ASSIGNMENT - 01

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SUBJECT: AUTOMATA THEORY & COMPUTABILITY

SUBJECT CODE: 18CS54

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1. Define the following terms:

a) Alphabet

A) An alphabet is a finite non-empty set of symbols of a language. The symbol 'Σ' denotes set of alphabets of a language.

Eg:
The Machine level language is made up of only two symbols 0 and 1. Therefore,

$$\Sigma = \{0,1\}$$

b) Power of an Alphabet

A) Power of an Alphabet is denoted as Σ^i, which is the set of words of length 'i'. There are two different types of variations in power of Alphabet,

(i) Kleen closure (or) kleen star (Σ^*)

(ii) Kleen plus (Σ^+)

KLEEN CLOSURE (Σ^*):

It is defined as,

$$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \cdots , \text{ for } \Sigma = \{0,1\}$$

i.e., $\Sigma^* = \{\varepsilon\} \cup \{0,1\} \cup \{00,01,10,11\} \cup \cdots$

$\Rightarrow \{0,1\}^* = \Sigma^* = \{\varepsilon, 0, 1, 00, 01, 10, 11, \ldots \ldots\}$

Eg: Let $\Sigma = \{a, bc\}$, the kleen star is defined as,

$\Sigma^* = \{\varepsilon\} \cup \{a, bc\} \cup \{abc, bca\} \cup \dots$

$\Rightarrow \{a, bc\}^* = \Sigma^* = \{\varepsilon, a, bc, abc, bca, \dots\}$

~~c) Language~~

~~A)~~ KLEEN PLUS (Σ^+):

It is a variation of Σ^*. It is defined as,

$\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \Sigma^4 \cup \dots$, For $\Sigma = \{0, 1\}$

$\Sigma^+ = \{0, 1\} \cup \{00, 01, 10, 11\} \cup \dots$

$\Rightarrow \Sigma^+ = \{0, 1, 00, 01, 10, 11\}$

Eg: Let $\Sigma = \{a, bc\}$, the kleen plus is defined as,

$\Sigma^+ = \{a, bc\} \cup \{abc, bca, aa, bcbc\} \dots$

$\Sigma^+ = \{a, bc, abc, bca, aa, bcbc\}$

c) Language

A) A language is a set of strings that are obtained from Σ^* (kleen star), where Σ is the alphabet set for a particular ~~s~~ language.

i.e., $L \subseteq \Sigma^*$

Eg: A language consisting of equal number of 0's followed by equal number of 1's is denoted as

$L = \{\varepsilon, 01, 0011, 000111, \dots\}$

d) Strings

A) A string is a finite sequence of symbols applied from the alphabets of a language.

Eg: Let $\Sigma = \{0,1\}$. Various strings that can be obtained are,

$\Sigma = \{\varepsilon, 0, 1, 00, 01, 10, 11, 000, 001, 010, \ldots\}$

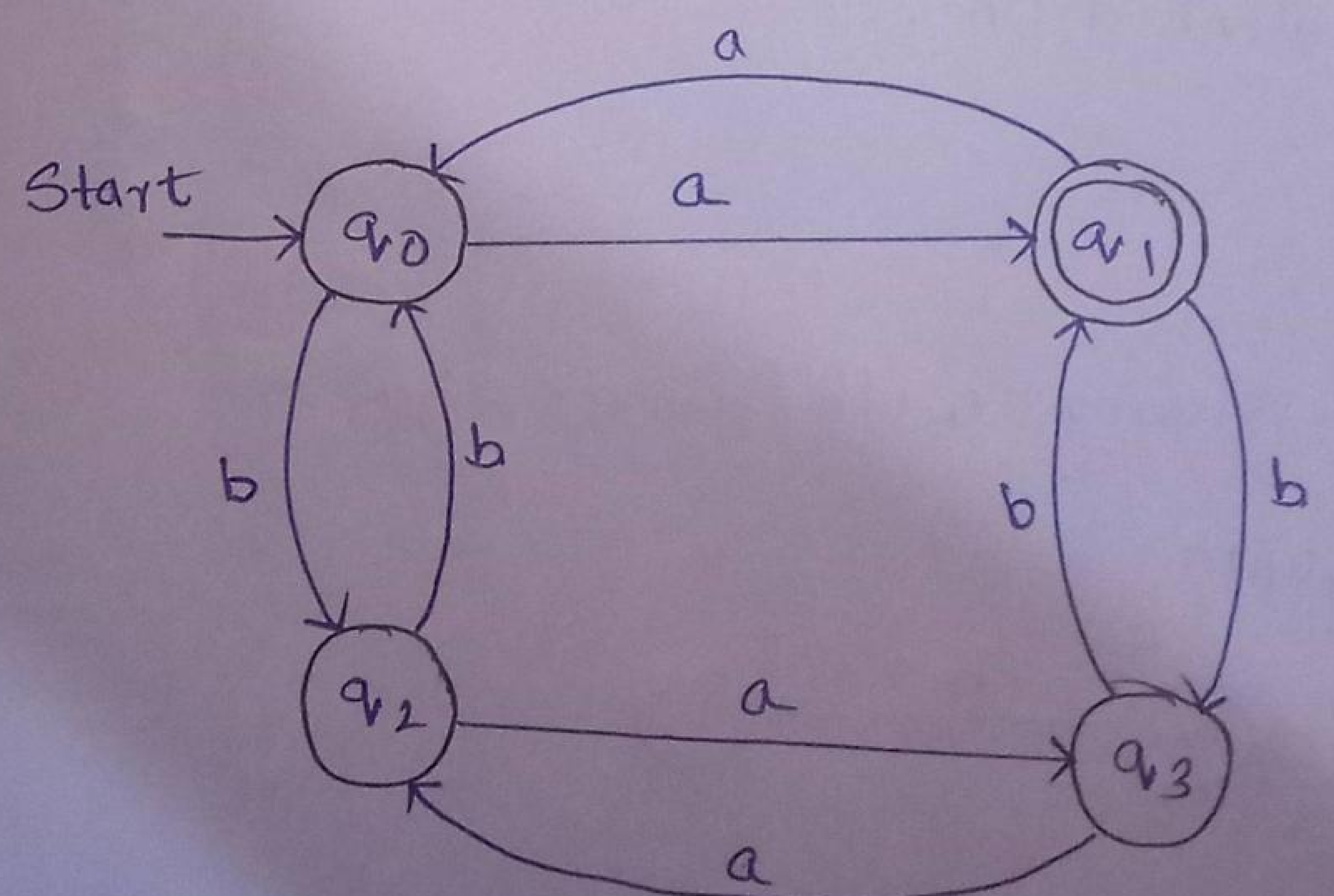
2. Draw a DFA to accept strings of a's and b's having odd no. of a's and even no. of b's.

