MODULE -4

BAYSEIAN LEARNING

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INTRODUCTION

Bayesian learning methods are relevant to study of machine learning for two different reasons.

- First, Bayesian learning algorithms that calculate explicit probabilities for hypotheses, such as the naive Bayes classifier, are among the most practical approaches to certain types of learning problems
- The second reason is that they provide a useful perspective for understanding many learning algorithms that do not explicitly manipulate probabilities.

Features of Bayesian Learning Methods

- Each observed training example can incrementally decrease or increase the estimated probability that a hypothesis is correct. This provides a more flexible approach to learning than algorithms that completely eliminate a hypothesis if it is found to be inconsistent with any single example
- Prior knowledge can be combined with observed data to determine the final probability of a hypothesis. In Bayesian learning, prior knowledge is provided by asserting (1) a prior probability for each candidate hypothesis, and (2) a probability distribution over observed data for each possible hypothesis.
- Bayesian methods can accommodate hypotheses that make probabilistic predictions
- New instances can be classified by combining the predictions of multiple hypotheses, weighted by their probabilities.
- Even in cases where Bayesian methods prove computationally intractable, they can provide a standard of optimal decision making against which other practical methods can be measured.

Practical difficulty in applying Bayesian methods

- One practical difficulty in applying Bayesian methods is that they typically require initial knowledge of many probabilities. When these probabilities are not known in advance they are often estimated based on background knowledge, previously available data, and assumptions about the form of the underlying distributions.

- A second practical difficulty is the significant computational cost required to determine the Bayes optimal hypothesis in the general case. In certain specialized situations, this computational cost can be significantly reduced.

BAYES THEOREM

Bayes theorem provides a way to calculate the probability of a hypothesis based on its prior probability, the probabilities of observing various data given the hypothesis, and the observed data itself.

Notations

- P(h) *prior probability of* h, reflects any background knowledge about the chance that h is correct
- P(D) *prior probability of* D, probability that D will be observed
- P(D|h) probability of observing D given a world in which h holds
- P(h|D) *posterior probability of* h, reflects confidence that h holds after D has been observed

Bayes theorem is the cornerstone of Bayesian learning methods because it provides a way to calculate the posterior probability ***P(h|D),*** from the prior probability ***P(h),*** together with ***P(D)*** and ***P(D(h).***

Bayes Theorem:

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

P(h|D) increases with ***P(h)*** and with ***P(D|h)*** according to Bayes theorem.

P(h|D) decreases as ***P(D)*** increases, because the more probable it is that ***D*** will be observed independent of ***h,*** the less evidence ***D*** provides in support of ***h.***

Maximum a Posteriori (MAP) Hypothesis

- In many learning scenarios, the learner considers some set of candidate hypotheses H and is interested in finding the most probable hypothesis $h \in H$ given the observed data D. Any such maximally probable hypothesis is called a ***maximum a posteriori (MAP) hypothesis***.
- Bayes theorem to calculate the posterior probability of each candidate hypothesis is h_{MAP} is a MAP hypothesis provided

$$h_{MAP} = \underset{h \in H}{argmax}\ P(h|D)$$

$$= \underset{h \in H}{argmax}\ \frac{P(D|h)P(h)}{P(D)}$$

$$= \underset{h \in H}{argmax}\ P(D|h)P(h)$$

- P(D) can be dropped, because it is a constant independent of h

Maximum Likelihood (ML) Hypothesis

In some cases, it is assumed that every hypothesis in H is equally probable a priori ***($P(h_i) = P(h_j)$*** for all ***h_i*** and ***h_j*** in H).

