Relation Q_k, Q_μ et Q_p, Q_s

- **Dahlen and Tromp 1998 – 3D**
  - Module d'élasticité isostatique (bulk modulus) : $k = \lambda + \frac{2}{n}\mu = \lambda + \frac{2}{3}\mu$
  - Module d'onde de compression : $M = k + 2\mu\left(1 - \frac{1}{n}\right) = k + \frac{4}{3}$

In some applications it may be more convenient to parameterize the anelasticity in terms of the compressional and shear wave speeds rather than the incompressibility and rigidity. In this case the reference Earth model is characterized by isotropic speeds $\alpha_0 = [(\kappa_0 + \cancel{\tfrac{4}{3}}\mu_0)/\rho]^{1/2}$ and $\beta_0 = (\mu_0/\rho)^{1/2}$, and we consider complex perturbations of the form

alpha here is for P and beta is for S.

$$\alpha_0 \to \alpha_0 + \delta\alpha(\omega) + \tfrac{1}{2} i \alpha_0 Q_\alpha^{-1}, \tag{9.57}$$

$$\beta_0 \to \beta_0 + \delta\beta(\omega) + \tfrac{1}{2} i \beta_0 Q_\beta^{-1}. \tag{9.58}$$

This for the 3D case is from the book of Dahlen and Tromp (1998).

The P-wave and S-wave quality factors Q_α and Q_β are related to the bulk and shear quality factors Q_κ and Q_μ by

$$Q_\alpha^{-1} = (1 - \cancel{\tfrac{4}{3}}\beta_0^2/\alpha_0^2) Q_\kappa^{-1} + \cancel{\tfrac{4}{3}}(\beta_0^2/\alpha_0^2) Q_\mu^{-1}, \tag{9.59}$$

$$Q_\beta^{-1} = Q_\mu^{-1}. \tag{9.60}$$

In 2D plane strain, the two 4/3 coefficients simply become 1. See e.g. equation (A9) of Carcione et al., Geophysical Journal vol. 95, p. 597-611 (1988).

- **Specfem2D : *2D plane strain***
  - $k = \lambda + \mu$
  - $M = k + \mu = \lambda + 2\mu$

See e.g. equation (A9) of Carcione et al., Geophysical Journal vol. 95, p. 597-611 (1988) for these two formulas in 2D plane strain.

$$\boxed{\begin{gathered} Q_p^{-1} = \left(1 - \frac{c_s^2}{c_p^2}\right) Q_k^{-1} + \left(\frac{c_s^2}{c_p^2}\right) Q_\mu^{-1} \\ Q_s^{-1} = Q_\mu^{-1} \end{gathered}}$$

Exemple :

$Q_k = 101.7, Q_\mu = 30$

=>

$Q_p = 57.3, Q_s = 30$