, $n_2 = 10$ (Girls)

we have $t = \frac{\bar{x}-\bar{y}}{S\sqrt{1/n_1 + 1/n_2}}$,

where $S^2 = \frac{1}{n_1+n_2-2}\left\{\sum_{i=1}^{n_1}(x_i-\bar{x})^2 + \sum_{j=1}^{n_2}(y_j-\bar{y})^2\right.$

(or) $$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{(12)(64) + 10(100)}{12 + 10 - 2}$$

$$= \frac{1768}{20}$$

$$S^2 = 88.4$$

$\therefore$ $S = 9.402 \approx 9.4$

Hence $$t = \frac{74 - 70}{9.4\sqrt{1/12 + 1/10}}$$

$$t = 0.994$$

& D.o.f $= \nu = n_1 + n_2 - 2$
$= 12 + 10 - 2$
$\nu = 20$

at 5% level $t_{0.05} = 2.086$ for 20 dof.

$\therefore$ $0.994 < 2.086$.

Thus the hypothesis that there is a difference b/w the means of the two group is accepted at 5% level of significance.

10) Two types of batteries are tested for their length of life & the following results we obtained.

Battery A: $n_1 = 10$, $\bar{x}_1 = 440$ hrs, $\sigma_1^2 = 100$

Battery B: $n_2 = 10$, $\bar{x}_2 = 500$ hrs, $\sigma_2^2 = 121$.

Compute students t & test whether there is a significant difference in the two means.

Sol^n:- $$S^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2 - 2}$$

$\nu = n_1 + n_2 - 2$
$= 10 + 10 - 2$
$\nu = 18$

$$S^2 = \frac{(10\times100) + (10\times121)}{18} = 122.78$$

$\therefore$ $S = 11.0805$

$$\therefore t = \frac{\bar{x}_2 - \bar{x}_1}{S\sqrt{1/n_1 + 1/n_2}} = \frac{500 - 440}{11.0805} = 12.1081 \simeq 12.11$$

& $t_{0.05}(18) = 2.101$
$t_{0.01}(18) = 1.330$.

Here calculated t value is greater than both the significant levels.

$\therefore$ Hypothesis is Rejected.

11) A group of 10 boys fed on a diet A & another group of 8 boys fed on a different diet B for a period of 6 months recorded the following increase in weight (lbs).

Diet A :	5	6	8	1	12	4	3	9	6	10
Diet B :	2	3	6	8	10	1	2	8		

Test whether diets A & B differ significantly regarding their effect on increase in weight.

Soln: Let x & y corresponds to diet A & B respectively.

$$\therefore \bar{x} = \frac{\Sigma x}{n_1} = \frac{64}{10} = 6.4, \quad \bar{y} = \frac{\Sigma y}{n_2} = \frac{40}{8} = 5$$

$$\sum_{i=1}^{n_1} (x_i - \bar{x})^2 = (5-6.4)^2 + (6-6.4)^2 + \cdots + (10-6.4)^2 = 102.4$$

$$\sum_{j=1}^{n_2} (y_j - \bar{y})^2 = (2-5)^2 + (3-5)^2 + \cdots + (8-5)^2 = 82.$$

$$\therefore S^2 = \frac{1}{n_1+n_2-2} \left\{ \sum_{i=1}^{n_1} (x_i-\bar{x})^2 + \sum_{j=1}^{n_2} (y_j - \bar{y})^2 \right\}$$

$$= \frac{1}{10+8-2} [102.4 + 82]$$

$$S^2 = \frac{184.4}{16} = 11.525$$

$$\therefore S = 3.395$$

Hence $t = \dfrac{\bar{x} - \bar{y}}{S\sqrt{1/n_1 + 1/n_2}}$ & DOF $(\nu) = n_1 + n_2 - 2$
$= 10 + 8 - 2$
$\nu = 16$

$$= \frac{6.4-5}{3.395\sqrt{1/10 + 1/8}} = \frac{1.4}{3.395\sqrt{1/10+1/8}}$$

$$t = 0.86935 \approx 0.87$$

But $t_{0.05}$ for 16 d.f $= 2.12$

$\therefore 0.87 < 2.12$.

Thus we can conclude that the two diets do not differ significantly regarding their effect are increases in weight.