In this case the below equation can be simplified and need only consider the term ***P(D|h)*** to find the most probable hypothesis.

$$h_{MAP} = \underset{h \in H}{argmax}\ P(D|h)P(h)$$

the equation can be simplified

$$h_{ML} = \underset{h \in H}{argmax}\ P(D|h)$$

P(D|h) is often called the ***likelihood*** of the data ***D*** given ***h,*** and any hypothesis that maximizes ***P(D|h)*** is called a ***maximum likelihood*** (ML) hypothesis

Example

Consider a medical diagnosis problem in which there are two alternative hypotheses

- The patient has a particular form of cancer (denoted by ***cancer***)
- The patient does not (denoted by ¬ ***cancer***)

The available data is from a particular laboratory with two possible outcomes: + (positive) and - (negative)

$$P(cancer) = .008 \qquad P(\neg cancer) = 0.992$$

$$P(\oplus|cancer) = .98 \qquad P(\ominus|cancer) = .02$$

$$P(\oplus|\neg cancer) = .03 \qquad P(\ominus|\neg cancer) = .97$$

- Suppose a new patient is observed for whom the lab test returns a positive (+) result.
- Should we diagnose the patient as having cancer or not?

$$P(\oplus|cancer)P(cancer) = (.98).008 = .0078$$
$$P(\oplus|\neg cancer)P(\neg cancer) = (.03).992 = .0298$$
$$\Rightarrow h_{MAP} = \neg cancer$$

BAYES THEOREM AND CONCEPT LEARNING

What is the relationship between Bayes theorem and the problem of concept learning?

Since Bayes theorem provides a principled way to calculate the posterior probability of each hypothesis given the training data, and can use it as the basis for a straightforward learning algorithm that calculates the probability for each possible hypothesis, then outputs the most probable.

Brute-Force Bayes Concept Learning

We can design a straightforward concept learning algorithm to output the maximum a posteriori hypothesis, based on Bayes theorem, as follows:

Brute-Force MAP LEARNING algorithm

1. For each hypothesis h in H calculate the posterior probability

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

2. Output the hypothesis h_{MAP} with the highest posterior probability

$$h_{MAP} = \underset{h \in H}{argmax}\ P(h|D)$$

In order specify a learning problem for the **BRUTE-FORCE MAP LEARNING** algorithm we must specify what values are to be used for ***P(h)*** and for ***P(D|h) ?***

Lets choose ***P(h)*** and for ***P(D|h)*** to be consistent with the following assumptions:

- The training data ***D*** is noise free (i.e., $d_i = c(x_i)$)
- The target concept ***c*** is contained in the hypothesis space H
- We have no a priori reason to believe that any hypothesis is more probable than any other.

What values should we specify for ***P(h)?***

- Given no prior knowledge that one hypothesis is more likely than another, it is reasonable to assign the same prior probability to every hypothesis ***h*** in ***H.***
- Assume the target concept is contained in H and require that these prior probabilities sum to 1.

$$P(h) = \frac{1}{|H|} \text{ for all } h \in H$$

What choice shall we make for ***P(D|h)***?

- ***P(D|h)*** is the probability of observing the target values D = ***(d_1 . . .d_m)*** for the fixed set of instances ***(x_1 . . . x_m),*** given a world in which hypothesis ***h*** holds
- Since we assume noise-free training data, the probability of observing classification ***d_i*** given ***h*** is just ***1*** if ***d_i*** = ***h(x_i)*** and 0 if ***d_i*** # ***h(x_i).*** Therefore,

$$P(D|h) = \begin{cases} 1 & \text{if } d_i = h(x_i) \text{ for all } d_i \in D \\ 0 & \text{otherwise} \end{cases}$$

Given these choices for ***P(h)*** and for ***P(D|h)*** we now have a fully-defined problem for the above **BRUTE-FORCE MAP LEARNING** algorithm.

In a first step, we have to determine the probabilities for P(h|D)

h is **inconsistent** with training data D

$$P(h|D) = \frac{0 \cdot P(h)}{P(D)} = 0$$

h is **consistent** with training data D

$$P(h|D) = \frac{1 \cdot \frac{1}{|H|}}{P(D)} = \frac{1 \cdot \frac{1}{|H|}}{\frac{|VS_{H,D}|}{|H|}} = \frac{1}{|VS_{H,D}|}$$