A) Given,

$\Sigma = \{a, b\}$

Minimal string: Odd number of a's, Even number of b's

TRANSITION DIAGRAM:



The resulting DFA is $M = (Q, \Sigma, \delta, q_0, F)$

~~M~~ $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{a, b\}$

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TRANSITION TABLE:

δ	a	b
q_0	q_1	q_2
q_1	q_0	q_3
q_2	q_3	q_0
q_3	q_2	q_1

$q_0 \in Q$ is start state

$F \subseteq Q = \{q_1\}$ is set of final state.

To accept string : aabba

$$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_0 \xrightarrow{b} q_2 \xrightarrow{b} q_0 \xrightarrow{a} q_1$$

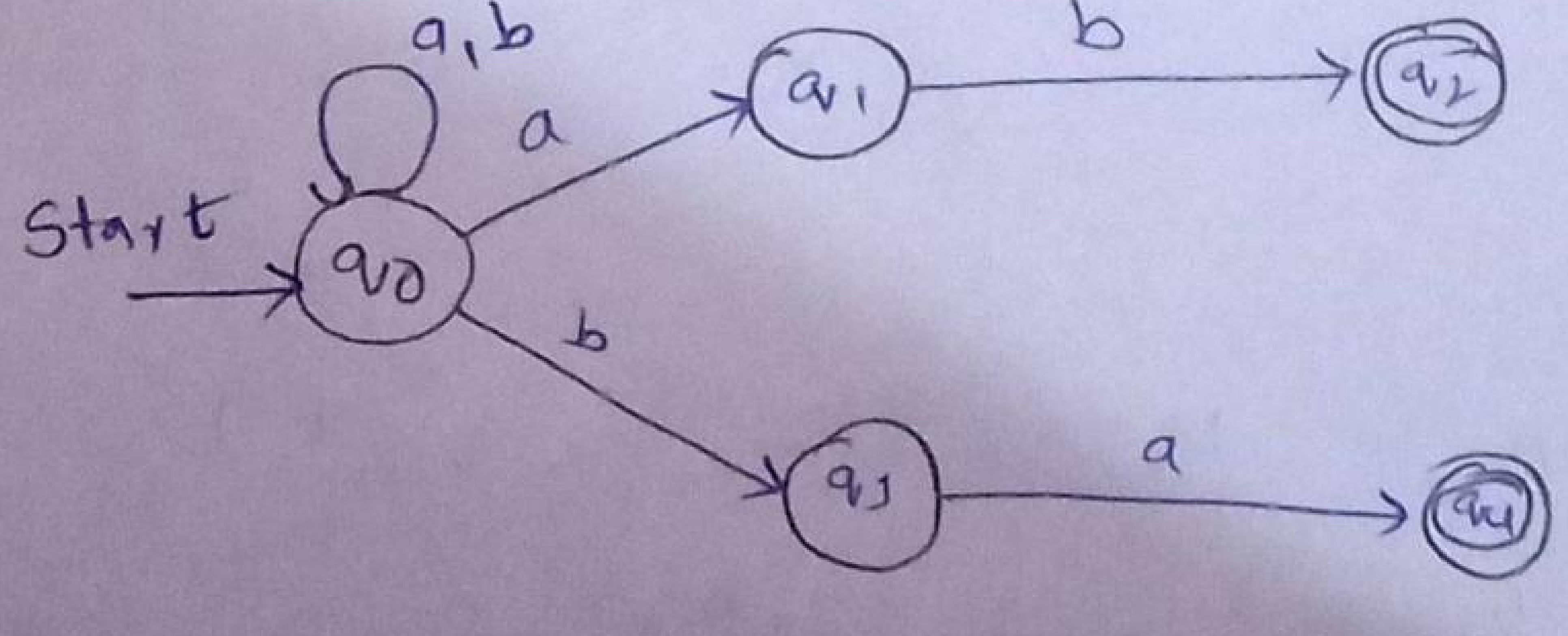
3. Obtain an NFA to accept strings of a's and b's ending with 'ab' or 'ba'. From this NFA obtain an Equivalent DFA.

A) Given,

$\Sigma = \{a, b\}$

Minimal string : Ending with ab or ba.

TRANSITION DIAGRAM:



The resulting NFA is $M = \{Q, \Sigma, \delta, q_0, F\}$

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{a, b\}$

TRANSITION TABLE:

δ	a	b
q_0	$\{q_0, q_1\}$	$\{q_0, q_3\}$
q_1	ϕ	$\{q_2\}$
q_2	ϕ	ϕ
q_3	q_4	ϕ
q_4	ϕ	ϕ

