For the following first-order ordinary equation：

 （1）

The fourth-order Runge-Kutta scheme:

 （2）

The corresponding recursion formula：

 (3)

where，





The first-order velocity-stress wave equations：



Define velocity and stress components on the following grid：



Pseudocode of fourth-order Runge-Kutta scheme of first-order velocity-stress wave equations：

!5 snapshots

vx(:,:) = 0.d0

vz(:,:) = 0.d0

sigmaxx(:,:) = 0.d0

sigmazz(:,:) = 0.d0

sigmaxz(:,:) = 0.d0

!temporary arrays for RK4

rk4vx(:,:,1:3) = 0.d0

rk4vz(:,:,1:3) = 0.d0

rk4sigmaxx(:,:,1:3) = 0.d0

rk4dsigmazz(:,:,1:3) = 0.d0

rk4sigmaxz(:,:,1:3) = 0.d0

do it = 1, nt, 1

!copy 5 snapshots at n-th time instant

rk4vx(:,:,3) = vx

rk4vz(:,:,3) = vz

rk4sigmaxx(:,:,3) = sigmaxx

rk4sigmazz(:,:,3) = sigmazz

rk4sigmaxz(:,:,3) = sigmaxz

rk4vx(:,:,2) = vx

rk4vz(:,:,2) = vz

rk4sigmaxx(:,:,2) = sigmaxx

rk4sigmazz(:,:,2) = sigmazz

rk4sigmaxz(:,:,2) = sigmaxz

!four loops of the Runge-Kutta scheme

do j = 1, 4, 1

!update the second terms in parenthesis of formua (3)

rk4vx(:,:,1) = rk4vx(:,:,3) + rk41(j)\*dt\*rk4vx(:,:,2)

rk4vz(:,:,1) = rk4vz(:,:,3) + rk41(j)\*dt\*rk4vz(:,:,2)

rk4sigmaxx(:,:,1) = rk4sigmaxx(:,:,3) + rk41(j)\*dt\*rk4sigmaxx(:,:,2)

rk4sigmazz(:,:,1) = rk4sigmazz(:,:,3) + rk41(j)\*dt\* rk4sigmazz(:,:,2)

rk4sigmaxz(:,:,1) = rk4sigmaxz(:,:,3) + rk41(j)\*dt\* rk4sigmaxz(:,:,2)

!update stress components

do iz = 2, nz-1, 1

do ix = 2, nx-1, 1

value\_dvx\_dx = (rk4vx(ix+1,iz,1) – rk4vx(ix,iz,1))/dx

value\_dvz\_dz = (rk4vz(ix,iz+1,1) – rk4vz(ix,iz,1))/dz

rk4sigmaxx(ix,iz,2) = lambda\_plus\_two\_mu\* value\_dvx\_dx +

lambda\* value\_dvz\_dz

rk4sigmazz(ix,iz,2) = lambda\* value\_dvx\_dx +

lambda\_plus\_two\_mu\* value\_dvz\_dz

value\_dvz\_dx = (rk4vz(ix,iz,1) – rk4vz(ix-1,iz,1))/dx

value\_dvx\_dz = (rk4vx(ix,iz,1) – rk4vx(ix,iz-1,1))/dz

rk4sigmaxz(ix,iz,2) = mu\*(value\_dvz\_dx + value\_dvx\_dz)

end do

end do

!update velocity components

do iz = 2, nz-1, 1

do ix = 2, nx-1, 1

value\_dsigmaxx\_dx = (rk4sigamxx(ix,iz,1) – rk4sigmaxx(ix-1,iz,1))/dx

value\_dsigmaxz\_dz = (rk4sigmaxz(ix,iz+1,1) – rk4sigmaxz(ix,iz,1))/dz

rk4vx(ix,iz,2) = value\_dsigmaxx\_dx + value\_dsigmaxz\_dz

value\_dsigmaxz\_dx = (rk4sigamxz(ix,iz,1) – rk4sigmaxz(ix-1,iz,1))/dx

value\_dsigmazz\_dz = (rk4sigmazz(ix,iz+1,1) – rk4sigmazz(ix,iz,1))/dz

rk4vz(ix,iz,2) = value\_dsigmaxz\_dx + value\_dsigmazz\_dz

end do

end do

!Update the first equation of formula (3)

vx = vx + rk42(j)\*dt\*rk4vx(:,:,2)

vz = vz + rk42(j)\*dt\*rk4vz(:,:,2)

sigmaxx = sigmaxx + rk42(j)\*dt\*rk4sigmaxx(:,:,2)

sigmazz = sigmazz + rk42(j)\*dt\*rk4sigmazz(:,:,2)

sigmaxz = sigmaxz + rk42(j)\*dt\*rk4sigmaxz(:,:,2)

end do !do j = 1, 4, 1

end do !do it = 1, nt, 1