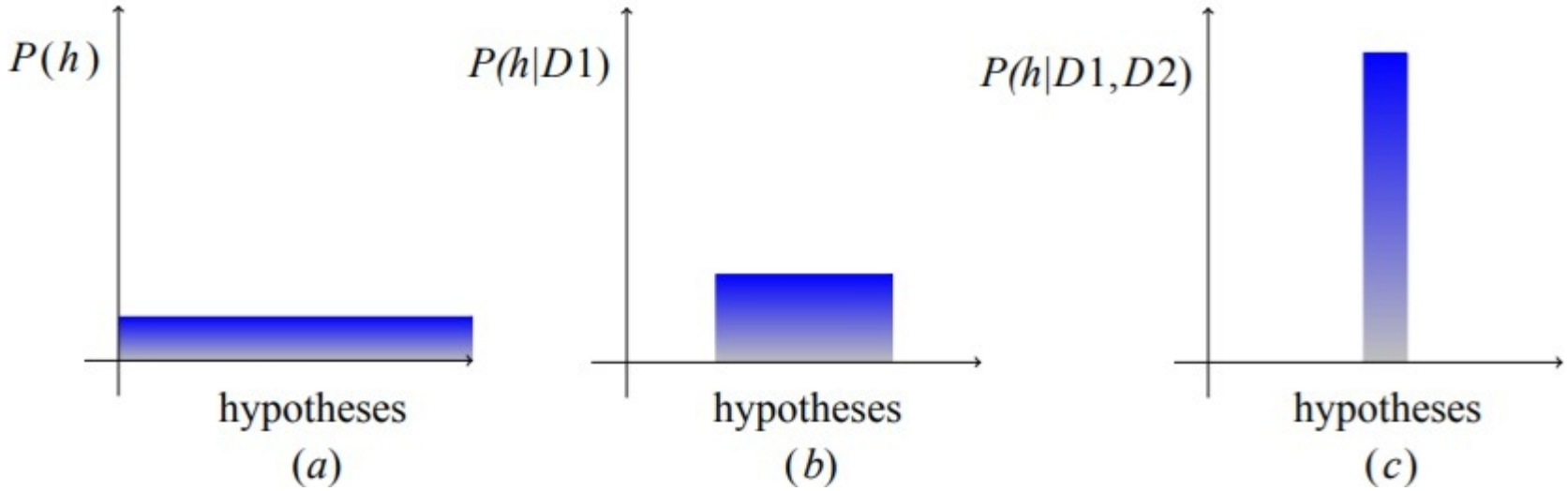
To summarize, Bayes theorem implies that the posterior probability P(h|D) under our assumed P(h) and P(D|h) is

$$P(h|D) = \begin{cases} \frac{1}{|VS_{H,D}|} & \text{if } h \text{ is consistent with } D \\ 0 & \text{otherwise} \end{cases}$$

where $|VS_{H,D}|$ is the number of hypotheses from H consistent with D

The Evolution of Probabilities Associated with Hypotheses

- Figure (a) all hypotheses have the same probability.
- Figures (b) and (c), As training data accumulates, the posterior probability for inconsistent hypotheses becomes zero while the total probability summing to 1 is shared equally among the remaining consistent hypotheses.



MAP Hypotheses and Consistent Learners

A learning algorithm is a consistent learner if it outputs a hypothesis that commits zero errors over the training examples.

Every consistent learner outputs a MAP hypothesis, if we assume a uniform prior probability distribution over H ($P(h_i) = P(h_j)$ for all i, j), and deterministic, noise free training data (P(D|h) =1 if D and h are consistent, and 0 otherwise).

Example:

- FIND-S outputs a consistent hypothesis, it will output a MAP hypothesis under the probability distributions P(h) and P(D|h) defined above.
- Are there other probability distributions for P(h) and P(D|h) under which FIND-S outputs MAP hypotheses? Yes.
- Because FIND-S outputs a maximally specific hypothesis from the version space, its output hypothesis will be a MAP hypothesis relative to any prior probability distribution that favours more specific hypotheses.