$q_0 \in Q$ is the start state

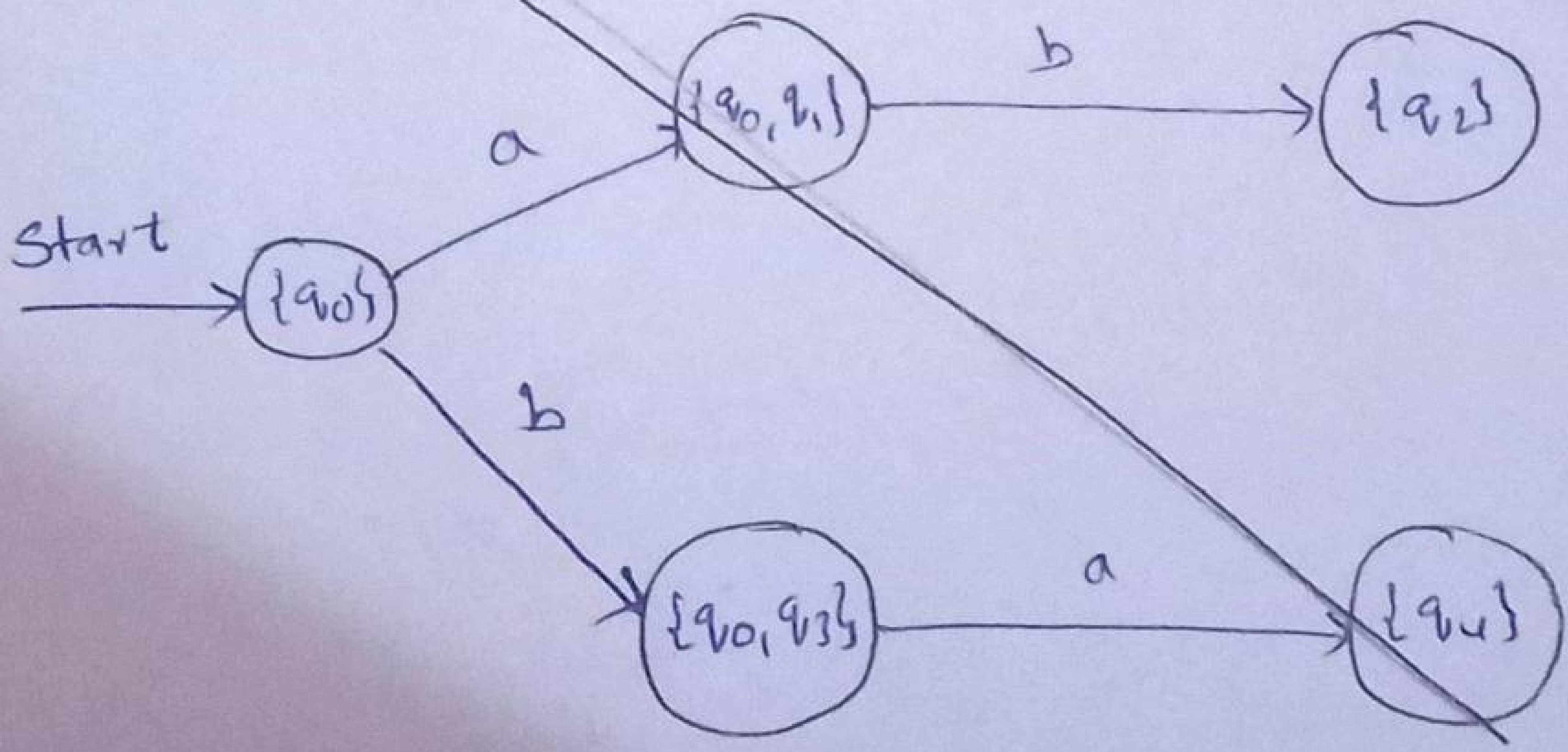
$F \subseteq Q = \{q_2, q_4)$ are set of final states.

To accept string abab

$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2$

NFA ~~TO~~ DFA CONVERSION:

From above transition table,



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NFA TO DFA CONVERSION:-

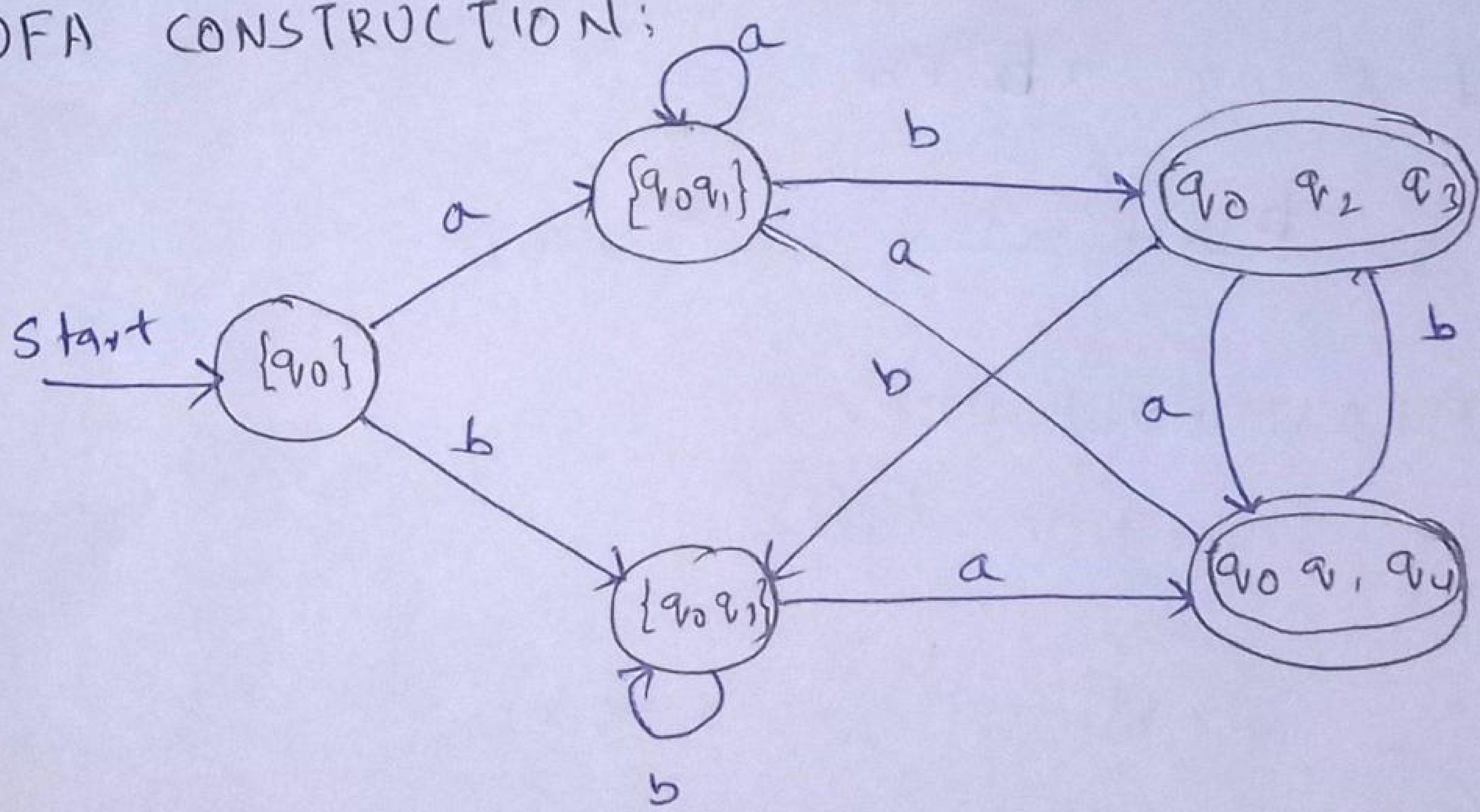
TRANSITION TABLES

NFA → DFA

δ	a	b
q_0	$\{q_0, q_1\}$	$\{q_0, q_3\}$
q_1	ϕ	$\{q_2\}$
q_2	ϕ	ϕ
q_3	q_4	ϕ
q_4	ϕ	ϕ

δ	a	b
$[q_0]$	$[q_0\ q_1]$	$[q_0\ q_3]$
$[q_0\ q_1]$	$[q_0\ q_1]$	$[q_0\ q_3\ q_2]$
$[q_0\ q_3]$	$[q_0\ q_1, q_4]$	$[q_0\ q_3]$
$[q_0\ q_2\ q_3]$	$[q_0\ q_1, q_4]$	$[q_0\ q_3]$
$[q_0\ q_1, q_4]$	$[q_0\ q_1]$	$[q_0\ q_3\ q_2]$

