APPENDIX B

Calculation of the 2-D Green's function using the correspondence principle

To find the Green's function $G(x, z, x_0, z_0, t)$ for a 2-D acoustic medium, we solve the inhomogeneous scalar wave equation

$$\frac{\partial^2 G}{\partial x^2}+\frac{\partial^2 G}{\partial z^2}-\frac{1}{c_a^2}\frac{\partial^2 G}{\partial t^2}=-4\pi\delta(x-x_0)\delta(z-z_0)\delta(t), \tag{B1}$$

where x, z are the observer coordinates, x_0, z_0 are the source coordinates, t is the time and c_a is the acoustic-wave velocity of the medium. The solution to equation (B1) is given by (Morse & Feshbach 1953),

$$G(x, z, x_0, z_0, t)=2H\left(t-\frac{r}{c_a}\right)\left(t^2-\frac{r^2}{c_a^2}\right)^{-1/2}, \tag{B2}$$

with

$$r=[(x-x_0)^2+(z-z_0)]^{1/2}, \tag{B3}$$

and H is the Heaviside function. Taking Fourier transform with respect to time in equation (B2) gives

$$\tilde{G}(x, z, x_0, z_0, \omega)=2\int_{r/c_a}^{\infty}\left(t^2-\frac{r^2}{c_a^2}\right)^{-1/2}e^{-i\omega t}\,dt, \tag{B4}$$

with ω real. By making a change of variable $\tau=c_a(t/r)$, equation (B4) reads

$$\tilde{G}(x, z, x_0, z_0, \omega)=2\int_1^{\infty}(t/c_a)\tau\,d\tau. \tag{B5}$$

This expression is merely the integral representation of the zero-order Hankel function of the second kind (Morse & Feshbach 1953):

$$\tilde{G}(x, z, x_0, z_0, \omega)=-i\pi H_0^{(2)}\left(\frac{\omega}{c_a}r\right). \tag{B6}$$

Using the correspondence principle, we replace the real wave number in equation (B6) by the complex wave number:

$$\frac{\omega}{c_a}\rightarrow\frac{\omega}{v(\omega)}, \tag{B7}$$

where $v(\omega)$ is the complex velocity of the medium given by equation (18).

With the wave number defined by equation (B7) we express the 2-D viscoacoustic Green's function in ω-space as

$$\tilde{G}_v(x, z, x_0, z_0, \omega)=-i\pi H_0^{(2)}\left(\frac{\omega}{v(\omega)}r\right), \tag{B8a}$$

for $\omega\geqslant 0$ and

$$\tilde{G}_v(x, z, x_0, z_0, \omega)=\tilde{G}_v^*(x, z, x_0, z_0, -\omega). \tag{B8b}$$

Equation (B8) ensures that the Fourier transform of the viscoacoustic Green's function is real.

Because the Hankel function in equation (B8) has a singularity at $\omega=0$, we multiply $\tilde{G}_v(x, z, x_0, z_0, \omega)$ by the Fourier transform of a shifted zero-phase Ricker wavelet defined by

$$F(t)=e^{-\eta f_0^2(t-t_0)^2}\cos\epsilon\pi f_0(t-t_0), \tag{B9}$$

where $f_0=2\pi\Omega_0$ is the cut-off frequency, t_0 is the time shift and η and ϵ are constants. The Fourier transform of $F(t)$ is

$$\tilde{F}(\omega)=\pi\left(\frac{\pi}{\eta}\right)^{1/2}\frac{1}{\Omega_0}e^{i\omega t_0}\times\left[\exp-\frac{\pi^2}{\eta}\left(\frac{\epsilon}{2}-\frac{\omega}{\Omega_0}\right)^2+\exp-\frac{\pi^2}{\eta}\left(\frac{\epsilon}{2}+\frac{\omega}{\Omega_0}\right)^2\right]. \tag{B10}$$

Multiplying the transformed Green's function (B8) by $\tilde{F}(\omega)$ we obtain

$$\tilde{\Phi}(r, \omega)=\begin{cases}\tilde{G}_v(x, z, x_0, z_0, \omega)\tilde{F}(\omega) & \omega\neq 0,\\ 0 & \omega=0,\end{cases} \tag{B11}$$

avoiding with this definition, the singularity (actually this is an approximation because strictly is $\tilde{F}(0)=0$). Because the inverse Fourier transform of $\tilde{\Phi}(r, \omega)$ has no exact analytical expression, we invert it numerically by using the discrete fast Fourier transform, obtaining the time function $\Phi(r, t)$.

There is a typo in equation (B10), the exp(i omega t0) should be exp(- i omega t0), i.e. a minus sign is missing.

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Erratum

GJ apologizes for the following errors that appeared in 'Wave propagation simulation in a linear viscoacoustic medium' by José M. Carcione, Dan Kosloff and Ronnie Kosloff (**93**, 393–401):

— line before equation (11) should read 'Heaviside function. Applying the convolution theorem to equation (9) we obtain . . .'.

— line 6 in right column on p. 395 should read 'representation of real materials to mechanical models, so we could'.

— equation (25) should read

$$\dot{e}_{1_l}(t) = -\frac{e_{1_l}(t)}{\tau_{\sigma l}} + \phi_l(0)e(t), \qquad l = 1, \ldots, L$$

— p. 396, left column, 4th line from bottom, should read 'becomes a coupled system of $(L+2)N = (L+2)NNN$'.

— equation (45) should read

$$e^{\gamma \mathbf{I} t}\mathbf{E}_N = e^{(\mathbf{M}_N + \gamma \mathbf{I})t}\mathbf{E}_N^0 + \int_0^t e^{(\mathbf{M}_N + \gamma \mathbf{I})\tau}\mathbf{S}_N'(t-\tau)\,d\tau, \quad (45)$$

— p. 398, left column, 14th line, should read 'The result for $K = 320$ shows that, . . .'

— the last reference is Wennschel not Enenschel

— equation (B5) should read

$$\tilde{G}(x, z, x_0, z_0, \omega) = 2\int_1^\infty (\tau^2 - 1)^{1/2} e^{-i\omega_{ca}'\tau}\,d\tau.$$