- Bayesian framework is a way to characterize the behaviour of learning algorithms
- By identifying probability distributions P(h) and P(D|h) under which the output is a optimal hypothesis, implicit assumptions of the algorithm can be characterized (Inductive Bias)
- Inductive inference is modelled by an equivalent probabilistic reasoning system based on Bayes theorem

MAXIMUM LIKELIHOOD AND LEAST-SQUARED ERROR HYPOTHESES

Consider the problem of learning a continuous-valued target function such as neural network learning, linear regression, and polynomial curve fitting

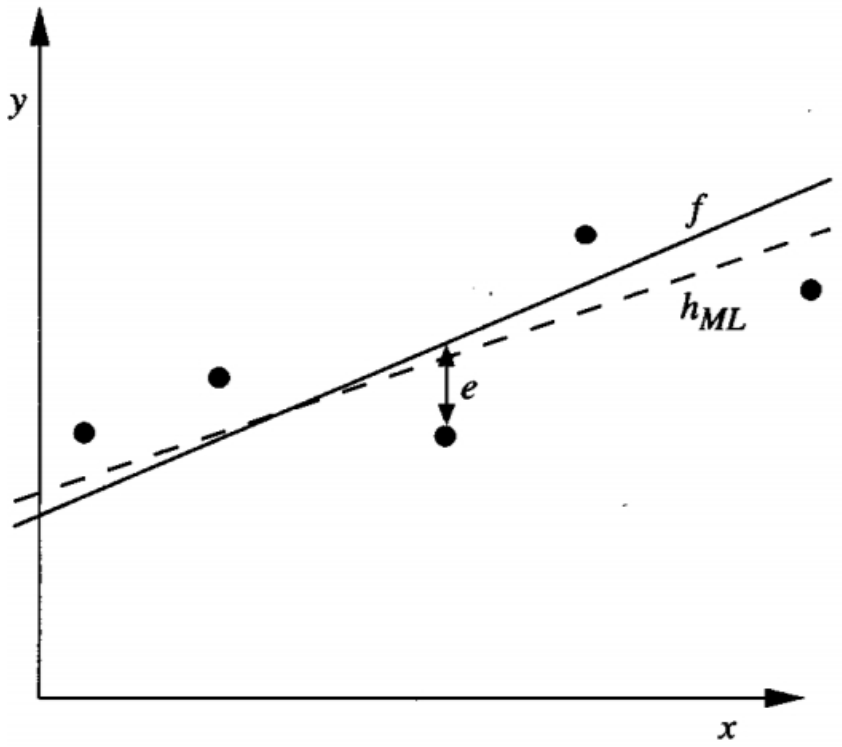
A straightforward Bayesian analysis will show that under certain assumptions any learning algorithm that minimizes the squared error between the output hypothesis predictions and the training data will output a maximum likelihood (ML) hypothesis

Learning A Continuous-Valued Target Function

- Learner L considers an instance space X and a hypothesis space H consisting of some class of real-valued functions defined over X, i.e., $(\forall\, h \in H)[\, h : X \rightarrow R]$ and training examples of the form $<x_i, d_i>$
- The problem faced by L is to learn an unknown target function $f : X \rightarrow R$
- A set of m training examples is provided, where the target value of each example is corrupted by random noise drawn according to a Normal probability distribution with zero mean ($d_i = f(x_i) + e_i$)
- Each training example is a pair of the form (x_i, d_i) where $di = f(x_i) + e_i$.
  - Here $f(x_i)$ is the noise-free value of the target function and e_i is a random variable representing the noise.
  - It is assumed that the values of the e_i are ***drawn independently*** and that they are distributed according to a ***Normal distribution*** with zero mean.
- The task of the learner is to output a ***maximum likelihood hypothesis***, or, equivalently, a MAP hypothesis assuming all hypotheses are equally probable a priori.