DFA CONSTRUCTION:



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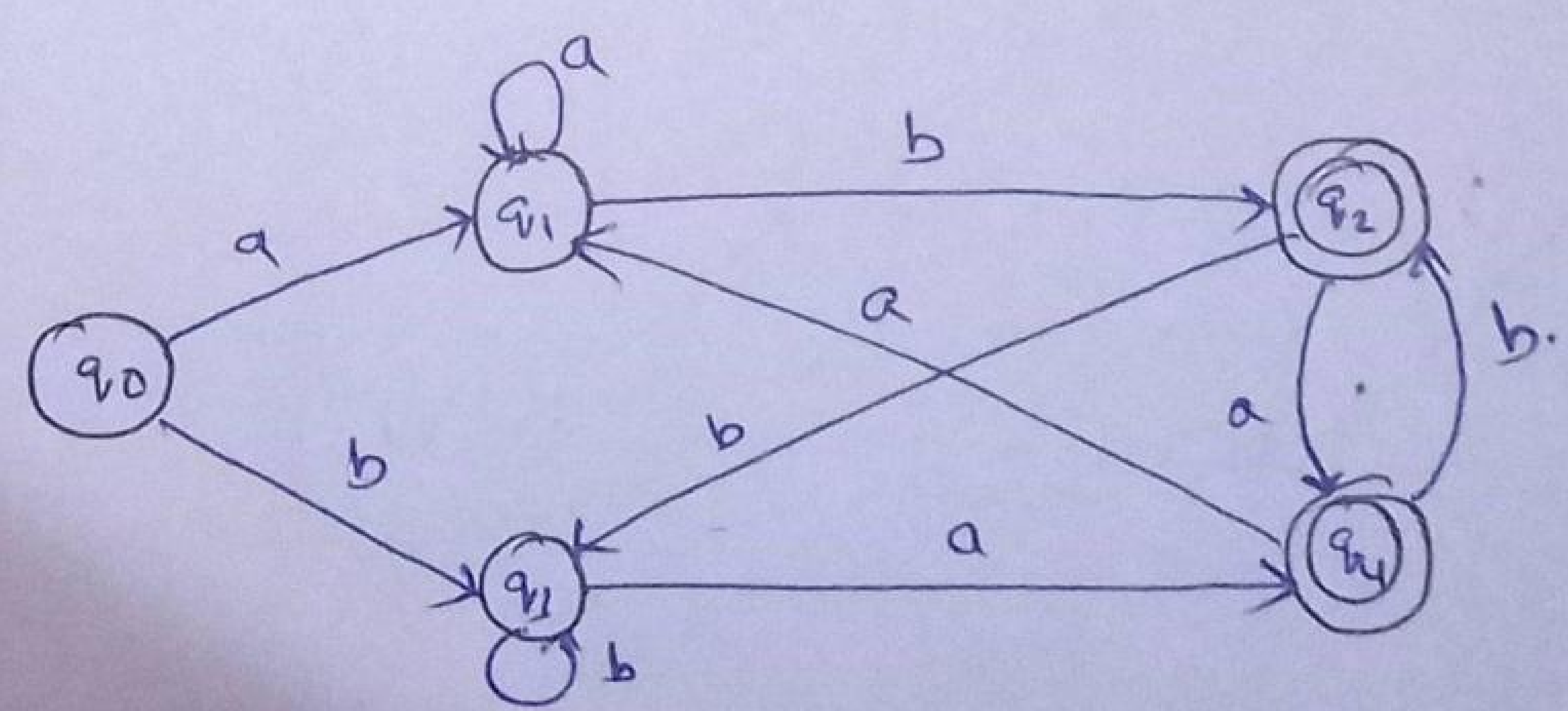


The resulting DFA is $M = \{Q, \Sigma, \delta, q_0, F)$

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{a, b\}$

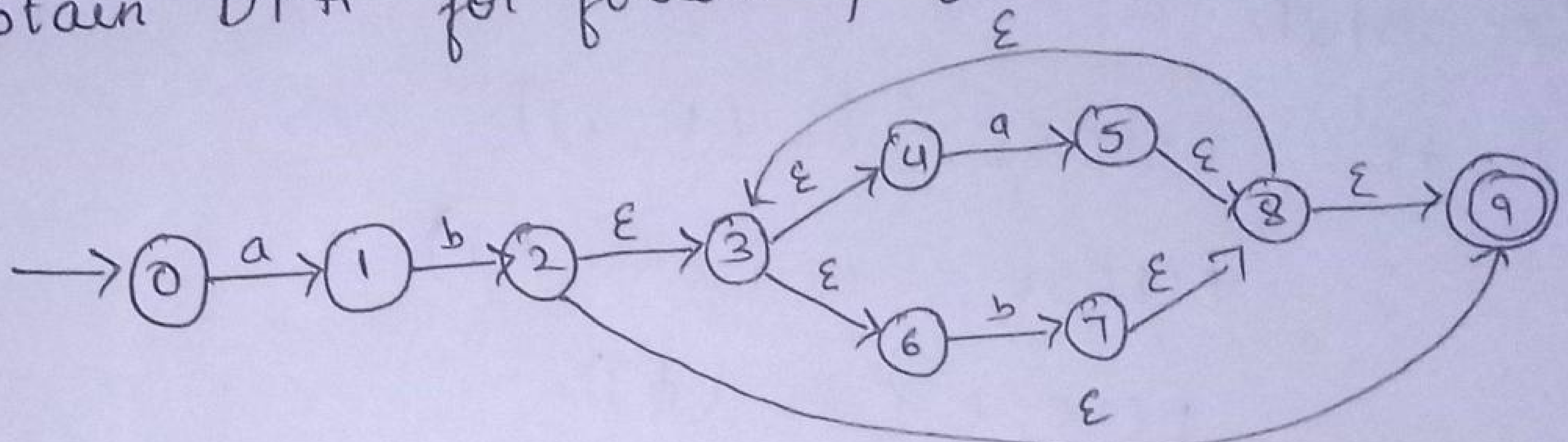
$q_0 \in Q$ is the start state

$F \subseteq Q = \{q_2, q_4\}$ are final states.

To accept string : abab

$q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_4 \xrightarrow{b} q_2$

4. Obtain DFA for following ε-NFA



A) From given ε-NFA

Eclose (0) = {0}

Eclose (1) = {1}

Eclose (2) = {2, 3, 4, 6, 9}

Eclose (3) = {3, 4, 6}

Eclose (4) = {4}

Eclose (5) = {5, 8, 3, 4, 6, 9}

Eclose (6) = {6}

Eclose (7) = {7, 8, 3, 4, 6, 9}

E close (8) = {8, 3, 4, 6, 9}

E close (9) = {9}

start state : E close (0) = {0} = A

consider state A

$\delta(A, a) = ECLOSE(\delta_\varepsilon(A, a))$

$= ECLOSE(\{0\}, a)$

$= ECLOSE(\{1\}) = B$

$\therefore \boxed{\delta(A, a) = B.}$

$\delta(A, B) = ECLOSE(\delta_\varepsilon(A, B)) = ECLOSE(\delta_\varepsilon(\{0\}, b)) = \phi.$

consider state B

$\delta(B, a) = ECLOSE(\delta_\varepsilon(B, a))$

$= ECLOSE(\delta_\varepsilon(\{1\}, a))$

$= ECLOSE(\delta_\varepsilon(\phi))$

$\boxed{\delta(B, a) = \phi}$

$\delta(B, b) = ECLOSE(\delta_\varepsilon(B, b))$

$= ECLOSE(\delta_\varepsilon(\{1\}, b))$

$= ECLOSE(\{2\}) = \{2, 3, 4, 6, 9\} - C.$

$\boxed{\delta(B, b) = C}$

~~consider state C~~

~~$\delta(C,a) = ECLOSE(\delta_E(C,a))$~~

~~$= ECLOSE(\delta_E(\{2\},a))$~~

~~$\boxed{\delta(C,a) = \phi}$~~

~~δ~~

consider state C

$\delta(C,a) = ECLOSE(\delta_E(C,a))$

$= ECLOSE(\delta_E(\{2,3,4,6,9\},a))$

$= ECLOSE(\{5\}) = \{3,4,6,5,8,9\} - D$

$\boxed{\delta(C,a) = D}$

$\delta(C,b) = ECLOSE(\delta_E(C,b)$

$= ECLOSE(\delta_E(\{2,3,4,6,9\},b)$

$= ECLOSE(7) = \{3,4,6,7,8,9\} = E$

$\boxed{\delta(C,b) = E}$

consider state D

$\delta(D,a) = ECLOSE(\delta_E(D,a))$

$= ECLOSE(\{3,4,5,6,8,9\},a)$

$= ECLOSE(5) = D$

$\boxed{\delta(D,a) = D}$

$\delta(D,b) = ECLOSE(\delta_E(D,b))$

$= ECLOSE(\{3,4,5,6,8,9\},b)$

$= ECLOSE(7) = E$

$\boxed{\delta(D,b) = E}$

Consider state E

$$\delta(E,a) = ECLOSE(\delta_\varepsilon(E,a))$$
$$= ECLOSE(\delta_\varepsilon(\{3,4,6,7,8,9\},a)$$
$$= ECLOSE(5) = D$$

$\boxed{\delta(E,a) = D}$

~~Consider state~~

$$\delta(E,b) = ECLOSE(\delta_\varepsilon(E,b))$$
$$= ECLOSE(\delta_\varepsilon(\{3,4,6,7,8,9\},b)$$
$$= ECLOSE(7) = E$$

$\boxed{\delta(E,b) = E}$

TRANSITION TABLE:

δ	a	b
A	B	ϕ
B	ϕ	C
C	D	E
D	D	E
E	D	E

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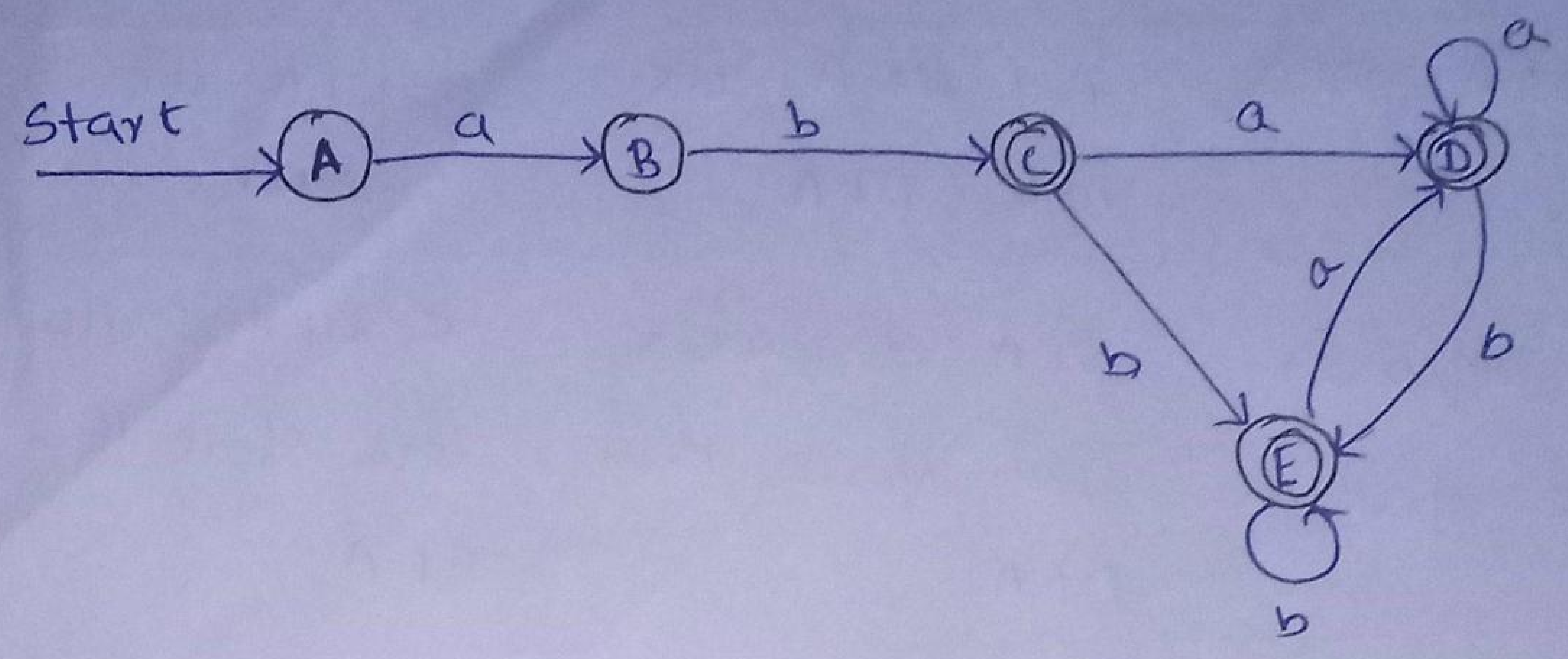
DFA CONSTRUCTION:



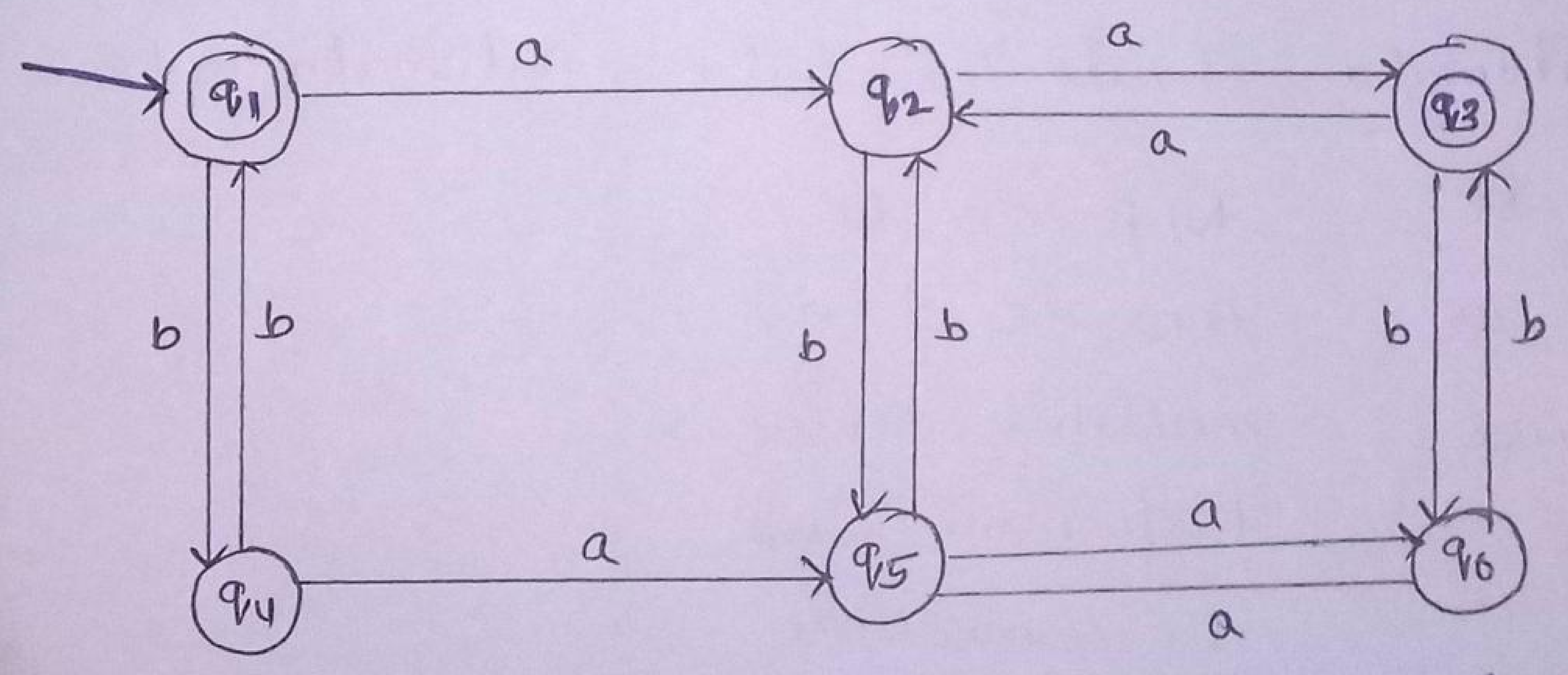
5. Write differences between DFA, NFA and ε-NFA

A)

DFA	NFA	ε-NFA
Stands for Deterministic Finite Automata	Stands for Non-deterministic Finite Automata	Stands for Non-deterministic Finite Automata with ε.
DFA can be considered as one machine.	NFA can be considered as multiple machines performing task at same time.	
Cannot use Empty string Transition	Can use Empty String Transition.	Can use Empty String Transition

DFA	NFA	ε-NFA
It is difficult to construct.	It is easier to construct.	It is easier to construct.
All DFA are NFA.	All NFA are not DFA.	ε-NFA are NFA
DFA requires more space	NFA requires less space than DFA	ε-NFA requires less space than DFA
Execution time is less.	Execution time is more.	Execution time is less compared to NFA

6. Make use of min DFSM to minimize M. Let M be following DFSM.



A) Initial class : $\{[q_1, q_3], [q_2, q_4, q_5, q_6]\}$

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$((q_1, a), [q_2, q_4, q_5, q_6])$