Learning A Linear Function

- The target function f corresponds to the solid line.
- The training examples (xi , di) are assumed to have Normally distributed noise ei with zero mean added to the true target value f (xi).
- The dashed line corresponds to the hypothesis h_{ML} with least-squared training error, hence the maximum likelihood hypothesis.
- Notice that the maximum likelihood hypothesis is not necessarily identical to the correct hypothesis, f, because it is inferred from only a limited sample of noisy training data



Before showing why a hypothesis that minimizes the sum of squared errors in this setting is also a maximum likelihood hypothesis, let us quickly review ***probability densities and Normal distributions***

Probability Density for continuous variables

$$p(x_0) \equiv \lim_{\epsilon \to 0} \frac{1}{\epsilon} P(x_0 \leq x < x_0 + \epsilon)$$

e: a random noise variable generated by a Normal probability distribution

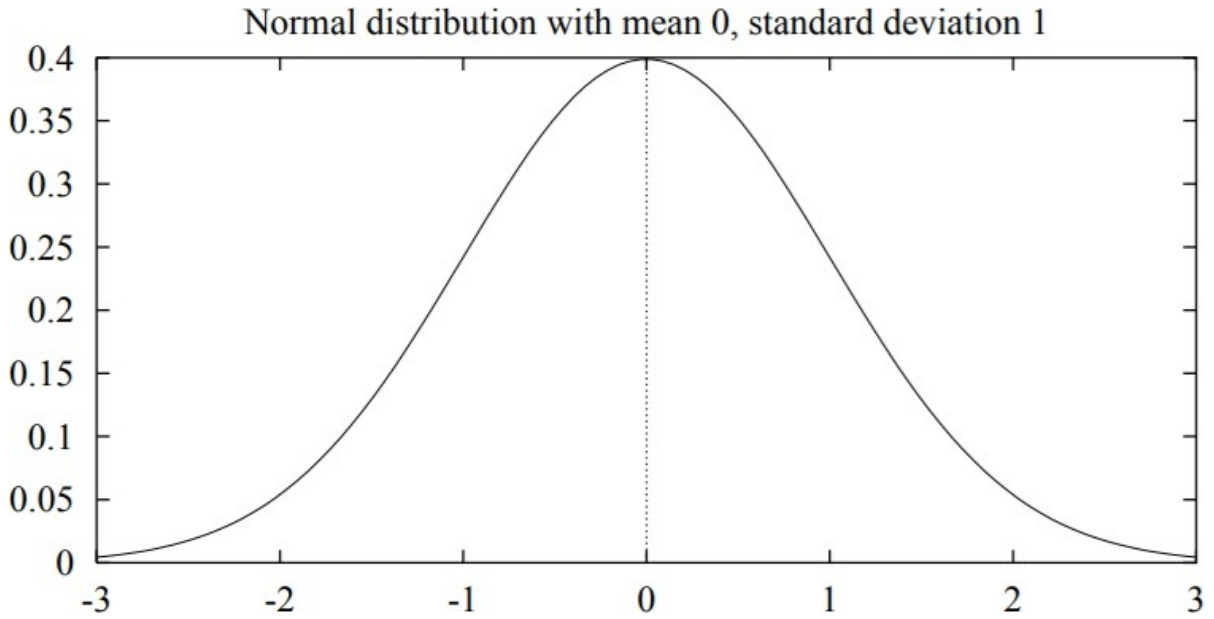
$<x_1 \ldots x_m>$: the sequence of instances (as before)

$<d_1 \ldots d_m>$: the sequence of target values with $d_i = f(x_i) + e_i$

Normal Probability Distribution (Gaussian Distribution)

A Normal distribution is a smooth, bell-shaped distribution that can be completely characterized by its mean μ and its standard deviation σ

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$



- A Normal distribution is fully determined by two parameters in the formula: μ and σ .
- If the random variable X follows a normal distribution:
  - The probability that X will fall into the interval (a, b) is $\int_a^b p(x)dx$
  - The expected, or ***mean value of X***, $E[X] = \mu$
  - The ***variance of X***, $Var(X) = \sigma^2$
  - The ***standard deviation of X***, $\sigma_x = \sigma$
- The ***Central Limit Theorem*** states that the sum of a large number of independent, identically distributed random variables follows a distribution that is approximately ***Normal***.