$((q_1, b), [q_2, q_4, q_5])$

$((q_3, a), [q_2, q_4, q_6])$

$((q_3, b), [q_2, q_4, q_6])$

$\Rightarrow [q_1, q_3]$

$((q_2, a), [q_1, q_3])$

$((q_2, b),$

$((q_1, a), [q_2, q_4, q_5, q_6])$

$((q_1, b), [q_2, q_4, q_5, q_6])$

$((q_3, a), [q_2, q_4, q_5, q_6])$

$((q_3, b), [q_2, q_4, q_5, q_6])$

$\Rightarrow [q_1, q_3]$

$((q_2, a), [q_1, q_3])$

$((q_2, b), [q_2, q_4, q_5, q_6])$

$((q_4, a), [q_2, q_4, q_5, q_6])$

$((q_4, b), [q_1, q_3])$

$((q_5, a), [q_2, q_4, q_5, q_6])$

$((q_5, b), [q_2, q_4, q_5, q_6])$

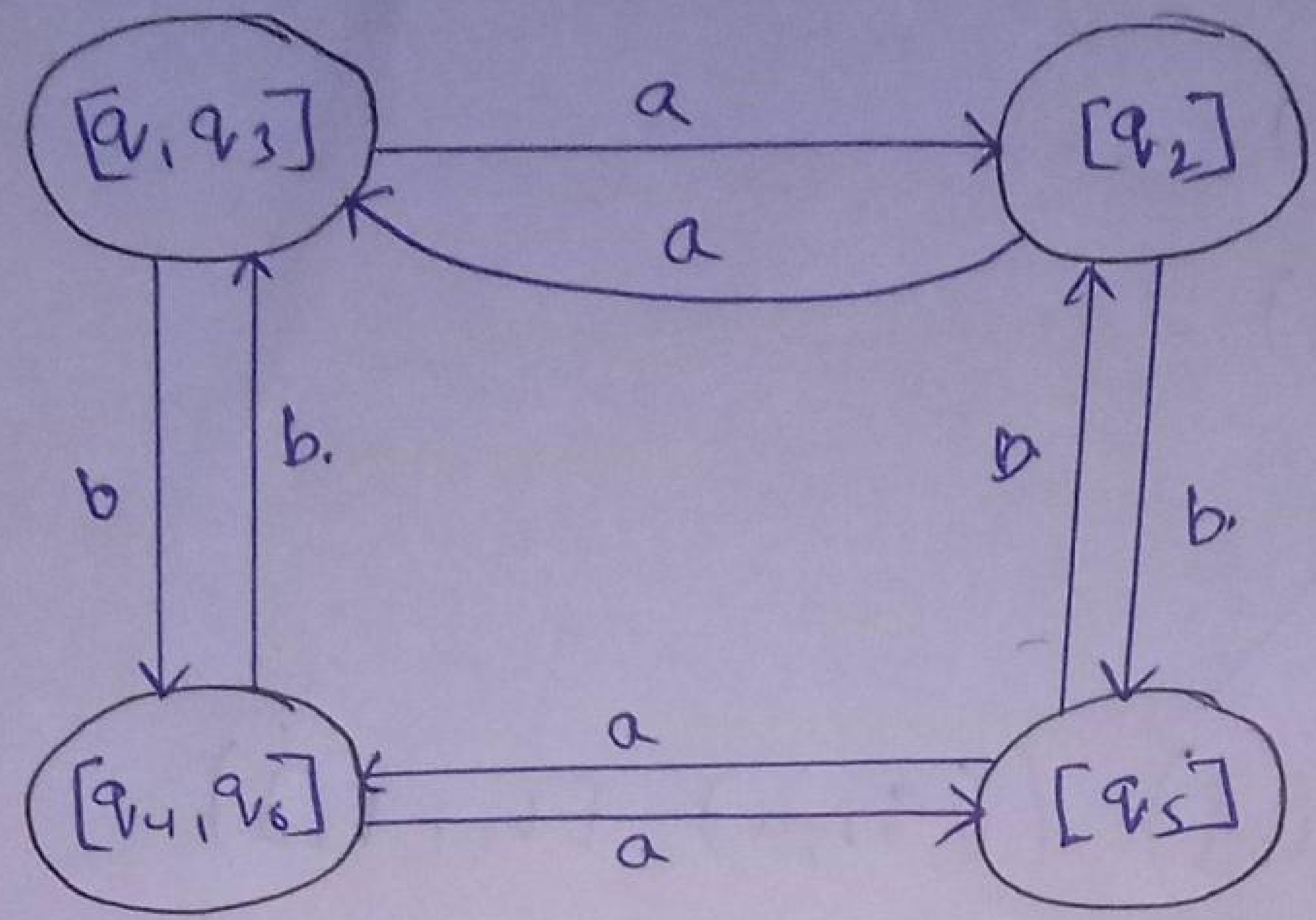
$((q_6, a), [q_2, q_4, q_5, q_6])$

$((q_6, b), [q_1, q_3])$

$\Rightarrow [q_2], [q_4, q_6] [q_5]$

Resulting class: $\{[q_1, q_3], [q_2], [q_5], [q_4, q_6]\}$

Resulting DFA is :-



7. Obtain regular expression for following languages.
$\Sigma = \{a, b, c\}$

a) All string containing exactly one a.

A) Regular Expression (R.E) = $(b+c)^* a (b+c)^*$

b) All strings containing more than three a's.

A) Regular Expression (R.E) :- $(b+c)^* (a+\varepsilon)(b+c)^* (a+\varepsilon)(b+c)^* (a+\varepsilon)$

c) All strings that contain at least one occurence of each symbol in Σ.

A) $(a+b+c)^* a (a+b+c)^* b (a+b+c)^* c (a+b+c)^* a (a+b+c)^* b (a+b+c)^* c (a+b+c)^* a (a+b+c)^* b (a+b+c)^* c$

8. Obtain a Regular Expression for the language

(i) $L = \{a^n b^m \mid n+m \text{ is even}\}$

(A) Regular Expression (R.E) = $(aa)^* (bb)^*$ | m, n are even
~~Regular Expression (R.E) = a(aa)*~~

~~(ii) $L = \{a^n b^m \mid m \geq 1, n \geq 1$ and~~ $nm \geq 0\}$

A) Regular Expression (R.E) = $a(aa)^* b(bb)^*$ | m, n are odd

$$\Rightarrow R.E = (aa)^*(bb)^* + a(aa)^* b(bb)^*$$

(ii) $L = \{a^n b^m \mid m \geq 1, n \geq 1$ and $nm \geq 3$

A) Regular expression (R.E) = $(a(bbbb))^*$ | $m = 1, n \geq 3 \Rightarrow mn \geq 3$

$R.E = ((aaaa)^* b)^*$ | $m \geq 3, n = 1 \Rightarrow mn \geq 3$

$R.E = (aaa)^* (bbb)^*$ | $m \geq 2, n \geq 2 \Rightarrow mn \geq 3$

$$\Rightarrow R.E = (a(bbbb))^* + ((aaaa)^* b + (aaa)^* (bbb)^*$$

$$R.E = abbbb^* + aaaab^* + aaa^* bbb^*$$

9. Define a Regular Expression. Find Regular Expression for the following languages.

A) A Regular Expression is a string or sequence of characters that define or generate a search pattern.

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(i) $L = \{a^{2m} b^{2n} \mid n \geq 0, m \geq 0\}$

A) Regular Expression (R.E) = $(aa)^*(bb)^*$

(ii) $L = \{\omega : |\omega| \bmod 3 = 0\}$, $\omega \in \{a,b\}^*$

A) Regular Expression (R.E) = $\{(a+b)^3\}^*$

10. Obtain Regular Expression for following languages.

(i) To accept strings of a's and b's such that every block of four consecutive symbols contains at least two a's.

A) ~~Regular Expression (R.E) = $((aa)^*(a+b)^*)^*$~~

(ii) ~~To accept strings of a's and b's whose length is either even or multiples of 3 or both.~~

(A) ~~Regular expression (R.E) = $(aa)^*(bb)^* + ((a+b)^3)^* + ((aa)^* + (bb)^*)^3)^*$~~

A) R.E = $((aa(a+b)(a+b)) + (a(a+b)a(a+b)) + (a(a+b)(a+b)a) + ((a+b)a(a+b)a) + ((a+b)(a+b)aa) + ((a+b)aa(a+b)))^+$

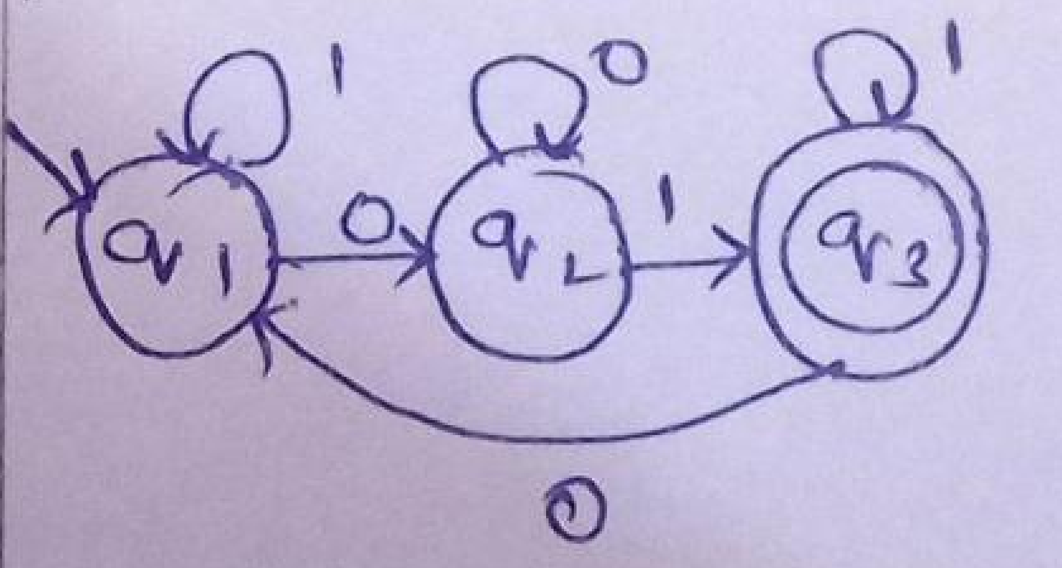
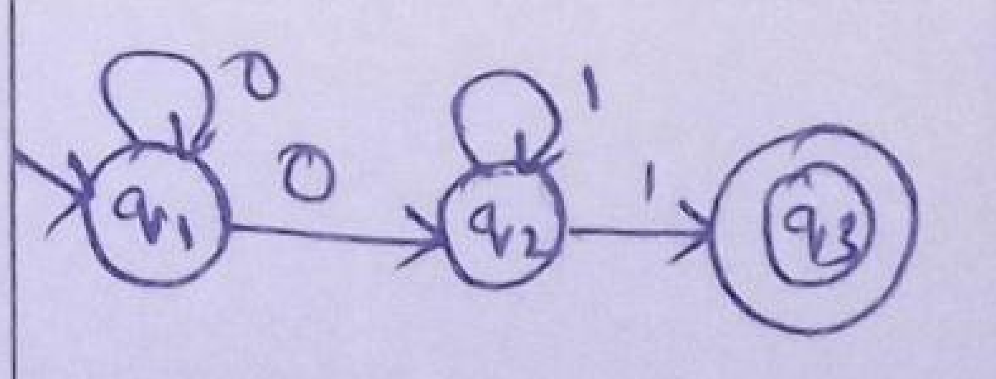
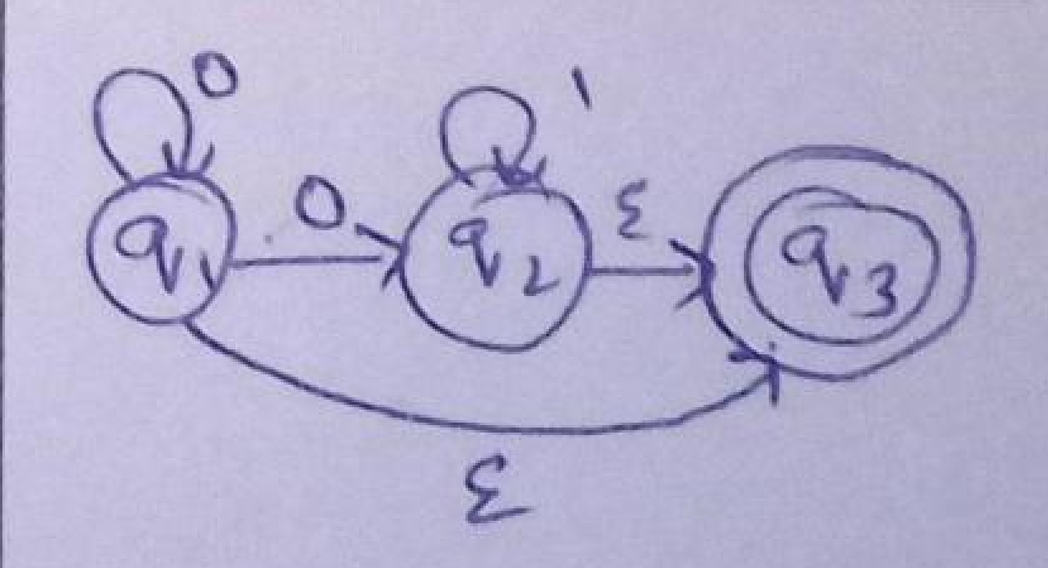
(ii) To accept strings of a's and b's whose length is either even or multiple of 3 or both.

A) Regular expression (R.E) = $((a+b)^2)^* + ((a+b)^3)^*$

$$R.E = ((a+b)^2)^* + ((a+b)^3)^*$$

5. Write differences between DFA, NFA and ε-NFA

A)

DFA	NFA	ε-NFA
Single transaction for each input symbol.	Two or more transition for any one Σ and Q	Two or more and ε transaction for any one of Σ and Q.
$M = (Q, \Sigma, \delta, q, F)$ $\delta = Q \times \Sigma \to Q$	$M = (Q, \Sigma, \delta, q, F)$ $\delta = Q \times \Sigma \to 2^Q$	$M = (Q, \Sigma, \delta, q, F)$ $\delta = Q \times \Sigma^* \to 2^Q$ $\Sigma^* = \Sigma \cup \varepsilon$
$\delta(q, \varepsilon) = \{q\}$	$\delta(q, \varepsilon) = \{q\}$	$\delta(q, \varepsilon) = ECLOSE(q)$
		

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δ	0	1
q_1	q_2	q_1
q_2	q_2	q_3
q_3	q_1	q_3

δ	0	1
q_1	$\{q_1, q_2\}$	q_1
q_2	$\{q_2, q_3\}$	ϕ
q_3	ϕ	ϕ

δ	0	1	ε
q_1	$\{q_1, q_2\}$	ϕ	q_3
q_2	ϕ	q_2	q_3
q_3	ϕ	ϕ	ϕ