Using the previous definition of h_{ML} we have

$$h_{ML} = \underset{h \in H}{argmax}\ p(D|h)$$

Assuming training examples are mutually independent given h, we can write P(D|h) as the product of the various (d_i|h)

$$h_{ML} = \underset{h \in H}{argmax} \prod_{i=1}^{m} p(d_i|h)$$

Given the noise e_i obeys a Normal distribution with zero mean and unknown variance σ^2 , each d_i must also obey a Normal distribution around the true targetvalue $f(x_i)$. Because we are writing the expression for P(D|h), we assume h is the correct description of f. Hence, $\mu = f(x_i) = h(x_i)$

$$h_{ML} = \underset{h \in H}{argmax} \prod_{i=1}^{m} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(d_i - h(x_i))^2}$$

$$h_{ML} = \underset{h \in H}{\operatorname{argmax}} \prod_{i=1}^{m} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(d_i - \mu)^2}$$

$$= \underset{h \in H}{\operatorname{argmax}} \prod_{i=1}^{m} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(d_i - h(x_i))^2}$$

It is common to maximize the less complicated logarithm, which is justified because of the monotonicity of function p.

$$= \underset{h \in H}{\operatorname{argmax}} \sum_{i=1}^{m} ln \frac{1}{\sqrt{2\pi\sigma^2}} - \frac{1}{2\sigma^2}(d_i - h(x_i))^2$$

The first term in this expression is a constant independent of $\boldsymbol{h}$ and can therefore be discarded

$$= \underset{h \in H}{\operatorname{argmax}} \sum_{i=1}^{m} -\frac{1}{2\sigma^2}(d_i - h(x_i))^2$$

Maximizing this negative term is equivalent to minimizing the corresponding positive term.

$$= \underset{h \in H}{\operatorname{argmin}} \sum_{i=1}^{m} \frac{1}{2\sigma^2}(d_i - h(x_i))^2$$

Finally Discard constants that are independent of ***h***

$$h_{ML} = \underset{h \in H}{argmin} \sum_{i=1}^{m} (d_i - h(x_i))^2$$

- the h_{ML} is one that minimizes the sum of the squared errors

Why is it reasonable to choose the Normal distribution to characterize noise?

- good approximation of many types of noise in physical systems
- Central Limit Theorem shows that the sum of a sufficiently large number of independent, identically distributed random variables itself obeys a Normal distribution

Only noise in the target value is considered, not in the attributes describing the instances themselves

MAXIMUM LIKELIHOOD HYPOTHESES FOR PREDICTING PROBABILITIES

Consider the setting in which we wish to learn a nondeterministic (probabilistic) function $f : X \rightarrow \{0, 1\}$, which has two discrete output values.

We want a function approximator whose output is the probability that $f(x) = 1$

In other words , learn the target function

$$f' : X \rightarrow [0, 1] \text{ such that } f'(x) = P(f(x) = 1)$$

How can we learn f' using a neural network?

Use of brute force way would be to first collect the observed frequencies of 1's and 0's for each possible value of x and to then train the neural network to output the target frequency for each x.

What criterion should we optimize in order to find a maximum likelihood hypothesis for f' in this setting?

- First obtain an expression for P(D|h)
- Assume the training data D is of the form $D = \{(x_1, d_1) \ldots (x_m, d_m)\}$, where d_i is the observed 0 or 1 value for $f(x_i)$.
- Both x_i and d_i as random variables, and assuming that each training example is drawn independently, we can write P(D|h) as

$$P(D \mid h) = \prod_{i=1}^{m} P(x_i, d_i \mid h)$$

Applying the product rule

$$P(D \mid h) = \prod_{i=1}^{m} P(d_i \mid h, x_i) P(x_i)$$

The probability $P(d_i|h, x_i)$

$$P(d_i|h, x_i) = \begin{cases} h(x_i) & \text{if } d_i = 1 \\ (1 - h(x_i)) & \text{if } d_i = 0 \end{cases} \qquad \text{equ (3)}$$

Re-express it in a more mathematically manipulable form, as

$$P(d_i|h, x_i) = h(x_i)^{d_i}(1 - h(x_i))^{1-d_i} \qquad \text{equ (4)}$$

Equation (4) to substitute for $P(d_i|h, x_i)$ in Equation (5) to obtain

$$P(D|h) = \prod_{i=1}^{m} h(x_i)^{d_i}(1 - h(x_i))^{1-d_i} P(x_i) \qquad \text{equ (5)}$$

We write an expression for the maximum likelihood hypothesis

$$h_{ML} = \underset{h \in H}{\operatorname{argmax}} \prod_{i=1}^{m} h(x_i)^{d_i} (1 - h(x_i))^{1-d_i} P(x_i)$$

The last term is a constant independent of h, so it can be dropped

$$h_{ML} = \underset{h \in H}{\operatorname{argmax}} \prod_{i=1}^{m} h(x_i)^{d_i} (1 - h(x_i))^{1-d_i} \qquad \text{equ (6)}$$

It easier to work with the log of the likelihood, yielding

$$h_{ML} = \underset{h \in H}{\operatorname{argmax}} \sum_{i=1}^{m} d_i \ln h(x_i) + (1 - d_i) \ln(1 - h(x_i)) \qquad \text{equ (7)}$$

Equation (7) describes the quantity that must be maximized in order to obtain the maximum likelihood hypothesis in our current problem setting

Gradient Search to Maximize Likelihood in a Neural Net

Derive a weight-training rule for neural network learning that seeks to maximize G(h, D) using gradient ascent

- The gradient of G(h, D) is given by the vector of partial derivatives of G(h, D) with respect to the various network weights that define the hypothesis h represented by the learned network
- In this case, the partial derivative of G(h, D) with respect to weight w_{jk} from input k to unit j is

$$\frac{\partial G(h,D)}{\partial w_{jk}} = \sum_{i=1}^{m} \frac{\partial G(h,D)}{\partial h(x_i)} \frac{\partial h(x_i)}{\partial w_{jk}}$$

$$= \sum_{i=1}^{m} \frac{\partial (d_i \ln h(x_i) + (1 - d_i) \ln(1 - h(x_i)))}{\partial h(x_i)} \frac{\partial h(x_i)}{\partial w_{jk}}$$

$$= \sum_{i=1}^{m} \frac{d_i - h(x_i)}{h(x_i)(1 - h(x_i))} \frac{\partial h(x_i)}{\partial w_{jk}} \qquad \text{equ (1)}$$

Suppose our neural network is constructed from a single layer of sigmoid units. Then,

$$\frac{\partial h(x_i)}{\partial w_{jk}} = \sigma'(x_i)x_{ijk} = h(x_i)(1 - h(x_i))x_{ijk}$$

where x_{ijk} is the k^{th} input to unit j for the i^{th} training example, and d(x) is the derivative of the sigmoid squashing function.

Finally, substituting this expression into Equation (1), we obtain a simple expression for the derivatives that constitute the gradient

$$\frac{\partial G(h, D)}{\partial w_{jk}} = \sum_{i=1}^{m} (d_i - h(x_i))\ x_{ijk}$$

Because we seek to maximize rather than minimize P(D|h), we perform ***gradient ascent*** rather than ***gradient descent search***. On each iteration of the search the weight vector is adjusted in the direction of the gradient, using the weight update rule

$$w_{jk} \leftarrow w_{jk} + \Delta w_{jk}$$

Where,

$$\Delta w_{jk} = \eta \sum_{i=1}^{m} (d_i - h(x_i))\ x_{ijk} \qquad \text{equ (2)}$$

where η is a small positive constant that determines the step size of the i gradient ascent search

It is interesting to compare this weight-update rule to the weight-update rule used by the BACKPROPAGATION algorithm to minimize the sum of squared errors between predicted and observed network outputs.

The BACKPROPAGATION update rule for output unit weights, re-expressed using our current notation, is

$$w_{jk} \leftarrow w_{jk} + \Delta w_{jk}$$

Where,

$$\Delta w_{jk} = \eta \sum_{i=1}^{m} h(x_i)(1 - h(x_i))(d_i - h(x_i))\ x_{ijk}$$

MINIMUM DESCRIPTION LENGTH PRINCIPLE

- A Bayesian perspective on Occam's razor
- Motivated by interpreting the definition of h_{MAP} in the light of basic concepts from information theory.

$$h_{MAP} = \underset{h \in H}{argmax}\ P(D|h)P(h)$$

which can be equivalently expressed in terms of maximizing the $\log_2$

$$h_{MAP} = \underset{h \in H}{argmax}\ \log_2 P(D|h) + \log_2 P(h)$$

or alternatively, minimizing the negative of this quantity

$$h_{MAP} = \underset{h \in H}{argmin}\ -\log_2 P(D|h) - \log_2 P(h) \qquad \text{equ (1)}$$

- This equation can be interpreted as a statement that short hypotheses are preferred, assuming a particular representation scheme for encoding hypotheses and data

Introduction to a basic result of information theory

- Consider the problem of designing a code to transmit messages drawn at random
- i is the message
- The probability of encountering message i is p_i
- Interested in the most compact code; that is, interested in the code that minimizes the expected number of bits we must transmit in order to encode a message drawn at random
- To minimize the expected code length we should assign shorter codes to messages that are more probable
- Shannon and Weaver (1949) showed that the optimal code (i.e., the code that minimizes the expected message length) assigns - log, pi bits to encode message i.
- The number of bits required to encode message i using code C as the ***description length of message i with respect to C***, which we denote by $L_c(i)$.

Interpreting the equation

$$h_{MAP} = \underset{h \in H}{argmin} \; -\log_2 P(D|h) - \log_2 P(h) \qquad \text{equ (1)}$$

- $-\log_2 P(h)$: the description length of h under the optimal encoding for the hypothesis space H $L_{CH}(h) = -\log_2 P(h)$, where C_H is the optimal code for hypothesis space H.
- $-\log_2 P(D \mid h)$: the description length of the training data D given hypothesis h, under the optimal encoding fro the hypothesis space H: $L_{CH}(D|h) = -\log_2 P(D| h)$, where $C_{D|h}$ is the optimal code for describing data D assuming that both the sender and receiver know the hypothesis h.

Rewrite Equation (1) to show that h_{MAP} is the hypothesis h that minimizes the sum given by the description length of the hypothesis plus the description length of the data given the hypothesis.

$$h_{MAP} = \underset{h \in H}{argmin} \; L_{C_H}(h) + L_{C_{D|h}}(D|h)$$

where C_H and $C_{D|h}$ are the optimal encodings for H and for D given h

The Minimum Description Length (MDL) principle recommends choosing the hypothesis that minimizes the sum of these two description lengths of equ.

$$h_{MAP} = \underset{h \in H}{argmin}\ L_{C_H}(h) + L_{C_{D|h}}(D|h)$$

Minimum Description Length principle:

$$h_{MDL} = \underset{h \in H}{\text{argmin}}\ L_{C_1}(h) + L_{C_1}(D \mid h)$$

Where, codes C_1 and C_2 to represent the hypothesis and the data given the hypothesis

The above analysis shows that if we choose C_1 to be the optimal encoding of hypotheses C_H, and if we choose C_2 to be the optimal encoding $C_{D|h}$, then $h_{MDL} = h_{MAP}$

Application to Decision Tree Learning

Apply the MDL principle to the problem of learning decision trees from some training data.

What should we choose for the representations C_1 and C_2 of hypotheses and data?

- **For C_1:** C_1 might be some obvious encoding, in which the description length grows with the number of nodes and with the number of edges
- **For C_2:** Suppose that the sequence of instances (x_1 . . .x_m) is already known to both the transmitter and receiver, so that we need only transmit the classifications ($f(x_1)$ . . . $f(x_m)$).

  Now if the training classifications ($f(x_1)$ . . .$f(x_m)$) are identical to the predictions of the hypothesis, then there is no need to transmit any information about these examples. The description length of the classifications given the hypothesis ZERO

  If examples are misclassified by h, then for each misclassification we need to transmit a message that identifies which example is misclassified as well as its correct classification

The hypothesis h_{MDL} under the encoding C_1 and C_2 is just the one that minimizes the sum of these description lengths.

- MDL principle provides a way for trading off hypothesis complexity for the number of errors committed by the hypothesis
- MDL provides a way to deal with the issue of overfitting the data.
- Short imperfect hypothesis may be selected over a long perfect hypothesis.